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Electroweak Precision Observables within a Fourth Generation Model with General Flavour Structure

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Abstract

We calculate the contributions to electroweak precision observables (EWPOs) due to a fourth generation of fermions with the most general (quark-) flavour structure (but assuming Dirac neutrinos and a trivial flavour structure in the lepton sector). We discuss the size of non-oblique contributions arising from Z–quark–anti-quark vertex corrections and the dependence of the EWPOs on all CKM mixing angles involving the fourth generation. We find that the electroweak precision observables are equally sensitive to all three fourth-generation mixing angles and that the corresponding constraints on these angles are competitive with those obtained from flavour physics. For non-trivial 4×4 flavour structures, the non-oblique contributions lead to relative corrections at the permille level and may well have a noticable effect in a global fit.

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1 Introduction

With the advent of LHC data the first direct tests for many models of new physics are within reach. Among the conceptionally simplest extensions of the Standard Model (SM3) are those which only add a minimal set of fermions to the SM particle content. This class encompasses both the additional vector-like quarks [? ? ? ?] and the fourth-generation scenario (SM4).

The SM4 was fairly popular in the 1980s until electroweak precision observables seemed to rule it out. In the last years models with an additional fourth generation experienced a renaissance as new analyses, e.g. [? ? ? ? ? ?], somewhat relaxed the electroweak tensions. This realisation also prompted numerous studies of the non-trivial flavour structure of the SM4 [? ? ? ?], as well as searches for specific signatures in new physics observables [? ? ? ? ? ?].

Recently, some effort has been directed towards providing an actual *fit* of the parameters of the model — or, to be precise, of one particular variant which restricts itself to an (almost) decoupled fourth Dirac-like neutrino.[1] This scenario requires merely seven additional parameters and fitting them simultaneously does not seem unrealistic. One of the first attempts in this direction primarily used the electroweak precision observables and restricted itself to only one CKM parameter [?]; still non-trivial correlations were found, for example, between the Higgs mass and the new mixing angle. More recent studies seek to contain [?] or even determine [?] the full 4×4 CKM matrix. In this case the main challenge is the fact that, if one allows for a generic CKM structure, the flavour and electroweak sector are intertwined and have to be treated simultaneously.

Usually the effects of new physics in the electroweak sector are parametrised by the *oblique electroweak parameters* S, T and U, as introduced by Peskin and Takeuchi [? ?]. These allow for fairly simple and straightforward estimates of new physics contribution to electroweak observables. However, the validity of this parametrisation relies on certain assumptions about the new physics model, which are, in principle, no longer satisfied in an SM4 with the most general flavour structure.

In this letter we discuss the contributions to electroweak precision observables (EWPOs) due to a fourth generation with general 4×4 flavour mixing. For the sake of simplicity we assume Dirac neutrinos and a trivial flavour structure in the lepton sector. Our calculation includes non-oblique contributions, i.e. those which are not captured by the S, T and U parameters, and can easily be combined with existing calculations of higher-order QCD and QED corrections within the SM3. We discuss the importance of the non-oblique contributions and the impact of flavour mixing between the fourth and the first three generation in several SM4 scenarios.

In section 2 we briefly review the oblique parameters and their range of applicability. In section 3 we introduce our notations for the SM4 parameters and explain our method

[1] See e.g. [?] for a discussion of fourth generation Majorana neutrinos.

for calculating the corrections to the EWPOs. In section 4 we describe our treatment of the Fermi constant G_F, which is an observable and not a parameter in our analysis. Our numerical results are presented in section **??**. We find that the EWPOs are equally sensitive to all three fourth-generation mixing angles and that the corresponding constraints on these angles are competitive with those obtained from flavour physics. For non-trivial 4×4 flavour structures, the non-oblique contributions lead to relative corrections of up to one permille for the hadronic Z width Γ_{had} and of several permille for the hadronic $Z \to b\bar{b}$ branching ratio R_b. A simultaneous fit of the SM4 masses, couplings and CKM matrix should therefore take into account all six SM4 CKM mixing angles and the non-oblique corrections to the EWPOs. We conclude in section **??**.

