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Next-to-Next-to-Leading Electroweak Logarithms for W-Pair Production at LHC

J.H. Kühn [a], F. Metzler [a], A.A. Penin [a,b,c], S. Uccirati [d]

Institut für Theoretische Teilchenphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany

Department of Physics, University Of Alberta, Edmonton, AB T6G 2J1, Canada

Institute for Nuclear Research, Russian Academy of Sciences, 117312 Moscow, Russia

Lehrstuhl für Theoretische Physik II, Universität Würzburg, 97074 Würzburg, Germany

Abstract

We derive the high energy asymptotic of one- and two-loop corrections in the next-to-next-to-leading logarithmic approximation to the differential cross section of W-pair production at the LHC. For large invariant mass of the W-pair the (negative) one-loop terms can reach more than 40%, which are partially compensated by the (positive) two-loop terms of up to 10%.

Key words: Electroweak radiative corrections, LHC

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1 Introduction

With the LHC now starting its operation, the experimental investigation of scattering processes at the TeV scale is within reach. Starting from these energies electroweak corrections are strongly enhanced by Sudakov logarithms of the form $(\alpha_{ew}^n \ln^{2n} s/M_W^2)^n$ [1]. The full evaluation of electroweak one-loop corrections to fermion- or W-pair production is by now a straightforward task. Two-loop corrections, however, can be obtained only in the high energy limit. By employing the evolution equation approach the analysis of the dominant logarithmically enhanced two-loop corrections for four-fermion processes has been pushed successfully from next-to-leading logarithmic (NLL) approximation [2–4] to NNLL [5] and even N^{3}LL approximation [6,7], which accounts for all the two-loop logarithmic terms (for additional work on this topic see e.g. [8–10]). Subsequent analysis performed in the effective theory framework [11] employing the two-loop anomalous dimensions calculated in Refs.[6,7] have confirmed the formentioned result.

In this paper we consider specifically pair production of W-bosons. Previously, the electroweak corrections were studied mainly in the context of the electron-positron annihilation. The one-loop corrections have been evaluated for the W-pair production [12–15] and the W-boson mediated $e^+e^- \to 4f$ processes [16–19]. For high energies the two-loop logarithmically enhanced terms have been obtained up to the NNLL approximation [3,4,20,21]. The one-loop contribution amounts to typically -20% for 1 TeV and -50% for 3 TeV while two-loop terms vary between 2 and 5% for 1 TeV, for 3 TeV they may even rise to 20%. For the W-pair production at the LHC the analysis of the one-loop electroweak logarithms to the NLL approximation is given in [22,23] with the realistic cuts and the effect of gauge boson decay included. Beyond one loop the logarithmic corrections to the partonic cross sections were considered in [24]. In view of the extremely large partonic energies and with the LHC eventually operating at full luminosity (not to speak of the SLHC) invariant masses of the W-pair exceeding 1 TeV and approaching 3 TeV seem within reach. Therefore the evaluation of the enhanced electroweak corrections is of particular interest. Here we present the explicit result for the one- and two-loop corrections to the partonic $q\bar{q} \to W^+W^-$ and hadronic $pp \to W^+W^-$ cross section in high energy limit in the NNLL approximation. Note that the cross section of W-pair hadronic production is a subject of large corrections due to the strong interaction of the initial states. Currently the analysis of QCD corrections is completed to the NLO and NLL approximation (see [25–27] and references therein). The size of the corrections depends strongly on a particular observable and in many cases the available approximation provides a few percent accuracy. As we will see the two-loop electroweak logarithms become essential at this level of precision and have to be included in the theoretical predictions.

Our paper is organized as follows: the partonic processes in Born approximation are introduced in Section 2. In Section 3 the evolution equation approach is outlined for the simplified case of a pure $SU(2)$ spontaneously broken gauge theory. The discussion closely follows Ref. [21]. However in the present paper we derive the explicit result for the one-loop corrections to scattering amplitudes given in Appendix A. The generalization to the $SU(2) \otimes U(1)$ Standard Model is presented in Section 4, which contains a more detailed analysis of the separation of infrared singularities connected with virtual photon emission. The results for the one- and the two-loop corrections to the partonic cross section in NNLL approximation are listed in the Appendix B. In Section 4.2 we present a numerical study of these corrections for $\sqrt{\hat{s}} = 1$ TeV and 3 TeV respectively. Based on these results, the corrections to the transverse and longitudinal W-pair production in proton-proton collisions at 14 TeV are presented in Section 4.3 together with the discussion of the anticipated statistical errors. Section 5 contains a brief summary and conclusions. In Appendix C we present the correction to the two-loop NNLL result for the transverse W-pair production in electron-positron annihilation [21].

2 The partonic process

The partonic processes relevant for the W-pair production at hadron colliders are gluon fusion and quark-antiquark annihilation. The gluon contribution to the total cross section is about 5% [28] and we focus on the process $q\,\bar{q} \to W^+\,W^-$. In the leading order it is described by the diagrams in Fig. 1.

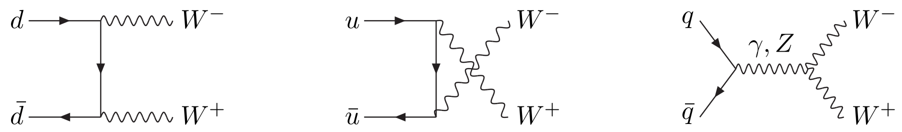

Figure 1: Tree level diagrams contributing to the partonic process

The kinematics at partonic level is defined by:

$$q(p_1, \lambda_+) + \bar{q}(p_2, \lambda_-) \to W^+(k_+, \kappa_+) + W^-(k_-, \kappa_-), \tag{1}$$

where $\lambda_\pm$ and $\kappa_\pm$ are the helicities of the incoming and outgoing particles respectively. For on-shell W-bosons, the matrix element can be then expressed as function of the Mandelstam variables:

$$\hat{s} = (p_1 + p_2)^2, \qquad \hat{t} = (p_1 - k_-)^2, \qquad \hat{u} = (p_1 - k_+)^2, \tag{2}$$

They are related to the scattering angle θ through the relations:

$$\hat{t} = M_W^2 - \frac{\hat{s}}{2}\Big(1 - \beta\,\cos\theta\Big), \qquad \hat{u} = M_W^2 - \frac{\hat{s}}{2}\Big(1 + \beta\,\cos\theta\Big), \qquad \beta^2 = 1 - 4\,\frac{M_W^2}{\hat{s}}. \tag{3}$$

In the high energy limit only final states where the W-bosons have the same polarization are not suppressed by a factor M_W^2/s or higher. In addition, the case where both W's are longitudinal can be reduced by means of the Goldstone equivalence theorem to the production of a pair of charged Goldstone bosons as shown in Fig. 2.

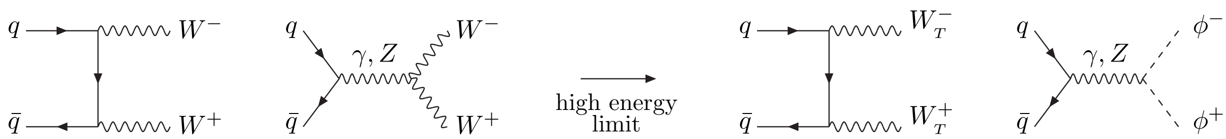

Figure 2: Goldstone equivalence theorem at Born level

3 Massive gauge boson production in $SU(2)$ model

Let us, in a first step, neglect the hypercharge and consider a simplified model with spontaneously broken gauge group $SU(2)$. The model retains the main features of the massive gauge boson sector of the Standard Model. In this case the result can be presented in a simple analytical

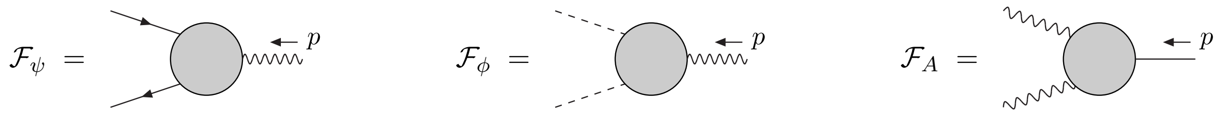

Figure 3: Fermion/scalar scattering in an external singlet vector field and scattering of a gauge boson in an external scalar field. The momentum of external field satisfies $p^2 = s = -Q^2$.

form and constitutes the basis for the further extension to the full electroweak theory. We study the process of gauge boson pair production in fermion-antifermion annihilation at high energy and fixed angle with all kinematical invariants of the same order and far larger than the gauge boson mass M, $|s| \sim |t| \sim |u| \gg M^2$. In this limit the asymptotic energy dependence of the amplitudes is dominated by *Sudakov* logarithms [29,30] and governed by the evolution equations [31–33]. The method of the evolution equations in the context of the electroweak corrections is described in detail for fermion pair production in Ref. [7] and for W-pair production in Ref. [21].

Following Ref. [21] we introduce the functions $\mathcal{Z}_{\psi,\phi,A}$ which describe the asymptotic dependence on the large momentum transfer Q of the scattering amplitude of the spinor (ψ) or scalar (ϕ) field in an external singlet vector field and of the vector boson (A) in an external singlet scalar field, *i.e.* of the respective form factors in the Euclidean region (see Fig. 3). In leading order in M^2/Q^2 these functions are known to satisfy the following linear evolution equation [31–33]

$$\frac{\partial}{\partial \ln Q^2}\mathcal{Z}_i = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\gamma_i(\alpha(x)) + \zeta_i(\alpha(Q^2)) + \xi_i(\alpha(M^2))\right]\mathcal{Z}_i\,, \tag{4}$$

with the solution

$$\mathcal{Z}_i = \exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma_i(\alpha(x')) + \zeta_i(\alpha(x)) + \xi_i(\alpha(M^2))\right]\right\}\,, \tag{5}$$

which satisfies the initial condition $\mathcal{Z}_i\big|_{Q^2=M^2} = 1$. Here the perturbative functions $\gamma_i(\alpha)$ *etc.* are given by the series in the coupling constant $\alpha(\mu^2)$, *e.g.* $\gamma_i(\alpha) = \sum_{n=1}^{\infty}(\alpha/4\pi)^n\gamma_i^{(n)}$. Then the amplitude of the transverse (longitudinal) gauge boson production $\boldsymbol{\mathcal{A}}_T$ ($\boldsymbol{\mathcal{A}}_L$) can be decomposed as follows

$$\boldsymbol{\mathcal{A}}_{T,L} = \alpha(\mu_{T,L})\mathcal{Z}_\psi\mathcal{Z}_{A,\phi}\tilde{\boldsymbol{\mathcal{A}}}_{T,L}\,. \tag{6}$$

where $\tilde{\boldsymbol{\mathcal{A}}}_{T,L}$ is the reduced amplitude and we factor out the Born coupling constant $\alpha(\mu_{T,L})$. The scale dependence of this factor is cancelled by the higher order renormalization group logarithms replacing $\mu_{T,L}$ by a physical scale of the process. In the case of the longitudinal W-pair the proper scale is $\mu_L = \sqrt{s}$ because it describes the interaction of far off-shell intermediate gauge boson with virtuality of order $\sqrt{s}$. For the transverse W-pair production it is $\mu_T = M_W$ corresponding to the coupling of the on-shell W-bosons. Note that in an alternative approach based on the soft-collinear effective theory [24] the normalization scale of the Born coupling constant for the transverse gauge bosons is set to $\sqrt{s}$. This is compensated by an additional β_0 contribution to the anomalous dimension $\zeta_A^{(1)}$ which effectively shift the normalization of the Born coupling constant to M_W, in agreement with our result.

