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Complete next-to-leading order gluino contributions to $b \to s\gamma$ and $b \to sg$

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Abstract

We present the first complete order α_s corrections to the Wilson coefficients (at the high scale) of the various versions of magnetic and chromomagnetic operators which are induced by a squark-gluino exchange. For this matching calculation, we work out the on-shell amplitudes $b \to s\gamma$ and $b \to sg$, both in the full and in the effective theory up to order α_s^2. The most difficult part of the calculation is the evaluation of the two-loop diagrams in the full theory; these can be split into two classes: a) diagrams with one gluino and a virtual gluon; b) diagrams with two gluinos or with one gluino and a four-squark vertex. Accordingly, the Wilson coefficients can be split into a part a) and a part b). While part b) of the Wilson coefficients is presented in this paper for the first time, part a) was given in [1]. We checked their results for the coefficients of the magnetic operators and found perfect agreement. Moreover, we work out the renormalization procedure in great detail.

Our results for the complete next-to-leading order Wilson coefficients are fully analytic, but far too long to be printed. We therefore publish them in the form of a C++ program. They constitute a crucial building block for the phenomenological next-to-leading logarithmic analysis of the branching ratio $\bar{B} \to X_s\gamma$ in a supersymmetric model beyond minimal flavor violation.

Contents

1 Introduction

Among the rare B meson decays those induced by radiative and electroweak penguin diagrams are of particular interest. Such flavor changing neutral current (FCNC) processes offer high sensitivity to new physics (NP) through potential new degrees of freedom beyond the Standard Model (SM). Additional contributions to the decay rate, in which SM particles in the loops are replaced by new particles such as the supersymmetric charginos or gluinos are not suppressed by the loop factor $\alpha/4\pi$ relative to the SM contribution. Thus, FCNC decays provide information about the SM and its extensions via virtual effects to scales presently not accessible otherwise. This approach is complementary to the direct production of new particles at collider experiments (for reviews see [2, 3]).

At the end of the first generation of the B factories at KEK (Belle experiment at the KEKB e^+e^- collider) [4] and at SLAC (BaBar experiment at the PEP-II e^+e^- collider) [5], all present measurements in flavor physics including those from the Tevatron B physics programs (CDF [6] and D0 [7] experiments) have not observed any unambiguous sign of new physics, in particular no $\mathcal{O}(1)$ NP effects in any FCNC process [8, 9]. This implies the famous flavor problem, namely why FCNC processes are suppressed. It has to be solved in any viable new physics model. It is well-known that the hypothesis of minimal flavor violation (MFV) [10,11], i.e. that the NP model has no flavor structures beyond the Yukawa couplings, solves the problem formally. However, new flavor structures beyond the Yukawa couplings are still compatible with the present data [12] because the flavor sector has been tested only at the 10% level, especially in the $b \to s$ transitions.

Among the penguin modes, the inclusive decay $\bar{B} \to X_s\gamma$ is the most important one, because it is theoretically well understood and at the same time it has been measured extensively at the B factories. While non-perturbative corrections to this decay mode are subleading and were recently estimated to be well below 10% [13], perturbative QCD corrections are the most important corrections. Within a global effort, a perturbative QCD calculation to the next-to-next-to-leading-logarithmic (NNLL) level has been performed and has led to the first NNLL prediction of the $\bar{B} \to X_s\gamma$ branching fraction [14, 15] which also includes the nonperturbative contributions. Using the photon energy cut $E_0 = 1.6$ GeV, the branching ratio reads

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm NNLL} = (3.15 \pm 0.23) \times 10^{-4}. \tag{1}$$

The combined experimental data according to the Heavy Flavor Averaging Group (HFAG) [16] leads to

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}\,, \tag{2}$$

where the first error is combined statistical and systematic, and the second is due to the extrapolation in the photon energy. Thus, the SM prediction and the experimental average are consistent at the 1.2σ level. This implies very stringent constraints on NP models like the bound on the charged Higgs mass in the two Higgs-doublet model (II) [17,18] ($M_{H^+} > 295$GeV at 95% CL) [14] or the bound on the inverse compactification radius of the minimal universal extra dimension model (mACD) ($1/R > 600$GeV at 95% CL) [19]. In both cases the bounds are much stronger than the ones derived from other measurements. Constraints within various supersymmetric extensions are analyzed in [20–31] (for overviews see [3, 35]). Bounds on the Little Higgs Model with T-parity have also been presented [36]. Finally, model-independent analyses in the effective field theory approach without [37] and with the assumption of minimal flavor violation [10, 11] also show the strong constraining power of the $\bar{B} \to X_s\gamma$ branching fraction.

While within the perturbative SM calculation NNLL precision has been achieved [14, 15], also within supersymmetric theories higher order calculations have been pushed forward in recent years. In particular, the complete NLL QCD calculation of the MSSM with the additional assumption of minimal flavor violation was presented in [28] including a published computer code [38]. Beyond minimal flavor violation, the most important role is played by the non-diagonal gluino-quark-squark vertex due to the large strong coupling which comes with this vertex. This flavor non-diagonal vertex is induced by squark-mixing to the extent as it is misaligned with quark mixing. It represents a new flavor structure beyond the SM Yukawa couplings. A complete LL analysis of the corresponding gluino contribution to the inclusive decay rate was presented in [23]. Some "beyond LL effects" were estimated [27], but a general NLL analysis of the gluino contribution is still missing.

Besides these NLL contributions due to the gluino vertex, there are of course more NLL corrections with non-minimal flavor violation; they involve electroweak (gaugino and higgsino) vertices. However, such contributions are in general suppressed compared to the corrections computed here. There are two types of such contributions at the NLL level: First, there are electroweak corrections to the non-minimal LL gluino contribution (in which the electroweak vertex is flavor-diagonal or MFV-like) which are naturally suppressed due to the smaller coupling constants and due to the CKM hierarchy. Second, there is also non-minimal flavor violation via squark-mixing in the electroweak vertices possible. But such contributions are already suppressed at the LL level compared to the gluino contribution due to the smaller coupling constant, apart from the chargino contributions in specific parts of the parameter space in which for example the trilinear coupling A^u_{23} is very large. These features do not change of course when gluon- and gluino-induced NLL corrections are added to such LL contributions.[1] Summing up, the complete NLL corrections induced by the gluino vertex represent the dominant contribution beyond MFV at this order in most parts of the MSSM parameter space. They are complementary to the MFV contributions at the NLL level which are given in [28].

In the present paper we work out for the first time the complete corrections of order α_s to the Wilson coefficients (at the matching scale μ_W) of the various versions of magnetic and chromomagnetic operators which are induced by a squark-gluino loop. In our procedure all the appearing heavy particles (which are the gluino, the squarks and the top quark) are simultaneously integrated out at the high scale, which we call μ_W. For this matching calculation, we work out the on-shell amplitudes $b \to s\gamma$ and $b \to sg$, both in the full and in the effective theory up to order α_s^2. The most difficult part of the calculation is the evaluation of the two-loop diagrams in the full theory; these can be split into two classes: a) diagrams with one gluino and a virtual gluon; b) diagrams with two gluinos or with one gluino and a squark-loop. Accordingly, the Wilson coefficients can be split into a part a) and a part b). While part b) of the Wilson coefficients is given in the present work for the first

[1] Still, the leading chirally enhanced corrections can be easily calculated by inserting the effective Feynman rules of [32] into the results of [24].

time, part a) was published by Bobeth et al. [1]. We explicitly checked their results for the Wilson coefficients of the magnetic operators, which were obtained using an off-shell matching procedure, and found perfect agreement.

Moreover, we work out all steps of the NLO renormalization procedure in great detail; to our knowledge, the complete set of renormalization constants, which we give in Appendices B to F, have not been presented before. We also discuss how the recently discussed chirally-enhanced terms [31] enter our scheme. Our results for the complete NLO Wilson coefficients are fully analytic, but far too long to be printed. We therefore publish them in the form of a C++ program.

The remainder of the paper is organized as follows: In the next section we present details on the framework used for our calculation. In particular, we list all relevant operators contributing to $\bar{B} \to X_s\gamma$. In Section 3, we describe in detail the calculation of the Wilson coefficients of the magnetic operators which is extended to the chromomagnetic case in Section 4. All our results are implemented in a computer code which is described in Section 5 and we present our conclusions in Section 6. In Appendix A, we present explicit analytic expressions for the LO result and Appendices B to F provide details on the NLO renormalization procedure.

2 Preliminaries

2.1 Squark-quark-gluino interaction

The minimal supersymmetric SM (MSSM) allows for generic new sources of flavor violation beyond the Yukawa structure in the SM, thus, beyond the minimal flavor violation hypothesis. Besides the usual quark mixing also the squarks induce flavor mixing due to a possible misalignment of quarks and squarks in flavor space.

More explicitly, in the MSSM without soft SUSY-breaking, flavor-violation is induced by the superpotential terms containing the Yukawa couplings of the matter superfields to the Higgs superfields

$$\mathcal{L}^{\text{Yukawa}} = -\bar{\mathcal{D}}_i^o (h_d)_{ij} \mathcal{Q}_j^o \mathcal{H}_d + \bar{\mathcal{U}}_i^o (h_u)_{ij} \mathcal{Q}_j^o \mathcal{H}_u - \mu \mathcal{H}_d \mathcal{H}_u. \tag{3}$$

The index "o" indicates that the current eigenstate basis is used. Following the notation of [39, 40], we define $\tilde{u}_R^{o*}$ as the scalar component and $u_R^{o\dagger}$ as the fermionic component of the superfield $\bar{U}^o$ (and for $\bar{D}^o$ analogously); the scalar and fermionic components of the superfield $\mathcal{Q}^o$ are defined as $\tilde{Q}^o = (\tilde{u}_L^o \tilde{d}_L^o)$ and $Q = (u_L^o d_L^o)$, respectively. $\mathcal{H}_{u,d}$ are the Higgs superfields. The matrices h_d and h_u act on flavor space. Diagonalizing these matrices via bi-unitary transformations defines the super-CKM basis.

In addition, there are *soft* SUSY-breaking terms leading to flavor transitions of the quarks. When restricted to the terms relevant for squark masses and quark-flavor transitions, the soft part of the Lagrangian can be expressed in terms of the scalar fields in the current eigenstate basis:

$$\begin{aligned} \mathcal{L}^{\text{soft}} = &- \tilde{Q}_i^{o\dagger} m_{\tilde{Q},ij} \tilde{Q}_j^o - \tilde{u}_{Ri}^o m^2_{\tilde{u},ij} \tilde{u}_{Rj}^{o*} - \tilde{d}_{Ri}^o m^2_{\tilde{d},ij} \tilde{d}_{Rj}^{o*} \\ &- \left(A_{ij}^d H_d \tilde{Q}_i^o \tilde{d}_{Rj}^{o*} + A_{ij}^u H_u \tilde{Q}_i^o \tilde{u}_{Rj}^{o*} + \text{h.c.} \right), \end{aligned} \tag{4}$$

where $\tilde{u}^o$, $\tilde{d}^o$ and $\tilde{Q}^o$ are the squark fields and $m_{\tilde{u}}$, $m_{\tilde{d}}$ and $m_{\tilde{Q}}$ the corresponding hermitian mass matrices. H_u and H_d are Higgs doublets and A_u and A_d are 3×3 matrices in flavor space. For the latter no proportionality to the Yukawa couplings is assumed; they are left completely general.

After the diagonalization of the Yukawa couplings in (3), i.e. in the super-CKM basis, the squark mass matrices are of the form ($q = u$ or $q = d$)

$$M_q^2 = \begin{pmatrix} m^2_{q,LL} & m^2_{q,LR} \\ m^2_{q,RL} & m^2_{q,RR} \end{pmatrix} + \begin{pmatrix} F_{q,LL} + D_{q,LL} & F_{q,LR} \\ F_{q,RL} & F_{q,RR} + D_{q,RR} \end{pmatrix}, \tag{5}$$

where the F-terms from the superpotential and the D-terms from the gauge sector are diagonal 3×3 submatrices of the 6×6 mass matrices M_q^2. This is in general not true for the additional terms, originating from the soft part of the Lagrangian, given in (4) (for more details see Chapter III of [18]). Thus, the (hermitian) squark mass terms

$$(\tilde{u}_L^\dagger, \tilde{u}_R^\dagger) M_u^2 \begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix} \quad \text{and} \quad (\tilde{d}_L^\dagger, \tilde{d}_R^\dagger) M_d^2 \begin{pmatrix} \tilde{d}_L \\ \tilde{d}_R \end{pmatrix}, \tag{6}$$

where the fields $\tilde{u}_{L,R}$ and $\tilde{d}_{L,R}$ now refer to the super-CKM basis, are not diagonal in general. Only when performing an additional appropriate unitary field transformation defined through

$$\begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} = \begin{pmatrix} \Gamma_{QL}^\dagger \\ \Gamma_{QR}^\dagger \end{pmatrix} \tilde{q}\,, \tag{7}$$

with 6×3 matrices $\Gamma_{(U,D)(L,R)}$, one obtains a diagonal matrix

$$(\Gamma_{QL}, \Gamma_{QR}) M_q^2 \begin{pmatrix} \Gamma_{QL}^\dagger \\ \Gamma_{QR}^\dagger \end{pmatrix}, \tag{8}$$

and the squark mass terms have the form

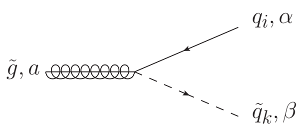
$$\tilde{q}^\dagger (\Gamma_{QL}, \Gamma_{QR}) M_q^2 \begin{pmatrix} \Gamma_{QL}^\dagger \\ \Gamma_{QR}^\dagger \end{pmatrix} \tilde{q}\,. \tag{9}$$

$\tilde{q}$ represents the six squark fields in the mass-eigenstate basis. Note that in (8) and (9) one either has $(q, Q) = (u, U)$ or $(q, Q) = (d, D)$. The Γ matrices satisfy the following unitarity relations:

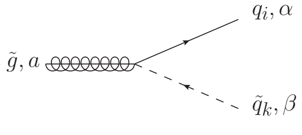
$$\sum_{i=1}^{3} (\Gamma_{QL}^{ki} \Gamma_{QL}^{hi*} + \Gamma_{QR}^{ki} \Gamma_{QR}^{hi}) = \delta^{kh}, \qquad \sum_{k=1}^{6} \Gamma_{QX}^{ki} \Gamma_{QY}^{kj*} = \delta^{ij} \delta_{XY}, \tag{10}$$

where $Q = U, D$ and $X, Y = L, R$.

The squark-quark-gluino coupling is flavor-diagonal in the super-CKM basis; thus, the last rotation of the squark fields of (9) gives rise to the following squark-quark-gluino vertices, which are non-diagonal in flavor-space



$$- i g_s \sqrt{2} T^a_{\beta\alpha} \left(\Gamma_{QL}^{ki} P_L - \Gamma_{QR}^{ki} P_R \right), \tag{11}$$



$$- i g_s \sqrt{2} T^a_{\alpha\beta} \left(\Gamma_{QL}^{ki*} P_R - \Gamma_{QR}^{ki*} P_L \right), \tag{12}$$

Further Feynman rules may be found in [41–43].

At this point, a comment on the super-CKM basis is appropriate. While in [27] and [28] a definition of the super-CKM basis is used in which the physical quark mass matrices are diagonal, we will apply a minimal renormalization scheme to the squark mixing matrices Γ appearing in (11) and (12). This corresponds to a definition of the super-CKM basis in which the Yukawa couplings remain diagonal also at the loop level [31, 33, 34].

In the following calculation, we will express our results for the Wilson coefficients in terms of the diagonal squark masses $m_{\tilde{q}_k}$ and the Γ matrices, renormalized in the $\overline{\mathrm{DR}}$ scheme.

2.2 Operator basis

In supersymmetric models, various combinations of the gluino–quark–squark vertex lead to $|\Delta(B)| = |\Delta(S)| = 1$ magnetic and chromomagnetic operators (of $\mathcal{O}_7$-type, $\mathcal{O}_8$-type) with an explicit factor α_s, and to four-quark operators, with a factor α_s^2 when integrating out squarks and gluinos [23]. These squark/gluino induced operators define the effective Hamiltonian $\mathcal{H}_{\mathrm{eff}}^{\tilde{g}}$, which is given in (13) below.

Using the standard effective field theory approach, the aim is to resum the following terms:

$$\begin{aligned} &\text{LO:} && \alpha_s\,(\alpha_s L)^N \\ &\text{NLO:} && \alpha_s\,\alpha_s(\alpha_s L)^N, \qquad (N = 0, 1, ...), \end{aligned}$$

respectively at the leading and next-to-leading order where L denotes large logarithms of the ratio $m_b^2/M_{\mathrm{susy}}^2$.

As discussed in [23], $\mathcal{H}_{eff}^{\tilde{g}}$ is unambiguous, but it is a matter of convention whether the α_s factors, peculiar to the gluino exchange, should be put into the definition of operators or into the Wilson coefficients. It is convenient to distribute the factors of α_s between operators and Wilson coefficients in such a way that the matching calculation and the evolution down to the low scale μ_b of the Wilson coefficients are organized exactly in the same way as in the SM such that the anomalous-dimension matrix indeed has the canonical expansion in α_s and starts with a term proportional to α_s^1.

The effective Hamiltonian $\mathcal{H}_{\mathrm{eff}}^{\tilde{g}}$ is of the form

$$\mathcal{H}_{\mathrm{eff}}^{\tilde{g}} = \sum_i C_{i\tilde{g}}(\mu)\mathcal{O}_{i\tilde{g}}(\mu) + \sum_i \sum_q C_{i\tilde{g}}^q(\mu)\mathcal{O}_{i\tilde{g}}^q(\mu). \tag{13}$$

In the second term, we sum over all light quark flavors $q = u, d, c, s, b$.

