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Phenomenological consequences of radiative flavor violation in the MSSM

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In this article we investigate the consequences of radiative flavor violation (RFV) in the Minimal Supersymmetric Standard Model (MSSM). In this framework the small off-diagonal elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the small quark masses of the first two generations are generated from the trilinear supersymmetry-breaking terms. The impact of RFV on flavor-physics observables is studied in detail. We focus on the limiting cases in which the CKM matrix is either generated in the down-sector, i.e. by the soft SUSY-breaking mass insertions $\delta^{d\,LR}_{i3}$ $(i = 1, 2)$, or in the up-quark sector, i.e. by the mass insertions $\delta^{u\,LR}_{i3}$. In the first case we find an enhancement of $b \to s\gamma$, which constrains the allowed range of sparticle masses (Fig. 3). In addition, neutral Higgs penguins significantly contribute to $B_{s,d} \to \mu^+\mu^-$ and, if also $\delta^{d\,LR}_{32}$ is different from zero, these Higgs effects are capable of explaining the observed CP phase in the B_s system. If, on the other hand, the CKM generation takes place in the up-sector, $|\epsilon_K|$ receives additional positive contributions enforcing large squark and gluino masses (see Fig. 8). In this case also the rare decay $K \to \pi\nu\bar{\nu}$ receives sizable contributions. In conclusion we find that for SUSY masses around 1 TeV RFV is an interesting alternative to Minimal Flavor Violation (MFV).

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I. INTRODUCTION

The smallness of the fermion masses of the first two generations and of the off-diagonal CKM elements suggest the idea that these quantities are perturbations, induced by quantum loop corrections. Already in 1972 S. Weinberg explored this idea [1]. Later this possibility has been studied in the context of Grand Unified Theories [2]. In 1982 and 1983 several authors realized that the trilinear soft SUSY-breaking terms [3] can indeed generate fermion masses radiatively in the MSSM [4–7]. This possibility was later worked out in more detail by T. Banks [8]. (A review of different mechanisms of radiative mass generation can be found in [9].) The fermion masses and off-diagonal CKM elements can be viewed to arise from *soft Yukawa couplings*, generated through loops involving a trilinear term A^q_{ij}, with $q = u, d$ and $i, j = 1, 2, 3$ labeling the fermion generation. While the usual hard Yukawa couplings in the superpotential are identical in Higgs and higgsino couplings to quarks and squarks, the corresponding soft Yukawa couplings are very different from each other. An extensive study of soft Yukawa couplings in supersymmetric theories was carried out in 1999 by F. Borzumati et al. [10]. An alternative possibility to generate CKM elements radiatively arises in left-right symmetric models [11]. In 2004 SUSY-breaking scenarios which can give rise to radiative masses have been studied [12]. The required non-minimal flavor structure of A^q_{ij} unavoidably affects flavor-changing neutral current (FCNC) processes. The precision studies of FCNC at B-factories and the Tevatron in the past decade therefore challenge the idea of RFV. However, recently two of us revisited RFV in Refs. [13, 14] and found that it is possible to generate each element of the CKM matrix separately without violating bounds from FCNC processes for squark masses above approximately 500 GeV.

The framework of Refs. [13, 14] is as follows: The $[U(3)]^3$ flavor symmetry of the gauge sector (we neglect leptons here) is broken to $[U(2)]^3 \times U(1)$ by non-zero hard Yukawa couplings $y_{b,t} \neq 0$ for bottom and top quarks, respectively. Then either A^u_{ij} or A^d_{ij} is chosen as the spurion breaking $[U(2)]^3 \times U(1)$ to $U(1)_B$, the baryon number symmetry. The bilinear SUSY-breaking terms are chosen universal, i.e. they respect the $[U(3)]^3$ symmetry of the gauge sector, up to renormalization effects from the A^q_{ij}. Our model of radiative mass and CKM generation has several advantages compared to the generic MSSM:

- The imposed $[U(2)]^3$ symmetry of the Yukawa sector protects the quarks of the first two generations from a tree-level mass term. The smallness of their masses is thus explained naturally via loop-suppression.

- The model is economical: Flavor violation and SUSY breaking have the same origin.

- Flavor universality holds for the first two generations. Thus our model is minimally flavor-violating with respect to the first two generations since the quark and the squark mass matrices are diagonal in the same basis [1]; i.e. the off-diagonal elements $\Delta^{q\,XY}_{12}$, $X,Y = L,R$, of the squark mass matrices vanish in the basis of the superfields in which the quark mass matrices are diagonal. This explains why K and D physics data comply well with the Standard Model predictions. However, double mass insertions involving the third generation affect the transitions between the first two generations (see section III for details) permitting small deviations from the CKM pattern.

- The SUSY flavor problem is reduced to the quantities $\delta^{q\,RL}_{13,23}$ ($\delta^{q\,XY}_{ij} = \Delta^{q\,XY}_{ij}/m^2_{\tilde{q}}$ with $m_{\tilde{q}}$ denoting the average squark mass) because they are the only flavor-changing SUSY-breaking terms which are not related to corresponding CKM elements. However, these parameters are less constrained from FCNCs than $\delta^{q\,LR}_{13,23}$ and can explain a potential new CP phase indicated by recent data on B_s mixing [16], as we will show below. Furthermore, as shown in Ref. [17] $\delta^{u\,RL}_{13,23}$ can also induce a right-handed W coupling which can explain discrepancies between inclusive and exclusive determinations of V_{ub} and V_{cb}.

- The SUSY CP problem is substantially alleviated by an automatic phase alignment [10] between the A-terms and the effective Yukawa couplings. In addition, the phase of μ essentially does not enter the EDMs of the light quarks at the one-loop level because the Yukawa couplings of the first two generations are zero.

- When our RFV framework is extended to leptons, the anomalous magnetic moment of the muon receives a contribution (independent of μ) which interferes constructively with the SM one. In this way the discrepancy between the SM prediction and experiment can be solved [10, 18].

[1] Note that our definition of MFV differs from the one of Ref. [15] in the sense that we refer to the effective Yukawa couplings and not the hard couplings of the superpotential.

In this article we investigate the implications of this model of RFV for flavor-changing processes in the quark sector and we complement the analyses of Refs. [13, 14] in some important points:

- As shown in Ref. [19], chirally enhanced corrections to FCNC processes are important in the presence of flavor-changing A-terms. Therefore, we include these sizable corrections into our analysis.

- Diagrams involving the double mass insertions $\delta^{q\,LR}_{13}\delta^{q\,LR}_{23}$ which contribute to K–$\overline{K}$ and D–$\overline{D}$ mixing are considered.

- Taking into account chirally enhanced corrections and multiple flavor changes we explicitly show the allowed regions in parameter space in the $m_{\tilde{g}} - m_{\tilde{q}}$ plane.

- Predictions for the rare decay $K \to \pi\nu\bar{\nu}$ are given.

- Off-diagonal A-terms induce (non-decoupling) flavor-changing neutral Higgs couplings proportional to $\tan\beta$ [20]. We study the effect of these couplings on $B_s \to \mu^+\mu^-$ and B_s-$\bar{B}_s$ mixing.

II. RADIATIVE MASS AND CKM GENERATION

As discussed in the introduction, the light quark masses (possibly also the bottom mass) and the off-diagonal CKM elements can be induced in the MSSM via self-energy diagrams involving the trilinear A-terms. These self-energies are chirally enhanced gaugino-sfermion loops which modify the relations between physical masses and Yukawa couplings significantly [2]. In this section we define our framework and quantify the size of the A-terms needed to generate the masses and the off-diagonal CKM elements radiatively.