Due to the factorization property of the Sudakov logarithms associated with the *collinear* divergences of the massless theory [35] the reduced amplitude satisfies the simple renormalization group like equation [36–38]

$$\frac{\partial}{\partial \ln Q^2}\tilde{\boldsymbol{\mathcal{A}}}_{T,L} = \boldsymbol{\chi}_{T,L}(\alpha(Q^2))\tilde{\boldsymbol{\mathcal{A}}}_{T,L}\,, \tag{7}$$

where $Q^2 = -s$ and $\boldsymbol{\chi}_{T,L}$ is the soft anomalous dimension matrix acting in the space of the isospin amplitudes. The solution of the above equation is given by the path-ordered exponent

$$\tilde{\boldsymbol{\mathcal{A}}}_{T,L} = \mathrm{Pexp}\left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\boldsymbol{\chi}_{T,L}(\alpha(x))\right]\boldsymbol{\mathcal{A}}_{0\,T,L}(\alpha(M^2))\,, \tag{8}$$

where $\boldsymbol{\mathcal{A}}_{0\,T,L}$ determines the initial conditions for the evolution equation at $Q = M$. By calculating the functions entering the evolution equations order by order in α one gets the logarithmic approximations for the amplitude. By expanding the exponents one gets the one- and two-loop corrections in the following form

$$\begin{aligned}
\boldsymbol{\mathcal{A}}^{(1)}_{T,L} &= \left[\frac{1}{2}\gamma^{(1)}L^2 + \left(\zeta^{(1)}+\xi^{(1)}+\boldsymbol{\chi}^{(1)}_{T,L}\right)L\right]\boldsymbol{\mathcal{A}}^{(0)}_{T,L} + \boldsymbol{\mathcal{A}}^{(1)}_{0,T,L}, \qquad f^{(1)} = f^{(1)}_{\psi} + f^{(1)}_{A,\phi}\,, \qquad f = \gamma,\zeta,\xi,\\
\boldsymbol{\mathcal{A}}^{(2)}_{T,L} &= \left\{\frac{1}{8}\left[\gamma^{(1)}\right]^2L^4 + \frac{1}{2}\gamma^{(1)}\left[\zeta^{(1)}+\xi^{(1)}+\boldsymbol{\chi}^{(1)}_{T,L} - \frac{1}{3}\beta_0\right]L^3 + \frac{1}{2}\left[\gamma^{(2)} + \left(\zeta^{(1)}+\xi^{(1)}+\boldsymbol{\chi}^{(1)}_{T,L}\right)^2\right.\right.\\
&\quad \left.\left. -\beta_0\left(\zeta^{(1)}+\boldsymbol{\chi}^{(1)}_{T,L}\right)\right]L^2\right\}\boldsymbol{\mathcal{A}}^{(0)}_{T,L} + \frac{1}{2}\gamma^{(1)}L^2\boldsymbol{\mathcal{A}}^{(1)}_{0,T,L} + \mathcal{O}(L), \qquad \gamma^{(2)} = \gamma^{(2)}_{\psi} + \gamma^{(2)}_{A,\phi}\,,
\end{aligned} \tag{9}$$

where $L = \ln(Q^2/M^2)$ and β_0 is the one-loop beta function. The anomalous dimensions $\gamma(\alpha)$, $\zeta(\alpha)$ and $\boldsymbol{\chi}(\alpha)$ are mass-independent and can be associated with the infrared divergences of the massless (unbroken) theory. At the same time the functions $\xi_i(\alpha)$ and $\mathcal{A}_{0,T,L}(\alpha)$ do depend on the infrared structure of the model and require the calculation in the spontaneously broken phase. All the perturbative coefficients in Eqs. (9) except $\boldsymbol{\mathcal{A}}^{(1)}_{0,T,L}$ are known [21]. In Ref. [21] the result for the one-loop correction to the cross section [19] has been used to obtain the two-loop NNLL terms. We complete this part of the calculation and present the explicit result for the one-loop corrections to the amplitude in Appendix A. Our result for the cross section agrees with Ref. [19].

The large Yukawa coupling of the third generation quarks to the scalar (Higgs and Goldstone) bosons results in specific logarithmic corrections proportional to m_t^2/M_W^2. This kind of Sudakov logarithms were studied in Ref. [32] and have universal structure for any renormalizable non-gauge theory. The factorization in this case is much simpler than in gauge theories and the logarithmic corrections are completely determined by the ultraviolet field renormalization of the external on-shell lines. Since the Yukawa coupling of the initial light quark states is suppressed the Yukawa enhanced Sudakov logarithms for hadronic production of W-pair are similar to those for W-pair production in electron-positron annihilation [21]. Thus the Yukawa enhanced corrections can be taken into account through the modification of the evolution equations for the corresponding $\mathcal{Z}_\phi$-function. The main complication is that the Yukawa interaction mixes the evolution of the quark and scalar boson form factors and in general does not commute with the $SU(2)$ coupling. Thus the evolution equation has a complicated matrix form:

$$\frac{\partial}{\partial \ln Q^2}\boldsymbol{\mathcal{Z}} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\boldsymbol{\gamma}(\alpha(x)) + \boldsymbol{\zeta}(\alpha(Q^2),\alpha_{Yuk}(Q^2)) + \boldsymbol{\xi}(\alpha(M^2))\right]\boldsymbol{\mathcal{Z}}\,, \tag{10}$$

with the solution

$$\boldsymbol{\mathcal{Z}} = \mathrm{Pexp}\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\boldsymbol{\gamma}(\alpha(x')) + \boldsymbol{\zeta}(\alpha(x),\alpha_{Yuk}(x)) + \boldsymbol{\xi}(\alpha(M^2))\right]\right\}\boldsymbol{\mathcal{Z}}_0\,, \tag{11}$$

where $\boldsymbol{\gamma}^{(1)} = (-3/2)\mathbf{1}$, $\boldsymbol{\xi} = 0$, $\alpha_{Yuk} = M_t^2/(2M_W^2)\,\alpha$, and we introduce the five-component vector

$$\boldsymbol{\mathcal{Z}} = (\mathcal{Z}_\phi,\ \mathcal{Z}_\chi,\ \mathcal{Z}_{b-},\ \mathcal{Z}_{t-},\ \mathcal{Z}_{t+})\,. \tag{12}$$

The subscript $+$ $(-)$ stand for the right (left) quark fields and $\mathcal{Z}_\chi$ corresponds to the transition of the Higgs boson into the neutral Goldstone boson in the external singlet vector field. The one-loop anomalous dimension matrix reads [21]

$$\boldsymbol{\zeta}^{(1)} = \frac{1}{4}\begin{pmatrix} 12 & 0 & 0 & 0 & 0\\ 0 & 12 & 0 & 0 & 0\\ 0 & 0 & 9 & 0 & 0\\ 0 & 0 & 0 & 9 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix} + \frac{\rho}{2}\begin{pmatrix} 0 & 0 & 6 & 0 & -6\\ 0 & 0 & 0 & 6 & -6\\ 1 & 0 & 0 & 0 & -1\\ 0 & 1 & 0 & 0 & -1\\ -1 & -1 & -1 & -1 & 0\end{pmatrix}\,, \tag{13}$$

where the first term represents the pure $SU_L(2)$ contribution, the second term represents the Yukawa contribution and we introduce the ratio $\rho = \alpha_{Yuk}/\alpha = \frac{M_t^2}{2M_W^2} \sim 1$. The proper initial condition for the evolution equation which corresponds to the $SU_L(2)$ Born amplitudes of the third generation quark and scalar boson production in light quark-antiquark annihilation is given by the vector $\boldsymbol{\mathcal{Z}}_0 = 2(T^3_\phi,\ T^3_\chi,\ T^3_{b-},\ T^3_{t-},\ T^3_{t+}) = (1,-1,-1,1,0)$ where T^3_i stands for the particle isospin and the overall factor of 2 is introduced for convenience. Since the Yukawa enhanced logarithmic corrections can be attributed to the external on-shell field renormalization we expect a diagonal form of the corrections. This is indeed the case due to a nontrivial matrix relation

$$\left(\boldsymbol{\zeta}^{(1)}_{Yuk}\right)^{2n}\cdot\boldsymbol{\mathcal{Z}}_0 = \left(\frac{3\rho^2}{2}\right)^n\boldsymbol{\mathcal{Z}}_0, \qquad \left(\boldsymbol{\zeta}^{(1)}_{Yuk}\right)^{2n+1}\cdot\boldsymbol{\mathcal{Z}}_0 = \left(\frac{3\rho^2}{2}\right)^n(-3\rho,3\rho,\rho/2,-\rho/2,0)\,, \tag{14}$$

where the vector on the right hand side of the second equation represents the one loop correction $\boldsymbol{\zeta}^{(1)}_{Yuk}\cdot\boldsymbol{\mathcal{Z}}_0$. By factorizing the components of $\boldsymbol{\mathcal{Z}}_0$ we can rewrite it as follows

$$\boldsymbol{\zeta}^{(1)}_{Yuk}\cdot\boldsymbol{\mathcal{Z}}_0 = 2\rho(-3T^3_\phi,-3T^3_\chi,-\frac{1}{2}T^3_{b-},-\frac{1}{2}T^3_{t-},-T^3_{t+})\,. \tag{15}$$

The coefficients of T^3_i in the above expression depend only on the field renormalization of the particle i as it has been explicitly shown in Ref. [39]. For example, for the hypercharge mediated Born amplitudes of the same process we have different initial conditions $\boldsymbol{\mathcal{Z}}_0 = (Y_\phi,\ Y_\chi,\ Y_{b-},\ Y_{t-},\ Y_{t+}) = (1,1,1/3,1/3,4/3)$ where Y_i stands for the particle hypercharge. However the one loop coefficients are the same as in Eq. (15)

$$\boldsymbol{\zeta}^{(1)}_{Yuk}\cdot\boldsymbol{\mathcal{Z}}_0 = \rho(-3Y_\phi,-3Y_\chi,-\frac{1}{2}Y_{b-},-\frac{1}{2}\ Y_{t-},-Y_{t+})\,. \tag{16}$$

The different form of the odd and even order corrections is dictated by the off-diagonal character of the matrix of field renormalization by Yukawa interaction.