At dimension five the following two-quark operators contribute

$$\begin{aligned} \mathcal{O}_{7\tilde{g},\tilde{g}} &= e\,g_s^2(\mu)\,(\bar{s}\sigma^{\mu\nu}P_R b)\,F_{\mu\nu}\,, & \mathcal{O}'_{7\tilde{g},\tilde{g}} &= e\,g_s^2(\mu)\,(\bar{s}\sigma^{\mu\nu}P_L b)\,F_{\mu\nu}\,, \\ \mathcal{O}_{8\tilde{g},\tilde{g}} &= g_s(\mu)\,g_s^2(\mu)\,(\bar{s}\sigma^{\mu\nu}T^a P_R b)\,G_{\mu\nu}^a\,, & \mathcal{O}'_{8\tilde{g},\tilde{g}} &= g_s(\mu)\,g_s^2(\mu)\,(\bar{s}\sigma^{\mu\nu}T^a P_L b)\,G_{\mu\nu}^a\,, \end{aligned} \tag{14}$$

with $P_L = \frac{1}{2}(1 - \gamma_5)$ and $P_R = \frac{1}{2}(1 + \gamma_5)$. In the SUSY contributions corresponding to these operators, the chirality-violating parameter is the gluino mass $m_{\tilde{g}}$ which is included in the corresponding Wilson coefficients:

At dimension six the two-quark operators with chirality violation coming from the b- or c-quark mass read:

$$
\begin{aligned}
\mathcal{O}_{7b,\tilde{g}} &= e\, g_s^2(\mu)\, \overline{m}_b(\mu)\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}\,, & \mathcal{O}'_{7b,\tilde{g}} &= e\, g_s^2(\mu)\, \overline{m}_b(\mu)\, (\bar{s}\sigma^{\mu\nu} P_L b)\, F_{\mu\nu}\,,\\
\mathcal{O}_{8b,\tilde{g}} &= g_s(\mu)\, g_s^2(\mu)\, \overline{m}_b(\mu)\, (\bar{s}\sigma^{\mu\nu} T^a P_R b)\, G^a_{\mu\nu}\,, & \mathcal{O}'_{8b,\tilde{g}} &= g_s(\mu)\, g_s^2(\mu)\, \overline{m}_b(\mu)\, (\bar{s}\sigma^{\mu\nu} T^a P_L b)\, G^a_{\mu\nu}\,,\\
\mathcal{O}_{7c,\tilde{g}} &= e\, g_s^2(\mu)\, \overline{m}_c(\mu)\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}\,, & \mathcal{O}'_{7c,\tilde{g}} &= e\, g_s^2(\mu)\, \overline{m}_c(\mu)\, (\bar{s}\sigma^{\mu\nu} P_L b)\, F_{\mu\nu}\,,\\
\mathcal{O}_{8c,\tilde{g}} &= g_s(\mu)\, g_s^2(\mu)\, \overline{m}_c(\mu)\, (\bar{s}\sigma^{\mu\nu} T^a P_R b)\, G^a_{\mu\nu}\,, & \mathcal{O}'_{8c,\tilde{g}} &= g_s(\mu)\, g_s^2(\mu)\, \overline{m}_c(\mu)\, (\bar{s}\sigma^{\mu\nu} T^a P_L b)\, G^a_{\mu\nu}\,.
\end{aligned}
\tag{15}
$$

The following four-quark (axial-)vector operators contribute:

$$
\begin{aligned}
\mathcal{O}^q_{11,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_L b)\,(\bar{q}\gamma^\mu P_L q)\,, & \mathcal{O}^{q\,\prime}_{11,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_R b)\,(\bar{q}\gamma^\mu P_R q)\,,\\
\mathcal{O}^q_{12,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_L b_\beta)\,(\bar{q}_\beta\gamma^\mu P_L q_\alpha)\,, & \mathcal{O}^{q\,\prime}_{12,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_R b_\beta)\,(\bar{q}_\beta\gamma^\mu P_R q_\alpha)\,,\\
\mathcal{O}^q_{13,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_L b)\,(\bar{q}\gamma^\mu P_R q)\,, & \mathcal{O}^{q\,\prime}_{13,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_R b)\,(\bar{q}\gamma^\mu P_L q)\,,\\
\mathcal{O}^q_{14,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_L b_\beta)\,(\bar{q}_\beta\gamma^\mu P_R q_\alpha)\,, & \mathcal{O}^{q\,\prime}_{14,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_R b_\beta)\,(\bar{q}_\beta\gamma^\mu P_L q_\alpha)\,.
\end{aligned}
\tag{16}
$$

where color indices are omitted for color-singlet currents. These operators arise from box diagrams through the exchange of two gluinos and from penguin diagrams through the exchange of a gluino and a gluon.

In addition there are also the following four-quark (pseudo-)scalar operators which are induced at leading order by box diagrams with two gluino exchanges

$$
\begin{aligned}
\mathcal{O}^q_{15,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_R b)\,(\bar{q}P_R q)\,, & \mathcal{O}^{q\,\prime}_{15,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_L b)\,(\bar{q}P_L q)\,,\\
\mathcal{O}^q_{16,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_R b_\beta)\,(\bar{q}_\beta P_R q_\alpha)\,, & \mathcal{O}^{q\,\prime}_{16,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_L b_\beta)\,(\bar{q}_\beta P_L q_\alpha)\,,\\
\mathcal{O}^q_{17,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_R b)\,(\bar{q}P_L q)\,, & \mathcal{O}^{q\,\prime}_{17,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_L b)\,(\bar{q}P_R q)\,,\\
\mathcal{O}^q_{18,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_R b_\beta)\,(\bar{q}_\beta P_L q_\alpha)\,, & \mathcal{O}^{q\,\prime}_{18,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_L b_\beta)\,(\bar{q}_\beta P_R q_\alpha)\,,\\
\mathcal{O}^q_{19,\tilde{g}} &= g_s^4(\mu)(\bar{q}_\alpha P_R b_\beta)\,(\bar{s}_\beta P_R q_\alpha)\,, & \mathcal{O}^{q\,\prime}_{19,\tilde{g}} &= g_s^4(\mu)(\bar{q}_\alpha P_L b_\beta)\,(\bar{s}_\beta P_L q_\alpha)\,,\\
\mathcal{O}^q_{20,\tilde{g}} &= g_s^4(\mu)(\bar{q}P_R b)\,(\bar{s}P_R q)\,, & \mathcal{O}^{q\,\prime}_{20,\tilde{g}} &= g_s^4(\mu)(\bar{q}P_L b)\,(\bar{s}P_L q)\,.
\end{aligned}
\tag{17}
$$

These operators are shown to mix at one-loop into the magnetic and chromomagnetic operators [23] and, thus, contribute to a LL analysis. Note that we defined $\mathcal{O}^q_{19,\tilde{g}}$ and $\mathcal{O}^q_{20,\tilde{g}}$ (and also $\mathcal{O}^{q\,\prime}_{19,\tilde{g}}$, $\mathcal{O}^{q\,\prime}_{20,\tilde{g}}$) differently from [23]. Denoting the operators from [23] by $\tilde{\mathcal{O}}^q_{19,\tilde{g}}$ and $\tilde{\mathcal{O}}^q_{20,\tilde{g}}$, the transformation between the operator bases reads:

$$
\mathcal{O}^q_{19,\tilde{g}} = -\frac{1}{2}\tilde{\mathcal{O}}^q_{15,\tilde{g}} - \frac{1}{8}\tilde{\mathcal{O}}^q_{19,\tilde{g}} \quad \text{and} \quad \mathcal{O}^q_{20,\tilde{g}} = -\frac{1}{2}\tilde{\mathcal{O}}^q_{16,\tilde{g}} - \frac{1}{8}\tilde{\mathcal{O}}^q_{20,\tilde{g}}. \tag{18}
$$

Correspondingly we get for the Wilson coefficients:

$$
\begin{aligned}
C^q_{15,\tilde{g}} &= \tilde{C}^q_{15,\tilde{g}} - 4\tilde{C}^q_{19,\tilde{g}}\,, & C^q_{16,\tilde{g}} &= \tilde{C}^q_{16,\tilde{g}} - 4\tilde{C}^q_{20,\tilde{g}}\,,\\
C^q_{19,\tilde{g}} &= -8\tilde{C}^q_{19,\tilde{g}}\,, & C^q_{20,\tilde{g}} &= -8\tilde{C}^q_{20,\tilde{g}}.
\end{aligned}
\tag{19}
$$

Analogous relations are valid for the primed operators.

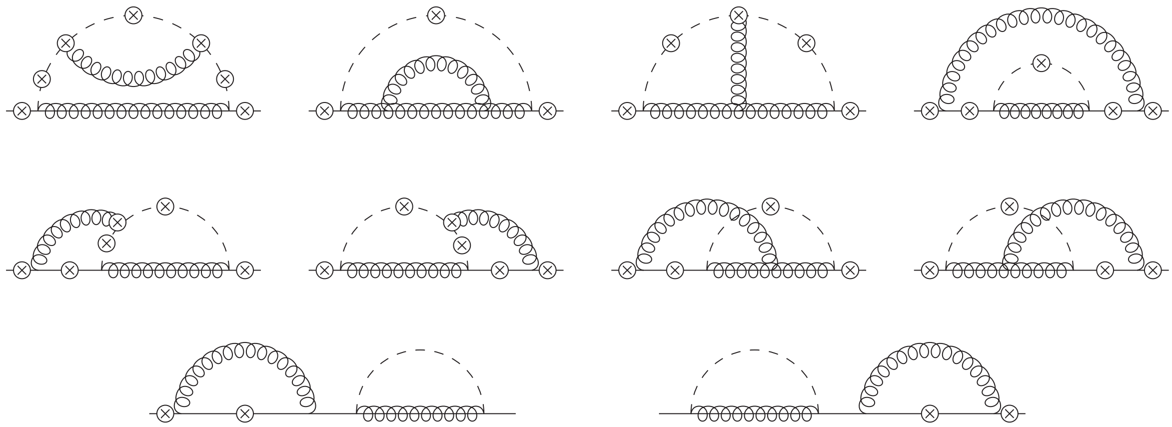


Figure 1: Gluon corrections: The solid, dashed, curly, and solid-curly lines represent quarks, squarks, gluons, and gluinos, respectively. The crosses denote the possible locations where the photon is emitted.

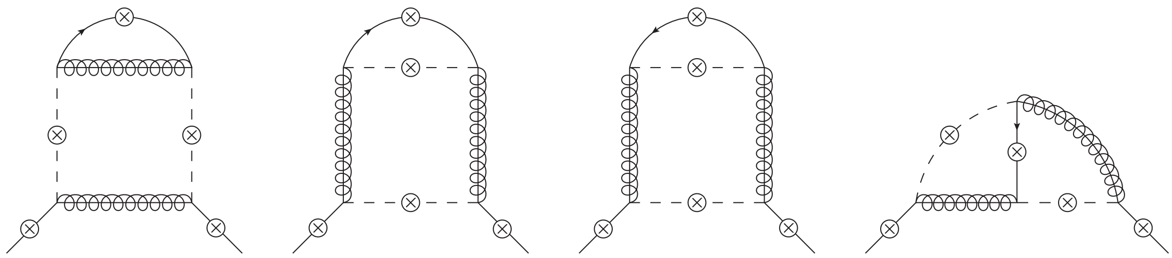


Figure 2: Gluino corrections: The crosses mark the places where a photon can be emitted.

3 Calculation of the Wilson coefficients of the magnetic operators

The various Wilson coefficients C_i appearing in the effective Hamiltonian $\mathcal{H}_{\text{eff}}^{\tilde{g}}$ can be expanded as follows:

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)} + \mathcal{O}(\alpha_s^2)\,. \tag{20}$$

In order to extract the Wilson coefficients $C_{7\tilde{g},\tilde{g}}$ $C_{7b,\tilde{g}}$ $C_{7c,\tilde{g}}$ of the magnetic operators $\mathcal{O}_{7\tilde{g},\tilde{g}}$, $\mathcal{O}_{7b,\tilde{g}}$ $\mathcal{O}_{7c,\tilde{g}}$ in (14) and (15), we calculate the on-shell decay amplitude $b \to s\gamma$ in the full theory and match the result with the corresponding one obtained by the effective

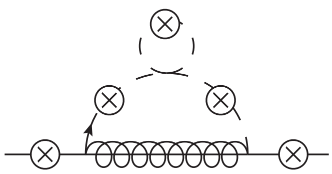


Figure 3: Diagrams containing a four-squark vertex.

Hamiltonian $\mathcal{H}_{\rm eff}^{\tilde{g}}$ in (13). We require the s quark, which we treat as a massless particle, to be left-handed, so that only the unprimed operators are involved. The Wilson coefficients of the primed operators (corresponding to a right-handed s quark) can be obtained at the very end by interchanging L and R in the results for the unprimed ones.

To get the LO pieces of the Wilson coefficients, we have to calculate the amplitude $\mathcal{A}(b \to s\gamma)$ up to order α_s. The corresponding result in the full theory (expanded up to second order in the external momenta and light masses) is given in Appendix A. In the effective theory, only the matrix elements associated with the magnetic operators contribute at the lowest order of α_s. Consequently, we get for the leading-order Wilson coefficients

$$C_{7\tilde{g},\tilde{g}}^{(0)} = K_{7\tilde{g},\tilde{g}}^{(0)}\Big|_{\epsilon\to 0}\,, \qquad C_{7b,\tilde{g}}^{(0)} = K_{7b,\tilde{g}}^{(0)}\Big|_{\epsilon\to 0}\,, \qquad C_{7c,\tilde{g}}^{(0)} = 0\,, \tag{21}$$

where the functions $K_{7\tilde{g},\tilde{g}}^{(0)}$ and $K_{7b,\tilde{g}}^{(0)}$ are given in Appendix A. They represent the d-dimensional versions of the LO Wilson coefficients.

We now discuss the calculation of the NLO pieces $C_{7\tilde{g},\tilde{g}}^{(1)}$, $C_{7b,\tilde{g}}^{(1)}$, $C_{7c,\tilde{g}}^{(1)}$ of these Wilson coefficients. There are two types of diagrams contributing to $b \to s\gamma$ in the full theory at $\mathcal{O}(\alpha_s^2)$:

Full a) Diagrams containing one virtual gluon and one gluino, shown in Figure 1. We call these contributions "gluon corrections".

Full b) Contributions with two gluinos or diagrams containing squark tadpoles, depicted in Figures 2 and 3. We refer to these contributions as "gluino corrections".

As we will see, these two contributions can be renormalized separately in the full theory. It is therefore convenient to split also the renormalized result obtained in the effective theory into two contributions. This leads to a decomposition of the NLO pieces of the Wilson coefficients of the magnetic operators into two pieces, as e.g. $C_{7\tilde{g},\tilde{g}}^{(1)} = C_{7\tilde{g},\tilde{g}}^{(1),a} + C_{7\tilde{g},\tilde{g}}^{(1),b}$.

The mentioned two contributions in the effective theory can be characterized as follows:

Effective a) Renormalized one-loop matrix elements $\langle s\gamma|C_i^{(0)}\mathcal{O}_i|b\rangle$ associated with the $\mathcal{O}_7$- and $\mathcal{O}_8$-type operators; one-loop matrix elements of the operators $\mathcal{O}_{11,\tilde{g}}^q$ to $\mathcal{O}_{14,\tilde{g}}^q$ (which are finite) weighted with the penguin part of the corresponding LO Wilson coefficients (see (31) in [23]); tree-level matrix elements associated with the operators $\frac{\alpha_s}{4\pi}\,C_{7i,\tilde{g}}^{(1),a}\,\langle s\gamma|\mathcal{O}_{7i,\tilde{g}}|b\rangle$, $i = b, \tilde{g}$.

Effective b) One-loop matrix elements of the operators $\mathcal{O}_{11,\tilde{g}}^q$ to $\mathcal{O}_{14,\tilde{g}}^q$ (which are finite) weighted with the box part of the corresponding LO Wilson coefficients (see (32) in [23]); renormalized one-loop matrix elements of $C_{15,\tilde{g}}^q\,\mathcal{O}_{15,\tilde{g}}^q$ to $C_{20,\tilde{g}}^q\,\mathcal{O}_{20,\tilde{g}}^q$ (see (33) in [23]); tree-level matrix elements associated with the operators $\frac{\alpha_s}{4\pi}\,C_{7i,\tilde{g}}^{(1),b}\,\langle s\gamma|\mathcal{O}_{7i,\tilde{g}}|b\rangle$, $i = b, c, \tilde{g}$.

For the precise definition of these two parts, see (30) and (51).

In the following, we separately work out $C_7^{(1),a}$ and $C_7^{(1),b}$ using dimensional regularization (DREG) both on the full theory and the effective theory side. Later we will take into account the important point, that one should use dimensional reduction (DRED) in the full theory to preserve supersymmetry. This is done in Section 3.4 by appropriately shifting the strong coupling constant g_s and the gluino mass in the leading order contributions of the full theory.