2 Oblique Corrections and Electroweak Observables

The constraints imposed on new physics by EWPOs measured at LEP have already been discussed extensively in the literature. In 1992 Peskin and Takeuchi presented a model-independent way of parametrising the new physics contributions to the Z pole observables [**?**]. Their analysis was based on three assumptions:

1. The electroweak gauge group of the new-physics model is $SU(2)_L \times U(1)_Y$.
2. The new-physics couplings to light fermions (i.e. all SM3 fermions except the top-quark) are negligible.
3. The scale of new physics is much larger than the electroweak scale.

The first assumption forbids the existence of additional gauge bosons coupling directly to leptons. The second assumption guarantees that there are no additional vertex or box-diagrams contributing to the Drell-Yan process. Thus, the only way the new physics contribute to the Z pole observables is through the renormalisation of weak gauge boson wave functions, the electric charge or the Weinberg angle. The third assumption is needed to justify a step in the discussion in [**?**], where the gauge boson self-energies are expanded to first order around $q^2 = 0$ (q being the momentum flowing through the self-energy graphs). In practice, it is usually sufficient to require that new particles coupling directly to weak gauge bosons are heavier than the Z boson.

In SM extensions that satisfy the criteria above, the new physics contributions to the Z pole observables can be expressed in terms of the *oblique electroweak parameters* S, T and U which were defined in [**? ?**] and represent different linear combinations of gauge boson self-energies and their derivatives. On the experimental side, the values of S, T and U can then be determined from data by performing a global fit of S, T, U and the SM3 parameters to the Z pole and possibly other low-energy observables. (See [**?**] for a recent analysis of this type.) On the theoretical side one can test to what

	experiment	theory (SM3)
$\Gamma_{\rm had}$ [GeV]	1.7444 ± 0.002	1.7418 ± 0.0009
R_b	0.21629 ± 0.00066	0.21578 ± 0.00005
A^b_{FB}	0.0992 ± 0.0016	0.1034 ± 0.0007
$\mathcal{A}_b$	0.923 ± 0.020	0.9348 ± 0.0001
$\mathcal{A}_e$	0.15138 ± 0.00216	0.1475 ± 0.0010
M_W [GeV]	80.420 ± 0.031	80.384 ± 0.014

Table 1: Experimental results and Standard Model predictions for selected electroweak observables. All numbers were taken from [**?**].

extent a given model of new physics agrees with low-energy observables by computing S, T and U in this model and comparing the results with the best-fit values.

This method of testing an SM extension against constraints from low-energy experiments is very convenient since it only requires the computation of three quantities. It has been applied to a number of models including the SM4 [**?**]. One should, however, keep in mind that the validity of this method depends on the validity of the assumptions listed above. In the SM4 the second assumption is no longer valid if the fourth generation quarks are allowed to mix with the quarks of the first three generations. Hence, the validity of the "oblique method" must be checked explicitly if one attempts to constrain the new mixing angles of the SM4 CKM matrix.