By expanding the solution for the component $\mathcal{Z}_\phi$ we obtain the Yukawa enhanced logarithmic corrections to the amplitude of the longitudinal W-pair production. Let us introduce the following notation

$$\langle \boldsymbol{\zeta} \rangle_{Yuk} = [\boldsymbol{\zeta} \cdot \boldsymbol{\mathcal{Z}}_0]_\phi \ , \tag{17}$$

where only the terms proportional to the second or fourth power of the top quark mass are kept on the right hand side. Then the Yukawa contribution to the amplitude (9) takes the following form

$$\begin{aligned}
\boldsymbol{\mathcal{A}}^{(1)}\big|_{Yuk} &= \langle \boldsymbol{\zeta}^{(1)} \rangle_{Yuk} L \, \boldsymbol{\mathcal{A}}^{(0)} + \boldsymbol{\mathcal{A}}_0^{(1)}\big|_{Yuk} \\
\boldsymbol{\mathcal{A}}^{(2)}|_{Yuk} &= \left\{ \frac{1}{2}\gamma^{(1)} \langle \boldsymbol{\zeta}^{(1)} \rangle_{Yuk} L^3 + \left[\left(\zeta^{(1)} + \xi^{(1)} + \boldsymbol{\chi}^{(1)} - \frac{1}{2}\beta_0^{Yuk} \right) \langle \boldsymbol{\zeta}^{(1)} \rangle_{Yuk}, \right.\right. \\
&\left.\left. + \frac{1}{2} \langle (\boldsymbol{\zeta}^{(1)})^2 \rangle_{Yuk} \right] L^2 \right\} \boldsymbol{\mathcal{A}}^{(0)} + \frac{1}{2}\gamma^{(1)} L^2 \boldsymbol{\mathcal{A}}_0^{(1)}|_{Yuk} + \mathcal{O}(L),
\end{aligned} \tag{18}$$

where $\beta_0^{Yuk} = 9/4 - 3\rho/2$ is the one-loop beta-function of the Yukawa coupling constant and $\boldsymbol{\mathcal{A}}_0^{(1)}|_{Yuk}$ is the one-loop nonlogarithmic Yukawa contribution given in the Appendix A.

4 W-pair production in the electroweak model

4.1 Analytic results

The electroweak Standard Model with the spontaneously broken $SU_L(2) \times U(1)$ gauge group involves both the massive W and Z-bosons and the massless photon. The corrections to the fully exclusive cross sections due to the virtual photon exchange are infrared divergent and should be combined with real photon emission to obtain infrared finite physical observables. The infrared divergences of the virtual corrections are regulated by giving the photon a small mass λ. In the limit $\lambda^2 \ll M_W^2 \ll Q^2$ the dependence of the amplitudes on λ in the full theory is the same as in QED. Thus the logarithmic corrections can be separated into "pure electroweak" Sudakov logarithms and QED Sudakov logarithms of the form $\ln(Q^2/\lambda^2)$ or $\ln(M_W^2/\lambda^2)$.

To disentangle the electroweak and QED logarithms we use the approach of Ref. [1,5,7]. While the dependence of the amplitudes on the large momentum transfer is governed by the *hard* evolution equations (*c.f.* Eqs. (4, 7)), their dependence on the photon mass is governed by the *infrared* evolution equations [1]. Two sets of equations completely determine the dependence of the amplitudes on two dimensionless variables Q/M_W and Q/λ up to the initial conditions which are fixed through the matching to the fixed-order result. For $\lambda^2 \ll M_W^2$ the singular dependence of the amplitudes on the infrared regulator is governed by the QED evolution equation. Its solution to NNLL accuracy in the massless fermion approximation $m_f = 0$ $(f \neq t)$ is given by the factor

$$\begin{aligned}
\mathcal{U} = U_0(\alpha_e) \exp \Bigg\{ & \frac{\alpha_e(\lambda^2)}{4\pi} \left[- \left(Q_q^2 + 1 \right) \ln^2 \frac{Q^2}{\lambda^2} + \left(3Q_q^2 - 4Q_q \ln \frac{\hat{u}}{\hat{t}} \right) \ln \frac{Q^2}{\lambda^2} + \ln^2 \frac{M_W^2}{\lambda^2} + 2 \ln \frac{M_W^2}{\lambda^2} \right] \\
+ & \frac{\alpha_e^2(\lambda^2)}{(4\pi)^2} \frac{8}{9} \left[-\frac{10}{3} \left(Q_q^2 + 1 \right) \ln^3 \frac{Q^2}{\lambda^2} + \left(\frac{95}{3} Q_q^2 + \frac{50}{3} - 20 Q_q \ln \frac{\hat{u}}{\hat{t}} \right) \ln^2 \frac{Q^2}{\lambda^2} + \mathcal{O}\left(\ln \frac{Q^2}{\lambda^2} \right) \right] + \mathcal{O}(\alpha_e^3) \Bigg\},
\end{aligned} \tag{19}$$

where α_e is the $\overline{\text{MS}}$ QED coupling constant, and Q_q is the quark electric charge. The NNLL approximation for $\mathcal{U}$ can be obtained from the result for the fermion-antifermion production [5] by proper modification of the QED anomalous dimensions. Note that we take into account the top quark decoupling and Eq. (19) corresponds to five light flavors in contrast to Ref. [5] where all the quarks were assumed to be massless. To exclude the top quark contribution in the expressions for the QED anomalous dimensions in [5] N_g should be replaced by $N_g - 1/2$. The preexponential factor U_0 in Eq. (19) is factorization scheme dependent. It is convenient to fix it by normalizing $\mathcal{U}(\alpha_e)\big|_{s=\lambda^2=M_W^2} = 1$. We factorize the QED factor and write the full theory amplitude as a product

$$\boldsymbol{\mathcal{A}} = \mathcal{U}\,\boldsymbol{\mathcal{A}}_{ew}. \tag{20}$$

where $\boldsymbol{\mathcal{A}}_{ew}$ includes only electroweak Sudakov logarithms. The logarithms of the photon mass in $\mathcal{U}$ are generated by loops with soft photons, photons collinear to the initial state fermions, and soft photons collinear to the final state gauge bosons, which result in the logarithmic dependence of the coefficients on M_W. In the physically motivated cross section which is inclusive in respect to the photons with the energy much less than electroweak scale the singular dependence of $\mathcal{U}$ on the photon mass is replaced by the experimental cuts on the soft photon energy or absorbed into the parton distribution functions. One may easily change the regularization scheme and use *e.g.* dimensional regularization which is more convenient for the analysis of the parton distribution functions. In the present paper we focus on the pure electroweak part of the amplitude $\boldsymbol{\mathcal{A}}_{ew}$. The factorization formula (20) implies that the anomalous dimensions corresponding to the electroweak Sudakov logarithms are obtained by subtracting the QED contribution from anomalous dimensions of the full theory. The functions γ, ζ, and $\boldsymbol{\chi}$ are mass-independent. Therefore the anomalous dimensions parametrizing the electroweak logarithms can be obtained by subtracting the QED contribution from the result of the unbroken symmetry phase calculation to all orders in the coupling constants. In particular in one loop we get

$$\begin{aligned}
&\gamma^{(1)}_{A,\phi} = \gamma^{(1)}_{A,\phi}\Big|_{SU(2)} - \frac{1}{2}Y^2_{A,\phi}\,t^2_W + 2\,Q^2_{A,\phi}\,s^2_W\,, &&\zeta^{(1)}_{A,\phi} = \zeta^{(1)}_{A,\phi}\Big|_{SU(2)} + Y^2_{A,\phi}\,t^2_W\,,\\
&\boldsymbol{\chi}^{(1)}_T = \boldsymbol{\chi}^{(1)}_T\Big|_{SU(2)} + 4\,Q_q\,s^2_W\,\ln\frac{u}{t}\,\mathbb{1}, &&\boldsymbol{\chi}^{(1)}_L = \boldsymbol{\chi}^{(1)}_L\Big|_{SU(2)} + \Big(Y_q\,Y_\phi\,t^2_W + 4\,Q_q\,s^2_W\Big)\ln\frac{u}{t}\,\mathbb{1},
\end{aligned} \tag{21}$$

where Y_q, $Y_A = 0$, $Y_\phi = -1$ are the hypercharges of quarks, gauge and Goldstone bosons, $s^2_W = \sin^2\theta_W$, $t^2_W = \tan^2\theta_W$, and θ_W is the electroweak mixing angle. The $SU(2)$ part of the anomalous dimensions can be found in [21] while the hypercharge contribution and QED subtraction term are given explicitly. The anomalous dimensions for the quark $\mathcal{Z}$-functions can be found in Ref. [5]. The only two-loop coefficients we need are

$$\gamma^{(2)}_{A,\phi} = \gamma^{(2)}_{A,\phi}\Big|_{SU(2)} + \frac{52}{9}Y^2_{A,\phi}\,t^4_W - \frac{800}{27}Q^2_{A,\phi}\,s^4_W\,, \tag{22}$$

in the $\overline{\text{MS}}$ scheme. On the other hand the functions ξ and $\mathcal{A}_0$ are infrared sensitive and require the use of the true mass eigenstates of the Standard Model in the perturbative calculation. In NNLL approximation one needs the one-loop contribution to these quantities which can be found by comparing the solution of the evolution equation with the explicit one-loop result for the amplitudes. In this way we find that the anomalous dimensions $\xi^{(1)}_i$ get contributions just from the mass difference between M_W and M_Z and obtain:

$$\xi^{(1)}_i = 2\left[(T^3_i)^2 + \left(\frac{Y_i}{2}\right)^2 t^2_W - Q^2_i\,s^2_W\right]\ln\frac{M^2_Z}{M^2_W}, \qquad i = \psi, A, \phi, \tag{23}$$

where $T_i^3 = Q_i - Y_i/2$ is the third component of the isospin. The expressions for the nonlogarithmic one-loop corrections to the amplitude $\mathcal{A}_0^{(1)}$ are rather cumbersome and we collect them in Appendix A. Note that $\mathcal{A}_0^{(1)}$ depends on the normalization of the QED factor. We use the normalization where all the nonlogarithmic one-loop corrections are contained in $\mathcal{A}_0^{(1)}$. With the above parameters of the evolution at hand we can write down the two-loop NNLL corrections to the amplitudes as in Eq. (9). The two-loop Yukawa contribution in the NLL approximation is given by the interference of the one-loop double logarithms and the one-loop Yukawa enhanced single logarithms. Thus it is straightforward to obtain this contribution exactly. For the NNLL two-loop Yukawa contribution we use the $SU(2)$ model of the previous section with $\rho = m_t^2/(2M_W^2)$, which approximates the exact result with the accuracy of order $\sin^2\theta_W \approx 0.2$.