3.1 Calculation of $C_{7\tilde{g},\tilde{g}}^{(1),a}$ and $C_{7b,\tilde{g}}^{(1),a}$

3.1.1 Full theory

To get the order α_s^2 corrections to the decay amplitude corresponding to the contributions of class a) in the full theory, we expand the diagrams shown in Figure 1 in inverse powers of the heavy mass M ($M = m_{\tilde{g}}$ or $M = m_{\tilde{q}}$), using the hard-mass procedure [44]. We systematically retain all terms up to order $1/M^2$. When evaluating a genuine two-loop diagram in Figure 1 according to this method, we get two contributions: The so-called hard contribution which is obtained by a naive Taylor expansion of the two-loop integral with respect to the external momenta, and the so-called soft contribution (for details how to get this contribution, see [44]). Adding the hard contributions of all diagrams defines the hard part $iA_{a,full,h}^{bare}$, while the sum of the soft contributions defines the part $iA_{a,full,s}$, so that

$$iA_{a,full}^{bare} = iA_{a,full,h}^{bare} + iA_{a,full,s}^{bare} \,. \tag{22}$$

Note that in this section the symbol A is used for the contribution of order α_s^2 of the relevant amplitude. Instead of calculating the soft part $iA_{a,full,s}^{bare}$ according to the rules of hard-mass procedure, it can be generated using the effective theory at LO (as we checked explicitly) in the following way:

$$\begin{aligned} i\,A_{a,full,s}^{bare} =& K_{7b,\tilde{g}}^{(0)} \,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{1-loop}} + K_{7\tilde{g},\tilde{g}}^{(0)} \,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{1-loop}} \\ &+ K_{8b,\tilde{g}}^{(0)} \,\langle s\gamma|\mathcal{O}_{8b,\tilde{g}}|b\rangle_{\text{1-loop}} + K_{8\tilde{g},\tilde{g}}^{(0)} \,\langle s\gamma|\mathcal{O}_{8\tilde{g},\tilde{g}}|b\rangle_{\text{1-loop}} \\ &+ \sum_{i=11}^{14} C_{i,\tilde{g}}^{q,a} \,\langle s\gamma|\mathcal{O}_{i,\tilde{g}}^{q}|b\rangle_{\text{1-loop}} \\ &+ (Z_{m_b}^{OS,a} - 1)\, K_{7b,\tilde{g}}^{(0)} \,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} \,. \end{aligned} \tag{23}$$

The various (d-dimensional Wilson coefficient) functions $K^{(0)}$, which in the limit $\epsilon \to 0$ coincide with the correponding ordinary LO Wilson coefficents $C^{(0)}$, are given in Appendix A. $C_{i,\tilde{g}}^{q,a}$ denote the penguin parts of the Wilson coefficients $C_{i,\tilde{g}}^{q}$ ($i = 11-14$); they are explicitly given in (31) of [23]. Note that it is important to keep the functions $K^{(0)}$ in $d = 4-2\epsilon$ dimensions, because they are multiplied with one-loop matrix elements which are singular in ϵ. On the other hand, the matrix elements of the operators $\mathcal{O}_{i,\tilde{g}}^{q}$ ($i = 11-14$) are finite, therefore it is sufficient to weight them with the corresponding (4-dimensional) Wilson coefficients $C_{i,\tilde{g}}^{q,a}$. Finally, $Z_{m_b}^{OS,a}$ denotes the renormalization constant of m_b due to gluonic corrections in the on-shell scheme:

$$Z_{m_b}^{OS,a} = 1 - \frac{\alpha_s}{4\pi}\, C_F \left[\frac{3}{\epsilon} + 3\ln\frac{\mu^2}{m_b^2} + 4\right] , \tag{24}$$

with $C_F = 4/3$.

The renormalized amplitude $iA_{a,full}^{ren}$ in the full theory is obtained by adding the counterterm $iA_{a,full}^{ct}$ induced by renormalizing the following quantities in the leading order expression for the amplitude given in Appendix A: The gluino-mass $m_{\tilde{g}}$, the down-type squark masses $m_{\tilde{d}_k}$ and the strong coupling constant of Yukawa type $g_{s,Y}$. Furthermore, the on-shell renormalization constants $\sqrt{Z_{2b}^{OS,a}}$ and $\sqrt{Z_{2s}^{OS,a}}$ have to be attached. As we are discussing gluon-corrections in this section, we only take into account the gluonic contributions of the involved renormalization constants. The counterterm amplitude then reads

$$
\begin{aligned}
i\,A^{ct}_{a,full} = &\left(2\,\delta Z^{a}_{g_{s,Y}} + \frac{1}{2}\,\delta Z^{OS,a}_{2b} + \frac{1}{2}\,\delta Z^{OS,a}_{2s}\right)\,\left(K^{(0)}_{7b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\rm tree}\right. \\
&\left. + \,K^{(0)}_{7\tilde{g},\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\rm tree}\right) + R^{a}_{\tilde{g}\tilde{d}_k}\,.
\end{aligned}
\tag{25}
$$

$Z^{a}_{g_{s,Y}}$, derived in the Appendix F.1, reads

$$
Z^{a}_{g_{s,Y}} = 1 - \frac{\alpha_s}{4\pi}\left[\frac{3(C_A + C_F)}{2\epsilon}\right]\,, \tag{26}
$$

with $C_A = 3$, while the on-shell renormalization constants for the b quark and the massless s quark, which will drop out in the matching equation (32), read (see Appendix B, (85))

$$
\begin{aligned}
Z^{OS,a}_{2b} &= 1 - \frac{\alpha_s}{4\pi}\,C_F\left[\frac{3}{\epsilon} + 4 + 3\,\ln\frac{\mu^2}{m_b^2}\right]\,, \\
Z^{OS,a}_{2s} &= 0.
\end{aligned}
\tag{27}
$$

In (25), the contribution $R^{a}_{\tilde{g}\tilde{d}_k}$ is obtained by shifting (according to the $\overline{\rm MS}$ scheme)

$$
\begin{aligned}
m_{\tilde{g}} &\to m_{\tilde{g}}\left[1 - \frac{\alpha_s}{4\pi}\,C_A\,\frac{3}{\epsilon}\right]\,, \\
m^2_{\tilde{d}_k} &\to m^2_{\tilde{d}_k}\left[1 - \frac{\alpha_s}{4\pi}\,C_F\,\frac{3}{\epsilon}\right]\,,
\end{aligned}
\tag{28}
$$

in the leading order expression for $i\,A_{full}$ in Appendix A, followed by an expansion in α_s and keeping the term proportional to α_s^2.

Finally, putting everything together, the renormalized result in the full theory reads

$$
\begin{aligned}
i\,A^{ren}_{a,full} =& i\,A^{bare}_{a,full,h} + i\,A^{bare}_{a,full,s} \\
&+ \left[2\,\delta Z^{a}_{g_{s,Y}} + \frac{1}{2}\,\delta Z^{OS,a}_{2b} + \frac{1}{2}\,\delta Z^{OS,a}_{2s}\right] \\
&\times\left(K^{(0)}_{7b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\rm tree} + K^{(0)}_{7\tilde{g},\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\rm tree}\right) + R^{a}_{\tilde{g}\tilde{d}_k}\,.
\end{aligned}
\tag{29}
$$

We now turn to the effective theory side.

3.1.2 Effective theory

After converting the running b-quark mass present in the operator $\mathcal{O}_{7b,\tilde{g}}$ to the pole mass, the renormalized amplitude $iA^{ren}_{a,eff}$ can be written as

$$\begin{aligned}
i\,A^{ren}_{a,eff} =& \frac{\alpha_s}{4\pi}\,C^{(1),a}_{7g,g}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}} + \frac{\alpha_s}{4\pi}\,C^{(1),a}_{7b,g}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} \\
&+ K^{(0)}_{7b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{1-loop}} + K^{(0)}_{7\tilde{g},\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{1-loop}} \\
&+ K^{(0)}_{8b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{8b,\tilde{g}}|b\rangle_{\text{1-loop}} + K^{(0)}_{8\tilde{g},\tilde{g}}\,\langle s\gamma|\mathcal{O}_{8\tilde{g},\tilde{g}}|b\rangle_{\text{1-loop}} \\
&+ \sum_{i=11}^{14} C^{q,a}_{i,\tilde{g}}\,\langle s\gamma|\mathcal{O}^{q}_{i,\tilde{g}}|b\rangle_{\text{1-loop}} \\
&+ (Z^{OS,a}_{m_b}-1)\,K^{(0)}_{7b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} \\
&+ \left[2\,\delta Z_{g_s} + \delta\,Z_{77,b\tilde{g}} + \frac{1}{2}\,\delta Z^{OS,a}_{2b} + \frac{1}{2}\,\delta Z^{OS,a}_{2s}\right] K^{(0)}_{7b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} \\
&+ \left[2\,\delta Z_{g_s} + \delta\,Z_{77,\tilde{g}\tilde{g}} + \frac{1}{2}\,\delta Z^{OS,a}_{2b} + \frac{1}{2}\,\delta Z^{OS,a}_{2s}\right] K^{(0)}_{7\tilde{g},\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}} \\
&+ \delta\,Z_{87,\tilde{g}\tilde{g}}\,K^{(0)}_{8g,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}} + \delta\,Z_{87,b\tilde{g}}\,K^{(0)}_{8b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}}\,,
\end{aligned} \tag{30}$$

where the terms in the lines 2–5 correspond to $i\,A^{bare}_{a,full,s}$ in (23) and will cancel in the matching equation (32). Those renormalization constants, whose effects do not drop out in the matching step (32), read

$$\begin{aligned}
Z_{g_s} &= 1 - \frac{\alpha_s}{4\pi}\,\frac{23}{6\,\epsilon}\,, \qquad & Z_{77,\tilde{g}\tilde{g}} &= 1 + \frac{\alpha_s}{4\pi}\,\frac{9}{\epsilon}\,, \\
Z_{77,b\tilde{g}} &= 1 + \frac{\alpha_s}{4\pi}\,\frac{13}{\epsilon}\,, \qquad & Z_{87,\tilde{g}\tilde{g}} &= Z_{87,b\tilde{g}} = 1 - \frac{\alpha_s}{4\pi}\,\frac{16}{9\,\epsilon}\,.
\end{aligned} \tag{31}$$

Note that in (30) we put the d-dimensional versions $K^0_{7\tilde{g},\tilde{g}}$, $K^0_{7b,\tilde{g}}$, $K^0_{8\tilde{g},\tilde{g}}$ and $K^0_{8b,\tilde{g}}$ of the LO Wilson coefficients, which implies that the infrared structure on the full and effective theory side are identical. Moreover, the use of the d-dimensional versions of the Wilson coefficients in the effective theory also implies that all contributions due to the on-shell renormalization constants of the b and s quark will cancel out in the matching condition (32). These features *manifestly* reflect the fact that the determination of the *short-distance* Wilson coefficients is independent of the infrared structure and of the choice of the external states.

3.1.3 Extracting $C^{(1),a}_{7\tilde{g},\tilde{g}}$ and $C^{(1),a}_{7b,\tilde{g}}$

The coefficients $C^{(1),a}_{7\tilde{g},\tilde{g}}$ and $C^{(1),a}_{7b,\tilde{g}}$ are obtained by requiring that

$$iA^{ren}_{a,full} = iA^{ren}_{a,eff}\,. \tag{32}$$

Using $x_{d_k} = m^2_{\tilde{d}_k}/m^2_{\tilde{g}}$ and $L_\mu = \ln(\mu^2/m^2_{\tilde{g}})$, we get

$$\begin{aligned}
C^{(1),a}_{7\tilde{g},\tilde{g}} &= \frac{4}{3}\,\frac{1}{16\pi^2}\,\frac{1}{m_{\tilde{g}}}\,\sum_{k=1}^{6}\Gamma^{kb}_{DR}\Gamma^{ks*}_{DL}\,h^{(1),a}_{7\tilde{g},\tilde{g}}(x_{d_k}), \\
C^{(1),a}_{7b,\tilde{g}} &= -\frac{4}{3}\,\frac{1}{16\pi^2}\,\frac{1}{m^2_{\tilde{g}}}\,\sum_{k=1}^{6}\Gamma^{kb}_{DL}\Gamma^{ks*}_{DL}\,h^{(1),a}_{7b,\tilde{g}}(x_{d_k}),
\end{aligned} \tag{33}$$

with

$$h_{7\tilde{g},\tilde{g}}^{(1),a}(x) = \left(-\frac{55x^2+69x+2}{9(x-1)^3} + \frac{2x\left(9x^2+40x+14\right)}{9(x-1)^4}\ln x\right) L_\mu + \frac{x\left(x^2-11x+13\right)}{18(x-1)^4}\ln^2 x$$
$$+ \frac{19x^2+60x-15}{9(x-1)^3}\mathrm{Li}_2(1-x) + \frac{28x^3+120x^2-15x-1}{9(x-1)^4}\ln x$$
$$+ \frac{-14x^2-333x+83}{18(x-1)^3},$$

$$h_{7b,\tilde{g}}^{(1),a}(x) = \left(\frac{-5x^3+384x^2+609x+20}{108(x-1)^4} - \frac{x\left(18x^2+107x+43\right)}{18(x-1)^5}\ln x\right) L_\mu + \frac{x(x+11)}{36(x-1)^4}\ln^2 x$$
$$+ \frac{-17x^2-86x+15}{18(x-1)^4}\mathrm{Li}_2(1-x) + \frac{87x^4-537x^3-2997x^2+265x+14}{324(x-1)^5}\ln x$$
$$+ \frac{-799x^3+1719x^2+10431x-1847}{972(x-1)^4}. \tag{34}$$

Our results are in full agreement with those of Bobeth et al. [1] who matched the corresponding off-shell Greens functions.

Finally, we emphasize that one could also insist to use the four-dimensional version of the Wilson coefficients in the effective theory, because one can work out the explicit infrared (IR) structure of the amplitudes in the effective and full theory by distinguishing IR poles ($1/\epsilon_{\rm ir}$) from ultraviolet (UV) poles ($1/\epsilon$). One then finds that the IR structures in $iA_{a,full}^{ren}$ and $iA_{a,eff}^{ren}$ are of the form

$$\left(\frac{A}{\epsilon_{\rm ir}^2} + \frac{B}{\epsilon_{\rm ir}}\right) K_7^{(0)}\,\langle O_7\rangle_{\rm tree} \quad \text{and} \quad \left(\frac{A}{\epsilon_{\rm ir}^2} + \frac{B}{\epsilon_{\rm ir}}\right) C_7^{(0)}\,\langle O_7\rangle_{\rm tree}\,,$$

respectively. In both cases the same singular factor multiplies the lowest order amplitude of the full theory and the effective theory, respectively. On general grounds, the corresponding statement about the IR structure holds for the bremsstrahlung contributions. So the correct matching conditions, to extract NLO pieces of the Wilson coefficients $C_{7\tilde{g},\tilde{g}}^{(1),a}$ and $C_{7b,\tilde{g}}^{(1),a}$, can be achieved in this case by discarding these explicitly identified IR sensitive terms in $iA_{a,full}^{ren}$ and $iA_{a,eff}^{ren}$.

3.2 Calculation of $C_{7\tilde{g},\tilde{g}}^{(1),b}$, $C_{7b,\tilde{g}}^{(1),b}$, and $C_{7c,\tilde{g}}^{(1),b}$

3.2.1 Full theory

In order to get the full theory NLO contributions to the decay amplitude belonging to class b), we have to calculate irreducible two-loop diagrams with two virtual gluinos depicted in Figure 2 and contributions with squark tadpoles shown in Figure 3. We denote the contributions of these diagrams by $iA_{b,irred,full}^{bare}$. Additionally, there are reducible diagrams shown in Figure 4 leading to a contribution to the decay amplitude denoted $iA_{b,red,full}^{bare}$.

First, we discuss the calculation of the irreducible diagrams. We expand the corresponding two-loop diagrams according to the rules of the hard-mass procedure [44] in inverse powers of the heavy masses, which are in our application $m_{\tilde{q}_k}$, $m_{\tilde{g}}$ and m_t. The class of diagrams

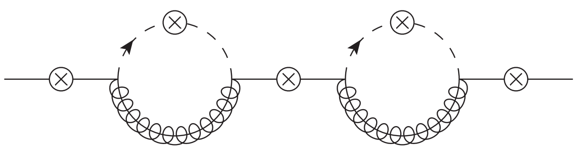


Figure 4: One-particle reducible diagrams, which are taken into account through the quark-field renormalization constants

containing squark tadpoles (Figure 3) does not contain any light propagators and can therefore just be naively Taylor-expanded in the external momenta. The diagrams with two virtual gluinos (Figure 2) can also contain propagators of light quarks. Thus, besides the naively expanded two-loop diagrams, we also have to take into account contributions with one-loop sub-diagrams containing all heavy lines which have to be expanded in the external momenta. The result is inserted in the second loop which contains only light propagators as an effective vertex. Logarithms of the small masses are only generated in the latter contributions and cancel in the matching procedure against equal terms on the effective side stemming from matrix elements of the four-Fermi operators $\mathcal{O}^q_{11,\tilde{g}}$ to $\mathcal{O}^q_{20,\tilde{g}}$.

The integrals which result from the naive Taylor expansion are two-loop vacuum integrals. Using integration-by-parts identities [51, 52] and partial fraction decompositions, we can reduce all these integrals to the following master integrals:

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2-m^2} = \frac{-i}{(4\pi)^{\frac{d}{2}}} \Gamma\left(1-\frac{d}{2}\right) m^{d-2},$$

$$\begin{aligned}
&\int \frac{d^dk_1}{(2\pi)^d} \frac{d^dk_2}{(2\pi)^d} \frac{1}{(k_1^2-m_1^2)\,(k_2^2-m_2^2)\,[(k_1+k_2)^2-m_3^2]} = \frac{1}{512\pi^4} m_3^{2-4\epsilon} \frac{\Gamma(1+\epsilon)^2}{(1-\epsilon)(1-2\epsilon)} \\
&\times \left[-\frac{1+x+y}{\epsilon^2} + \frac{2(x\ln x + y\ln y)}{\epsilon} - x\ln^2 x - y\ln^2 y + (1-x-y)\ln x \ln y - \lambda^2 \Phi(x,y) \right],
\end{aligned} \tag{35}$$

where $d = 4-2\epsilon$, $\lambda^2 = (1-x-y)^2 - 4xy$, $x = m_1^2/m_3^2$, and $y = m_2^2/m_3^2$. The function $\Phi(x,y)$ has been evaluated in [50]. For $\lambda^2 > 0$ it is represented by dilogarithms as

$$\begin{aligned}
\Phi(x,y) = \frac{1}{\lambda}\Bigg[& 2\ln\frac{1+x-y-\lambda}{2} \ln\frac{1-x+y-\lambda}{2} - \ln x \ln y \\
& - 2\mathrm{Li}_2 \frac{1+x-y-\lambda}{2} - 2\mathrm{Li}_2 \frac{1-x+y-\lambda}{2} + \frac{\pi^2}{3}\Bigg],
\end{aligned} \tag{36}$$

whereas for $\lambda^2 < 0$ it can be written in terms of Clausen's functions:

$$\begin{aligned}
\Phi(x,y) = & \frac{2}{\sqrt{-\lambda^2}} \left[\mathrm{Cl}_2\left(2\arccos\frac{-1+x+y}{2\sqrt{xy}}\right) + \mathrm{Cl}_2\left(2\arccos\frac{1+x-y}{2\sqrt{x}}\right) \right. \\
& \left. + \mathrm{Cl}_2\left(2\arccos\frac{1-x+y}{2\sqrt{y}}\right) \right].
\end{aligned} \tag{37}$$

We now turn to the reducible diagrams. Their contribution to the decay amplitude can be obtained by amputating the external fermion legs (with squark-gluino self-energy insertions) followed by attaching the LSZ-factor, i.e. the gluino parts of the renormalization constants of the b- and s-quark fields in the on-shell scheme which are derived in detail in Appendix B. This amounts to replace the $\Gamma_{DL/R}$ matrices in the LO amplitude, given in Appendix A, according to

$$\Gamma^{ki}_{DL/R} \to \Gamma^{kj}_{DL/R}\,\frac{1}{2}\,\delta Z^{b,L/R}_{ji} = \Gamma^{kj}_{DL/R}\,\frac{1}{2}\,\left(\delta Z^{b,L/R,H}_{ji} + \delta Z^{b,L/R,AH}_{ji}\right), \tag{38}$$

where a summation over the index j is understood.