While in the MSSM the light fermion masses can arise from loop-induced Higgs couplings involving virtual squarks and gluinos [3], the heaviness of the top quark requires a special treatment for Y^t. The successful bottom-tau Yukawa unification suggests to keep the tree-level Yukawa couplings for the third generation lepton and down-type quark, as well. At large $\tan\beta$, this idea gets even more support from the successful unification of the top and bottom Yukawa couplings, as suggested by some SO(10) GUTs. In the modern language of Refs. [15, 22] the global $[U(3)]^5$ flavor symmetry of the gauge sector is

[2] By using 't Hooft's naturalness argument very strong bounds on the mass-insertions $\delta^{f\,LR}_{11,22}$ can be derived by requiring that the supersymmetric corrections do not exceed the measured masses [13, 21]

[3] Of course also the bino diagram contributes to the quark masses, but it is suppressed by a factor $\frac{3\alpha_1}{8\alpha_s}$

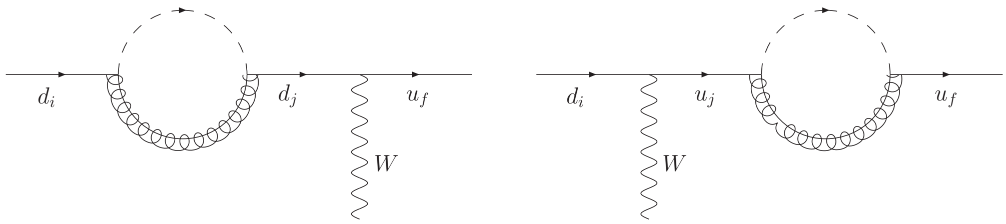

FIG. 1: One-loop contributions to the CKM matrix from the down-sector and from the up-sector, respectively.

broken down to $[U(2)]^5 \times [U(1)]^2$ by the Yukawa couplings of the third generation. Here the five $U(2)$ factors correspond to rotations of the left-handed doublets and the right-handed singlets of the first two generation quarks and leptons in flavor space, respectively. Imposing this symmetry on the Yukawa sector implies

$$Y^{f(0)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y^f \end{pmatrix}, \quad V^{(0)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

in the bare Lagrangian. The absence of tree-level Yukawa couplings of the light fermions as well as of off-diagonal CKM elements requires that these quantities have to be generated via radiative corrections.

While the trilinear SUSY-breaking terms A^u and A^d are the spurions breaking the $[U(2)]^3 \times U(1)$ symmetry of the hard quark Yukawa sector, we assume that the bilinear squark mass terms $\mathbf{M}_{\tilde{q}\,LL,RR}$ possess the full $[U(3)]^3$ flavor symmetry of the (s)quark sector at some high scale. At the low electroweak scale the $[U(3)]^3$ symmetry of $\mathbf{M}_{\tilde{q}\,LL,RR}$ is broken by two renormalisation effects: First, there are renormalization-group (RG) effects proportional to Yukawa couplings. These effects split the third eigenvalue from the first two ones, but do not induce off-diagonal terms in $\mathbf{M}_{\tilde{q}\,LL,RR}$. Second, the trilinear terms A^u and A^d will also renormalize $\mathbf{M}_{\tilde{q}\,LL,RR}$. The off-diagonal terms induced by the RG evolution lead to flavor-changing elements in $\mathbf{M}_{\tilde{q}\,LL,RR}$. In a given FCNC loop diagram these terms lead to additional contributions, which, however, are governed by the same element of A^q which enters the considered loop directly. Therefore the RG effects in $\mathbf{M}_{\tilde{q}\,LL,RR}$ simply shift the numerical value of A^q_{jk} determined from the condition that RFV reproduces the measured CKM elements. In our phenomenological study, treating the trilinear terms as independent, one can therefore neglect the RG effects in $\mathbf{M}_{\tilde{q}\,LL,RR}$. The situation is different, once GUT boundary conditions are placed on the RFV model.

Let us first have a look at the consequences of this symmetry-breaking pattern for the first two quark generations. Here we can exploit the $SU(2)$ flavor symmetry of the gauge- and Yukawa-sector to choose a basis for the left- and right-handed quark fields in which for example the upper left 2×2 block of A^d, denoted by $A^d_{2\times2}$, is diagonal. Choosing four different rotations for left- and right-handed up and down quark fields (so that the $SU(2)_L$ gauge symmetry is no more manifest), we can even diagonalize $A^d_{2\times2}$ and $A^u_{2\times2}$ simultaneously. In this step the tree-level Cabibbo matrix $(V_C^{(0)})_{2\times2}$ is generated. Since in this basis no sources of flavor-violating (1,2) elements are present in the squark-mass matrices, the so-obtained $(V_C^{(0)})_{2\times2}$ equals the Cabibbo matrix $V_{2\times2}$ known from experiment (up to negligible corrections arising from loops involving a $1 \to 3 \to 2$ transition). This observation implies that $A^q_{2\times2}$ is proportional to the corresponding effective (loop-induced) Yukawa matrix $(Y^q_{\text{eff}})_{2\times2}$. Note that even with respect to the first two generations the model is not minimally flavor-violating in the literal sense of Ref. [15]: The A-terms cannot be constructed out of the (vanishing) tree-level Yukawa couplings and vice versa. However, our model obeys the MFV definition with respect to the first two generations, if one defines MFV with $(Y^q_{\text{eff}})_{2\times2}$ instead, and the Cabibbo matrix is the only source of flavor violation.

For the third generation the situation is different. The direction of the third generation in flavor space is already fixed from the Yukawa sector, by the requirement of diagonal Yukawa matrices $Y^{q(0)}$. Therefore, the elements A^q_{3i} and A^q_{i3} ($i = 1, 2$) cannot be eliminated by a redefinition of the flavor basis. The effect of these A-terms is twofold: On one hand, they have to generate the effective CKM elements V_{3i} and V_{i3}. On the other hand, they act as sources of non-minimal flavor violation and thus they are constrained from FCNC processes.

It is common to choose a basis for the quark fields in which the Yukawa couplings are diagonal and, in order to have manifest supersymmetry in the superpotential, to subject the squarks to the same rotations as the quarks. The resulting basis for the super-fields is called the super-CKM basis. However, since the Yukawa couplings of the first two generations are zero in our scenario, the super-CKM basis is not defined unambiguously. We fix this ambiguity with the additional requirement of diagonal $A^d_{2\times2}$ and $A^u_{2\times2}$ (as described above). The (bare) CKM

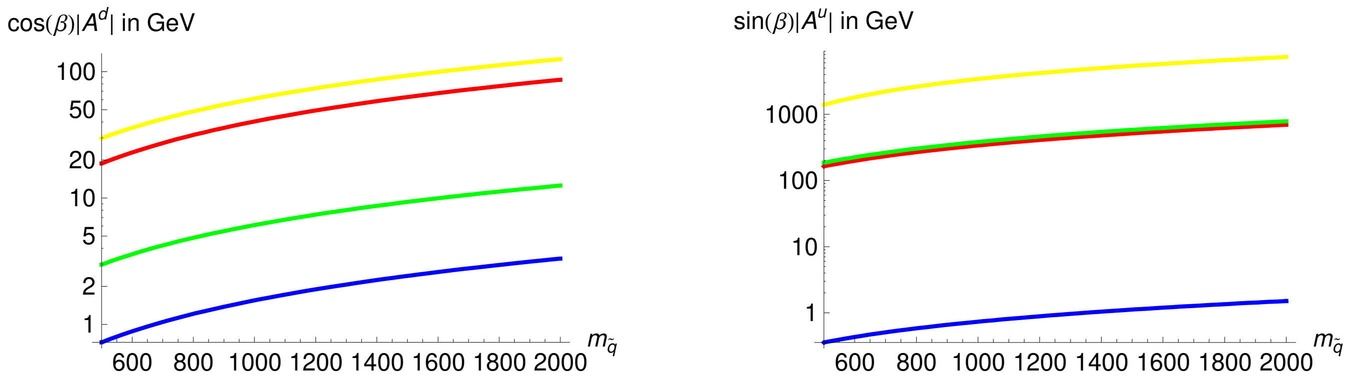

FIG. 2: Left: Absolute values of A^d_{ij} (at scale $M_{\rm SUSY}$) needed to generate the down and strange quark masses and the off-diagonal CKM elements involving the third generation. Blue(darkest): A^d_{11}, red: A^d_{22}, green: A^d_{13}, yellow(lightest): A^d_{23}. Right: Absolute values of A^u_{ij} (at scale $M_{\rm SUSY}$) needed to generate the up and the charm quark mass and the off-diagonal CKM elements involving the third generation. blue(darkest): A^u_{11}, red: A^u_{22}, green: A^u_{13}, yellow(lightest): A^u_{23}.

matrix and the A-terms then take the following form:

$$V_C^{(0)} = \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (2)$$

$$A_C^q = \begin{pmatrix} A^q_{11} & 0 & A^q_{13} \\ 0 & A^q_{22} & A^q_{23} \\ A^q_{31} & A^q_{32} & A^q_{33} \end{pmatrix}. \qquad (3)$$

The subscript C denotes the Cabbibo rotation performed for this choice of the super-CKM basis. The Cabbibo angle stems from the misalignment between the A^u-terms and the A^d-terms of the first two generations in any weak basis. In the basis corresponding to Eqs. (2) and (3), however, $A^d_{2\times2}$ and $A^u_{2\times2}$ are simultaneously diagonal and the familiar Cabibbo angle θ_C appears in the W couplings to (s)quarks.