Now we are in the position to present the final result for the cross sections. We define the perturbative series as follows

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \left[1 + \left(\frac{\alpha}{4\pi}\right)\delta^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2\delta^{(2)} + \dots\right]\frac{\mathrm{d}\sigma_{LO}}{\mathrm{d}\cos\theta}\,. \tag{24}$$

The coefficients for the one and two-loop NNLL terms are listed in the Appendix B. Below we present the numerical analysis of the corrections to the partonic and hadronic cross sections.

4.2 The partonic cross section

For the numerical estimates we adopt the following input values

$$M_W = 80.41\ \text{GeV}, \qquad M_Z = 91.19\ \text{GeV}, \qquad M_H = 117\ \text{GeV}, \qquad m_t = 172.7\ \text{GeV}, \tag{25}$$
$$\alpha(M_Z^2) = \frac{1}{128.1}, \qquad s_W^2 = 0.231,$$

and take $\sqrt{s} = 1$ TeV as characteristic example. The one and two-loop corrections for left-handed u-quarks in the initial state are plotted in Fig. 4 showing a sizable NNLL contribution[1]. The structure of the corrections for the left-handed d-quarks is similar, see Fig. 5. To facilitate the comparison of the u- and d-quarks cases related by crossing symmetry in the Born approximation, we plot the cross section for u-quarks as a function of $-\cos\theta$. In the Born cross section we always use the physically motivated normalizarion scale of the coupling constants, which is $\mu = M_W$ for the transverse and $\mu = \sqrt{s}$ for the longitudinal boson production.

The contribution of the right-handed quarks vanishes for transversally polarized W-bosons, and for longitudinally polarized bosons it is significantly smaller than the one of left-handed quarks, see Fig. 6. In one as well as in two-loop approximation one observes large compensations between LL, NLL and NNLL terms. Evidently the LL approximation, even when combined with NLL terms only, does not lead to an adequate description of the full result. In Ref. [15] the quality of the high energy approximation has been studied at one-loop level. The error turns out to be less than a few percents for a partonic center of mass energy above 500 GeV and a scattering angle in the range $30^o < \theta < 150^o$.

[1]Numerical results for the partonic cross section have been presented in Ref. [24] and qualitatively agree with our analysis. However a direct comparison of the results is not possible since the authors of [24] use different power counting and QED subtraction prescription.

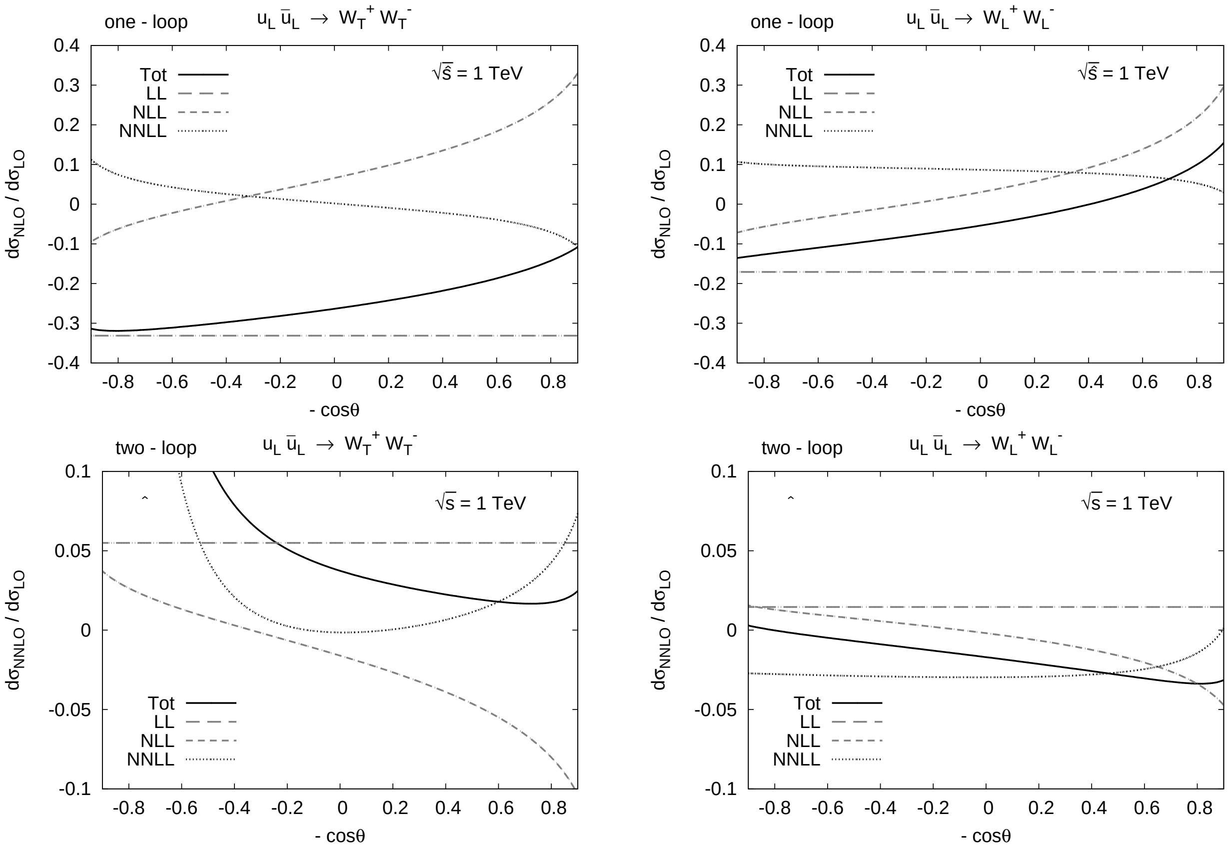

Figure 4: One and two-loop corrections to the partonic cross section for left-handed u-quarks in the initial state, transverse (left panel) and longitudinal (right panel) W-bosons at $\sqrt{\hat{s}} = 1$ TeV.

4.3 Hadronic cross section

To obtain transverse momentum and invariant mass distributions for the process $pp \to W^+W^- + X$ the partonic cross section must be convoluted with the parton distribution functions $f_{h_1,i}(x_1, \mu_F^2)$ and $f_{h_2,j}(x_2, \mu_F^2)$, where μ_F is the factorization scale, x_1 and x_2 are the momentum fractions carried by the parton i in the hadron h_1 and by the parton j in the hadron h_2 respectively. The p_T-distribution is given by

$$\frac{d\sigma}{dp_T} = \frac{1}{N_c^2} \sum_{ij} \int_0^1 dx_1 \int_0^1 dx_2\, f_{h_1,i}(x_1, \mu_F^2)\, f_{h_2,j}(x_2, \mu_F^2)\, \theta(x_1 x_2 - \tau_{\min})\, \frac{d\hat{\sigma}_{ij}}{dp_T}\,, \tag{26}$$

where N_c is the number of colors, the sum is over all possible $q\bar{q}$ partonic initial state, $p_{\mathrm{T}} = \sin\theta \sqrt{\hat{s} - 4M_W^2}\Big/2$ is the transverse momentum of the W-bosons and we adopt $\mu_F = p_{\mathrm{T}}$. The quantity

$$\tau_{\min} = \frac{4(p_T^2 + M_W^2)}{s} \tag{27}$$

is related to the minimal partonic energy that is needed to produce two W-bosons with a given transverse momentum p_{T}. The partonic differential cross section $d\hat{\sigma}_{ij}/dp_T$ are given in terms of

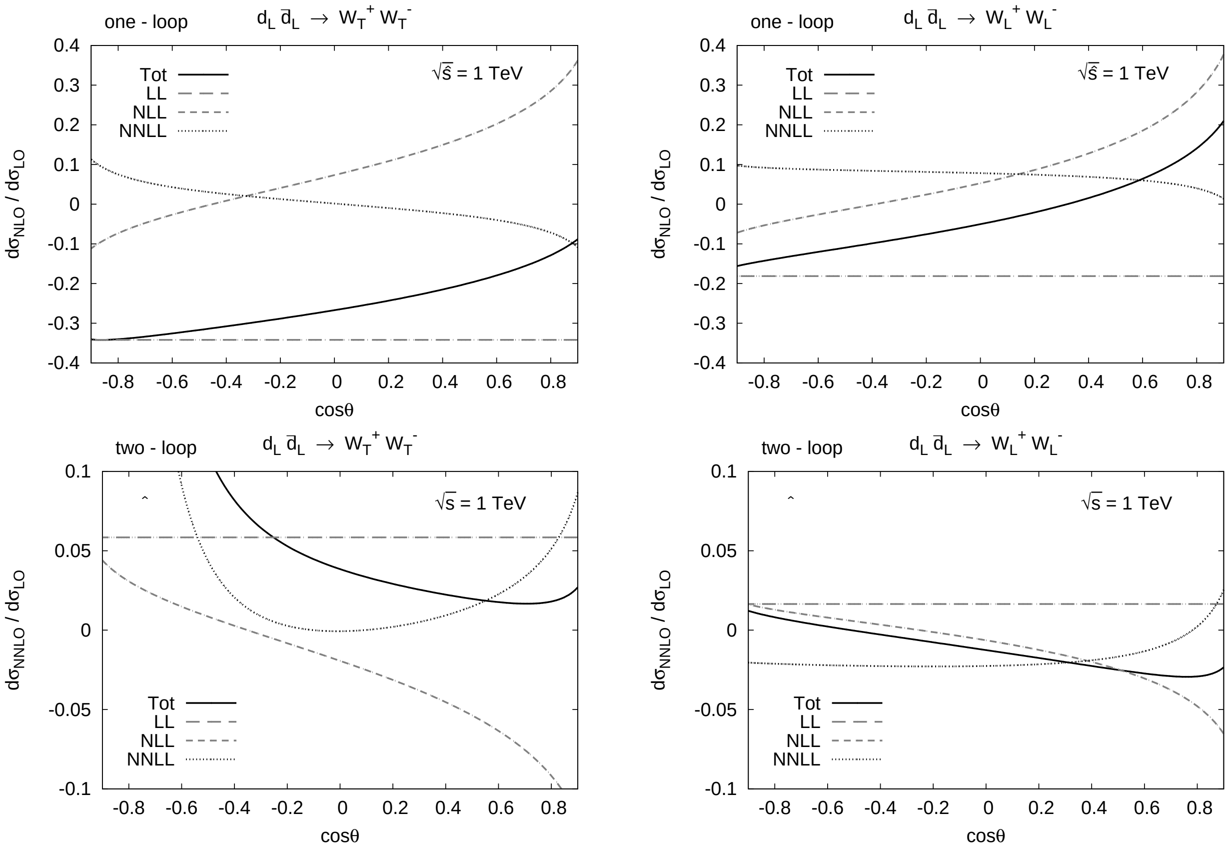

Figure 5: One and two-loop corrections to the partonic cross section for left-handed d-quarks in the initial state, transverse (left panel) and longitudinal (right panel) W-bosons at $\sqrt{\hat{s}} = 1$ TeV.