In (38), we split the renormalization constants into a hermitian and an antihermitian part as

$$\delta Z^{b,L/R} = \delta Z^{b,L/R,H} + \delta Z^{b,L/R,AH}. \tag{39}$$

Explicitly, they read in the off-diagonal case (using $x_{d_k} = m^2_{\tilde{d}_k}/m^2_{\tilde{g}}$)

$$\begin{aligned}\delta Z^{b,L,AH}_{ji} =& \frac{\alpha_s}{4\pi}\frac{C_F}{m_i^2 - m_j^2}\sum_{k=1}^{6}\Bigg\{(\Gamma^{ki}_{DL}\Gamma^{kj*}_{DR}m_j + \Gamma^{kj*}_{DL}\Gamma^{ki}_{DR}m_i)\left(\frac{m_i^2+m_j^2}{m_{\tilde{g}}}f_1(x_{d_k}) - 4m_{\tilde{g}}f_2(x_{d_k})\right)\\ &+\left(\Gamma^{ki}_{DL}\Gamma^{kj*}_{DL}\left(m_i^2+m_j^2\right) + 2\Gamma^{ki}_{DR}\Gamma^{kj*}_{DR}m_i m_j\right)f_3(x_{d_k})\Bigg\},\end{aligned} \tag{40}$$

$$\delta Z^{b,L,H}_{ji} = \frac{\alpha_s}{4\pi}C_F\sum_{k=1}^{6}\left(\frac{1}{m_{\tilde{g}}}(\Gamma^{kj*}_{DL}\Gamma^{ki}_{DR}m_i + \Gamma^{ki}_{DL}\Gamma^{kj*}_{DR}m_j)f_1(x_{d_k}) + \Gamma^{ki}_{DL}\Gamma^{kj*}_{DL}f_3(x_{d_k})\right), \tag{41}$$

where m_i are the masses of the down-type quarks. The corresponding results for the right-handed versions are obtained by interchanging the labels R and L. The functions $f_i(x)$ are given in Appendix B. In the diagonal case, we get (using $L_\mu = \ln(\mu^2/m^2_{\tilde{g}})$)

$$\begin{aligned}\delta Z^{b,L,AH}_{ii} =& 0,\\ \delta Z^{b,L,H}_{ii} =& \frac{\alpha_s}{4\pi}C_F\sum_{k=1}^{6}\left(\frac{m_i}{m_{\tilde{g}}}(\Gamma^{ki*}_{DL}\Gamma^{ki}_{DR} + \Gamma^{ki}_{DL}\Gamma^{ki*}_{DR})f_1(x_{d_k}) + \Gamma^{ki}_{DL}\Gamma^{ki*}_{DL}f_3(x_{d_k})\right)\\ &-\frac{\alpha_s}{4\pi}C_F\left(\frac{1}{\epsilon} + L_\mu\right).\end{aligned} \tag{42}$$

Performing the shifts (38) in the LO result yields $iA^{bare}_{b,red,full}$ at the NLO level. Since the hermitian parts of the Z factors do not contain any inverse powers of the light quark masses, the corresponding contribution to $iA^{bare}_{b,red,full}$ can be expanded up to linear order in m_b. The contributions proportional to m_b^0 and m_b^1 will end up in the Wilson coefficients of the magnetic operators of dimension five and six, respectively.

For the contributions induced by the antihermitian parts of the Z-factors this splitting does not work since they also contain inverse powers of the light quark masses, i.e. *chirally enhanced* terms which have been recently discussed in [31]. Instead, we proceed in such a way that the contributions involving the LO functions $K^{(0)}_{7\tilde{g},\tilde{g}}$ and $K^{(0)}_{7b,\tilde{g}}$ will end up in the Wilson coefficients of the magnetic operators of dimension five and six, respectively. This ends the discussion for constructing $iA^{bare}_{b,red,full}$. Note that in principle one could renormalize the Γ matrices in a way that the antihermitian parts of the quark field renormalization constants get exactly cancelled. However, the terms induced by the antihermitian parts of quark field renormalization constants remain in the decay amplitude in our scheme in which we renormalize the Γ matrices minimally (see counterterms induced by the renormalization of the Γ matrices below and also Appendix F.1).

We now discuss the counterterm contribution $iA^{ct}_{b,full}$ which gets induced by renormalizing the strong coupling constant of Yukawa-type $g_{s,Y}$, the gluino mass $m_{\tilde{g}}$, the squark masses $m_{\tilde{d}_k}$, and the $\Gamma_{D,L/R}$ matrices in the LO amplitude. We get

$$i\,A^{ct}_{b,full} = 2\,\delta Z^b_{g_{s,Y}}\;\left(K^{(0)}_{7b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} + K^{(0)}_{7\tilde{g},\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}}\right) + R^b_{\tilde{g}\tilde{d}_k\Gamma}\,. \tag{43}$$

$\delta Z^b_{g_{s,Y}}$ is derived in Appendix F.1 and reads

$$\delta Z^b_{g_{s,Y}} = \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\left(\frac{3}{2}C_F + tr\,n_f\right), \tag{44}$$

with $tr = 1/2$ and $n_f = 6$ denoting the number of flavors. The term $R^b_{\tilde{g}\tilde{d}_k\Gamma}$ is obtained by replacing

$$m_{\tilde{g}} \to m_{\tilde{g}}\left[1 + \frac{\alpha_s}{4\pi}\,2\,tr\,\frac{n_f}{\epsilon}\right], \tag{45}$$

$$\begin{aligned} m^2_{\tilde{d}_k} \to m^2_{\tilde{d}_k} &+ \frac{\alpha_s}{\pi}C_F\left[\sum_d\left(m_{\tilde{g}}\,m_d\,(\Gamma^{kd*}_{DR}\Gamma^{kd}_{DL} + \Gamma^{kd*}_{DL}\Gamma^{kd}_{DR})\right)\right]\frac{1}{\epsilon} + \frac{\alpha_s}{2\pi}C_F\,m^2_{\tilde{g}}\,(x_{d_k} - 2)\frac{1}{\epsilon} \\ &+ \frac{\alpha_s}{3\pi}\,m^2_{\tilde{g}}\,X^{kl}_D\,x_{d_l}\,X^{lk}_D\,\frac{1}{\epsilon}, \end{aligned} \tag{46}$$

and

$$\Gamma^{ki}_{DL/R} \to \Gamma^{ki}_{DL/R} + \delta\Gamma^{kl}_D\,\Gamma^{li}_{DL/R}, \tag{47}$$

in the leading order expression for iA_{full} in Appendix A, followed by an expansion in α_s and keeping the term proportional to α_s^2. The explicit sum in (46) runs over all down-type quark flavors of mass m_d, while the one over $l = 1, ..., 6$ is implicitly understood. Again, $x_{d_k} = m^2_{\tilde{d}_k}/m^2_{\tilde{g}}$ and the matrix X_D is given by

$$X^{kk'}_D = \Gamma^{ki}_{DR}\Gamma^{k'i*}_{DR} - \Gamma^{ki}_{DL}\Gamma^{k'i*}_{DL}, \tag{48}$$

with i summed over $i = 1, 2, 3$. The shift in the coupling matrices, described by the matrix $\delta\Gamma^{kl}_D$ is derived in Appendix F.1 and reads

$$\begin{aligned} \delta\Gamma^{kl}_D = -\frac{\alpha_s}{\pi}\,\frac{1}{x_{d_l} - x_{d_k}}\;\Bigg\{C_F\;&\left[\sum_d\left(\frac{m_d}{m_{\tilde{g}}}\,(\Gamma^{kd}_{DL}\Gamma^{ld*}_{DR} + \Gamma^{kd}_{DR}\Gamma^{ld*}_{DL})\right)\right]\frac{1}{\epsilon} \\ &+\frac{1}{3}X^{kn}_D\,x_{d_n}\,X^{nl}_D\,\frac{1}{\epsilon}\Bigg\} \qquad \text{for } k \neq l\,. \end{aligned} \tag{49}$$

For $k = l$, we have $\delta\Gamma^{kl}_D = 0$. Detailed derivations of the renormalization of the squark and gluino masses are given in Appendices C and D, respectively.

Finally, we can put everything together to form the complete contributions of class b) in the full theory:

$$\begin{aligned} iA^{ren}_{b,full} =& iA^{bare}_{b,irred,full} + iA^{bare}_{b,red,full} \\ &+ 2\,\delta Z^b_{g_{s,Y}}\;\left(K^{(0)}_{7b,\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} + K^{(0)}_{7\tilde{g},\tilde{g}}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}}\right) + R^b_{\tilde{g}\tilde{d}_k\Gamma}\,. \end{aligned} \tag{50}$$

3.2.2 Effective theory

Since the contributions of class b) do not contain any gluon corrections, the only one-loop contributions in the effective theory are due to the box-operators in (17) and the box-induced part of the operators in (16) (given in expression (32) in [23]). The renormalized amplitude $iA^{ren}_{b,eff}$ can be written as

$$
\begin{aligned}
i\,A^{ren}_{b,eff} =& \frac{\alpha_s}{4\pi}\,C^{(1),b}_{7g,g}\,\langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}} + \frac{\alpha_s}{4\pi}\,C^{(1),b}_{7b,g}\,\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} + \frac{\alpha_s}{4\pi}\,C^{(1),b}_{7c,g}\,\langle s\gamma|\mathcal{O}_{7c,\tilde{g}}|b\rangle_{\text{tree}} \\
&+ \sum_{i=11}^{14}\sum_q C^{(0),q,b}_{i,\tilde{g}}\,\langle s\gamma|\mathcal{O}^q_{i,\tilde{g}}|b\rangle_{\text{1-loop}} + \sum_{i=15}^{20}\sum_q C^{(0),q}_{i,\tilde{g}}\,\langle s\gamma|\mathcal{O}^q_{i,\tilde{g}}|b\rangle_{\text{1-loop}} \\
&+ \left(C^{(0),b}_{15,\tilde{g}}\delta Z_{15,7b} + C^{(0),b}_{16,\tilde{g}}\delta Z_{16,7b}\right)\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} \\
&+ \left(C^{(0),b}_{19,\tilde{g}}\delta Z_{19,7b} + C^{(0),b}_{20,\tilde{g}}\delta Z_{20,7b}\right)\langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} \\
&+ \left(C^{(0),c}_{19,\tilde{g}}\delta Z_{19,7c} + C^{(0),c}_{20,\tilde{g}}\delta Z_{20,7c}\right)\langle s\gamma|\mathcal{O}_{7c,\tilde{g}}|b\rangle_{\text{tree}},
\end{aligned}
\tag{51}
$$

with

$$
\begin{aligned}
\delta Z_{15,7b} &= -\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{6}, & \delta Z_{16,7b} &= -\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{2}, \\
\delta Z_{19,7b} &= -\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{2}, & \delta Z_{20,7b} &= -\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{6}, \\
\delta Z_{19,7c} &= \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}, & \delta Z_{20,7c} &= \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{3}.
\end{aligned}
\tag{52}
$$

3.2.3 Extracting $C^{(1),b}_{7\tilde{g},\tilde{g}}$, $C^{(1),b}_{7b,\tilde{g}}$ and $C^{(1),b}_{7c,\tilde{g}}$

From the requirement that (50) and (51) produce the same results, we can extract the next-to-leading order Wilson coefficients $C^{(1),b}_{7\tilde{g},\tilde{g}}$, $C^{(1),b}_{7b,\tilde{g}}$ and $C^{(1),b}_{7c,\tilde{g}}$. Unfortunately, the results are much too long to be printed. We therefore provide a computer code (see Section 5), which enables the user to evaluate our results numerically for arbitrary input parameters.

3.3 Results for $C^{(1)}_{7\tilde{g},\tilde{g}}$, $C^{(1)}_{7b,\tilde{g}}$, and $C^{(1)}_{7c,\tilde{g}}$

We are now ready to recombine the contributions to the Wilson coefficients of both the gluon corrections of class a) and the gluino corrections of class b) as

$$
\begin{aligned}
C^{(1)}_{7\tilde{g},\tilde{g}} &= C^{(1),a}_{7\tilde{g},\tilde{g}} + C^{(1),b}_{7\tilde{g},\tilde{g}}, \\
C^{(1)}_{7b,\tilde{g}} &= C^{(1),a}_{7b,\tilde{g}} + C^{(1),b}_{7b,\tilde{g}}, \\
C^{(1)}_{7c,\tilde{g}} &= C^{(1),a}_{7c,\tilde{g}} + C^{(1),b}_{7c,\tilde{g}}.
\end{aligned}
\tag{53}
$$

These results were, however, obtained by using dimensional regularization (DREG) throughout the calculation followed by renormalizing $m_{\tilde{g}}$, $m^2_{\tilde{d}_k}$, $g_{s,Y}$ and $\Gamma_{DL/R}$ in the $\overline{\text{MS}}$ scheme. But, since DREG introduces a mismatch between the bosonic and fermionic degrees of freedom, the present calculation in the full theory has to be worked out in dimensional reduction (DRED) [45, 46], which is expected to preserve supersymmetry. In principle this requires either the calculation of additional graphs featuring ϵ-scalars or working out the Dirac algebra in $d = 4$ dimensions.

At leading order in α_s, the results in DREG and DRED are identical, while at next-to-leading order the difference between DREG and DRED can be obtained by shifting the strong coupling g_s and the gluino mass $m_{\tilde{g}}$ in the leading order contribution. Exploiting this, we chose in a first step to evaluate all diagrams in the full theory strictly in DREG, followed by renormalizing $m_{\tilde{d}_k}$, $m_{\tilde{g}}$ and g_s in the $\overline{\text{MS}}$ scheme. In Section 3.4, we will discuss the shifts in g_s and $m_{\tilde{g}}$.

Additionally, the calculation in the full theory was performed without taking into account the decoupling of the heavy particles in the running of the strong coupling. This will be done in Section 3.5.

3.4 Transition to the $\overline{\text{DR}}$ scheme

Since supersymmetry is broken in DREG, we have to distinguish between the Yukawa-type coupling $g_{s,Y}$ appearing in the squark-quark-gluino vertex and the gauge coupling $g_{s,G}$. Only supersymmetry guarantees that both versions of the strong coupling are equal, which is the case in DRED accompanied by minimal subtraction (which is usually called $\overline{\text{DR}}$ scheme).

The result for the NLO full theory contributions obtained in the $\overline{\text{MS}}$ scheme is expressed in terms of $g_{s,Y}^{\overline{\text{MS}}}$ and $m_{\tilde{g}}^{\overline{\text{MS}}}$. The transition to the expression depending on the corresponding $\overline{\text{DR}}$ quantities amounts to performing the following replacements in the leading order contributions [48, 49]:[2]

$$\begin{aligned} g_{s,Y}^{\overline{\text{MS}}} &= g_s^{\overline{\text{DR}}}\left[1+\frac{\alpha_s^{\overline{\text{DR}}}}{4\pi}\frac{(C_A-C_F)}{2}\right], \\ m_{\tilde{g}}^{\overline{\text{MS}}} &= m_{\tilde{g}}^{\overline{\text{DR}}}\left[1+\frac{\alpha_s^{\overline{\text{DR}}}}{4\pi}C_A\right]. \end{aligned} \tag{54}$$

This leads to finite shifts of the NLO Wilson coefficients which we denote by $\delta C_{7\tilde{g},\tilde{g}}^{(1),\overline{\text{DR}}}$ and $\delta C_{7b,\tilde{g}}^{(1),\overline{\text{DR}}}$. Since we treat the external b and s quarks in the on-shell scheme, their masses do not need to be shifted.

3.5 Decoupling of heavy particles

After performing the shifts in (54), the result on the full theory side depends on $\alpha_{s,\overline{\text{DR}}}^{(\text{full})}$. However, on the effective side, there are only five active quark flavors and the coupling is renormalized in the $\overline{\text{MS}}$ scheme. Hence, we have to express $\alpha_{s,\overline{\text{DR}}}^{(\text{full})}$ in terms of $\alpha_{s,\overline{\text{MS}}}^{(5)}$ on the full theory side. The relation between $\alpha_{s,\overline{\text{DR}}}^{(\text{full})}$ and $\alpha_{s,\overline{\text{DR}}}^{(5)}$ can be found in [47] as

$$\alpha_{s,\overline{\text{DR}}}^{(\text{full})} = \left(\zeta_{g_s}^{\text{SUSY}}\right)^{-2}\alpha_{s,\overline{\text{DR}}}^{(5)}, \tag{55}$$

with the decoupling coefficient

$$\left(\zeta_{g_s}^{\text{SUSY}}\right)^{-2} = 1-\frac{\alpha_{s,\overline{\text{DR}}}^{(5)}}{\pi}\left[-\frac{1}{6}\ln\frac{\mu^2}{m_t^2}-\frac{1}{24}\left(\sum_{k=1}^{6}\ln\frac{\mu^2}{m_{\tilde{u}_k}^2}+\sum_{k=1}^{6}\ln\frac{\mu^2}{m_{\tilde{d}_k}^2}\right)-\frac{1}{2}\ln\frac{\mu^2}{m_{\tilde{g}}^2}\right]. \tag{56}$$

[2] Note that $g_{s,G}$ does not appear in our LO expressions.