Since the bare Yukawa couplings of quarks of the first two generations are zero, these quarks do not develop tree-level mass terms. Because of the non-vanishing A-terms of the first two generations they can, however, couple to the Higgs fields and to their vevs via a loop with SUSY-particles. The corresponding self-energy diagrams generate effective masses for the quarks.

At this point we recall some results concerning quark self-energies, the renormalization of masses and flavor-valued field rotations. It is possible to decompose any self-energy into its chirality-flipping and its chirality-conserving parts as

$$\Sigma^q_{ij}(p) = \left(\Sigma^{q\,LR}_{ij}(p^2) + \not{p}\,\Sigma^{q\,RR}_{ij}(p^2)\right) P_R + \left(\Sigma^{q\,RL}_{ij}(p^2) + \not{p}\,\Sigma^{q\,LL}_{ij}(p^2)\right) P_L\,. \qquad (4)$$

Only the chirality-flipping self-energies $\Sigma^{q\,RL}_{ij} = \Sigma^{q\,LR*}_{ji}$ are capable of generating sizable effective mass terms in the absence of tree-level Yukawas. Since we are working with quarks we can concentrate on the contributions from gluino-squark loops.[4] At vanishing external momentum, the SQCD self-energy is given by (the conventions are defined in the appendix of Ref. [13])

$$\Sigma^{q\,LR}_{ij} = \frac{2\alpha_s}{3\pi} m_{\tilde{g}} \sum_{s=1}^{6} W^{\tilde{q}}_{is} W^{\tilde{q}*}_{(j+3)s} B_0(m^2_{\tilde{g}}, m^2_{\tilde{q}_s}). \qquad (5)$$

At first order in the mass insertion approximation this simplifies to

$$\Sigma^{q\,LR}_{ij} = \frac{2\alpha_s}{3\pi} m_{\tilde{g}}\, \Delta^{q\,LR}_{ij}\, C_0\left(M_1^2, m^2_{\tilde{q}_{iL}}, m^2_{\tilde{q}_{jR}}\right) \qquad (6)$$

with

$$\begin{aligned} \Delta^{u\,LR}_{ij} &= -v_u A^u_{ij} - v_d\,\mu\, Y^{u(0)}_{ij}\,, \\ \Delta^{d\,LR}_{ij} &= -v_d A^d_{ij} - v_u\,\mu\, Y^{d(0)}_{ij}\,. \end{aligned} \qquad (7)$$

Now we turn to the renormalization of quark masses and to the rotations in flavor-space which are induced by the self-energies. Including chirally enhanced corrections, the physical masses m_{q_i}, which are extracted from experiment using the SM prescription with ordinary QCD corrections renormalized in the $\overline{\rm MS}$ scheme, are given as

$$m_{q_i} = m^{(0)}_{q_3}\,\delta_{i3} + \Sigma^{q\,LR}_{ii}. \qquad (8)$$

[4] This is a good approximation for the self-energies. However, when we later consider FCNC processes also the chargino contributions are important. In principle also chirally enhanced chargino self-energies (in the down-quark sector) contribute to the CKM renormalization if $A^u_{13,23}$ are unequal to zero. Even though these contributions are enhanced by a factor m_t/m_b they are small due to a suppression by $v^2/M^2_{\rm SUSY} \times g_2^2/g_s^2$, especially at smaller values of $\tan\beta$.

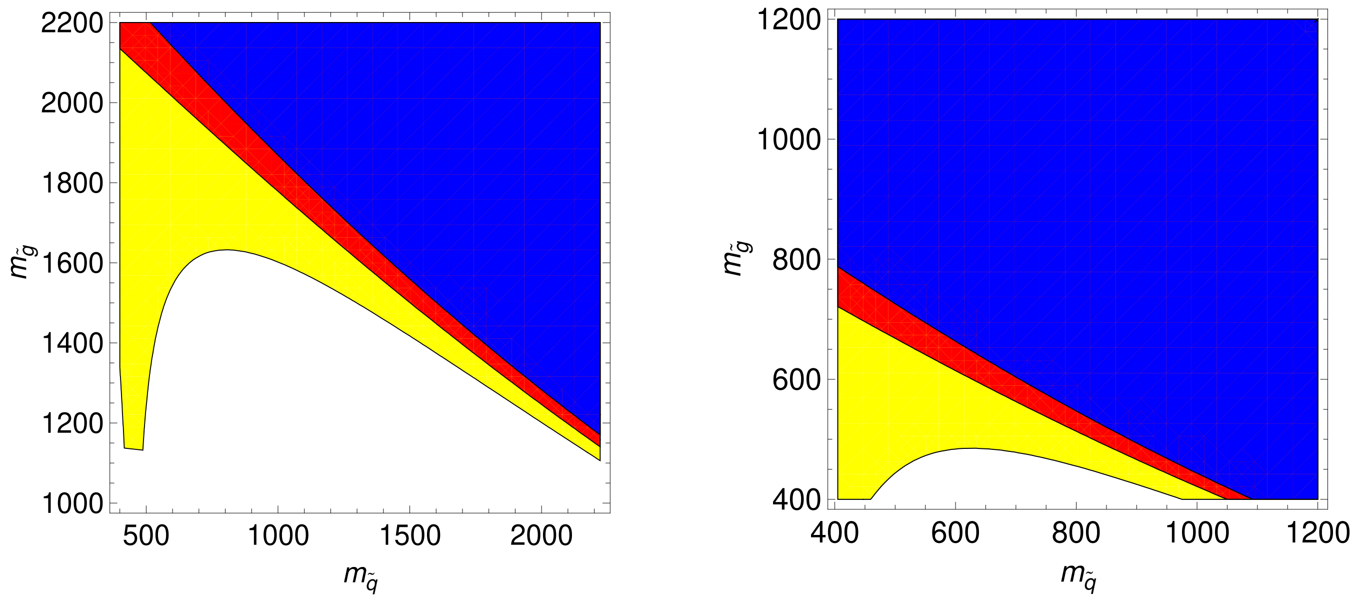

FIG. 3: Left: Allowed regions in the $m_{\tilde q} - m_{\tilde g}$ plane. Constraints from $b \to s\gamma$ for different values of $\mu \tan\beta$ assuming that the CKM matrix is generated in the down sector. We demand that the calculated branching ratio, with the SM and gluino contributions, lies within the $2\,\sigma$ range of the measurement. Yellow(lightest): allowed region for $\mu \tan\beta = 30\,\mathrm{TeV}$, red: $\mu \tan\beta = 0\,\mathrm{TeV}$ and blue(darkest): $\mu \tan\beta = -30\,\mathrm{TeV}$.
Right: Same as the left plot, but with the weaker requirement that the gluino contribution should not exceed the SM one.