the angular differential cross section as follows

$$\frac{d\hat{\sigma}_{ij}}{dp_T} = \frac{4p_T}{\sqrt{\hat{s} - 4M_W^2}\sqrt{\hat{s} - s\,\tau_{\min}}} \left[\frac{d\hat{\sigma}_{ij}}{d\cos\theta} + (\hat{t} \leftrightarrow \hat{u}) \right], \qquad \hat{s} = x_1\, x_2\, s\,. \tag{28}$$

The numerical results are obtained by using the MRST parton distributions [40] and the integration routine *CUHRE* from the *CUBA* library [41]. The upper panel of Fig. 7 shows the NNLO p_{T}-distributions for the production of transverse and longitudinal W-bosons in the NNLL approximation. Transverse bosons production is evidently dominant, with the cross section being about twenty times larger than the one of the longitudinal bosons. The lower panel of Fig. 7 shows the NLO and NNLO corrections separately. For the production of transversely polarized W-pairs the one-loop correction reaches 40% at $p_{\mathrm{T}} = 1$ TeV and 60% at $p_{\mathrm{T}} = 2$ TeV. The two-loop contribution amounts up to 10% at $p_{\mathrm{T}} = 1$ TeV and 20% at $p_{\mathrm{T}} = 2$ TeV and partially compensate the one-loop corrections. For the longitudinal boson production the one-loop correction is about 15% (30%) at p_{T} =1 TeV (p_{T} =2 TeV), while the two-loop contribution does not exceed a few percent up to $p_{\mathrm{T}} = 2$ TeV. As anticipated above the radiative corrections for the longitudinal case are smaller than those for transverse W bosons. This is because the value of the quadratic Casimir operator of the $SU_L(2)$ electroweak group, which govern the leading logarithmic contribution, is smaller for the fundamental representation of the longitudinal degrees of freedom than for the adjoint representation of the transversely polarized W-bosons.

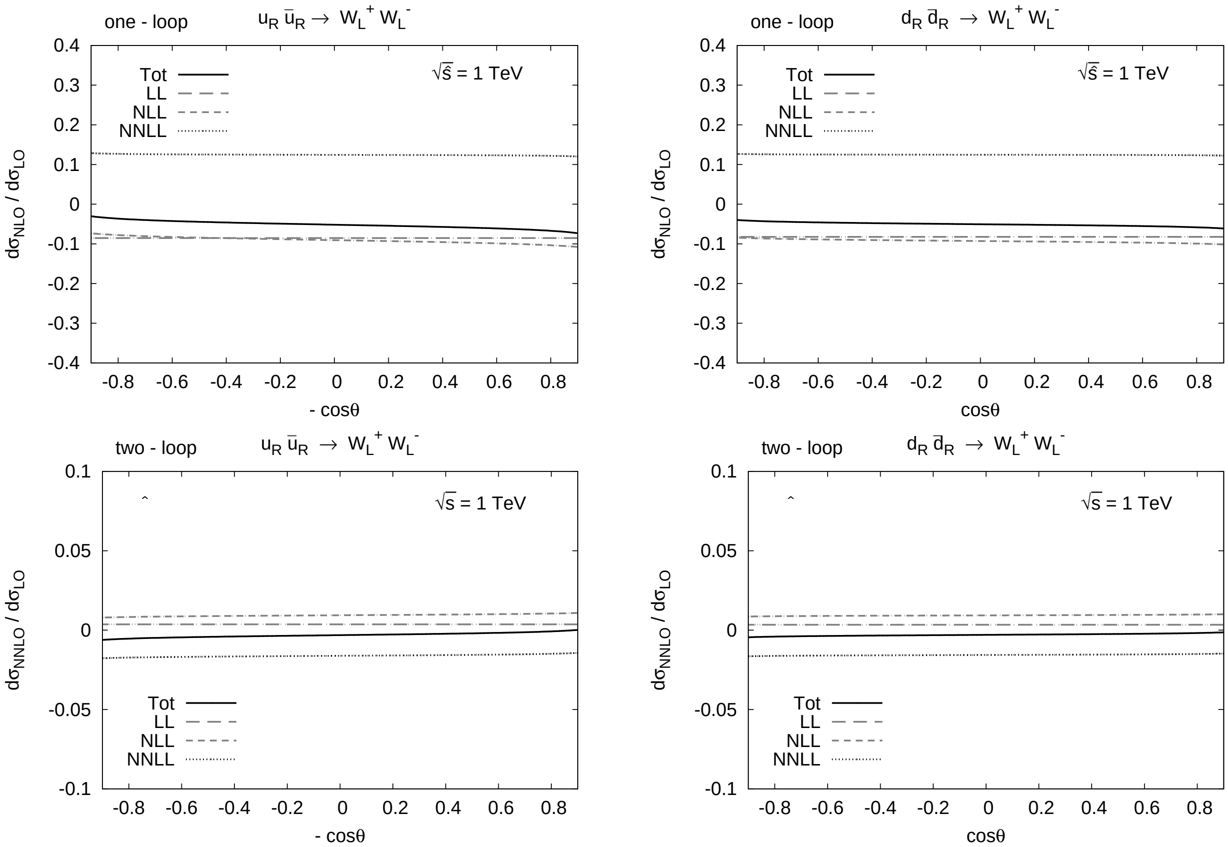

Figure 6: One and two-loop corrections to the partonic cross section for right-handed u-quarks (left panel) and d-quarks (right panel) in the initial state, and longitudinal W-bosons at $\sqrt{\hat{s}} = 1$ TeV.

The invariant mass distribution for the W-pair production is defined as follows

$$\frac{d\sigma}{dM_{WW}} = \frac{1}{N_c^2} \sum_{ij} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,i}(x_1, \mu_F^2) f_{h_2,j}(x_2, \mu_F^2) \frac{\mathrm{d}\hat{\sigma}_{ij}(M_{WW}^2, \alpha)}{\mathrm{d}M_{WW}}\,, \tag{29}$$

where $M_{WW} = \sqrt{(k_+ + k_-)^2} = \sqrt{\hat{s}}$ is the invariant mass of the W-pair system and we adopt $\mu_F = M_{WW}$. Here the partonic differential cross section $\mathrm{d}\hat{\sigma}_{ij}/\mathrm{d}M_{WW}$ is obtained by integrating the angular differential cross section in the region $-\cos\theta_{\min} < \cos\theta < \cos\theta_{\min}$

$$\frac{\mathrm{d}\hat{\sigma}_{ij}(M_{WW}^2)}{\mathrm{d}M_{WW}} = \int_{-\cos\theta_{\min}}^{\cos\theta_{\min}} \mathrm{d}\cos\theta \frac{\mathrm{d}\hat{\sigma}_{ij}(M_{WW}^2)}{\mathrm{d}\cos\theta} \delta(\sqrt{x_1 x_2 s} - M_{WW})\,, \tag{30}$$

which excludes the range of small angles where the high energy and the Sudakov approximations are not valid. The results for the invariant mass distribution are plotted in Fig. 8 with an angular cutoff $\theta_{\min} = 30^o$. To estimate the potential statistical sensitivity, the corresponding plots are shown for the production cross section of W pairs with $p_T \geq p_T^{\mathrm{cut}}$. Taking, as crude estimate, an integrated luminosity of 200 fb^{-1}, about 1200 W pairs with $p_T > 600$ GeV would be produced. Assuming that the experimental analysis would be based on the final state with one W-boson decaying leptonically and the other hadronically a fraction of about 4/9 of the pairs could be observed, leading to a nominal statistical error of about 4%. Under this (optimistic) assumption the one-loop terms would be clearly relevant and the two-loop terms start to contribute.

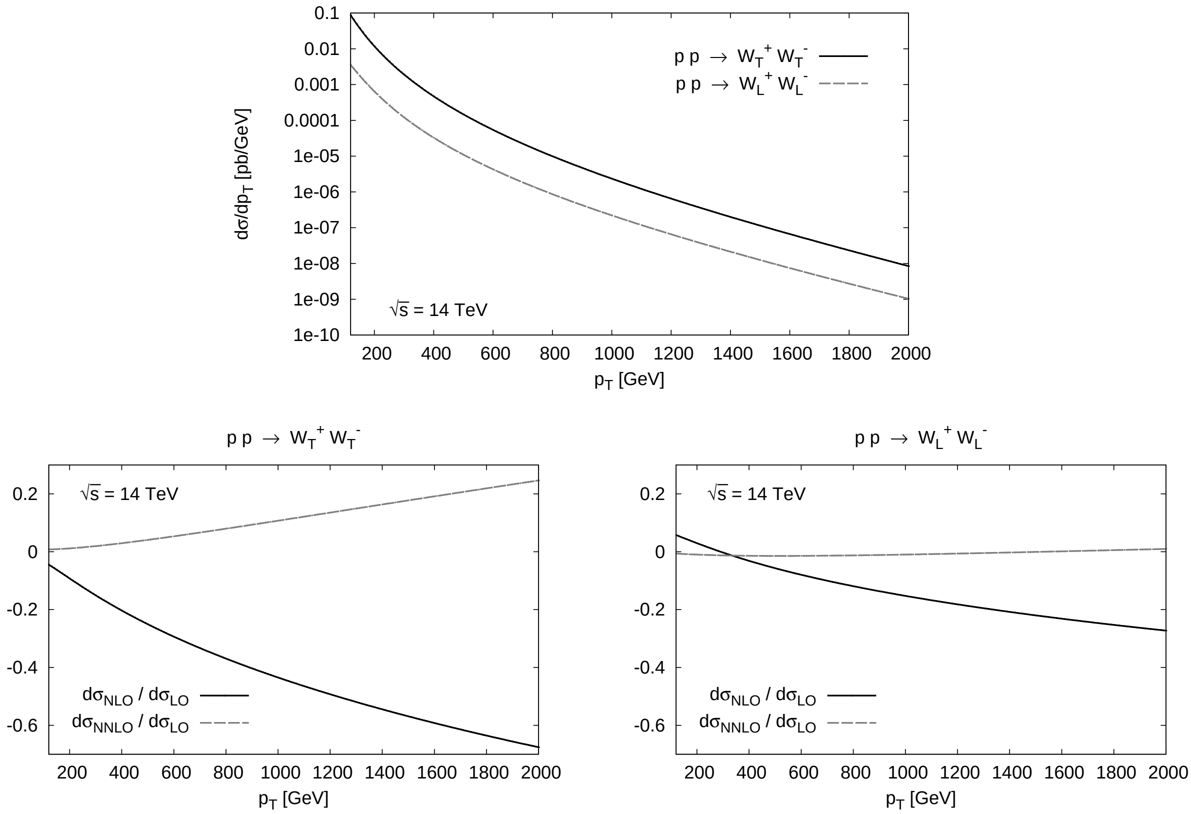

Figure 7: Transverse momentum distribution (including corrections) of transverse and longitudinal W pairs and relative corrections for proton-proton collisions at $\sqrt{s} = 14$ TeV.