We now express $\alpha_{s,\overline{\rm DR}}^{(5)}$ in terms of $\alpha_{s,\overline{\rm MS}}^{(5)}$ according to

$$\alpha_{s,\overline{\rm DR}}^{(5)} = \alpha_{s,\overline{\rm MS}}^{(5)} \left[1 + \frac{\alpha_{s,\overline{\rm MS}}^{(5)}}{4\pi} \frac{C_A}{3} \right]. \tag{57}$$

Combining (55) and (57) we can finally give the relation between $\alpha_{s,\overline{\rm DR}}^{(\rm full)}$ and $\alpha_{s,\overline{\rm MS}}^{(5)}$ as

$$\alpha_{s,\overline{\rm DR}}^{(\rm full)} = \alpha_{s,\overline{\rm MS}}^{(5)} \left\{ 1 + \frac{\alpha_{s,\overline{\rm MS}}^{(5)}}{\pi} \left[\frac{C_A}{12} + \frac{1}{6} \ln \frac{\mu^2}{m_t^2} + \frac{1}{24} \left(\sum_{k=1}^{6} \ln \frac{\mu^2}{m_{\tilde{u}_k}^2} + \sum_{k=1}^{6} \ln \frac{\mu^2}{m_{\tilde{d}_k}^2} \right) + \frac{1}{2} \ln \frac{\mu^2}{m_{\tilde{g}}^2} \right] \right\}. \tag{58}$$

This replacement leads to a finite shift of the NLO Wilson coefficients which we denote by $\delta C_{7\tilde{g},\tilde{g}}^{(1),dec.}$ and $\delta C_{7b,\tilde{g}}^{(1),dec.}$.

3.6 Final result for $C_{7\tilde{g},\tilde{g}}^{(1)}$, $C_{7b,\tilde{g}}^{(1)}$, and $C_{7c,\tilde{g}}^{(1)}$

We now have all ingredients to give the final result for all NLO Wilson coefficients of the magnetic operators. They read

$$\begin{aligned} C_{7\tilde{g},\tilde{g}}^{(1),\overline{\rm DR}} &= C_{7\tilde{g},\tilde{g}}^{(1),a} + C_{7\tilde{g},\tilde{g}}^{(1),b} + \delta C_{7\tilde{g},\tilde{g}}^{(1),\overline{\rm DR}} + \delta C_{7\tilde{g},\tilde{g}}^{(1),dec.}, \\ C_{7b,\tilde{g}}^{(1),\overline{\rm DR}} &= C_{7b,\tilde{g}}^{(1),a} + C_{7b,\tilde{g}}^{(1),b} + \delta C_{7b,\tilde{g}}^{(1),\overline{\rm DR}} + \delta C_{7b,\tilde{g}}^{(1),dec.}, \\ C_{7c,\tilde{g}}^{(1),\overline{\rm DR}} &= C_{7c,\tilde{g}}^{(1),a} + C_{7c,\tilde{g}}^{(1),b}. \end{aligned} \tag{59}$$

4 Calculation of the Wilson coefficients of the chromomagnetic operators

In order to extract the NLO Wilson coefficients of the chromomagnetic operators, we have to match the full theory NLO result for the amplitude of $b \to sg$ to the one in the effective theory. Since the Dirac structure of the photon vertices is the same as the one of the gluon vertices, we can use the results for the diagrams already calculated for $b \to s\gamma$ and supplement them with the appropriate color factor. In addition to these diagrams, we have to take into account all those where the gluon couples to gluino and gluon lines.

In full analogy to the procedure for extracting the Wilson coefficients of the magnetic operators, we also divide the calculation into the classes a) and b) introduced in Section 3.

4.1 Results for $C_{8\tilde{g},\tilde{g}}^{(1),a}$ and $C_{8b,\tilde{g}}^{(1),a}$

Since the calculation for the gluon corrections has been presented in [1] and we cross-checked the results for $b \to s\gamma$, we do not repeat the calculation and directly take the result from [1]. It reads

$$\begin{aligned} C_{8\tilde{g},\tilde{g}}^{(1),a} &= \frac{4}{3} \frac{1}{16\pi^2} \frac{1}{m_{\tilde{g}}} \sum_{k=1}^{6} \Gamma_{DR}^{kb} \Gamma_{DL}^{ks*} h_{8\tilde{g},\tilde{g}}^{(1),a}(x_{d_k}), \\ C_{8b,\tilde{g}}^{(1),a} &= -\frac{4}{3} \frac{1}{16\pi^2} \frac{1}{m_{\tilde{g}}^2} \sum_{k=1}^{6} \Gamma_{DL}^{kb} \Gamma_{DL}^{ks*} h_{8b,\tilde{g}}^{(1),a}(x_{d_k}), \end{aligned} \tag{60}$$

with

$$h^{(1),a}_{8\tilde{g},\tilde{g}}(x) = \left(\frac{7\left(79x^2-12x+5\right)}{12(x-1)^3} - \frac{7x\left(18x^2+19x-1\right)}{6(x-1)^4}\ln(x)\right)L_\mu$$
$$+\frac{\left(-359x^2+339x-204\right)}{24(x-1)^3}\mathrm{Li}_2(1-x) + \frac{x\left(181x^2-236x+31\right)}{48(x-1)^4}\ln^2(x)$$
$$-\frac{\left(2419x^3-771x^2+453x+11\right)}{48(x-1)^4}\ln(x) + \frac{1667x^2-990x+379}{24(x-1)^3},$$

$$h^{(1),a}_{8b,\tilde{g}}(x) = \left(\frac{x\left(747x^2+640x-43\right)}{48(x-1)^5}\ln(x) - \frac{779x^3+7203x^2+93x-11}{288(x-1)^4}\right)L_\mu$$
$$+\frac{\left(-45x^3+1208x^2-901x+570\right)}{96(x-1)^4}\mathrm{Li}_2(1-x) - \frac{x\left(45x^2+358x-49\right)}{192(x-1)^4}\ln^2(x)$$
$$+\frac{\left(-183x^4+30027x^3-10692x^2+6115x+77\right)}{864(x-1)^5}\ln(x)$$
$$+\frac{5359x^3-241425x^2+143253x-59251}{5184(x-1)^4}. \tag{61}$$

4.2 Calculation of $C^{(1),b}_{8\tilde{g},\tilde{g}}$, $C^{(1),b}_{8b,\tilde{g}}$, and $C^{(1),b}_{8c,\tilde{g}}$

Apart from additional diagrams, there are two new features in this calculation on the full theory side: Firstly, the emitted gluon couples with the gauge version of the strong coupling $g_{s,G}$, which, at leading order, is different from the Yukawa version. Secondly, the gluon field has to be renormalized. The required renormalization constants are derived in Appendices F.2 and D, respectively. They read

$$\delta Z^b_{g_s,G} = \frac{\alpha_s}{4\pi}\left[\frac{1}{3}\,tr\,n_f + \frac{C_A}{3}\right]\frac{1}{\epsilon},$$
$$\delta Z^b_3 = -2\frac{\alpha_s}{4\pi}\left[\frac{1}{3}\,tr\,n_f + \frac{C_A}{3}\right]\frac{1}{\epsilon}. \tag{62}$$

Multiplying the leading order contribution on the full theory side, they both induce counterterms at the next-to-leading order. However, they appear in the combination

$$\delta Z^b_{g_s,G} + \frac{1}{2}\delta Z^b_3 = 0, \tag{63}$$

and therefore their effect cancels.

The calculation of the decay amplitude for $b \to sg$ on the effective side is very similar to the one for the $b \to s\gamma$ transition. The non-vanishing renormalization constants for the operators read

$$\delta Z_{15,8b} = \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{2}, \qquad \delta Z_{20,8b} = \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{2}, \qquad \delta Z_{20,8c} = \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{1}{2}. \tag{64}$$

After the extraction of the Wilson coefficients, the results are expressed in $\overline{\mathrm{MS}}$ entities and we need to replace them by their $\overline{\mathrm{DR}}$ counterparts. In contrast to the $b \to s\gamma$ case, the

Parameter	C++ variable
$(\Gamma_{UL}, \Gamma_{UR})$	`std::complex<double> GU[6][6]`
$(\Gamma_{DL}, \Gamma_{DR})$	`std::complex<double> GD[6][6]`
$(m^2_{\tilde{u}_1}, \ldots, m^2_{\tilde{u}_6})$	`double mupsquark[6]`
$(m^2_{\tilde{d}_1}, \ldots, m^2_{\tilde{d}_6})$	`double mdownsquark[6]`
$m_{\tilde{g}}$	`double mg`
m_t	`double mt`
m_b	`double mb`
m_s	`double ms`
m_d	`double md`

Table 1: Input parameters.

leading order contribution now also contains the gauge coupling $g_{s,g}^{\overline{\mathrm{MS}}}$ which has to be replaced according to [48]

$$g_{s,G}^{\overline{\mathrm{MS}}} = g_s^{\overline{\mathrm{DR}}} \left[1 - \frac{\alpha_s^{\overline{\mathrm{DR}}}}{4\pi} \frac{C_A}{6} \right] . \tag{65}$$

The decoupling of the heavy particles and the shift back to the $\overline{\mathrm{MS}}$ scheme works exactly the same way as for $b \to s\gamma$.

5 Results for the Wilson coefficients at the matching scale μ_W

As mentioned above, the analytic results for the NLO Wilson coefficients are too long to be printed. We therefore provide a C++ code available at `arXiv.org` which enable the user to evaluate our results numerically for arbitrary input parameters. We also include the results for the LO Wilson coefficients that were given in [23].

The file `bsg2gluino.h` contains the definitions of the functions for all LO and NLO Wilson coefficients and can be included into any C++ code.[3] All input parameters, namely the rotation matrices of the squark fields $\Gamma_{(U,D)(L,R)}$ defined in (9), the squark masses, the gluino mass $m_{\tilde{g}}$, the top mass m_t, the bottom mass m_b, the strange mass m_s, and the down mass m_d, are declared as global variables within `bsg2gluino.h` according to the naming scheme shown in Table 1.

The names for the provided functions for the Wilson coefficients are listed in Tables 2 and 3. We define the expansion in α_s as usual by

$$C_i = C_i^{(0)} + \frac{\alpha_s}{4\pi} C_i^{(1)} . \tag{66}$$

The NLO Wilson coefficients explicitly depend on the matching scale μ_W. Hence, these functions have to be provided with a value for μ_W as an argument.

[3] An equivalent FORTRAN version is available from C.G.

Wilson coefficient	C++ function
$C^{(0)}_{7\tilde{g}\tilde{g}}$	`std::complex<double> C7ggLO()`
$C'^{(0)}_{7\tilde{g}\tilde{g}}$	`std::complex<double> C7ggLOPrime()`
$C^{(0)}_{7b\tilde{g}}$	`std::complex<double> C7bgLO()`
$C'^{(0)}_{7b\tilde{g}}$	`std::complex<double> C7bgLOPrime()`
$C^{(0)}_{8\tilde{g}\tilde{g}}$	`std::complex<double> C8ggLO()`
$C'^{(0)}_{8\tilde{g}\tilde{g}}$	`std::complex<double> C8ggLOPrime()`
$C^{(0)}_{8b\tilde{g}}$	`std::complex<double> C8bgLO()`
$C'^{(0)}_{8b\tilde{g}}$	`std::complex<double> C8bgLOPrime()`
$C^{q,(0)}_{i\tilde{g}}$	`std::complex<double> C‘i’g‘q’LO()`, e.g. `C11guLO()`
$C^{q,\prime(0)}_{i\tilde{g}}$	`std::complex<double> C‘i’g‘q’LOPrime()`

Table 2: C++ functions for the LO Wilson coefficients with $i = 11, \ldots, 20$ and $q = u, d, c, s, b$.

Wilson coefficient	C++ function
$C^{\overline{\rm DR},(1)}_{i\tilde{g}\tilde{g}}(\mu)$	`std::complex<double> C‘i’ggDRbar(double)`
$C^{\overline{\rm DR},(1)'}_{i\tilde{g}\tilde{g}}(\mu)$	`std::complex<double> C‘i’ggDRbarPrime(double)`
$C^{\overline{\rm DR},(1)}_{ib\tilde{g}}(\mu)$	`std::complex<double> C‘i’bgDRbar(double)`
$C^{\overline{\rm DR},(1)'}_{ib\tilde{g}}(\mu)$	`std::complex<double> C‘i’bgDRbarPrime(double)`
$C^{\overline{\rm DR},(1)}_{ic\tilde{g}}(\mu)$	`std::complex<double> C‘i’cgDRbar(double)`
$C^{\overline{\rm DR},(1)'}_{ic\tilde{g}}(\mu)$	`std::complex<double> C‘i’cgDRbarPrime(double)`

Table 3: C++ functions for the NLO Wilson coefficients with $i = 7, 8$.

To illustrate the use of `bsg2gluino.h`, we also provide the compilable file `template.cc` in which the header `bsg2gluino.h` is included and all input parameters are set to some specific values. The compiled program will then generate output for all included Wilson coefficients as shown in the file `output.dat`.

6 Conclusion

The main purpose of this paper is the two-loop matching calculation of the gluino-induced contributions to the processes $b \to s\gamma$ and $b \to sg$. While the contribution involving one gluino has already been available in the literature, the two-gluino part is new. We discuss in detail our renormalization procedure and the issues related to dimensional reduction and the decoupling of heavy particles from the running of α_s. Our results are presented in the C++ file `bsg2gluino.h` which comes together with this paper and allows for a convenient numerical evaluation.

The results obtained in this paper constitute a crucial building block for the phenomenological NLL analysis of the branching ratio $\bar{B} \to X_s\gamma$ in a supersymmetric model beyond MFV because the NLL gluino contributions are the most important corrections beyond MFV at this order in most parts of the parameter space in such models.

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Appendix

In Appendix A we give the LO expressions for the Wilson coefficients (in d dimensions) which are used at various places in this paper. Appendix B is devoted to the evaluation of the renormalization constants of the quark fields and quark masses in the on-shell scheme. Appendices C to E deal with the corresponding renormalization constants related to the squarks, the gluino and the gluon. For completeness we give all these quantities in the on-shell scheme, although for some of them only the singular piece (in ϵ) is needed in the main text. Finally, in Appendix F we derive the $1/\epsilon$ pieces of the renormalization constants for the strong coupling constants $g_{s,Y}$ and $g_{s,G}$, as well as for the $\Gamma_{DL/R}$ matrices. All the results in the Appendix are obtained by using DREG.

A Leading order result for $b \to s\gamma$ in the full theory

Using the abbreviations $x_{d_k} = m^2_{\tilde{d}_k}/m^2_{\tilde{g}}$ and $L_\mu = \ln(\mu^2/m^2_{\tilde{g}})$, the decay amplitude $\mathcal{A}(b \to s\gamma)$ in the full theory to leading order in QCD (corresponding to the diagrams with one-loop gluino/squark exchange) can be written for a massless s quark as

$$\begin{aligned} iA_{full}(b \to s\gamma) =& K^{(0)}_{7b,\tilde{g}} \langle s\gamma|\mathcal{O}_{7b,\tilde{g}}|b\rangle_{\text{tree}} + K^{(0)}_{7\tilde{g},\tilde{g}} \langle s\gamma|\mathcal{O}_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}} \\ &+ K^{(0)\prime}_{7b,\tilde{g}} \langle s\gamma|\mathcal{O}'_{7b,\tilde{g}}|b\rangle_{\text{tree}} + K^{(0)\prime}_{7\tilde{g},\tilde{g}} \langle s\gamma|\mathcal{O}'_{7\tilde{g},\tilde{g}}|b\rangle_{\text{tree}} . \end{aligned} \tag{67}$$

The first two terms on the r.h.s. of (67) correspond to a left handed s quark and the last two terms to a right handed s quark. The various coefficients $K^{(0)}$ read (retaining terms of order ϵ^1)

$$\begin{aligned} K^{(0)}_{7b,\tilde{g}} =& \sum_{k=1}^{6} \frac{C_F \Gamma^{kb}_{DL}\Gamma^{ks*}_{DL} Q_d}{16\pi^2 m^2_{\tilde{g}} \left(x_{d_k}-1\right)^4} \Bigg\{ \frac{1}{12}\left(-x^3_{d_k} + 6x^2_{d_k} - 3x_{d_k} - 2\right) - \frac{1}{2} x_{d_k} \ln(x_{d_k}) \\ &+\epsilon \Bigg[-\frac{(x_{d_k}-1)}{72}\left(5x^2_{d_k} - 49x_{d_k} - 22 + 6\left(x^2_{d_k} - 5x_{d_k} - 2\right) L_\mu\right) \\ &+\frac{1}{12} x_{d_k} \ln(x_{d_k})\left(x^2_{d_k} - 6x_{d_k} - 6 - 6L_\mu\right) + \frac{1}{4} x_{d_k} \ln^2(x_{d_k}) \Bigg] \Bigg\} , \end{aligned} \tag{68}$$

$$\begin{aligned} K^{(0)}_{7\tilde{g},\tilde{g}} =& \sum_{k=1}^{6} \frac{C_F \Gamma^{kb}_{DR}\Gamma^{ks*}_{DL} Q_d}{16\pi^2 m_{\tilde{g}} \left(x_{d_k}-1\right)^3} \Bigg\{ \frac{1}{2}\left(x^2_{d_k} - 1\right) - x_{d_k} \ln(x_{d_k}) + \epsilon \Bigg[\frac{1}{4}\left(x^2_{d_k}-1\right)\left(3 + 2L_\mu\right) \\ &-\frac{1}{2} x_{d_k} \ln(x_{d_k})\left(x_{d_k} + 2 + 2L_\mu\right) + \frac{1}{2} x_{d_k} \ln^2(x_{d_k}) \Bigg] \Bigg\} . \end{aligned} \tag{69}$$

The corresponding primed coefficients $K^{(0)\prime}_{7b,\tilde{g}}$ and $K^{(0)\prime}_{7\tilde{g},\tilde{g}}$ can be obtained by interchanging $L \leftrightarrow R$ in the unprimed versions. Note that we explicitly kept terms of order ϵ^1. For the individual equations (23), (29) and (30) even the ϵ^2 contributions of these coefficients would be required. In the matching equation (32), they drop out, however.