3 The $Zq\bar{q}$ Vertex in the SM4

The properties of the Z boson and its couplings to fermions have been measured at LEP 1 with a very high accuracy. Table 1 shows the experimental values and accuracies for a selection of Z-pole observables as well as their theoretical predictions within the SM3. The observables are: the partial width for $Z \to$ hadrons ($\Gamma_{\rm had}$), the hadronic branching fraction for $Z \to b\bar{b}$ (R_b), the forward-backward asymmetry for $Z \to b\bar{b}$ ($A^b_{\rm FB}$) and the mass of the W (M_W). In the Z-pole approximation, the forward-backward asymmetry can be written as $\frac{3}{4}\mathcal{A}_e\mathcal{A}_b$, where the quantities $\mathcal{A}_e$ and $\mathcal{A}_b$ only depend on the Ze^+e^- and $Zb\bar{b}$ couplings, respectively. The relative precision of $\Gamma_{\rm had}$ is approximately 0.1% and R_b is known to an accuracy of 0.3%. The measured value of $A^b_{\rm FB}$ deviates from its SM3 prediction by more than two standard deviations. The discrepancy originates mainly from the factor $\mathcal{A}_e$. Oblique corrections due to a fourth generation of fermions affect all Z-pole observables, but only observables related to the Z–quark–anti-quark vertex are subject to non-oblique corrections; of the observables from table 1, only $\Gamma_{\rm had}$, R_b and $\mathcal{A}_b$ receive non-oblique contributions. Our discussion will therefore mainly focus on these quantities.[2]

Within the SM3 the couplings of Z bosons to quarks have been studied in great detail. Electroweak and QCD corrections to the gauge boson self-energies and the Z–quark–anti-quark vertex have been calculated at two-loop order [?] and the results have been implemented in public codes such as `TOPAZ0` [?] or `ZFITTER` [? ?]. Radiative corrections to the partial widths are of the order of 0.1% (QED) and 4% (QCD). To match the experimental accuracy of the Z-pole observables they must therefore be included in theoretical calculations. In this section we explain how predictions for the Z pole observables within the SM4 can be calculated at the required level of accuracy without the need to re-visit the SM3 calculations.

Before we begin, let us briefly explain our notations for the SM3 and SM4 parameters. For the SM3 CKM matrix we use the *standard parametrisation*. In this parametrisation the independent parameters are the three mixing angles θ_{12}, θ_{13} and θ_{23} and one complex phase δ_{13}. The explicit form of the SM3 CKM matrix in terms of the phase and mixing angles is given in appendix **??**.

In the SM4 the CKM matrix is a unitary 4×4 matrix. After absorbing unphysical complex phases into the definitions of the quark fields, its parametrisation requires only three additional mixing angles θ_{14}, θ_{24} and θ_{34} and two additional complex phases δ_{14} and δ_{24}. The explicit form of the SM4 CKM matrix is also given in appendix **??**. For the discussion below it is only important to know that for $\theta_{14} = \theta_{24} = \theta_{34} = \delta_{14} = \delta_{24} = 0$ the SM4 CKM matrix assumes a block-diagonal form with the SM3 CKM matrix in the first 3×3 block and a one in the last block.

To distinguish the phase δ_{13} and the mixing angles θ_{12}, θ_{13} and θ_{23} of the SM4 CKM matrix from their SM3 counterparts we will use superscripts 'SM4' and 'SM3', respectively. The same applies to other parameters like m_H or M_W, which exist in both models. We will also use the shorthands s_{ij} and c_{ij} for the sines and cosines of the mixing angles θ_{ij}. Finally, we denote the lepton, neutrino, up and down-type quark of the fourth generation as ℓ_4, ν_4, t' and b', respectively. Their masses m_{ℓ_4}, m_{ν_4}, $m_{t'}$ and $m_{b'}$ are independent parameters of the SM4.

Let us now proceed with the discussion of higher order corrections to the $Zq\bar{q}$ vertex. In the limit of vanishing external quark masses m_q, the on-shell $Zq\bar{q}$ vertex function only contains two Lorentz structures:

$$\Gamma_\mu^q = ie\gamma_\mu[F_V^q - F_A^q\gamma_5] \quad . \tag{1}$$

Here and in the following, $q = u, d, s, c, b$ denotes the quark flavour. The form factors F_V^q and F_A^q depend on the quark flavour, the external masses and the parameters of the model under consideration (SM3 or SM4). Following the discussion in [?], we express

[2] The branching fraction R_c and asymmetry factor $\mathcal{A}_c$ for the charm quark also receive non-oblique corrections, but these observables are less constraining due to their larger experimental error.