5 Summary

In the present paper we derived the one and two-loop electroweak corrections to W-pair production at the LHC in NNLL approximation in high energy limit. We present the analytical result for the amplitudes, differential partonic cross sections, hadronic p_T- and invariant mass distributions. The structure of the corrections is similar to the W-pair production in e^+e^- annihilation [21]. In the case of the transverse boson production we observe the cancellation between the huge NLL and NNLL contributions so that the sum is dominated by the LL term. For the longitudinal bosons the corrections exhibit significant cancellation between the LL, NLL and NNLL terms. The maximal effect of the corrections is on the p_T-distribution of the transverse W-pair production and reaches 60% and 20% at $p_T = 2$ TeV in one and two loops, respectively. To push the theoretical error below 1% the evaluation of the two-loop linear logarithmic terms should be completed, which requires the calculation of the two-loop mass-dependent anomalous dimensions [7].

Acknowledgments

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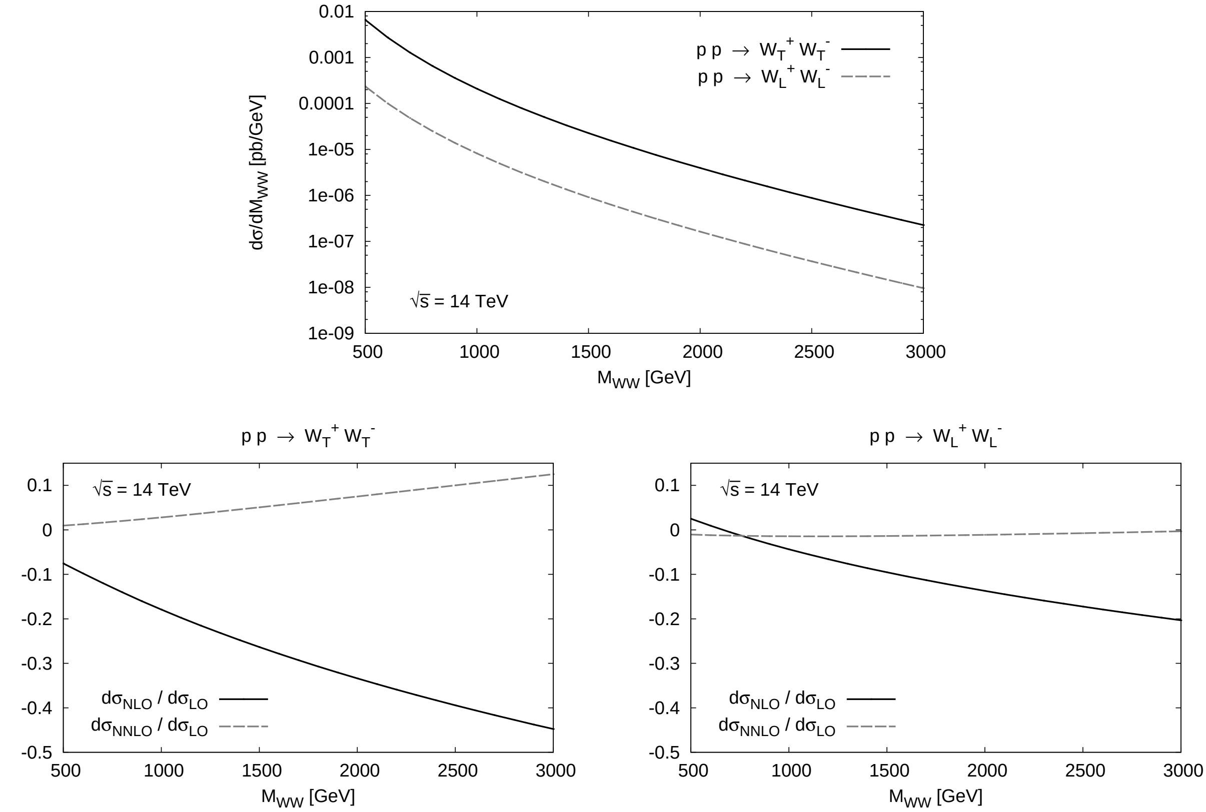

Figure 8: Total invariant mass distribution (including corrections) of transverse and longitudinal W-pairs (upper panel) and the corrections to the invariant mass distribution (lower panel) for proton-proton collisions at $\sqrt{s} = 14$ TeV.

Appendix A

In this appendix we give the explicit analytical result, valid in the high energy limit, for the one and two-loop NNLL amplitudes of the processes:

$$\begin{aligned} A_{Tq_-}: &\quad q_-(p_1) + \bar{q}_+(p_2) \to W_T^+(k_+) + W_T^-(k_-)\,, \\ A_{Lq_\mp}: &\quad q_\mp(p_1) + \bar{q}_\pm(p_2) \to \phi^+(k_+) + \phi^-(k_-)\,, \end{aligned} \tag{31}$$

where $q_\mp$ are the left/right handed fermions in the initial state, which can be either u or d quarks. The amplitude A_T describes the production of transversely polarized W-bosons and vanishes for the right-handed initial quarks. The amplitude $A_{L\mp}$ describes through the one-loop Goldstone equivalence theorem the production of the longitudinally polarized W-bosons. The one-loop corrections to the Goldstone equivalence theorem of Fig. 2 can be properly described by introducing the following effective wave function counterterm for the $\phi^\pm$ field:

$$Z_\phi^{1/2} = 1 - \frac{\Sigma_L^W(M_W^2)}{M_W^2} - \frac{\Sigma^{W\phi}(M_W^2)}{M_W} + \frac{1}{2}\frac{\delta M_W^2}{M_W^2} + \frac{1}{2}\delta Z_W + \mathcal{O}(\alpha^2)\,, \tag{32}$$

where Σ_L^W is the longitudinal part of the W self-energy, $\Sigma^{W\phi}$ is the $W\!-\!\phi$ self-energy, δM_W^2 and δZ_W are the mass and wave function counterterms of the W boson (see also [42]).

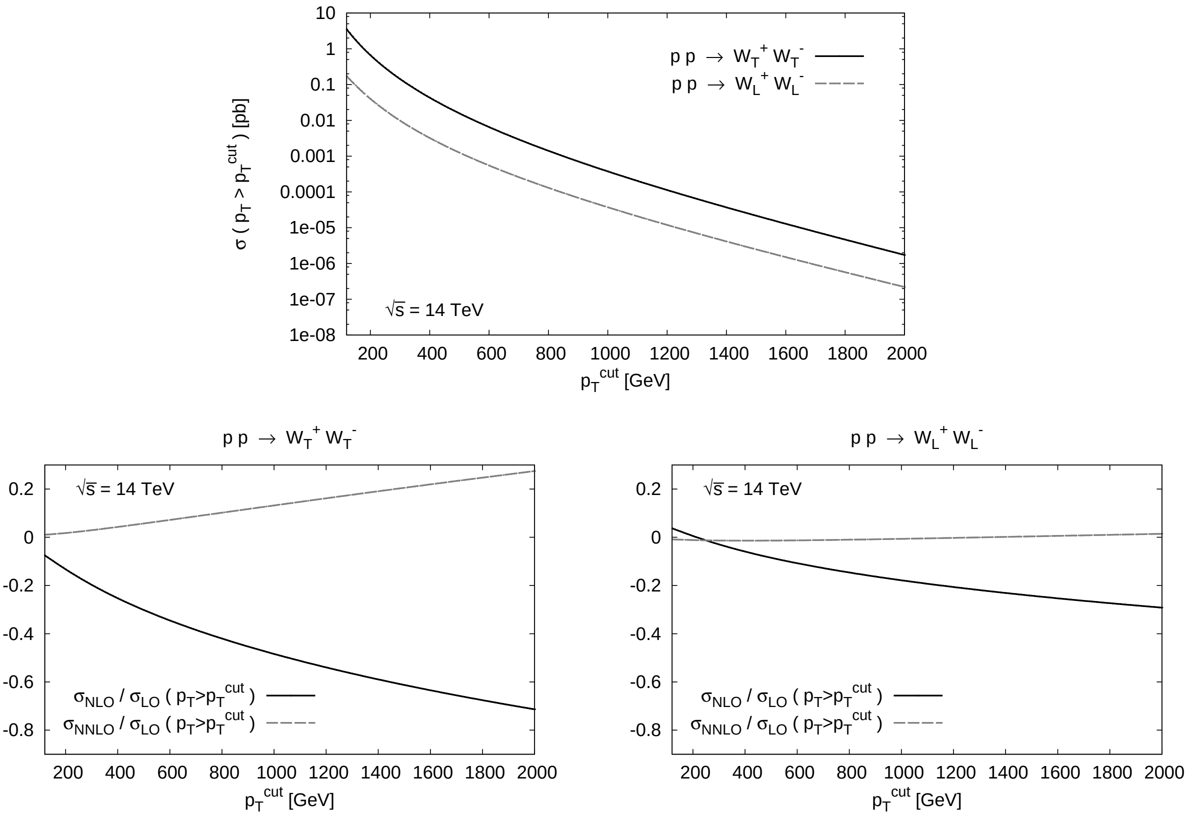

Figure 9: Cross section (including corrections) for transverse and longitudinal W pair production and relative corrections for proton-proton collisions at $\sqrt{s} = 14$ TeV (see text).