The decay amplitude $\mathcal{A}(b \to sg)$ in the full theory to leading order in QCD (corresponding to the diagrams with one-loop gluino/squark exchange) can be written in complete analogy to (67). The corresponding coefficients read

$$
\begin{aligned}
K^{(0)}_{8b,\tilde g} =& \sum_{k=1}^{6} \frac{\Gamma^{kb}_{DL}\Gamma^{ks*}_{DL}}{16\pi^2 m^2_{\tilde g}\left(x_{d_k}-1\right)^4} \left\{ \left(C_F - \frac{1}{2}C_A\right) \left[\frac{1}{12}\left(-x^3_{d_k}+6x^2_{d_k}-3x_{d_k}-2\right) - \frac{1}{2}x_{d_k}\ln(x_{d_k}) \right.\right. \\
&+\epsilon \left(\frac{1}{12}x_{d_k}\ln(x_{d_k})\left(-6L_\mu + x^2_{d_k} - 6x_{d_k} - 6\right) + \frac{1}{4}x_{d_k}\ln^2(x_{d_k}) \right. \\
&\left.\left. -\frac{1}{72}\left(x_{d_k}-1\right)\left(6\left(x^2_{d_k}-5x_{d_k}-2\right)L_\mu + 5x^2_{d_k} - 49x_{d_k} - 22\right)\right)\right] \\
&+\frac{1}{2}C_A \left[-\frac{1}{2}x^2_{d_k}\ln(x_{d_k}) + \frac{1}{12}\left(2x^3_{d_k}+3x^2_{d_k}-6x_{d_k}+1\right) \right. \\
&+\epsilon\left(-\frac{1}{12}x^2_{d_k}\ln(x_{d_k})\left(6L_\mu + 2x_{d_k} + 9\right) + \frac{1}{4}x^2_{d_k}\ln^2(x_{d_k}) \right. \\
&\left.\left.\left. +\frac{1}{72}\left(x_{d_k}-1\right)\left(6\left(2x^2_{d_k}+5x_{d_k}-1\right)L_\mu + 22x^2_{d_k} + 49x_{d_k} - 5\right)\right)\right]\right\},
\end{aligned}
\tag{70}
$$

$$
\begin{aligned}
K^{(0)}_{8\tilde g,\tilde g} =& \sum_{k=1}^{6} \frac{\Gamma^{kb}_{DR}\Gamma^{ks*}_{DL}}{16\pi^2 m_{\tilde g}\left(x_{d_k}-1\right)^3} \left\{ \left(C_F - \frac{1}{2}C_A\right) \left[\frac{1}{2}\left(x^2_{d_k}-1\right) - x_{d_k}\ln(x_{d_k}) \right.\right. \\
&\left. +\epsilon \left(\frac{1}{4}\left(x^2_{d_k}-1\right)\left(2L_\mu+3\right) - \frac{1}{2}x_{d_k}\ln(x_{d_k})\left(2L_\mu + x_{d_k} + 2\right) + \frac{1}{2}x_{d_k}\ln^2(x_{d_k})\right)\right] \\
&+\frac{1}{2}C_A \left[-x^2_{d_k}\ln(x_{d_k}) + \frac{1}{2}\left(3x^2_{d_k}-4x_{d_k}+1\right) \right. \\
&+\epsilon\left(\frac{1}{2}x^2_{d_k}\ln^2(x_{d_k}) - \frac{1}{2}x^2_{d_k}\ln(x_{d_k})\left(2L_\mu+3\right) \right. \\
&\left.\left.\left. +\frac{1}{4}\left(x_{d_k}-1\right)\left(\left(6x_{d_k}-2\right)L_\mu + 7x_{d_k} - 1\right)\right)\right]\right\}.
\end{aligned}
\tag{71}
$$

The corresponding primed coefficients $K^{(0)'}_{8b,\tilde g}$ and $K^{(0)'}_{8\tilde g,\tilde g}$ can again be obtained by interchanging $L \leftrightarrow R$ in the unprimed versions.

B Quark-field and quark-mass renormalization in the on-shell scheme

We start with the free (bare) quark Lagrangian for the down-type quarks

$$
\mathcal{L} = i\,\bar\psi^0_{i,L}\,\gamma^\mu\partial_\mu\delta_{ij}\psi^0_{j,L} + i\,\bar\psi^0_{i,R}\,\gamma^\mu\partial_\mu\delta_{ij}\psi^0_{j,R} - \bar\psi^0_{i,R}m^0_i\psi^0_{i,L} - \bar\psi^0_{i,L}(m^0_i)^*\psi^0_{i,R}\,, \tag{72}
$$

and express the bare quantities (marked with the superscript (0)) in terms of renormalized ones:

$$
\begin{aligned}
&\psi^0_{i,L} = \left(\delta_{ij} + \frac{1}{2}\delta Z^L_{ij}\right)\psi_{j,L}\,, \qquad \psi^0_{i,R} = \left(\delta_{ij} + \frac{1}{2}\delta Z^R_{ij}\right)\psi_{j,R}\,, \\
&m^0_i = m_i - \delta m_i\,.
\end{aligned}
\tag{73}
$$

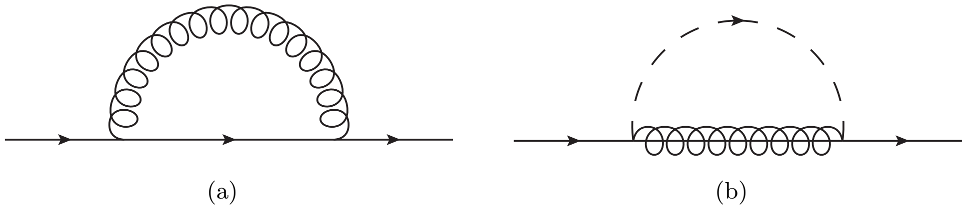

Figure 5: Contributions to the bare quark self-energies Σ_{ji}.

For the renormalized quark self-energies Σ^{ren}_{ji} we get

$$\begin{aligned}\Sigma^{ren}_{ji} =& \Sigma_{ji} - \frac{1}{2}(\delta Z^L + \delta Z^{L\dagger})_{ji}\, \not{p}\, P_L - \frac{1}{2}(\delta Z^R + \delta Z^{R\dagger})_{ji}\, \not{p}\, P_R \\ &+ \frac{1}{2}(\delta Z^{R\dagger})_{ji}\, m_i\, P_L + \frac{1}{2}(\delta Z^{L\dagger})_{ji}\, m_i\, P_R + \frac{1}{2}\, m_j\, \delta Z^L_{ji}\, P_L + \frac{1}{2}\, m_j\, \delta Z^R_{ji}\, P_R \\ &- \delta m_i\, \delta_{ji}\, P_L - \delta m_i^*\, \delta_{ji}\, P_R\,,\end{aligned} \tag{74}$$

where Σ_{ji} denotes the bare quark self-energies which can be decomposed as

$$\Sigma_{ji} = \not{p}\, P_L\, \Sigma^L_{ji}(p^2) + \not{p}\, P_R\, \Sigma^R_{ji}(p^2) + P_L\, A^L_{ji}(p^2) + P_R\, A^R_{ji}(p^2)\,. \tag{75}$$

We note that for $j \neq i$ the bare self-energies at order α_s^1 receive only contributions from diagram (b) in Figure 5 with a squark and a gluino in the loop, while in the diagonal case $j = i$ also diagram (a) with an internal quark and a gluon contributes.

B.1 Off-diagonal case

For $j \neq i$ there are only contributions from the gluino-squark loops shown in Figure 5 (b). Because of this, we supplement the renormalization constants with the label b. The on-shell renormalization conditions read

$$\begin{aligned}\widetilde{\mathrm{Re}}\, \Sigma^{ren}_{ji}\, u(p_i) \stackrel{!}{=} 0\Big|_{p_i^2=m_i^2}, \\ \widetilde{\mathrm{Re}}\, \bar{u}(p_j)\Sigma^{ren}_{ji} \stackrel{!}{=} 0\Big|_{p_j^2=m_j^2}\ ,\end{aligned} \tag{76}$$

where the symbol $\widetilde{\mathrm{Re}}$ stands for taking the dispersive part. From these conditions, the wave function renormalization constants can be fixed uniquely. The generic formulas for the hermitian and antihermitian parts of $\delta Z^{b,L}_{ji}$ read (written in terms of Σ^L_{ji}, Σ^R_{ji}, A^L_{ji}, and A^R_{ji})

$$\begin{aligned}\delta Z^{b,L,H}_{ji} =& \frac{1}{m_i^2 - m_j^2} \widetilde{\mathrm{Re}}\left\{ m_i[A^R_{ji}(m_i^2) - A^R_{ji}(m_j^2)] + m_j[A^L_{ji}(m_i^2) - A^L_{ji}(m_j^2)] \right. \\ & \left. + m_i m_j[\Sigma^R_{ji}(m_i^2) - \Sigma^R_{ji}(m_j^2)] + m_i^2 \Sigma^L_{ji}(m_i^2) - m_j^2 \Sigma^L_{ji}(m_j^2)\right\}, \\ \delta Z^{b,L,AH}_{ji} =& \frac{1}{m_i^2 - m_j^2} \widetilde{\mathrm{Re}}\left\{ m_i[A^R_{ji}(m_j^2) + A^R_{ji}(m_i^2)] + m_j[A^L_{ji}(m_i^2) + A^L_{ji}(m_j^2)] \right. \\ & \left. + m_i m_j[\Sigma^R_{ji}(m_i^2) + \Sigma^R_{ji}(m_j^2)] + m_i^2 \Sigma^L_{ji}(m_i^2) + m_j^2 \Sigma^L_{ji}(m_j^2)\right\}\ .\end{aligned} \tag{77}$$

The right handed versions can be obtained by replacing all labels $R \leftrightarrow L$. Due to the unitarity of the Γ matrices, the off-diagonal renormalization constants turn out to be finite for $d = 4$.

They are needed to order ϵ^0 and up to linear order in the quark masses m_i and m_j. Explicitly, we find

$$\begin{aligned}
\delta Z_{ji}^{b,L,AH} =& \frac{\alpha_s}{4\pi}\frac{C_F}{m_i^2-m_j^2}\sum_{k=1}^{6}\Bigg\{(\Gamma_{DL}^{ki}\Gamma_{DR}^{kj*}m_j+\Gamma_{DL}^{kj*}\Gamma_{DR}^{ki}m_i)\left(\frac{m_i^2+m_j^2}{m_{\tilde g}}f_1(x_{d_k})-4m_{\tilde g}f_2(x_{d_k})\right)\\
&+\left(\Gamma_{DL}^{ki}\Gamma_{DL}^{kj*}\left(m_i^2+m_j^2\right)+2\Gamma_{DR}^{ki}\Gamma_{DR}^{kj*}m_im_j\right)f_3(x_{d_k})\Bigg\}\,,
\end{aligned} \tag{78}$$

$$\delta Z_{ji}^{b,L,H} = \frac{\alpha_s}{4\pi}C_F\sum_{k=1}^{6}\left(\frac{1}{m_{\tilde g}}(\Gamma_{DL}^{kj*}\Gamma_{DR}^{ki}m_i+\Gamma_{DL}^{ki}\Gamma_{DR}^{kj*}m_j)f_1(x_{d_k})+\Gamma_{DL}^{ki}\Gamma_{DL}^{kj*}f_3(x_{d_k})\right)\,, \tag{79}$$

where the functions $f_i(x)$ read

$$\begin{aligned}
f_1(x) &= \frac{x^2-2x\ln(x)-1}{(x-1)^3}\,,\\
f_2(x) &= \frac{x\ln(x)}{x-1}-1\,,\\
f_3(x) &= \frac{2x(x-2)\ln(x)-x^2+4x-3}{2(x-1)^2}\,.
\end{aligned} \tag{80}$$

B.2 Diagonal case

For $j=i$ the on-shell renormalization conditions read

$$\begin{aligned}
&\widetilde{\mathrm{Re}}\,\Sigma_{ii}^{ren}\,u(p_i)\stackrel{!}{=}0\Big|_{p_i^2=m_i^2},\\
&\widetilde{\mathrm{Re}}\,\bar u(p_i)\Sigma_{ii}^{ren}\stackrel{!}{=}0\Big|_{p_i^2=m_i^2},\\
&\widetilde{\mathrm{Re}}\,\frac{\not p+m_i}{p^2-m_i^2}\,\Sigma_{ii}^{ren}\,u(p_i)\stackrel{!}{=}0\,,\\
&\widetilde{\mathrm{Re}}\,\bar u(p_i)\,\Sigma_{ii}^{ren}\,\frac{\not p+m_i}{p^2-m_i^2}\stackrel{!}{=}0\,.
\end{aligned} \tag{81}$$

We note that these conditions do not uniquely fix the renormalization constants. We are therefore free to choose the antihermitian parts $\delta Z_{ii}^{L,AH}$ and $\delta Z_{ii}^{R,AH}$ to vanish:

$$\delta Z_{ii}^{L,AH}=0\,,\qquad \delta Z_{ii}^{R,AH}=0\,. \tag{82}$$

This is the only choice where the δm_i do not depend on the first derivative of Σ_{ji}^L, Σ_{ji}^R, A_{ji}^L, A_{ji}^R with respect to p^2; in other words, the δm_i are only related to the pole position of the renormalized propagators, as it should be. With this choice, the other renormalization constants are uniquely fixed by the renormalization conditions (81).

The results for $\delta Z_{ji}^{L,H}$ ($\delta Z_{ji}^{R,H}$ is obtained by exchanging $R\leftrightarrow L$) and for the mass shifts δm_i, again written in terms of Σ_{ji}^L, Σ_{ji}^R, A_{ji}^L ,A_{ji}^R and their derivatives with respect to p^2 (denoted by a dot) read

$$\begin{aligned}
\delta m_i &= \widetilde{\mathrm{Re}}\left[\frac{m_i}{2}\left(\Sigma_{ii}^L(m_i^2)+\Sigma_{ii}^R(m_i^2)\right)+A_{ii}^L(m_i^2)\right],\\
\delta Z_{ii}^{L,H} &= \widetilde{\mathrm{Re}}\,\Sigma_{ii}^L(m_i^2)+m_i\,\widetilde{\mathrm{Re}}\left[m_i\,\dot\Sigma_{ii}^L(m_i^2)+m_i\,\dot\Sigma_{ii}^R(m_i^2)+\dot A_{ii}^L(m_i^2)+\dot A_{ii}^R(m_i^2)\right].
\end{aligned} \tag{83}$$

These quantities are again needed up to order ϵ^0 and up to linear order in the quark mass m_i.

As already mentioned, the diagonal quantities Σ^L_{ii}, Σ^R_{ii}, A^L_{ii} and A^R_{ii} get contributions both from squark-gluino loops and from quark-gluon loops. We therefore write $\delta Z^{L,H}_{ii}$ and δm_i as a sum of two contributions:

$$\delta Z^{L,H}_{ii} = \delta Z^{a,L,H}_{ii} + \delta Z^{b,L,H}_{ii} \,, \qquad \delta m_i = \delta m_i^a + \delta m_i^b \,, \tag{84}$$

where the first and second term on the corresponding r.h.s. result from the quark-gluon and the squark-gluino loops, respectively. Note that $\delta Z^{a,L,H}_{ii}$, $\delta Z^{b,L,H}_{ii}$, δm_i^a and δm_i^b contain $1/\epsilon$ poles. Explicitly, we get (using the Feynman gauge for the gluon propagator and $L_\mu = \ln(\mu^2/m^2_{\tilde{g}})$)

$$\begin{aligned}
\delta Z^{a,L,H}_{bb} =& \delta Z^{a,R,H}_{bb} = -\frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} + \frac{2}{\epsilon_{\rm ir}} + 4 + 3\,\ln(\mu^2/m_b^2) \right) , \\
\delta Z^{a,L,H}_{ss} =& \delta Z^{a,R,H}_{ss} = -\frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} - \frac{1}{\epsilon_{\rm ir}} \right) , \\
\delta Z^{b,L,H}_{ii} =& \frac{\alpha_s}{4\pi} C_F \sum_{k=1}^{6} \left(\frac{m_i}{m_{\tilde{g}}} (\Gamma^{ki*}_{DL}\Gamma^{ki}_{DR} + \Gamma^{ki}_{DL}\Gamma^{ki*}_{DR}) f_1(x_{d_k}) + \Gamma^{ki}_{DL}\Gamma^{ki*}_{DL} f_3(x_{d_k}) \right) \\
& - \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} + L_\mu \right) .
\end{aligned} \tag{85}$$

$$\begin{aligned}
\delta m_i^a =& \frac{\alpha_s}{4\pi} C_F\, m_i \left(\frac{3}{\epsilon} + 4 + 3\,\ln(\mu^2/m_i^2) \right) , \\
\delta m_i^b =& \frac{1}{2}\frac{\alpha_s}{4\pi}\, C_F \sum_{k=1}^{6} \left(m_i (\Gamma^{ki}_{DL}\Gamma^{ki*}_{DL} + \Gamma^{ki}_{DR}\Gamma^{ki*}_{DR})\, f_3(x_{d_k}) - 4\, m_{\tilde{g}}\, \Gamma^{ki}_{DL}\Gamma^{ki*}_{DR}\, f_2(x_{d_k}) \right) \\
& - \frac{\alpha_s}{4\pi}\, C_F\, m_i \left(\frac{1}{\epsilon} + L_\mu \right) .
\end{aligned} \tag{86}$$

In (85) the symbol $\epsilon_{\rm ir}$ has been introduced in order to mark the infrared poles and distinguish them from the ultraviolet ones.