QCD and QED radiative corrections to F_V^q and F_A^q in terms of *radiator functions* $\mathcal{R}_V^q$ and $\mathcal{R}_A^q$ and write

$$F_V^q = g_V^q \mathcal{R}_V^q \quad , \quad F_A^q = g_A^q \mathcal{R}_A^q \quad . \tag{2}$$

In doing this, we neglect the non-factorisable contributions [**?** **?**], whose effect is below the permille level. The *effective couplings* g_V^q and g_A^q now only contain infrared finite contributions. At leading order $\mathcal{R}_V^q = \mathcal{R}_A^q = 1$ and g_V^q and g_A^q are the tree-level vector and axial couplings of the Z boson.

In this paper we are interested in the difference between predictions for Z pole observables within the SM3 and SM4. For this purpose we denote, for any quantity X, the *new physics correction* by

$$\delta X = X^{\rm SM4} - X^{\rm SM3} \quad , \tag{3}$$

where the superscripts 'SM4' and 'SM3' indicate that X is evaluated with a given set of SM4 or SM3 parameters, respectively. In principle, the two sets of parameters can be completely unrelated. It is, however, extremely convenient to use the same values of M_Z, M_W, m_t, α and α_s in both sets.[3] In this case, $\delta\mathcal{R}_V^q = \delta\mathcal{R}_A^q = 0$ and the new physics corrections to any Z pole observable can be obtained by only computing the infrared finite quantities δg_V^q and δg_A^q. The form factors $F_V^{q,\rm SM4}$ and $F_A^{q,\rm SM4}$ (and thus for the Z-pole observables within the SM4) may then be calculated by scaling the corresponding SM3 form factors with the ratios $g_V^{q,\rm SM4}/g_V^{q,\rm SM3}$ and $g_A^{q,\rm SM4}/g_A^{q,\rm SM3}$, respectively. This way, factorisable QCD and QED corrections are included in $F_V^{\rm SM4}$ and $F_A^{\rm SM4}$ if they were included in the SM3 'reference values' $F_V^{\rm SM3}$ and $F_A^{\rm SM3}$. As we will see below, the ratios $\delta g_V^q/g_V^{q(0)}$ and $\delta g_A^q/g_A^{q(0)}$ (with $g_V^{q(0)}$ and $g_A^{q(0)}$ being the tree-level couplings) are typically below 1%. Thus, the approximation

$$F_V^{q,\rm SM4} \approx F_V^{q,\rm SM3}\left(1+\frac{\delta g_V^q}{g_V^{q,(0)}}\right) \quad , \quad F_A^{q,\rm SM4} \approx F_A^{q,\rm SM3}\left(1+\frac{\delta g_A^q}{g_A^{q,(0)}}\right) \tag{4}$$

(with $g_V^{q(0)}$ and $g_A^{q(0)}$ being the tree-level couplings) is generally valid with a relative precision of the order of 10^{-4}.

The difference between $\mathcal{R}_V^q$ and $\mathcal{R}_A^q$ is of the order of a few percent [**?**]. Thus, to estimate the size of the new physics contributions to the EWPOs we use the approximation

$$\mathcal{R}_V^q \approx \mathcal{R}_A^q \equiv \mathcal{R}^q \tag{5}$$

and obtain

$$\delta\Gamma_\mu^q = ie\mathcal{R}^q\gamma_\mu[\delta g_V^q - \delta g_A^q\gamma_5] \quad . \tag{6}$$

The hadronic Z partial widths and asymmetries are then given by

$$\Gamma(Z\to q\bar{q}) = \alpha M_Z \mathcal{R}^q[(g_V^q)^2 + (g_A^q)^2] \quad , \quad \mathcal{A}_q = \frac{g_V^q g_A^q}{(g_V^q)^2+(g_A^q)^2} \tag{7}$$

[3] These are independent SM3 input parameters in the *on-shell* renormalisation scheme [**?**], which is the scheme we used in our calculations.