The results given in this section are obtained by adopting the $\overline{\mathrm{MS}}$ renormalization for the couplings and the weak mixing angle and on-shell renormalization for the masses. As before the renormalization scale in the Born amplitudes is fixed to $\mu_T^2 = M_W^2$ for the transverse and $\mu_L^2 = s$ for the longitudinal case. The Lorentz-Dirac structure of the amplitudes in the high energy limit takes a simple form:

$$\begin{aligned} A_{Tq_-} &= \bar{q}(p_2)\left[\not\epsilon^*_\kappa\, p_1\!\cdot\!\epsilon^*_\kappa\, \mathcal{A}_{Tq_-} \;+\; \not k_+\,(p_1\!\cdot\!\epsilon^*_\kappa)^2\,\mathcal{B}_{Tq_-}\right]\omega_-\, q(p_1), \\ A_{Lq_\mp} &= \bar{q}(p_2)\,\not k_+\,\omega_\mp\, q(p_1)\,\mathcal{A}_{Lq_\mp}, \end{aligned} \tag{33}$$

where $\omega_\pm = \frac{1\pm\gamma^5}{2}$ and we use the relation between the polarization vectors $\epsilon^\mu_\kappa(k_\pm)$ of the transversely polarized $W^\pm$ in the center of mass frame

$$\epsilon^\mu_\kappa(k_+) = -\epsilon^\mu_{-\kappa}(k_-) \;\equiv\; \epsilon^\mu_\kappa, \tag{34}$$

where $\kappa = \pm 1$ stands for the polarization. The perturbative series for the amplitudes read

$$\mathcal{A}_{Pq_\mp} = 4\pi\alpha(\mu_P^2)\sum_{n=0}^{\infty}\left(\frac{\alpha}{4\pi}\right)^n \mathcal{A}^{(n)}_{Pq_\mp} \quad (P = T, L), \qquad \mathcal{B}_{Tq_-} = 4\pi\alpha\sum_{n=0}^{\infty}\left(\frac{\alpha}{4\pi}\right)^n \mathcal{B}^{(n)}_{Tq_-}\,, \tag{35}$$

where $\alpha(\mu^2) = \alpha_e(\mu^2)/s_W^2(\mu^2)$ and the coupling constants are supposed to be normalized at $\mu = M_W$ unless the normalization point is indicated explicitly. The Born amplitudes read:

$$\mathcal{A}^{(0)}_{Tu_-} = \frac{1}{\hat{u}}, \qquad \mathcal{A}^{(0)}_{Td_-} = \frac{1}{\hat{t}}, \qquad \mathcal{B}^{(0)}_{Tq_-} = 0,$$

$$\mathcal{A}^{(0)}_{Lu_-} = \frac{1}{2\,\hat{s}}\left[1 + \frac{t_W^2(\mu_L^2)}{3}\right], \quad \mathcal{A}^{(0)}_{Ld_-} = \frac{1}{2\,\hat{s}}\left[-1 + \frac{t_W^2(\mu_L^2)}{3}\right], \quad \mathcal{A}^{(0)}_{Lu_+} = \frac{2}{3\,\hat{s}} t_W^2(\mu_L^2), \quad \mathcal{A}^{(0)}_{Ld_+} = -\frac{1}{3\,\hat{s}} t_W^2(\mu_L^2). \tag{36}$$

The one-loop contribution to the second transverse Lorentz-Dirac structure of Eq.(33) is particularly simple and does not contain Sudakov logarithms

$$\mathcal{B}^{(1)}_{Tu_-} = \left[\beta^{(1)}_{T,SU(2)} + t_W^2 \beta^{(1)}_{T,Y}\right]\frac{1}{\hat{t}\,\hat{u}}, \qquad \mathcal{B}^{(1)}_{Td_-} = -\,\mathcal{B}^{(1)}_{Tu_-}(\hat{t} \leftrightarrow \hat{u}), \tag{37}$$

$$\beta^{(1)}_{T,SU(2)} = -\left(4 - \frac{5}{2}\frac{\hat{u}}{\hat{t}}\right)\frac{\hat{u}}{\hat{t}}\left[L_{us}^2 + \pi^2\right] + 3\frac{\hat{t}}{\hat{u}}\left[L_{ts}^2 + \pi^2\right] - \left(\frac{9}{2} - 5\frac{\hat{u}}{\hat{t}}\right)L_{us} + \frac{5}{2},$$

$$\beta^{(1)}_{T,Y} = -\frac{1}{9}\left(1 + \frac{1}{2}\frac{\hat{u}}{\hat{t}}\right)\frac{\hat{u}}{\hat{t}}\left[L_{us}^2 + \pi^2\right] - \frac{1}{9}\left(\frac{3}{2} + \frac{\hat{u}}{\hat{t}}\right)L_{us} - \frac{1}{18}. \tag{38}$$

All the notations are explained at the end of the section. The one-loop corrections to the remaining Lorentz-Dirac structure can be formally decomposed according to the gauge coupling constant factor

$$\mathcal{A}^{(1)}_{Pq_\mp} = \mathcal{A}^{(1)}_{Pq_\mp,\,SU(2)} + t_W^2\,\mathcal{A}^{(1)}_{Pq_\mp,Y} + s_W^2\left(\mathcal{A}^{(1)}_{Pq_\mp,\,QED} - \mathcal{A}^{(1)}_{Pq_\mp,\,sub}\right), \qquad (P = T, L), \tag{39}$$

where the last term $\mathcal{A}^{(1)}_{Pq_\mp,\,sub}$ corresponds to the first order term of the expansion of the singular QED factor (19) and cancels the infrared logarithms coming from soft photons and from photons collinear to the incoming quarks[2]. The QED correction factorizes with respect to the Born amplitude and reads

$$\mathcal{A}^{(1)}_{Pq_\mp,QED} = \left[2\left(Q_q^2+1\right)L_\gamma L + 4Q_q L_\gamma L_{ut} - Q_q^2 L_\gamma^2 - 2\,Q_q^2 L_\gamma L_Z - \left(3Q_q^2+2\right)L_\gamma + \Delta^{(1)}_{P,\lambda}\right]\mathcal{A}^{(0)}_{Pq_\mp},$$

$$\Delta^{(1)}_{T,\lambda} = \left(\frac{9}{2} - \frac{7}{2}w_Z + \frac{2}{3}w_Z^2\right)w_Z L_Z - \left[\frac{4}{\beta_Z} - \left(\frac{3}{2} - \frac{13}{6}w_Z + \frac{2}{3}w_Z^2\right)w_Z\beta_Z\right]L_{\chi_Z} - L_{\chi_Z}^2 - \pi^2 + 5w_Z - \frac{4}{3}w_Z^2,$$

$$\Delta^{(1)}_{L,\lambda} = \left(\frac{9}{2} - 11w_Z + 3w_Z^2\right)w_Z L_Z - \left[\frac{4}{\beta_Z} + \left(\frac{1}{2} + 5w_Z - 3w_Z^2\right)w_Z\beta_Z\right]L_{\chi_Z} + L_{\chi_Z}^2 + \pi^2 + 11w_Z - 6w_Z^2,$$

$$\mathcal{A}^{(1)}_{Pq_\mp,sub} = \left[-\left(Q_q^2+1\right)\ln^2\frac{Q^2}{\lambda^2} + \left(3Q_q^2 - 4Q_q L_{ut}\right)\ln\frac{Q^2}{\lambda^2} + \ln^2\frac{M_W^2}{\lambda^2} + 2\ln\frac{M_W^2}{\lambda^2}\right]\mathcal{A}^{(0)}_{Pq_\mp}. \tag{40}$$

where $Q^2 = -\hat{s} - i\,0^+$ and Q_q is the electric charge of the quark. Note that $\mathcal{A}^{(1)}_{Pq_\mp,QED}$ vanishes for $M_Z \sim \lambda \to 0$. After the subtraction the λ-dependence disappears and we get the QED contribution in terms of the parameters of the evolution equation

$$\mathcal{A}^{(1)}_{Pq_\mp,QED} - \mathcal{A}^{(1)}_{Pq_\mp,sub} = -\left[\frac{\gamma^{(1)}_{q,QED}}{2}L^2 + \left(\zeta^{(1)}_{q,QED} + \xi^{(1)}_{q,QED} + \chi^{(1)}_{q,QED}\right)L + \Delta^{(1)}_{Pq,QED}\right]\mathcal{A}^{(0)}_{Pq_\mp},$$

[2] In contrast to Section 4.1, we normalize here $\mathcal{U}(\alpha_e)\big|_{Q^2=\lambda^2=M_W^2} = 1$. In this case the amplitude is manifestly λ independent. The numerical estimates are obtained with the normalization of Section 4.1, where the λ dependence survives in the imaginary part of the one-loop amplitude, but does not contribute to the cross section up to N^{3}LL approximation.

$$\gamma^{(1)}_{q,QED} = -2\Big(Q_q^2+1\Big), \qquad \zeta^{(1)}_{q,QED} = 3Q_q^2, \qquad \chi^{(1)}_{q,QED} = -4Q_q L_{ut}, \qquad \xi^{(1)}_{q,QED} = 2\Big(Q_q^2+1\Big)L_Z,$$
$$\Delta^{(1)}_{Pq,QED} = -\Big(3Q_q^2+2\Big)L_Z + 4Q_q L_Z L_{ut} - Q_q^2 L_Z^2 - \Delta^{(1)}_{P,\lambda}. \tag{41}$$

In order to present the result of the remaining $SU(2)$ and Y components, in terms of the coefficients of the evolution equations, it is necessary to analyze the isospin structure of the amplitude for left handed quarks. The general $SU(2)$ basis for the amplitude A_T of the left-handed quark-antiquark pair transition into two transverse gauge bosons reads

$$\bar{q}_L\, q_L \to B_a\, B_b: \qquad (\bar{u}_+\ \ \bar{d}_+)\left(\mathcal{A}_1\,\frac{\boldsymbol{\sigma}_a}{2}\frac{\boldsymbol{\sigma}_b}{2} + \mathcal{A}_2\,\frac{\boldsymbol{\sigma}_b}{2}\frac{\boldsymbol{\sigma}_a}{2} + \mathcal{A}_3\,\delta_{ab}\mathbb{1}\right)\begin{pmatrix} u_- \\ d_- \end{pmatrix}, \qquad a,b=1,2,3, \tag{42}$$

where B_a are $SU(2)$ gauge fields and σ_a are the Pauli matrices. From the definition $W^\pm_T = (B_1 \mp iB_2)/\sqrt{2}$ we get the following structure for the production of $W^+_T W^-_T$

$$\begin{aligned}\bar{q}_L\, q_L \to W^+_T W^-_T: \qquad & (\bar{u}_+\ \ \bar{d}_+)\left(\mathcal{A}_1\,\frac{\boldsymbol{\sigma}_-}{2}\frac{\boldsymbol{\sigma}_+}{2} + \mathcal{A}_2\,\frac{\boldsymbol{\sigma}_+}{2}\frac{\boldsymbol{\sigma}_-}{2} + \mathcal{A}_3\,\mathbb{1}\right)\begin{pmatrix} u_- \\ d_- \end{pmatrix} = \\ & (\bar{u}_+\ \ \bar{d}_+)\left[\frac{\mathcal{A}_1}{2}\begin{pmatrix} 0\,0 \\ 0\,1 \end{pmatrix} + \frac{\mathcal{A}_2}{2}\begin{pmatrix} 1\,0 \\ 0\,0 \end{pmatrix} + \mathcal{A}_3\begin{pmatrix} 1\,0 \\ 0\,1 \end{pmatrix}\right]\begin{pmatrix} u_- \\ d_- \end{pmatrix},\end{aligned} \tag{43}$$

where $\boldsymbol{\sigma}_\pm = (\boldsymbol{\sigma}_1 \pm i\,\boldsymbol{\sigma}_2)/\sqrt{2}$. Thus

$$\mathcal{A}_{Tu_-} = \frac{1}{2}\mathcal{A}_2 + \mathcal{A}_3, \qquad \mathcal{A}_{Td_-} = \frac{1}{2}\mathcal{A}_1 + \mathcal{A}_3. \tag{44}$$