C Squark-field and squark-mass renormalization in the on-shell scheme

We start with the free (bare) Lagrangian for the down-type squarks

$$\mathcal{L} = -\tilde{\phi}_k^{0\,\dagger} \delta_{kk'} \Box \tilde{\phi}^0_{k'} - (m^0_{\tilde{d}_k})^2 \tilde{\phi}_k^{0\,\dagger} \tilde{\phi}^0_k , \tag{87}$$

and express the bare quantities (marked with the superscript "0") in terms of renormalized ones:

$$\begin{aligned}
\tilde{\phi}^0_k &= \left(\delta_{kk'} + \frac{1}{2} \delta \tilde{Z}_{kk'} \right) \tilde{\phi}_{k'} \,, \\
(m^0_{\tilde{d}_k})^2 &= (m_{\tilde{d}_k})^2 - \delta m^2_{\tilde{d}_k} \,.
\end{aligned} \tag{88}$$

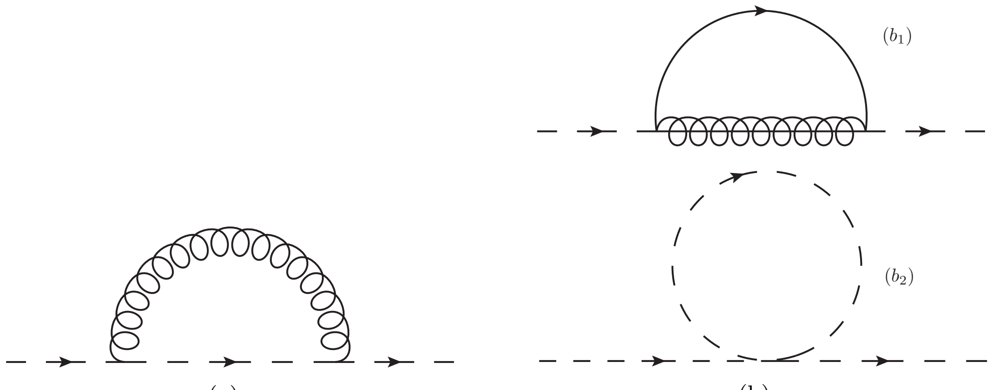

Figure 6: Contributions to the bare squark self-energies $\tilde{\Sigma}_{k'k}$.

For the renormalized squark self-energies $\tilde{\Sigma}^{ren}_{k'k}$ we get

$$\tilde{\Sigma}^{ren}_{k'k} = \tilde{\Sigma}_{k'k} - \frac{1}{2}\left(\delta\tilde{Z} + \delta\tilde{Z}^\dagger\right)_{k'k} p^2 + \frac{1}{2}(\delta\tilde{Z}^\dagger)_{k'k} m^2_{\tilde{d}_k} + \frac{1}{2} m^2_{\tilde{d}_{k'}} (\delta\tilde{Z})_{k'k} - \delta_{k'k}\, \delta m^2_{\tilde{d}_k}\,, \tag{89}$$

where $\tilde{\Sigma}_{k'k}$ denotes the bare squark self-energies. We note that for $k' \neq k$ the bare self-energies at order α_s^1 receive contributions from diagram (b_1) (see Figure 6) with a quark and a gluino in the loop and from diagram (b_2) with a squark tadpole. In the diagonal case $k' = k$ diagram (a) with an internal squark and a gluon also contributes.

C.1 Off-diagonal case

For $k' \neq k$ the on-shell renormalization conditions read

$$\widetilde{\mathrm{Re}}\, \Sigma^{ren}_{k'k}(p^2 = m^2_{\tilde{d}_k}) \stackrel{!}{=} 0\,, \qquad \widetilde{\mathrm{Re}}\, \Sigma^{ren}_{k'k}(p^2 = m^2_{\tilde{d}_{k'}}) \stackrel{!}{=} 0\,, \tag{90}$$

where the symbol $\widetilde{\mathrm{Re}}$ stands for taking the dispersive part. From these conditions and from (89) we get

$$\begin{aligned} \delta\tilde{Z}_{k'k} &= \frac{2}{m^2_{\tilde{d}_k} - m^2_{\tilde{d}_{k'}}}\, \widetilde{\mathrm{Re}}\, \tilde{\Sigma}_{k'k}(m^2_{\tilde{d}_k})\,, \\ (\delta\tilde{Z}^\dagger)_{k'k} &= \frac{2}{m^2_{\tilde{d}_{k'}} - m^2_{\tilde{d}_k}}\, \widetilde{\mathrm{Re}}\, \tilde{\Sigma}_{k'k}(m^2_{\tilde{d}_{k'}})\,. \end{aligned} \tag{91}$$

When looking at the explicit result for the off-diagonal bare self-energies $\tilde{\Sigma}_{k'k}$ below, one finds that these two equations are compatible with each other. Therefore only one of them is needed.

The off-diagonal bare self-energies $\tilde{\Sigma}_{k'k}(p^2)$ can be decomposed as

$$\tilde{\Sigma}_{k'k}(p^2) = \tilde{\Sigma}^{(b_1)}_{k'k}(p^2) + \tilde{\Sigma}^{(b_2)}_{k'k}\,, \tag{92}$$

where $\tilde{\Sigma}^{(b_1)}_{k'k}(p^2)$ and $\tilde{\Sigma}^{(b_2)}_{k'k}$ correspond to diagrams (b_1) and (b_2) in Figure 6. As indicated in the notation, $\tilde{\Sigma}^{(b_2)}_{k'k}$ is independent of the external momentum. Explicitly, one obtains for

$\tilde{\Sigma}^{(b_1)}_{k'k}(p^2)$ (retaining only linear terms in the down-type quark mass m_d)

$$\begin{aligned}\tilde{\Sigma}^{(b_1)}_{k'k}(p^2) = &-\frac{\alpha_s}{\pi}C_F\left[\sum_d\left(m_{\tilde{g}}\,m_d\,(\Gamma^{kd*}_{DR}\Gamma^{k'd}_{DL}+\Gamma^{kd*}_{DL}\Gamma^{k'd}_{DR})\right)\right]\\ &\times\left[\frac{1}{\epsilon}+2+L_\mu+\left(\frac{m^2_{\tilde{g}}}{p^2}-1\right)\ln\left(1-\frac{p^2}{m^2_{\tilde{g}}}-i\delta\right)\right]\\ &-\frac{\alpha_s}{2\pi}C_F\,\delta_{kk'}\left[\frac{1}{\epsilon}(p^2-2m^2_{\tilde{g}})+2p^2-3m^2_{\tilde{g}}+(p^2-2m^2_{\tilde{g}})L_\mu\right.\\ &\left.+\left(2m^2_{\tilde{g}}-p^2-\frac{m^4_{\tilde{g}}}{p^2}\right)\ln\left(1-\frac{p^2}{m^2_{\tilde{g}}}-i\delta\right)\right],\end{aligned}\tag{93}$$

while $\tilde{\Sigma}^{(b_2)}_{k'k}$ reads (using $X^{kk'}_D=\Gamma^{ki}_{DR}\Gamma^{k'i*}_{DR}-\Gamma^{ki}_{DL}\Gamma^{k'i*}_{DL}$ with i summed over $i=1,2,3$)

$$\tilde{\Sigma}^{(b_2)}_{k'k} = -\frac{\alpha_s}{3\pi}X^{k'l}_D\,m^2_{\tilde{d}_l}\,X^{lk}_D\left[\frac{1}{\epsilon}+1+\ln\frac{\mu^2}{m^2_{\tilde{d}_l}}\right].\tag{94}$$

These expressions for $\tilde{\Sigma}^{(b_1)}_{k'k}$ and $\tilde{\Sigma}^{(b_2)}_{k'k}$ hold for both the off-diagonal and the diagonal case. The explicit result for $\delta\tilde{Z}_{k'k}$ reads (for $k'\neq k$, the label b indicates that there are only contributions from class b))

$$\begin{aligned}\delta\tilde{Z}^b_{k'k} = &-\frac{\alpha_s}{\pi}\,\frac{2}{x_{d_k}-x_{d_{k'}}}\left[C_F\left[\sum_d\left(\frac{m_d}{m_{\tilde{g}}}(\Gamma^{kd*}_{DR}\Gamma^{k'd}_{DL}+\Gamma^{kd*}_{DL}\Gamma^{k'd}_{DR})\right)\right]\right.\\ &\times\left(\frac{1}{\epsilon}+2+L_\mu-\frac{x_{d_k}-1}{x_{d_k}}\ln|1-x_{d_k}|\right)\\ &\left.+\frac{1}{3}X^{k'l}_D\,x_{d_l}\,X^{lk}_D\left(\frac{1}{\epsilon}+1+\ln\frac{\mu^2}{m^2_{\tilde{d}_l}}\right)\right].\end{aligned}\tag{95}$$

The hermitian and antihermitian parts are:

$$\begin{aligned}\delta\tilde{Z}^{b,H}_{k'k} = &-\frac{\alpha_s}{\pi}\,C_F\,\frac{2}{x_{d_k}-x_{d_{k'}}}\left[\sum_d\left(\frac{m_d}{m_{\tilde{g}}}(\Gamma^{kd*}_{DR}\Gamma^{k'd}_{DL}+\Gamma^{kd*}_{DL}\Gamma^{k'd}_{DR})\right)\right]\\ &\times\left(\frac{x_{d_{k'}}-1}{2x_{d_{k'}}}\ln|1-x_{d_{k'}}|-\frac{x_{d_k}-1}{2x_{d_k}}\ln|1-x_{d_k}|\right),\end{aligned}\tag{96}$$

$$\begin{aligned}\delta\tilde{Z}^{b,AH}_{k'k} = &-\frac{\alpha_s}{\pi}\,\frac{2}{x_{d_k}-x_{d_{k'}}}\left\{C_F\left[\sum_d\left(\frac{m_d}{m_{\tilde{g}}}(\Gamma^{kd*}_{DR}\Gamma^{k'd}_{DL}+\Gamma^{kd*}_{DL}\Gamma^{k'd}_{DR})\right)\right]\right.\\ &\times\left(\frac{1}{\epsilon}+2+L_\mu-\frac{x_{d_{k'}}-1}{2x_{d_{k'}}}\ln|1-x_{d_{k'}}|-\frac{x_{d_k}-1}{2x_{d_k}}\ln|1-x_{d_k}|\right)\\ &\left.+\frac{1}{3}X^{k'l}_D\,x_{d_l}\,X^{lk}_D\left(\frac{1}{\epsilon}+1+\ln\frac{\mu^2}{m^2_{\tilde{d}_l}}\right)\right\},\end{aligned}\tag{97}$$

where we used the shorthand notation $x_{d_k}=m^2_{\tilde{d}_k}/m^2_{\tilde{g}}$. The result for $\delta\tilde{Z}_{kk}$ is given in the following subsection.

C.2 Diagonal case

For $k' = k$ the on-shell renormalization conditions read

$$\begin{aligned} \widetilde{\mathrm{Re}}\,\Sigma^{ren}_{kk}(p^2) &\stackrel{!}{=} 0, \ \text{ for } p^2 \to m^2_{\tilde{d}_k}, \\ \frac{1}{p^2 - m^2_{\tilde{d}_k}}\,\widetilde{\mathrm{Re}}\,\Sigma^{ren}_{kk}(p^2) &\stackrel{!}{=} 0, \ \text{ for } p^2 \to m^2_{\tilde{d}_k}. \end{aligned} \tag{98}$$

These conditions fix $\delta m^2_{\tilde{d}_k}$ to be

$$\delta m^2_{\tilde{d}_k} = \widetilde{\mathrm{Re}}\,\tilde{\Sigma}_{kk}(m^2_{\tilde{d}_k})\,. \tag{99}$$

However, they only fix the hermitian part (real part) of $\delta\tilde{Z}_{kk}$. We choose the antihermitian part (imaginary part) to vanish and get

$$\delta\tilde{Z}_{kk} = \left.\frac{\partial\widetilde{\mathrm{Re}}\,\tilde{\Sigma}_{kk}(p^2)}{\partial p^2}\right|_{p^2\to m^2_{\tilde{d}_k}}\,. \tag{100}$$

As already mentioned, the diagonal bare squark self-energies $\tilde{\Sigma}_{kk}$ get contributions from diagrams (a) and diagram (b) in Figure 6. It turns out to be convenient to decompose $\delta m^2_{\tilde{d}_k}$ and $\delta\tilde{Z}_{kk}$ accordingly:

$$\delta\tilde{Z}_{kk} = \delta\tilde{Z}^a_{kk} + \delta\tilde{Z}^b_{kk}\,, \qquad \delta m^2_{\tilde{d}_k} = \delta m^{2,a}_{\tilde{d}_k} + \delta m^{2,b}_{\tilde{d}_k}\,. \tag{101}$$

Explicitly, we find

$$\delta m^{2,a}_{\tilde{d}_k} = \frac{\alpha_s}{4\pi} C_F\, m^2_{\tilde{d}_k}\left(\frac{3}{\epsilon} + 7 + 3\,\ln\frac{\mu^2}{m^2_{\tilde{d}_k}}\right), \tag{102}$$

$$\begin{aligned} \delta m^{2,b}_{\tilde{d}_k} = &-\frac{\alpha_s}{\pi} C_F \left[\sum_d \left(m_{\tilde{g}}\, m_d\,(\Gamma^{kd*}_{DR}\Gamma^{kd}_{DL} + \Gamma^{kd*}_{DL}\Gamma^{kd}_{DR})\right)\right]\left[\frac{1}{\epsilon} + 2 + L_\mu - \frac{x_{d_k}-1}{x_{d_k}}\ln|1-x_{d_k}|\right] \\ &-\frac{\alpha_s}{2\pi} C_F\, m^2_{\tilde{g}}\left[\frac{1}{\epsilon}(x_{d_k}-2) + 2x_{d_k} - 3 + (x_{d_k}-2)L_\mu - \frac{(x^2_{d_k} - 2x_{d_k} + 1)}{x_{d_k}}\ln|1-x_{d_k}|\right] \\ &-\frac{\alpha_s}{3\pi}\, m^2_{\tilde{g}}\, X^{kl}_D\, x_{d_l}\, X^{lk}_D\left[\frac{1}{\epsilon} + 1 + \ln\frac{\mu^2}{m^2_{\tilde{d}_l}}\right], \end{aligned} \tag{103}$$

$$\delta\tilde{Z}^a_{kk} = \frac{\alpha_s}{4\,\pi} C_F\left[\frac{2}{\epsilon} - \frac{2}{\epsilon_{\mathrm{ir}}}\right], \tag{104}$$

$$\begin{aligned} \delta\tilde{Z}^b_{kk} = &-\frac{\alpha_s}{2\,\pi} C_F\left[\frac{1}{\epsilon} + 2 + L_\mu - \frac{(x^2_{d_k}-1)}{x^2_{d_k}}\ln|1-x_{d_k}| - \frac{(2x_{d_k} - x^2_{d_k} - 1)}{x_{d_k}\,(1-x_{d_k})}\right] \\ &+\frac{\alpha_s}{\pi} C_F\left[\sum_d\left(\frac{m_d}{m_{\tilde{g}}}\,(\Gamma^{*kd}_{DR}\Gamma^{kd}_{DL} + \Gamma^{*kd}_{DL}\Gamma^{kd}_{DR})\right)\right]\frac{(x_{d_k} + \ln|1-x_{d_k}|)}{x^2_{d_k}}\,. \end{aligned} \tag{105}$$

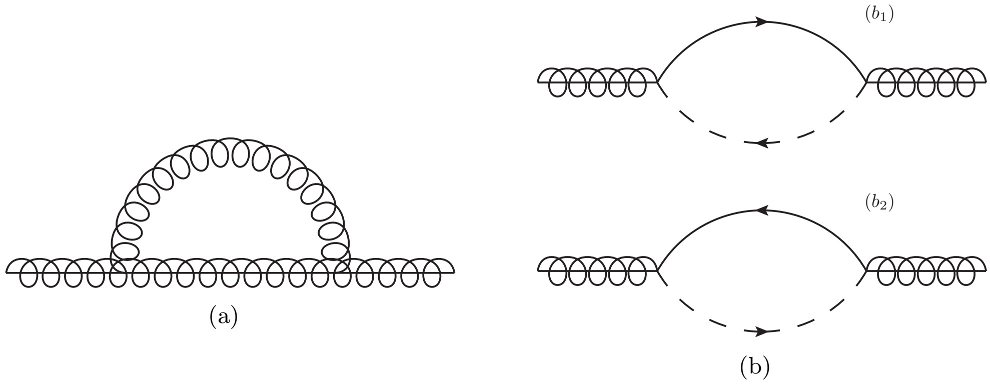

Figure 7: Contributions to the bare gluino self-energy Σ.