Here $m_{q_3}^{(0)} = y^q v_q$ denotes the bare quark mass generated by the Yukawa coupling y^q defined in Eq. (1). Since $\Sigma_{ii}^{q\,LR}$ is finite, there is no need to renormalize $m_{q_3}^{(0)}$ by splitting it into a renormalized part and into a counterterm. For the first two quark generations, we directly read off the requirement of radiative mass generation from Eq. (8):

$$\Sigma_{ii}^{q\,LR} = m_{q_i} \qquad (i = 1, 2). \tag{9}$$

The flavor-changing self-energies $\Sigma_{ij}^{q\,LR}$ induce field rotations

$$\psi_i^{q\,L} \longrightarrow U_{ij}^{q\,L}\,\psi_j^{q\,L} \tag{10}$$

in flavor space. To leading order in small quark mass ratios $U^{q\,L}$ reads

$$U^{q\,L} = \begin{pmatrix} 1 & \dfrac{\Sigma_{12}^{q\,LR}}{m_{q_2}} & \dfrac{\Sigma_{13}^{q\,LR}}{m_{q_3}} \\ -\dfrac{\Sigma_{21}^{q\,RL}}{m_{q_2}} & 1 & \dfrac{\Sigma_{23}^{q\,LR}}{m_{q_3}} \\ -\dfrac{\Sigma_{31}^{q\,RL}}{m_{q_3}} + \dfrac{\Sigma_{21}^{q\,RL}}{m_{q_2}}\dfrac{\Sigma_{32}^{q\,RL}}{m_{q_3}} & -\dfrac{\Sigma_{32}^{q\,RL}}{m_{q_3}} & 1 \end{pmatrix}. \tag{11}$$

Note that the quark masses m_{q_i} appearing in this equation are the physical $\overline{\mathrm{MS}}$ - masses which have to be evaluated at the same scale as the self-energies [13, 23] and that $U_{31}^{q\,L}$ contains a two-loop contribution which can be numerically important [21]. The formula for $U^{q\,L}$ given in Eq. (11) is valid irrespective of the basis chosen for the (s)quark superfields. In our super-CKM basis (with diagonal $A_{2\times2}^{d}$ and $A_{2\times2}^{u}$), $\Sigma_{12}^{q\,LR} = \Sigma_{21}^{q\,RL*}$ vanishes and the corresponding terms in $U^{q\,L}$ are absent:

$$U_C^{q\,L} = \begin{pmatrix} 1 & 0 & \dfrac{\Sigma_{13}^{q\,LR}}{m_{q_3}} \\ 0 & 1 & \dfrac{\Sigma_{23}^{q\,LR}}{m_{q_3}} \\ -\dfrac{\Sigma_{31}^{q\,RL}}{m_{q_3}} & -\dfrac{\Sigma_{32}^{q\,RL}}{m_{q_3}} & 1 \end{pmatrix}. \tag{12}$$

The flavor-changing self-energies $\Sigma_{ij}^{q\,LR}$ induce corrections to the CKM matrix $V_C^{(0)}$ as depicted in Fig. 1. In this way they generate the physical CKM matrix V, measured in low-energy experiments. In terms of the field rotations $U_C^{q\,L}$ the physical CKM matrix V can be expressed as

$$V \;=\; U_C^{u\,L\dagger}\, V_C^{(0)}\, U_C^{d\,L}\,. \tag{13}$$

The CKM matrix can be generated in the up-sector, in the down-sector or in both sectors at the same time (see Fig. 1). In the following we concentrate on the two limiting cases in which either the up-squark or the down-squark sector is flavor-diagonal in our super-CKM basis.

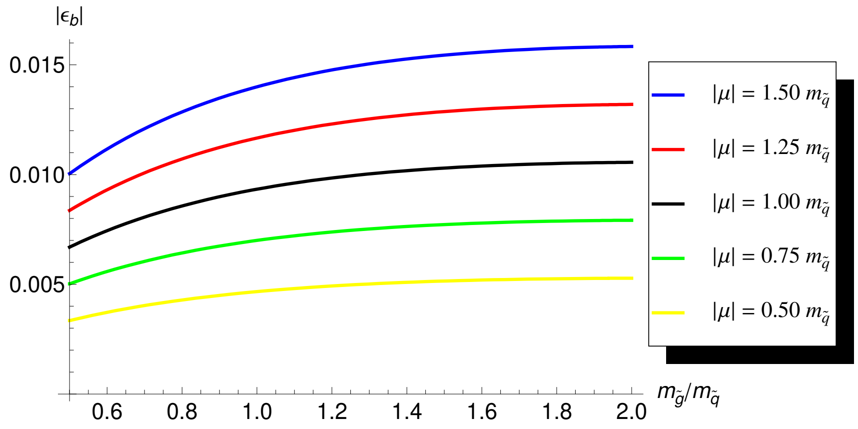

FIG. 4: $|\epsilon_b|$ as a function of $m_{\tilde{g}}/m_{\tilde{q}}$ for different values $|\mu|/m_{\tilde{q}}$. In the decoupling limit $|\epsilon_b|$ depends only on these two ratios.

We refer to these two scenarios as "CKM generation in the down-sector" and "CKM generation in the up sector", respectively. For "CKM generation in the down-sector" we obtain from Eq. (13) the following conditions:

$$\begin{aligned} \Sigma_{23}^{d\,LR} &= m_b V_{cb} \approx -m_b V_{ts}^*\,, \\ \Sigma_{13}^{d\,LR} &= m_b V_{ub}\,. \end{aligned} \tag{14}$$

In principle, the self-energy $\Sigma_{13}^{d\,LR}$ can equivalently be determined from the CKM element V_{td}. Note, however, that concerning V_{td} the situation is a bit more complicated due to the additional doubly flavor-changing contribution $V_{us}^* \dfrac{\Sigma_{32}^{q\,RL}}{m_{q_3}}$. For "CKM generation in the up sector" we have

$$\begin{aligned} \Sigma_{23}^{u\,LR} &= -m_t V_{cb} \approx m_t V_{ts}^*\,, \\ \Sigma_{13}^{u\,LR} &= m_t V_{td}^*\,. \end{aligned} \tag{15}$$

For illustration we show in Fig. 2 the size of the A-terms needed to generate the quark masses and the CKM mixing angles according to Eqs. (9), (14) or (15). To this end we have set the gluino mass and the left-handed and right-handed squark masses to a common value $m_{\tilde{q}}$.

III. PHENOMENOLOGICAL CONSEQUENCES FOR FLAVOR CHANGING PROCESSES

Although the B factories have confirmed the CKM mechanism as the dominant source of flavor violation with high precision, leaving little room for new sources of FCNCs in $b \to d$ and $s \to d$ transitions, we show in this section that radiative generation of quark masses and of the CKM matrix still remains a valid scenario. While flavor-changing transitions among the first two generations are CKM-like, this is no longer true once the third generation is involved, because the A-terms are not diagonal in the same basis as the bare Yukawa couplings. It is evident from Eq. (3) that A_{i3}^q and A_{3i}^q are non-minimal sources of flavor-violation. In the following we will concentrate on the two simple limiting cases in which either A^u is diagonal (in the same basis as $Y^{u(0)}$) and the CKM elements are generated by the off-diagonal elements of A^d, or on the opposite case in which A^d is diagonal but A^u is not. Even though the elements $A_{31,32}^q$ are not needed for the generation of the CKM matrix, no symmetry argument requires them to be zero. Note that it is in principle also possible to generate the fermion masses with non-holomorphic trilinear terms. Such a scenario (as proposed in Ref. [10, 24]) will lead to additional effects in the Higgs sector [20].

A. CKM generation in the down-sector

If the CKM matrix is generated in the down sector, the off-diagonal elements of the squark mass matrix $\Delta_{13,23}^{d\,LR}$ are determined by the requirement that they generate the observed CKM matrix via Eq. (14). Since the off-diagonal elements needed to generate the CKM matrix in the down-sector are very small (cf. Fig. 2), the mass-insertion approximation excellently reproduces the exact result. Therefore, we can solve for $\Delta_{13,23}^{d\,LR}$ analytically by using Eq. (6) in Eq. (14). In the following we investigate the consequences of the so-determined $\Delta_{13,23}^{d\,LR}$ on FCNC processes.