The new physics corrections to these quantities are readily obtained by expanding the effective couplings to first order in δg_V^q and δg_A^q:

$$\begin{aligned}
\frac{\delta\Gamma(Z\to q\bar{q})}{\Gamma^{\rm SM3}(Z\to q\bar{q})} &= 2\,\frac{g_V^{q(\rm SM3)}\,{\rm Re}\,\delta g_V^q + g_A^{q(\rm SM3)}\,{\rm Re}\,\delta g_A^q}{(g_V^{q(\rm SM3)})^2+(g_A^{q(\rm SM3)})^2} \\
&\approx 2\,\frac{g_V^{q(0)}\,{\rm Re}\,\delta g_V^q + g_A^{q(0)}\,{\rm Re}\,\delta g_A^q}{(g_V^{q(0)})^2+(g_A^{q(0)})^2} \quad , \qquad (8a)\\
\frac{\delta\mathcal{A}_q}{\mathcal{A}_q^{\rm SM3}} &= \frac{{\rm Re}\,\delta g_V^q}{g_V^{q(\rm SM3)}} + \frac{{\rm Re}\,\delta g_A^q}{g_A^{q(\rm SM3)}} - 2\,\frac{g_V^{q(\rm SM3)}\,{\rm Re}\,\delta g_V^q + g_A^{q(\rm SM3)}\,{\rm Re}\,\delta g_A^q}{(g_V^{q(\rm SM3)})^2+(g_A^{q(\rm SM3)})^2} \\
&\approx \frac{{\rm Re}\,\delta g_V^q}{g_V^{q(0)}} + \frac{{\rm Re}\,\delta g_A^q}{g_A^{q(0)}} - 2\,\frac{g_V^{q(0)}\,{\rm Re}\,\delta g_V^q + g_A^{q(0)}\,{\rm Re}\,\delta g_A^q}{(g_V^{q(0)})^2+(g_A^{q(0)})^2} \quad . \qquad (8b)
\end{aligned}$$

Note that, as a result of approximating $\mathcal{R}_V^q \approx \mathcal{R}_A^q$, the radiator functions cancel in the ratios above.

If mixing between the fourth generation quarks and the quarks of the first three generations is neglected, the new physics corrections can be expressed in terms of the oblique electroweak parameters S, T and U [**? ?**]. In this case, the relations between δg_V^q, δg_A^q and S, T and U are

$$\delta g_V^q = \frac{\alpha}{16 c_W s_W^3}\left[2 I_3^q S - 4[(c_W^2 - s_W^2) I_3^q + 2 s_W^2 Q^q] T - \Big(\frac{c_W^2 - s_W^2}{s_W^2} I_3^q + 2Q^q\Big) U\right] \quad , \qquad (9a)$$

$$\delta g_A^q = \frac{\alpha}{16 c_W s_W^3}\left[2S - \frac{c_W^2 - s_W^2}{s_W^2}(4 s_W^2 T + U)\right] I_3^q \quad , \qquad (9b)$$

where Q^q and I_3^q are the electric charge and weak isospin of the quark q and s_W and c_W are the sine and cosine of the Weinberg angle, defined by $s_W^2 = 1 - M_W^2/M_Z^2$.

If the fourth generation quarks are allowed to mix with the quarks of the first three generations one also needs to compute the vertex diagrams contributing to δg_V^q and δg_A^q. We used the `FeynArts/FormCalc` package [**? ? ?**] to compute δg_V^q and δg_A^q to one-loop order. The renormalisation of the $Zq\bar{q}$ vertex was done in the on-shell scheme [**?**]. At the one-loop level only diagrams involving W bosons, charged Goldstone bosons or Higgs bosons contribute to δg_V^q and δg_A^q, as long as α, α_s, M_Z, M_W and m_t are chosen to be the same in the SM3 and SM4. The SM3 parameters and corresponding values for $\Gamma(Z\to q\bar{q})$ and $\mathcal{A}_q$ were taken from [**?**]. Specifically, we use