D Gluino-field and Gluino-mass renormalization in the on-shell scheme

We start with the free (bare) Lagrangian for a gluino, which is described by a four-component Majorana field $\psi^0_{\tilde g}$:

$$\mathcal{L} = \frac{i}{2}\,\bar\psi^0_{\tilde g,L}\,\gamma^\mu\partial_\mu\psi^0_{\tilde g,L} + \frac{i}{2}\,\bar\psi^0_{\tilde g,R}\,\gamma^\mu\partial_\mu\psi^0_{\tilde g,R} - \frac{1}{2}\,\bar\psi^0_{\tilde g,R} m^0_{\tilde g}\psi^0_{\tilde g,L} - \frac{1}{2}\,\bar\psi^0_{\tilde g,L}(m^0_{\tilde g})^*\psi^0_{\tilde g,R}\,, \tag{106}$$

and express the bare quantities (marked with the superscript (0)) in terms of renormalized ones:

$$\begin{aligned} &\psi^0_{\tilde g,L} = \left(1+\frac{1}{2}\delta\tilde Z^L_3\right)\,\psi_{\tilde g,L}\,, \qquad \psi^0_{\tilde g,R} = \left(1+\frac{1}{2}\delta\tilde Z^R_3\right)\,\psi_{\tilde g,R}\,,\\ &m^0_{\tilde g} = m_{\tilde g} - \delta m_{\tilde g}\,. \end{aligned} \tag{107}$$

Due to the Majorana nature of the gluino, we have $\psi^{(0)}_{\tilde g,R} = \left(\psi^{(0)}_{\tilde g,L}\right)^c$ and therefore $\delta\tilde Z^R_3 = (\delta\tilde Z^L_3)^*$. For the renormalized gluino self-energy Σ^{ren}, we get

$$\Sigma^{ren} = \Sigma - \frac{1}{2}\,(\delta\tilde Z^L_3 + \delta\tilde Z^{L,*}_3)\,\not p + m_{\tilde g}\,\delta\tilde Z^L_3\,P_L + m_{\tilde g}\,\delta\tilde Z^{L,*}_3\,P_R - \delta m_{\tilde g}\,P_L - \delta m^*_{\tilde g}\,P_R\,. \tag{108}$$

The bare gluino self-energy Σ, which gets contributions from the three diagrams in Figure 7, can be decomposed as

$$\Sigma = \not p\,P_L\,\Sigma^L(p^2) + \not p\,P_R\,\Sigma^R(p^2) + P_L\,A^L(p^2) + P_R\,A^R(p^2)\,. \tag{109}$$

The on-shell renormalization conditions are formulated in the same way as for the quarks in the diagonal case $j = i$ in Subsection B.2. The generic formulae for the hermitian and antihermitian part of $\tilde Z^L_3$ and for the mass shift $\delta m_{\tilde g}$ read

$$\begin{aligned} \delta\tilde Z^{L,H}_3 &= \Sigma^L(m^2_{\tilde g}) + 2\,m^2_{\tilde g}\,\dot\Sigma^L(m^2_{\tilde g}) + m_{\tilde g}\,\dot A^L(m^2_{\tilde g}) + m_{\tilde g}\,\dot A^R(m^2_{\tilde g})\,,\\ \delta\tilde Z^{L,AH}_3 &= \frac{1}{2\,m_{\tilde g}}\,\left(A^R(m^2_{\tilde g}) - A^L(m^2_{\tilde g})\right)\,,\\ \delta m_{\tilde g} &= m_{\tilde g}\,\Sigma^L(m^2_{\tilde g}) + \frac{1}{2}\left[A^L(m^2_{\tilde g}) + A^R(m^2_{\tilde g})\right]\,. \end{aligned} \tag{110}$$

We now turn to the explicit results which we decompose as

$$\delta m_{\tilde{g}} = \delta m_{\tilde{g}}^a + \delta m_{\tilde{g}}^b \,, \qquad \delta \tilde{Z}_3^L = \delta \tilde{Z}_3^{a,L} + \delta \tilde{Z}_3^{b,L} \,, \tag{111}$$

according to the contributions shown in Figure 7. We find (using the Feynman gauge for the gluon propagator)

$$\begin{aligned}
\delta \tilde{Z}_3^{a,L,H} &= -\frac{\alpha_s}{4\pi} C_A \frac{1}{\epsilon} + \text{finite}, \\
\delta \tilde{Z}_3^{a,L,AH} &= 0, \\
\delta \tilde{Z}_3^{b,L,H} &= -\frac{\alpha_s}{4\pi} 2\, tr \frac{n_f}{\epsilon} + \text{finite}, \\
\delta \tilde{Z}_3^{b,L,AH} &= \text{finite} \,,
\end{aligned} \tag{112}$$

where 'finite' denotes ultraviolet finite pieces not needed in this work. Numerically, we have $tr = 1/2$, $C_A = 3$, $n_f = 6$. For the mass shift $\delta m_{\tilde{g}}$ the explicit results read in the on-shell scheme

$$\begin{aligned}
\delta m_{\tilde{g}}^a =& \frac{\alpha_s}{4\pi} C_A\, m_{\tilde{g}} \left[\frac{3}{\epsilon} + 3 \ln \frac{\mu^2}{m_{\tilde{g}}^2} + 4 \right] , \\
\delta m_{\tilde{g}}^b =& -\frac{\alpha_s}{4\pi} 2\, tr \frac{n_f}{\epsilon} m_{\tilde{g}} + \frac{\alpha_s}{4\pi} tr\, m_{\tilde{g}} \sum_{k,i,Q} \left(\Gamma_{QL}^{ki} \Gamma_{QL}^{ki*} + \Gamma_{QR}^{ki} \Gamma_{QR}^{ki*} \right) \left[\frac{m_{Q_i}^2}{m_{\tilde{g}}^2} \left(1 + \ln \frac{\mu^2}{m_{Q_i}^2} \right) \right. \\
& \left. - \frac{m_{\tilde{Q}_k}^2}{m_{\tilde{g}}^2} \left(1 + \ln \frac{\mu^2}{m_{\tilde{Q}_k}^2} \right) + \frac{m_{\tilde{Q}_k}^2 - m_{Q_i}^2 - m_{\tilde{g}}^2}{m_{\tilde{g}}^2} \widetilde{\mathrm{Re}} \hat{B}_0(m_{\tilde{g}}^2; m_{\tilde{Q}_k}, m_{Q_i}) \right] \\
& + \frac{\alpha_s}{4\pi} 2\, tr \sum_{k,i,Q} m_{Q_i} \left(\Gamma_{QL}^{ki} \Gamma_{QR}^{ki*} + \Gamma_{QR}^{ki} \Gamma_{QL}^{ki*} \right) \widetilde{\mathrm{Re}} \hat{B}_0(m_{\tilde{g}}^2; m_{\tilde{Q}_k}, m_{Q_i}) \,.
\end{aligned} \tag{113}$$

The indices in the sums within the expression for $\delta m_{\tilde{g}}^b$ run over the following ranges: $k = 1, ..., 6$, $i = 1, 2, 3$, and $Q = D, U$. For $Q = D$ ($Q = U$) the symbols m_{Q_i} and $m_{\tilde{Q}_k}$ denote the masses of the down-type (up-type) quarks and down-type (up-type) squarks, respectively. The function $\hat{B}_0$ appearing in (113) is defined as

$$\hat{B}_0(p^2; m_1, m_2) = -\int_0^1 dx \, \ln \frac{-p^2\, x\,(1-x) + m_1^2\, x + m_2^2\,(1-x) - i\delta}{\mu^2} \,. \tag{114}$$

E Gluon-field renormalization

To fix the Wilson coefficients of the chromomagnetic operators $\mathcal{O}_8$, which are related to the process $b \to sg$, we need the on-shell renormalization constant (LSZ-factor) for the gluon field. The diagrams which define the bare gluon two-point function at order α_s are shown in Figure 8. The corresponding gluon field renormalization constant Z_3, defined through $A_\mu^0 = \sqrt{Z_3}\, A_\mu$ (where A_μ^0 and A_μ denote the bare and renormalized gluon field, respectively), can be written as

$$Z_3 = 1 + \delta Z_3^a + \delta Z_3^b \,, \tag{115}$$

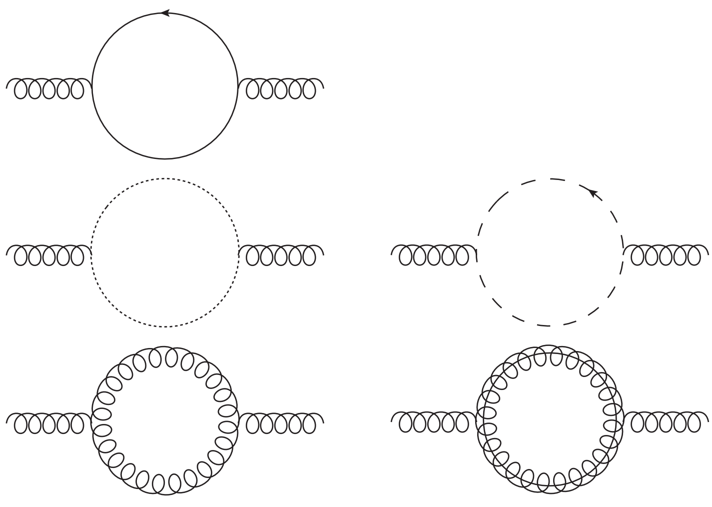

Figure 8: Contributions to the bare gluino two-point function.

according to the contributions shown in Figure 8. As we do not want to reproduce the order α_s correction to the Wilson coefficients C_8 due to *gluons* (which were computed in [1]), we only need δZ_3^b in our work. We obtain

$$\delta Z_3^b = \frac{\alpha_s}{4\pi}\frac{2}{3}\left[\frac{1}{2}\,tr\left(\sum_{k=1}^{6}\ln\frac{m^2_{\tilde{d}_k}}{m^2_{\tilde{g}}} + \sum_{k=1}^{6}\ln\frac{m^2_{\tilde{u}_k}}{m^2_{\tilde{g}}} - 2\,n_f\ln\frac{\mu^2}{m^2_{\tilde{g}}} - \frac{2\,n_f}{\epsilon}\right) - C_A\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2_{\tilde{g}}}\right)\right], \tag{116}$$

where $n_f = 6$.

F Renormalization of $g_{s,Y}$, $\Gamma^{ki}_{DL/R}$ and $g_{s,G}$ in the full theory

F.1 Renormalization of $g_{s,Y}$ and $\Gamma^{ki}_{DL/R}$

We use the squark-quark-gluino-vertex in order to compute the one-loop counterterms for the strong coupling constant $g_{s,Y}$ of Yukawa type and the $\Gamma_{DL/R}$ matrices in the $\overline{\mathrm{MS}}$ scheme. At tree-level, this vertex for a down-type squark of flavor k (color β), a down-type quark of flavor i (color α) and a gluino (color a) reads (see (11))

$$V^{tree}_{ki} = -ig_s\sqrt{2}T^a_{\beta\alpha}\left(\Gamma^{ki}_{DL}P_L - \Gamma^{ki}_{DR}P_R\right). \tag{117}$$

At one-loop order, there are four one-particle irreducible diagrams contributing to the vertex correction. Three of them contain a virtual gluon in the loop, while the remaining one contains a quark-squark-gluino loop. It turns out that the latter contribution is finite (after using the unitarity relations) and thus there is no contribution in the $\overline{\mathrm{MS}}$ scheme. For the vertex correction δV^{irred}_{ki} due to these irreducible diagrams we obtain (retaining only the UV-singularities)

$$\delta V^{irred}_{ki} = \frac{\alpha_s}{4\pi}\frac{2C_A + C_F}{\epsilon}V^{tree}_{ki}. \tag{118}$$

Additionally, there is a counterterm contribution δV_{ki}^{ct} which contains the following sources: renormalization of the quark, squark and gluino fields, as well as the renormalization of the parameters $g_{s,Y}$ and $\Gamma_{DL/R}^{ki}$. In the $\overline{\rm MS}$ scheme the renormalization constants for the various fields can be extracted from the previous sections in this appendix. Splitting them again into class a) and class b) contributions, the non-vanishing ones read for the quarks

$$\begin{aligned} \delta Z^a \equiv \delta Z_{ii}^{a,L} &= \delta Z_{ii}^{a,R} = -\frac{\alpha_s}{4\pi}\, C_F\, \frac{1}{\epsilon}\,, \\ \delta Z^b \equiv \delta Z_{ii}^{b,L} &= \delta Z_{ii}^{b,R} = -\frac{\alpha_s}{4\pi}\, C_F\, \frac{1}{\epsilon}\,, \end{aligned} \tag{119}$$

and for the squarks

$$\begin{aligned} \delta \tilde{Z}^a \equiv \delta \tilde{Z}_{kk}^{a} &= \frac{\alpha_s}{4\pi}\, C_F\, \frac{2}{\epsilon}\,, \\ \delta \tilde{Z}^b \equiv \delta \tilde{Z}_{kk}^{b} &= -\frac{\alpha_s}{4\pi}\, C_F\, \frac{2}{\epsilon}\,, \\ \delta \tilde{Z}_{kk'}^{b,AH} &= -\frac{\alpha_s}{\pi}\, \frac{2}{x_{d_{k'}} - x_{d_k}} \left[C_F \sum_d \left(\frac{m_d}{m_{\tilde{g}}} \left(\Gamma_{DL}^{kd}\Gamma_{DR}^{k'd*} + \Gamma_{DR}^{kd}\Gamma_{DL}^{k'd*}\right)\right) \right. \\ &\quad \left. + \frac{1}{3} X_D^{kl}\, x_{d_l}\, X_D^{lk'} \right] \frac{1}{\epsilon}, \qquad (k \neq k'), \end{aligned} \tag{120}$$

and finally for the gluino ($\tilde{Z}_3 = 1 + \delta\tilde{Z}_3^a + \delta\tilde{Z}_3^b$)

$$\delta\tilde{Z}_3^a = -\frac{\alpha_s}{4\pi}\, C_A\, \frac{1}{\epsilon}\,, \qquad \delta\tilde{Z}_3^b = -\frac{\alpha_s}{4\pi}\, 2\, tr\, n_f\, \frac{1}{\epsilon}. \tag{121}$$

The counterterm contribution to the vertex due to the renormalization of the fields can be written as

$$\begin{aligned} \delta V_{ki}^{ct,fields} =& \frac{1}{2}\left(\delta Z^a + \delta Z^b + \delta\tilde{Z}^a + \delta\tilde{Z}^b + \delta\tilde{Z}_3^a + \delta\tilde{Z}_3^b\right) V_{ki}^{tree} \\ & - \frac{1}{2}\sum_{k'} \delta\tilde{Z}_{kk'}^{b,AH}\, V_{k'i}^{tree}\,. \end{aligned} \tag{122}$$

We now turn to the renormalization of the parameters $g_{s,Y}$ and $\Gamma_{DL/R}$ matrices. For the former we write the connection between the bare- and renormalized version in the form $g_{s,Y}^{bare} = (1 + \delta Z_{g_s,Y}^a + \delta Z_{g_s,Y}^b)\, g_{s,Y}$ while for the latter the corresponding connection reads

$$\Gamma_{DL/R}^{bare,ki} = \sum_{k'} \left(\delta^{kk'} + \delta\Gamma_D^{kk'}\right) \Gamma_{DL/R}^{k'i}\,, \tag{123}$$

where $\delta\Gamma_D^{kk'}$ is an antihermitian 6×6 matrix. This procedure corresponds to a rotation of the squarks only. Therefore the renormalized $\Gamma_{DL/R}$ matrices still describe the transition between the super-CKM basis and the mass eigenstate basis. The counterterm contribution to the vertex due to the renormalization of the parameters then reads

$$\delta V_{ki}^{ct,param} = \delta Z_{g_s,Y} V_{ki}^{tree} + \sum_{k'} \delta\Gamma_D^{kk'}\, V_{k'i}^{tree}\,. \tag{124}$$

Requiring $\delta V_{ki}^{irred} + \delta V_{ki}^{ct,fields} + \delta V_{ki}^{ct,param}$ to be finite, fixes $\delta Z^a_{g_s,Y}$, $\delta Z^b_{g_s,Y}$ and $\delta\Gamma_D^{kk'}$ in the $\overline{\mathrm{MS}}$ scheme to be ($Z_{g_s,Y} = 1 + \delta Z^a_{g_s,Y} + \delta Z^b_{g_s,Y}$)

$$\begin{aligned} \delta Z^a_{g_s,Y} &= -\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\frac{3}{2}(C_A + C_F), \\ \delta Z^b_{g_s,Y} &= \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\left(\frac{3}{2}C_F + tr\, n_f\right), \\ \delta\Gamma_D^{kk'} &= \frac{1}{2}\delta\tilde{Z}^{b,AH}_{kk'}\,, \end{aligned} \tag{125}$$

where $\delta\tilde{Z}^{b,AH}_{kk'}$ is given in (120).

F.2 Renormalization of $g_{s,G}$

We use the squark-squark-gluon-vertex to compute the one-loop renormalization constant for the strong coupling constant $g_{s,G}$ of gauge type in the $\overline{\mathrm{MS}}$ scheme.

At the one-loop order, there are six one-particle irreducible diagrams contributing to the vertex correction. Two of them contain a gluino and a quark in the loop, one consists of a squark loop, while the remaining three feature a virtual gluon. For the vertex correction $\delta\tilde{V}_{kk}^{irred}$ due to these irreducible diagrams we obtain (keeping only the UV-singularities)

$$\delta\tilde{V}_{kk}^{irred} = -\frac{\alpha_s}{4\pi}\left(2\,C_F - C_A\right)\frac{1}{\epsilon}\,V_{kk}^{tree} + \frac{\alpha_s}{4\pi}\,2\,C_F\,\frac{1}{\epsilon}\,V_{kk}^{tree}\,, \tag{126}$$

where the first term on the r.h.s. is due to gluon corrections, while the second one results from gluinos and squarks. Additionally, there is a counterterm contribution $\delta\tilde{V}_{kk}^{ct}$ which contains the following sources: renormalization of the squark and the gluino field, as well as the renormalization of $g_{s,G}$. Again, the renormalization constants for the various fields in the $\overline{\mathrm{MS}}$ scheme can be extracted from the previous sections in this appendix. Unlike for the squark-quark-gluino-vertex, only the diagonal renormalization constants for the squark fields are involved in the squark-squark-gluon-vertex. The gluon-field renormalization constant reads ($Z_3 = 1 + \delta Z_3^a + \delta Z_3^b$)

$$\delta Z_3^a = \frac{\alpha_s}{4\pi}\left[\frac{5}{3}C_A - \frac{4}{3}\,tr\,n_f\right]\frac{1}{\epsilon}\,, \qquad \delta Z_3^b = -\frac{\alpha_s}{4\pi}\left[\frac{2}{3}\,tr\,n_f + \frac{2}{3}C_A\right]\frac{1}{\epsilon}\,, \tag{127}$$

where the first expression stems from gluon, quark and ghost contributions in the corresponding gluon self-energy, while the second one is due to gluino and squark loops. Note that the second term corresponds to the UV-singular part of δZ_3^b in (116). The diagonal renormalization constants for the squark fields are contained in (120).

The counterterm contribution to the vertex due to the renormalization of the fields and of $g_{s,G}$ can be written as ($Z_{g_s,G} = 1 + \delta Z^a_{g_s,G} + \delta Z^b_{g_s,G}$)

$$\delta\tilde{V}_{kk}^{ct} = \left(\delta\tilde{Z}^a + \delta\tilde{Z}^b + \frac{1}{2}\,\delta Z_3^a + \frac{1}{2}\,\delta Z_3^b + \delta Z^a_{g_s,G} + \delta Z^b_{g_s,G}\right)\,\tilde{V}_{kk}^{tree}\,. \tag{128}$$

Requiring $\delta\tilde{V}_{kk}^{irred} + \delta\tilde{V}_{kk}^{ct}$ to be finite, fixes $\delta Z^a_{g_s,G}$ and $\delta Z^b_{g_s,G}$ in the $\overline{\mathrm{MS}}$ scheme to be

$$\begin{aligned} \delta Z^a_{g_s,G} =& \frac{\alpha_s}{4\pi}\left[\frac{2}{3}\,tr\,n_f - \frac{11}{6}C_A\right]\frac{1}{\epsilon}\,, \\ \delta Z^b_{g_s,G} =& \frac{\alpha_s}{4\pi}\left[\frac{1}{3}\,tr\,n_f + \frac{C_A}{3}\right]\frac{1}{\epsilon}\,. \end{aligned} \tag{129}$$

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