1. $b \to s\gamma$

To leading order in the mass insertion approximation, the flavor off-diagonal elements $\delta_{13,23}^{d\,LR}$ ($\delta_{ij}^{q\,XY} = \Delta_{ij}^{q\,XY}/m_{\tilde{q}}^2$) enter FCNC processes involving the third generation. Furthermore, also Kaon and D mixing are affected by diagrams containing the combination $\delta_{13}^{d\,LR} \times \delta_{32}^{d\,RL}$. Even though Kaon mixing is very sensitive to NP,

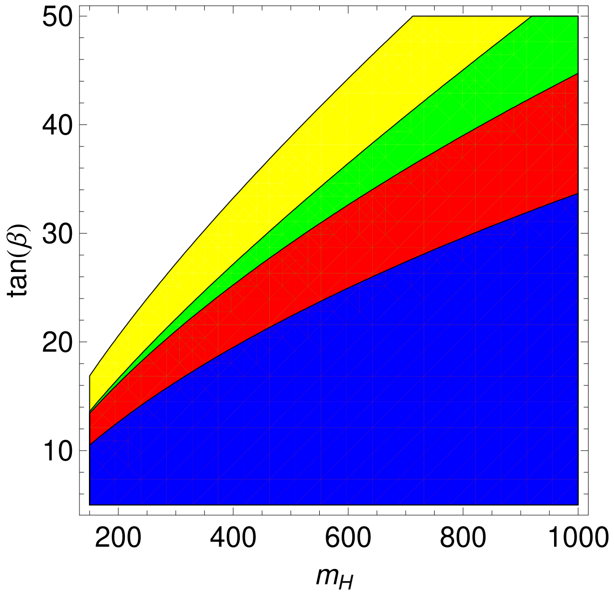

FIG. 5: Allowed region in the m_H–$\tan\beta$ plane for different values of ϵ_b from $\mathrm{Br}[\mathrm{B_s} \to \mu^+\mu^-] \le 3.2{\cdot}10^{-8}$[95%CL]. Yellow: $\epsilon_b = 0.005$, green: $\epsilon_b = 0.01$, red: $\epsilon_b = -0.005$, blue: $\epsilon_b = -0.01$ (light to dark).

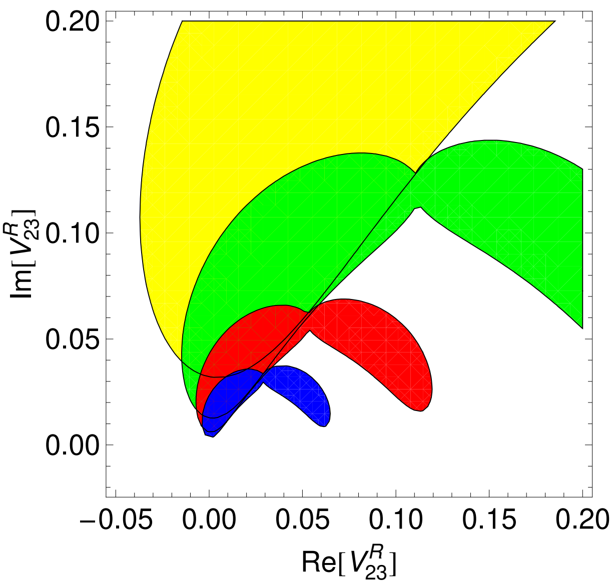

FIG. 6: Allowed region (95% confidence level) in the $\mathrm{Re}[\mathrm{V^R_{23}}]$–$\mathrm{Im}[\mathrm{V^R_{23}}]$ plane for $\epsilon_b = 0.0075$, $m_H = 400\mathrm{GeV}$ and different values of $\tan\beta$ from $B_s - \overline{B}_s$ mixing. Yellow: $\tan\beta = 11$, green: $\tan\beta = 14$, red: $\tan\beta = 17$, blue: $\tan\beta = 20$ (light to dark). The peculiar shape stems from the fact that the data permit two solutions for the $B_s - \overline{B}_s$ mixing phase. The solution with the smaller values of $\mathrm{Re}\, V^R_{23}$ corresponds to the solution closer to the SM value.

especially to new sources of CP violation, the product $\delta^{d\,LR}_{13} \times \delta^{d\,RL}_{32}$ is too small to give sizable effects. The contribution to D mixing is even further suppressed since it is generated by a chargino diagram. However, $b \to s(d)\gamma$ is very sensitive to $\delta^{d\,LR}_{23}$ ($\delta^{d\,LR}_{13}$) since these parameters violate both flavor and chirality. Even though the relative effect (compared to the SM contribution) in $b \to s\gamma$ and $b \to d\gamma$ is approximately equal, $b \to s\gamma$ turns out to be the process which is most sensitive to RFV stemming from the down sector since it is measured more precisely than $b \to d\gamma$. The new contributions affect the Wilson coefficients C_7 and C_8 of the magnetic and chromomagnetic operators. The interference of the gluino contribution with the SM contribution is necessarily constructive. It is important to take into account the chirally enhanced corrections to the Wilson coefficients discussed in Ref. [19]. Due to the inclusion of these effects also the gluino constraints depend on μ and $\tan\beta$ (see Fig. 3).

The allowed region in the left plot of Fig. 3 is obtained under the assumption that the calculated branching ratio (including only the SM contribution taken from Ref. [25, 26] and the gluino contribution involving $\delta^{d\,LR}_{23}$) is less than 2σ away from the measured central value of the branching ratio $\mathrm{Br}\,[\mathrm{b} \to \mathrm{s}\gamma] = (3.55 \pm 0.26) \times 10^{-4}$ [27]. However, there are more contributions involving additional free parameters. First there is the charged Higgs contribution which depends to a very good approximation only on $m^2_{H^+}$ and interferes constructively with the SM [28, 29]. In addition, since no symmetry argument forbids a non-vanishing term A^d_{32} (and therefore $\delta^{d\,RL}_{23}$) in our model there is another possible contribution to C'_7 and C'_8 which can enhance the branching ratio. However, there is also the chargino contribution which grows with $\tan\beta$ and can have either sign depending on the product μA^u_{33}. Therefore, this contribution can interfere destructively with the SM, the gluino, and the charged Higgs contribution. In total the branching ratio can be in agreement with experiment. This is possible for a wide range of parameters, however some degree of fine-tuning is necessary. In order to avoid very large cancellations, one can demand that none of the various NP contributions contribution should exceed the SM one. Under this assumption the allowed region in the right plot of Fig. 3 is obtained.

2. *Non-decoupling Higgs-mediated effects*

At moderate-to-large $\tan\beta$ Higgs-mediated effects become important. These effects are non-decoupling; this means that they do not vanish for heavy SUSY masses but only decouple like $1/M^2_{\mathrm{H}}$ (for large $\tan\beta$ the CP-even Higgs, the charged Higgs and the CP-odd Higgs have approximately equal masses $m_H \equiv m_{H^0} \approx m_{A^0} \approx m_{H^+}$). As shown recently [20, 30], if one consistently includes all chirally enhanced effects into the calculation of the effective Higgs vertices, the trilinear A-terms induce effective flavor-changing neutral Higgs couplings. These effects are important in our model already for moderate $\tan\beta$