$$\begin{aligned}
&1/\alpha(m_Z) = 128.892 \quad , \quad \alpha_s(m_Z) = 0.1185 \quad , \quad M_Z = 91.1875\,{\rm GeV} \quad ,\\
&M_W = 80.384\,{\rm GeV} \quad , \quad m_t = 173.2\,{\rm GeV} \quad , \quad m_H^{\rm SM3} = 90\,{\rm GeV} \quad ,\\
&\Gamma_{\rm had}^{\rm SM3} = 1.7418\,{\rm GeV} \quad , \quad R_b^{\rm SM3} = 0.21578 \quad , \quad \mathcal{A}_b^{\rm SM3} = 0.9348 \quad , \qquad (10)
\end{aligned}$$

where

$$\Gamma_{\text{had}} = \sum_{q=u,d,s,c,b} \Gamma(Z \to q\bar{q}) \quad , \quad R_q = \frac{\Gamma(Z \to q\bar{q})}{\Gamma_{\text{had}}} \quad . \tag{11}$$

The phase and mixing angles of the SM3 CKM matrix were also taken from [?]:

$$\theta_{12}^{\text{SM3}} = 0.2273 \quad , \quad \theta_{13}^{\text{SM3}} = 0.003466 \quad , \quad \theta_{23}^{\text{SM3}} = 0.04103 \quad , \quad \delta_{13}^{\text{SM3}} = 1.2020 \quad . \tag{12}$$

Note that the numerical values for $\Gamma_{\text{had}}^{\text{SM3}}$ and R_b^{SM3} are for a fixed "reference" Higgs mass $m_H^{\text{SM3}} = 90\,\text{GeV}$. In the SM4 the Higgs mass is treated as a free parameter.

4 A Note on G_F

As mentioned above, we use in this work the on-shell renormalisation scheme for the computation of new physics corrections. In this scheme, the quantities $\alpha(M_Z)$, M_Z and M_W are independent parameters. This parametrisation is very convenient for the computation of higher order corrections, but it has its disadvantages if one wants to compare it with experimental data. The Fermi constant G_F, which is determined from the muon lifetime, is a non-trivial function of $\alpha(M_Z)$, M_Z, M_W and the other model parameters. Since G_F is measured very accurately (namely, to a relative precision of 10^{-5}) it constrains the model to a non-trivial hyper-surface in its parameter space. In other words, one parameter of the model is fixed by the requirement that G_F assumes its measured value. Typically, one adjusts the value of M_W to obtain the correct value of G_F.

The relation between G_F and M_W is conventionally written as [?]

$$G_F = \frac{\pi\alpha}{\sqrt{2} s_W^2 M_W^2} \frac{1}{1 - \Delta r} \quad , \tag{13}$$

where Δr encodes higher order corrections and is, in general, a function of all other parameters. New physics, like the existence of a fourth generation of fermions, changes the function Δr. Denoting, as before, the new physics correction to Δr as $\delta\Delta r$ and writing the solutions of (13) in the SM3 and SM4 as M_W^{SM3} and $M_W^{\text{SM4}} \equiv M_W^{\text{SM3}} + \delta M_W$, respectively, we find

$$\frac{\delta M_W}{M_W^{\text{SM3}}} = -\frac{s_W^2}{2(c_W^2 - s_W^2)} \delta\Delta r \quad . \tag{14}$$

However, since the parameters (10) already satisfy the G_F constraint we have $M_W^{\text{SM3}} = M_W$ with M_W from (10). If the SM4 is to agree with the measured value of the W mass, the ratio $\delta M_W / M_W^{\text{SM3}}$ cannot be much larger than one permille. Hence, the shift in M_W is unimportant for the purpose of computing $\delta\Delta r$ and the new physics corrections (8) of the hadronic Z partial widths and asymmetries. We can therefore safely use the value from (10) in these calculations.