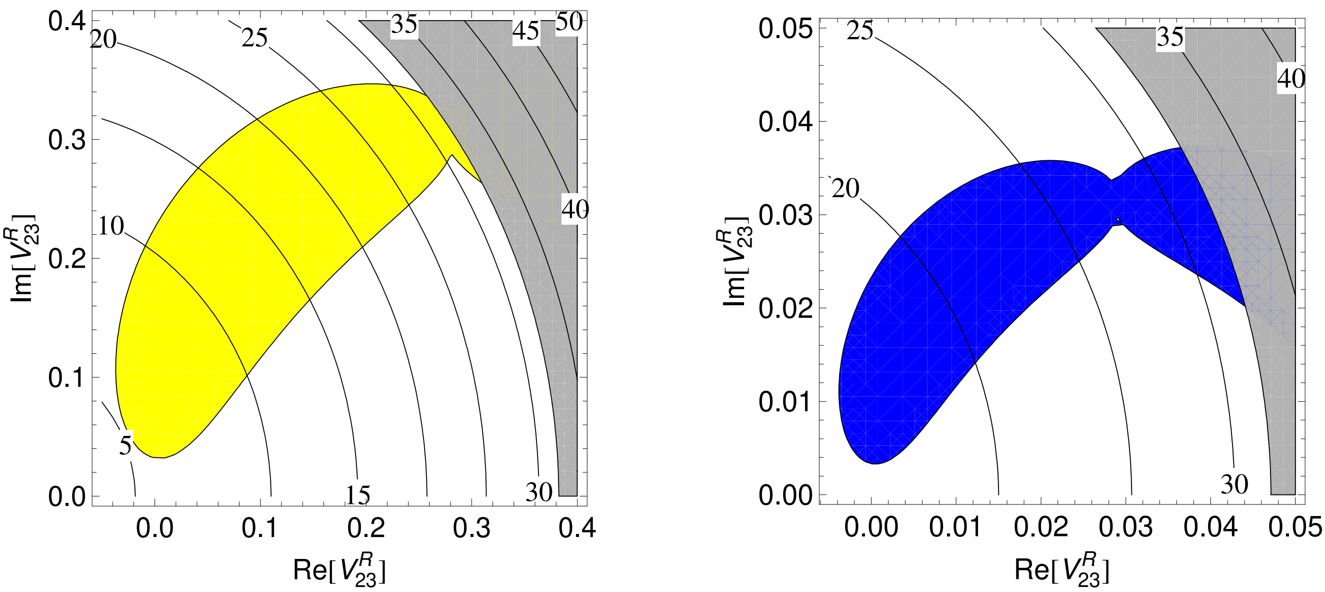

FIG. 7: Correlations between $B_s \to \mu^+\mu^-$ and $B_s - \overline{B}_s$ mixing for $\epsilon_b = 0.0075$, $m_H = 400\text{GeV}$. Left plot, yellow: Allowed region from $B_s - \overline{B}_s$ mixing (95% confidence level) for $\tan\beta = 11$. The contour-lines show $\text{Br}[\text{B}_\text{s} \to \mu^+\mu^-] \times 10^9$. The grey area at the right side is excluded by the Tevatron bound on $\text{Br}[\text{B}_\text{s} \to \mu^+\mu^-]$. Right plot, blue: same for $\tan\beta = 20$.

since we necessarily have flavor off-diagonal A-terms in order to generate the CKM matrix.

Following Refs. [20, 30] and neglecting terms proportional to $\cos\beta$, a Feynman rule for the effective neutral Higgs coupling mediating b-s transitions (The corresponding formula for b-d transitions are simply obtained by exchanging s and d) is given by

$$i\left(\Gamma_{sb}^{H_k^0\,LR} P_R + \Gamma_{sb}^{H_k^0\,RL} P_L\right) \tag{16}$$

with

$$\Gamma_{sb}^{H_k^0\,LR} = \Gamma_{bs}^{H_k^0\,RL*} = x_d^k \frac{1}{v_d} \frac{\Sigma_{23}^{d\,LR}}{m_b} \Sigma_{33}^{\prime d\,LR} . \tag{17}$$

Here H_k^0 denotes the three physical neutral Higgs bosons: the heavy CP-even Higgs $H_1^0 = H^0$, the light CP even Higgs $H_2^0 = h^0$ and the CP-odd Higgs $H_3^0 = A^0$. For $H_k^0 = (H^0, h^0, A^0)$ the coefficients x_d^k are given by

$$x_d^k = \left(\frac{-1}{\sqrt{2}}\cos(\alpha), \frac{1}{\sqrt{2}}\sin(\alpha), \frac{i}{\sqrt{2}}\sin(\beta)\right). \tag{18}$$

Furthermore, $\Sigma_{33}^{\prime d\,LR}$ denotes the non-holomorphic part of the self-energy which is proportional to the μ-term originating from $\Delta_{33}^{d\,LR}$ in Eq. (7). It is given by

$$\Sigma_{33}^{\prime d\,LR} = m_b \frac{\varepsilon_b \tan\beta}{1 + \varepsilon_b \tan\beta} \tag{19}$$

with

$$\varepsilon_b = -\frac{2\alpha_s}{3\pi} m_{\tilde{g}} \mu\, C_0\left(m_{\tilde{g}}^2, m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2\right). \tag{20}$$

The loop function C_0 can for example be found in Ref. [23]. Due to the mass eigenstates in the loop-function, Eq. (20) is also valid beyond the decoupling limit in the absence of flavor violation [23, 31]. In the decoupling limit (which is an excellent approximation [20]) with degenerate diagonal squark mass terms $m_{\tilde{q}}^2$, ε_b is only a function of the two ratios $m_{\tilde{g}}/m_{\tilde{q}}$ and $\mu/m_{\tilde{q}}$. We see from Fig. 4 that typical values for $|\varepsilon_b|$ range from 0.005 to 0.01.

With $\Sigma_{23}^{d\,LR}$ being fixed by Eq. (14), $\Gamma_{sb}^{H_k^0\,LR}$ becomes

$$\Gamma_{sb}^{H_k^0\,LR} = x_d^k \frac{V_{cb}}{v_d} \Sigma_{33}^{\prime d\,LR} . \tag{21}$$

This effective Higgs coupling induces a contribution to $B_s \to \mu^+\mu^-$ and therefore gets constrained from this process. Fig. 5 shows the allowed region compatible with the experimental bound of Refs. [27, 32, 33] in the $\tan\beta$–m_H plane, where m_H is the heavy Higgs mass. We see that $\text{Br}[\text{B}_\text{s} \to \mu^+\mu^-]$ can be significantly enhanced even for moderate values of $\tan\beta$.

As already stated, no symmetry argument forbids a non-vanishing value of $\Delta_{32}^{d\,LR}$. Such a term would generate a chirally enhanced self-energy $\Sigma_{32}^{d\,LR}$ which would in turn lead to an effective Higgs coupling $\Gamma_{bs}^{H_k^0\,LR}$. In this case we also get a contribution to $B_s - \overline{B}_s$ mixing. This additional contribution might explain the observed deviation of the measured $B_s - \overline{B}_s$ mixing phase from the SM expectation [16, 34–37]. The SM contributions to the width difference appearing in the angular analysis of the $B_s \to J/\psi\phi$ data and to the CP asymmetry in flavor-specific decays (equivalent to the dimuon asymmetry) have been calculated in Refs. [38–40]. In Fig. 6 we

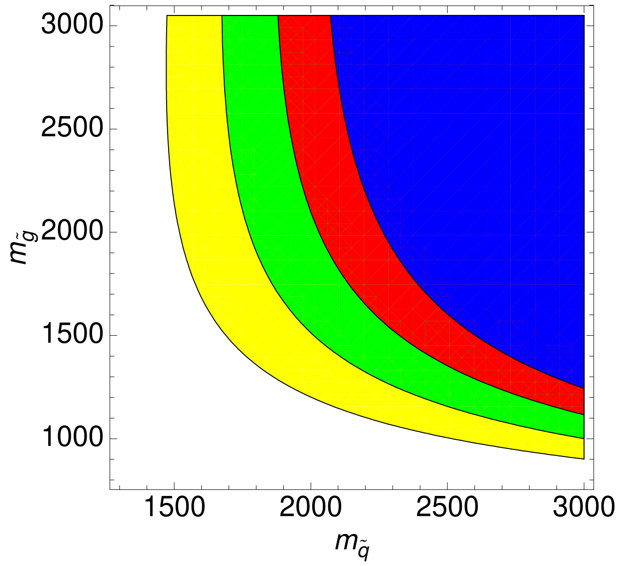

FIG. 8: Allowed regions in the $m_{\tilde{q}} - m_{\tilde{g}}$ plane. Constraints from K–$\overline{K}$ mixing for different values of M_2 assuming that the CKM matrix is generated in the up sector. Yellow(lightest): $M_2 = 1000\,\text{GeV}$, green: $M_2 = 750\,\text{GeV}$, red: $M_2 = 500\,\text{GeV}$ and blue(darkest): $M_2 = 250\,\text{GeV}$.

show the regions in parameter space which are consistent with $B_s - \overline{B}_s$ mixing at the 95% confidence level using the fit result of Ref. [16]. In order to simplify the notation and to allow for an easy comparison with the size of $\Sigma^{d\,LR}_{23} = m_b V_{cb}$ we have defined the quantity

$$V^R_{23} = \Sigma^{d\,RL}_{23}/m_b. \qquad (22)$$

In Fig. 7 we show the correlation between B_s–$\overline{B}_s$ mixing and $B_s \to \mu^+\mu^-$ for $m_H = 400\text{GeV}$, $\epsilon_b = 0.0075$ and two values of $\tan\beta$. Note that the region in parameter space which can explain the phase in $B_s - \overline{B}_s$ mixing is well compatible with the current limits on $\text{Br}[\text{B}_\text{s} \to \mu^+\mu^-]$. Moreover, if the hints for a sizable new-physics contribution to B_s–$\overline{B}_s$ mixing persist, $B_s \to \mu^+\mu^-$ will necessarily be enhanced. LHCb will be able to probe this correlation in the near future. The global analysis in Ref. [16] also finds a preference for a new CP phase $\phi^\Delta_d \approx -13°$ in $B_d - \overline{B}_d$ mixing adding to the SM phase of 2β. Since a smaller new-physics contribution than in the B_s system is needed, it is easy to accomodate this with the free parameter $\delta^{d\,RL}_{13}$. (The experimental bound on the relevant ratio $\text{Br}[\text{B}_\text{d} \to \mu^+\mu^-]/|\text{V}_\text{td}|^2$ is weaker than on $\text{Br}[\text{B}_\text{s} \to \mu^+\mu^-]/|\text{V}_\text{ts}|^2$.) In summary, if the CKM matrix is generated in the down-sector, sizable Higgs-induced effects are generated (even in the decoupling limit) which can enhance $B_s \to \mu^+\mu^-$ and can accomodate the observed evidence for new CP-violating physics in $B_s - \overline{B}_s$ mixing.

Finally we want to discuss the correlation between the bounds from $b \to s\gamma$ and the non-decoupling constraints. The constraints on the SUSY-masses shown in Fig. 3 are weakened for positive μ and large values of $\tan\beta$. In addition, at large $\tan\beta$ also the chargino contribution to $b \to s\gamma$ becomes important and can interfere destructively with the gluino and the SM contribution lowering the bound on the SUSY masses. Thus the constraint on the SUSY masses can only be lowered for large values of $\tan\beta$ which then leads to sizable contributions to $B_s \to \mu^+\mu^-$ (see Fig. 5). Since the bounds from $B_s \to \mu^+\mu^-$ do not decouple with the SUSY scale but only with the Higgs mass they do not vanish for heavy SUSY masses like the constraints from $b \to s\gamma$.

B. CKM generation in the up-sector

If the CKM matrix is generated in the up-sector, the off-diagonal elements of the squark mass matrix $\Delta^{u\,LR}_{13,23}$ are determined by Eq. (15). Due to the large top mass these off-diagonal elements are much larger than in the case of CKM generation in the down sector. Therefore, already the requirement that the lighter stop mass does not violate the bounds from direct searches requires the diagonal elements of the squark mass matrix to be heavier than approximately $(700\,\text{GeV})^2$. Furthermore, the mass-insertion approximation (MIA) does not necessarily hold for such large off-diagonal elements. One cannot solve the exact expressions analytically for $\Delta^{u\,LR}_{13,23}$ but rather has to determine these elements numerically. However, for squark masses above 700 GeV we find the off-diagonal elements determined in MIA larger than the ones obtained by exact diagonalization by just ten percent or less. Therefore, it is still possible to rely on MIA for a qualitative understanding of the flavor structure.

If the CKM matrix is generated in the up-sector, one naively expects the chargino contributions to $b \to s\gamma$, $b \to d\gamma$ and $B_{s,d} - \overline{B}_{s,d}$ mixing to give relevant bounds on $\delta^{u\,LR}_{13,23}$. However, $B_{s,d}$–$\overline{B}_{s,d}$ mixing does not give useful constraints on $\delta^{u\,LR}_{13,23}$ and $\text{Br}[b \to s\gamma, d\gamma]$ also heavily depends on μ and $\tan\beta$. Furthermore, we again have to take into account the chirally enhanced effects by using the effective chargino vertices given in Ref. [13]. In the present case of RFV in the up-sector, these effective vertices read

$$\begin{aligned}
\Gamma^{\tilde{\chi}^\pm_k L}_{d_i \tilde{u}_s} &= \sum_{j=1}^{3} V^{(0)}_{C\,ji} \left(Y^{t(0)} \delta_{j3}\, V^{\tilde{\chi}^\pm *}_{k2} W^{\tilde{u}*}_{j+3,s} - g_2 V^{\tilde{\chi}^\pm *}_{k1} W^{\tilde{u}*}_{js} \right), \\
\Gamma^{\tilde{\chi}^\pm_k R}_{d_i \tilde{u}_s} &= Y^{b(0)} \delta_{i3}\, U^{\tilde{\chi}^\pm}_{k2} \sum_{j=1}^{3} V^{(0)}_{C\,ji} W^{\tilde{u}*}_{js}. \qquad (23)
\end{aligned}$$

Note that it is the bare CKM matrix $V^{(0)}_C$ (with vanishing elements connecting the third with the first two generations) and not the physical CKM matrix V which appears in these couplings to external down-type quarks. This is easy to understand since the physical CKM matrix V is generated in the up-sector meaning that the down-type

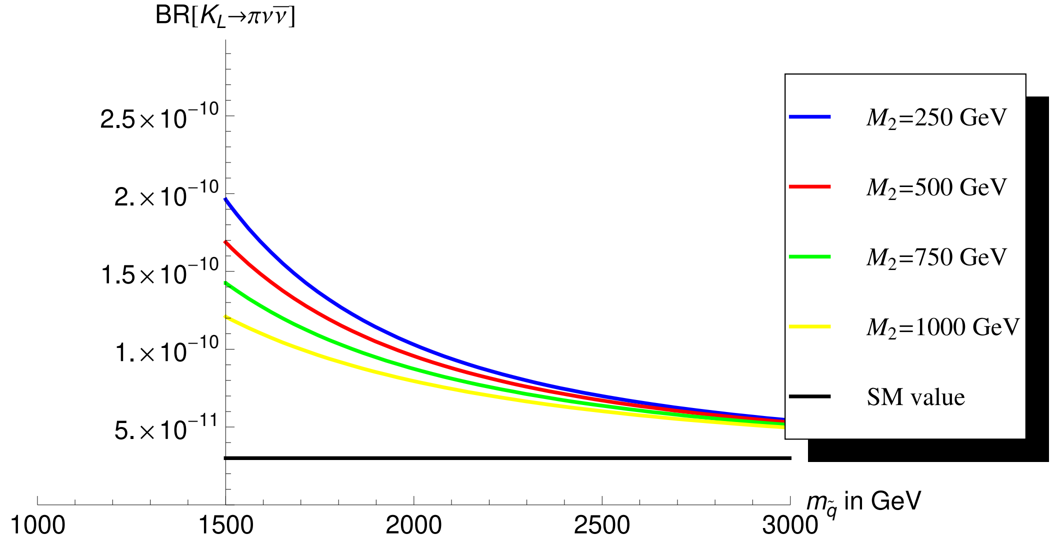

FIG. 9: Predicted branching ratio for the rare Kaon decay $K_L \to \pi\nu\overline{\nu}$ assuming that the CKM matrix is generated in the up-sector for $m_{\tilde{q}} = m_{\tilde{g}}$. The branching ratio is enhanced due to a constructive interference with the SM contribution.

quarks are not rotated by flavor-changing self-energies. Note further that the Yukawa couplings of the quarks of the first two generations are zero in our scenario of RFV.

While $b \to s\gamma$, $b \to d\gamma$ and $B_{s,d} - \overline{B}_{s,d}$ mixing do not give severe constraints on our model of RFV, there exist other (less obvious) contributions to $K - \overline{K}$ and $D - \overline{D}$ mixing which must be taken into account. An effective element $\delta^{u\,LL}_{12\,\text{eff}}$ is induced through the double mass insertion $\delta^{u\,LR}_{13} \times \delta^{u\,LR*}_{23}$. Note that this element is proportional to two powers of an electroweak vev and is therefore not subjected to the $SU(2)$ relation which relates $\delta^{u\,LL}_{ij}$ to $\delta^{d\,LL}_{ij}$. Therefore, on the one hand only chargino diagrams contribute to $K - \overline{K}$ mixing while on the other hand only gluino diagrams contributes to $D - \overline{D}$ mixing mixing. However, in the case of $K - \overline{K}$ mixing we have very precise experimental information on CP violation, the corresponding quantity ϵ_K is well understood in the SM [41–43]. Since moreover $\delta^{u\,LL}_{12\,\text{eff}}$ carries the CKM phase γ (because it is proportional to $\delta^{u\,LR}_{13}$ which generates V_{ub}), the constraint from ϵ_K turns out to be stronger than the constraints from D mixing [44]. The allowed regions in the $m_{\tilde{q}} - m_{\tilde{g}}$ plane for different values of M_2 are shown in Fig. 8. Note that the constraints are nearly independent of μ and $\tan\beta$ since the quark-squark coupling involves the gaugino component of the charginos.

Another process which is sensitive to the combination $\delta^{u\,LR}_{13} \times \delta^{u\,LR*}_{23}$ via chargino loops is $K \to \pi\nu\nu$ [45–48]. Even though, at present, this process does not give useful bounds, but NA62 results will change this situation in the future. Fig. 9 and Fig. 10 show the predicted branching ratios for $K_L \to \pi\nu\overline{\nu}$ and $K^+ \to \pi^+\nu\overline{\nu}$. In a wide range of squark and gluino masses both quantities largely deviate from the SM predictions [49–51], the effect on the charged mode should be detectable at NA62. Our values for $\text{Br}[\text{K}_\text{L} \to \pi\nu\overline{\nu}]$ plotted in Fig. 9 overlap with the region probed by the KOTO experiment at JPARC. Note that the branching ratios are again to a very good approximation independent of μ and $\tan\beta$.

In principle $\delta^{u\,LR}_{23}$ also contributes to $B \to K\nu\overline{\nu}$ via a Z penguin (and at the same time also to $B_s \to \mu^+\mu^-$ which is strongly correlated to $B \to K\nu\overline{\nu}$ in the MSSM at low $\tan\beta$) [52–54]. Even though the branching ratios are slightly enhanced, they also depend on A^u_{33}, μ and $\tan\beta$. Furthermore, $B \to K\nu\overline{\nu}$ is also correlated to $b \to s\gamma$ which forbids large effects [53]. Of course also in the up-sector no symmetry argument forbids non-zero elements $\delta^{u\,LR}_{31,32,33}$. While, as already discussed, $\delta^{u\,LR}_{33}$ affects $b \to s\gamma$ and $B \to K\nu\overline{\nu}$, the elements $\delta^{u\,LR}_{31,32}$ are rather unconstrained from FCNC processes, since they enter these processes only in combination with small quark masses and small chargino mixing. Their mere effect is to correct the eigenvalues of the up-type squark mass matrix. However, as shown in Ref. [17], they can induce a sizable right-handed W-coupling if at the same time also $\delta^{d\,LR}_{33}$ is large.

IV. CONCLUSIONS

Radiative generation of light fermion masses and CKM mixing angles is an appealing and very predictive concept. Within the MSSM this approach can solve the SUSY CP and flavor problems [10, 13]. In this article we have studied a model with radiative flavor violation (RFV) using the trilinear terms as spurions break-

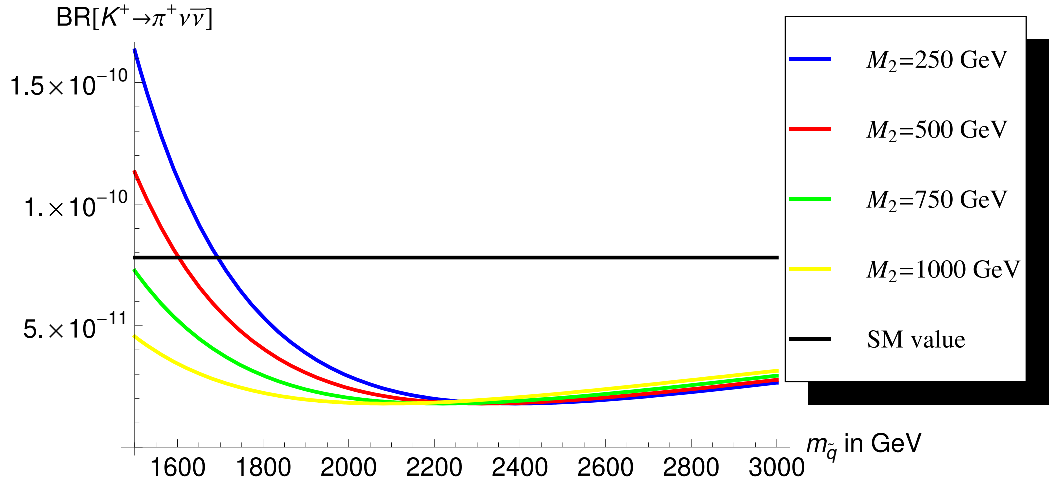

FIG. 10: Predicted branching ratio for the rare Kaon decay $K^+ \to \pi^+\nu\overline{\nu}$ assuming that the CKM matrix is generated in the up-sector for $m_{\tilde{q}} = m_{\tilde{g}}$. The branching ratio is enhanced for light SUSY masses but suppressed if the scale of SUSY breaking is higher.

ing flavor symmetries [13] and we have analyzed its phenomenological implications on FCNC processes. Keeping the third-generation fermion Yukawa couplings, the CKM matrix can either be induced in the up or in the down sector. (In principle also a mixed scenario is possible, however, we did not further investigate this possibility.) If the CKM matrix is generated in the up-sector, Kaon mixing severely constrains the allowed values of $m_{\tilde{g}}$ and $m_{\tilde{q}}$ (see Fig. 8). However, the rare Kaon decays $K \to \pi\nu\nu$ can still receive sizable contributions. If the other possibility is realized and the CKM matrix is generated in the down sector, $b \to s\gamma$ restricts the allowed range for the SUSY masses. Fig. 3 shows our results which take into account the chirally enhanced correction discussed in Refs. [19, 30]. In the case of CKM generation in the down sector $B_s \to \mu^+\mu^-$ receives sizable contributions (even in the decoupling limit) from flavor-changing effective Higgs couplings, which are already sizable at moderate $\tan\beta$ [20]. If in addition $\delta^{d\,RL}_{23} \neq 0$, $B_s-\overline{B}_s$ mixing is affected by double Higgs penguins as well. In this way the $B_s-\overline{B}_s$ mixing phase which disagrees with the SM expectation [16] can be explained.

Having shown that all CKM elements and small quark masses can be generated from loop diagrams while simultaneously obeying all FCNC constraints, we conclude that the MSSM with RFV is a viable alternative to the popular MFV variant of the MSSM. As opposed to the MFV-MSSM our RFV model is capable to explain the large $B_s-\overline{B}_s$ mixing phase favored by current data.

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