We introduce now the isospin vector amplitude $\boldsymbol{\mathcal{A}}_T$ of Section 3

$$\boldsymbol{\mathcal{A}}_{Tq_-} = \begin{pmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \end{pmatrix}. \tag{45}$$

The Born amplitudes $\mathcal{A}^{(0)}_{Tu_-}$ and $\mathcal{A}^{(0)}_{Td_-}$ of Eq.(36) correspond to the vector

$$\boldsymbol{\mathcal{A}}^{(0)}_{Tq_-} = 2\begin{pmatrix} 1/\hat{t} \\ 1/\hat{u} \\ 0 \end{pmatrix}. \tag{46}$$

For the amplitude A_{Lq_-} of the left-handed quark-antiquark pair transition into Goldstone bosons, the isospin basis is:

$$\mathcal{A}_\sigma\,(\bar{u}_+\ \ \bar{d}_+)\frac{\boldsymbol{\sigma}_a}{2}\begin{pmatrix} u_- \\ d_- \end{pmatrix}(\Phi_0\ \ \Phi_+)\frac{\boldsymbol{\sigma}_a}{2}\begin{pmatrix} \Phi_0^* \\ \Phi_- \end{pmatrix} + \mathcal{A}_{\mathbb{1}}\,(\bar{u}_+\ \ \bar{d}_+)\mathbb{1}\begin{pmatrix} u_- \\ d_- \end{pmatrix}(\Phi_0\ \ \Phi_+)\mathbb{1}\begin{pmatrix} \Phi_0^* \\ \Phi_- \end{pmatrix}. \tag{47}$$

In the first term the sum over a goes from 1 to 3, but only σ_3 contributes to the production of charged ϕ pair so that

$$\mathcal{A}_{Lu_-} = -\frac{1}{4}\,\mathcal{A}_\sigma + \mathcal{A}_{\mathbb{1}}, \qquad \mathcal{A}_{Ld_-} = \frac{1}{4}\,\mathcal{A}_\sigma + \mathcal{A}_{\mathbb{1}}. \tag{48}$$

As for the transverse case we introduce the isospin vector amplitude $\boldsymbol{\mathcal{A}}_L$ of Section 3

$$\boldsymbol{\mathcal{A}}_{Lq_-} = \begin{pmatrix} \mathcal{A}_\sigma \\ \mathcal{A}_{\mathbb{1}} \end{pmatrix}. \tag{49}$$

The Born amplitudes $\mathcal{A}^{(0)}_{Lu-}$ and $\mathcal{A}^{(0)}_{Ld-}$ of Eq.(36) correspond to the vector

$$\boldsymbol{\mathcal{A}}^{(0)}_{Lq-} = -\frac{2}{\hat{s}}\begin{pmatrix} 1 \\ \frac{Y_i Y_f}{4} t^2_{W}(\mu^2_L) \end{pmatrix} = \frac{2}{\hat{s}}\begin{pmatrix} -1 \\ \frac{1}{12} t^2_{W}(\mu^2_L) \end{pmatrix}, \tag{50}$$

where Y_i (Y_f) is the hypercharge of the initial (final) state. The isospin vector amplitudes can also be decomposed according to the gauge couplings

$$\boldsymbol{\mathcal{A}}^{(1)}_{Pq-} = \boldsymbol{\mathcal{A}}^{(1)}_{Pq-,SU(2)} + t^2_W\, \boldsymbol{\mathcal{A}}^{(1)}_{Pq-,Y} + s^2_W \left(\boldsymbol{\mathcal{A}}^{(1)}_{Pq-,\,QED} - \boldsymbol{\mathcal{A}}^{(1)}_{Pq-,\,sub}\right), \qquad (P = T, L). \tag{51}$$

The expressions for the $SU(2)$ and Y components of the vectorial amplitude are then written in terms of the parameters of the evolution equation

$$\boldsymbol{\mathcal{A}}^{(1)}_{Pq-,i} = \left[\frac{\gamma^{(1)}_{Pq-,i}}{2} L^2 + \left(\zeta^{(1)}_{Pq-,i} + \xi^{(1)}_{Pq-,i} + \boldsymbol{\chi}^{(1)}_{Pq-,i}\right) L + \boldsymbol{\Delta}^{(1)}_{Pq-,i},\right] \boldsymbol{\mathcal{A}}^{(0)}_{Pq-,i}, \qquad (i = SU(2), Y). \tag{52}$$

The coefficients $\gamma^{(1)}_{P,i}$, $\zeta^{(1)}_{P,i}$ and $\boldsymbol{\chi}^{(1)}_{P,i}$ are universal, the other are obtained by explicit calculation.

The expression for the amplitudes $\mathcal{A}^{(1)}_{Tq-,i}$ and $\mathcal{A}^{(1)}_{Lq-,i}$ in terms of the anomalous dimensions can be then obtained by using Eqs.(44, 48) and takes the form:

$$\mathcal{A}^{(1)}_{Pq-,i} = \left[\frac{\gamma^{(1)}_{Pq-,i}}{2} L^2 + \left(\zeta^{(1)}_{Pq-,i} + \xi^{(1)}_{Pq-,i} + \chi^{(1)}_{Pq-,i}\right) L + \Delta^{(1)}_{Pq-,i}\right] \mathcal{A}^{(0)}_{Pq-} + \left[\bar{\chi}^{(1)}_{Pq-,i} L + \bar{\Delta}^{(1)}_{Pq-,i}\right] \bar{\mathcal{A}}^{(0)}_{Pq-}, \tag{53}$$

where we introduced the notations

$$\bar{\mathcal{A}}^{(0)}_{Pu\mp} = \mathcal{A}^{(0)}_{Pd\mp}, \qquad \bar{\mathcal{A}}^{(0)}_{Pd\mp} = \mathcal{A}^{(0)}_{Pu\mp}. \tag{54}$$

The coefficient are given by:

Transverse W, left-handed quarks, SU(2) component:

$$\gamma^{(1)}_{Tq-,SU(2)} = -\frac{11}{2}, \qquad \zeta^{(1)}_{Tq-,SU(2)} = \frac{9}{4}, \qquad \xi^{(1)}_{Tq-,SU(2)} = \frac{5}{2} L_Z, \qquad \boldsymbol{\chi}^{(1)}_{Tq-,SU(2)} = \begin{pmatrix} -4L_{ts} & 0 & 4L_{ut} \\ 0 & -4L_{us} & 4L_{tu} \\ L_{us} & L_{ts} & 0 \end{pmatrix},$$

$$\chi^{(1)}_{Tu-,SU(2)} = 2L_{ts} - 4L_{us}, \qquad \chi^{(1)}_{Td-,SU(2)} = 2L_{us} - 4L_{ts}, \qquad \bar{\chi}^{(1)}_{Tu-,SU(2)} = 2L_{us}, \qquad \bar{\chi}^{(1)}_{Td-,SU(2)} = 2L_{ts},$$

$$\Delta^{(1)}_{Tu-,SU(2)} = \Delta^{(1)}_{T,SU(2)}, \qquad \Delta^{(1)}_{Td-,SU(2)} = \Delta^{(1)}_{T,SU(2)}(\hat{t} \leftrightarrow \hat{u}), \qquad \bar{\Delta}^{(1)}_{Tq-,SU(2)} = 0,$$

$$\begin{aligned}
\Delta^{(1)}_{T,SU(2)} = &-\frac{5}{4}\left(1 - \frac{3}{5}\frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2}\right) L^2_{us} + \frac{3}{2} L^2_{ts} + 2\left(1 + \frac{\hat{u}}{\hat{t}}\right) L_{ts} L_{us} + \left(\frac{9}{4} - \frac{5}{2}\frac{\hat{u}}{\hat{t}}\right) L_{us} + 2 L_Z L_{ut} + \frac{485}{72} \\
&+ \frac{\pi^2}{4}\left(7 + 3\frac{\hat{u}}{\hat{t}} - 5\frac{\hat{u}^2}{\hat{t}^2}\right) - \frac{11}{2} w_Z + \frac{3}{2} w^2_Z - \frac{1}{2} w_H + \frac{1}{6} w^2_H - \frac{1}{2} w_t - w^2_t + L_t + \left(1 - w^3_t\right) L_{tW} \\
&- \frac{1}{4}\left(\frac{1}{3} + 17 w_Z - \frac{31}{2} w^2_Z + 3 w^3_Z\right) L_Z - \frac{1}{4} L^2_Z + \frac{1}{2}\left(1 - \frac{3}{2} w_H + \frac{3}{4} w^2_H - \frac{1}{6} w^3_H\right) L_H + L^2_{\chi_Z} \\
&+ \left[\frac{9}{2}\frac{1}{\beta_Z} - \left(1 - \frac{19}{8} w_Z + \frac{3}{4} w^2_Z\right) w_Z \beta_Z\right] L_{\chi_Z} - \frac{1}{2}\left[\frac{1}{\beta_H} + \left(1 - \frac{5}{12} w_H + \frac{1}{6} w^2_H\right) w_H \beta_H\right] L_{\chi_H}.
\end{aligned} \tag{55}$$

Transverse W, left-handed quarks, Y component:

$$\gamma^{(1)}_{Tq-,Y} = -\frac{1}{18}, \qquad \zeta^{(1)}_{Tq-,Y} = \frac{1}{12}, \qquad \xi^{(1)}_{Tq-,Y} = \frac{1}{18} L_Z, \qquad \boldsymbol{\chi}^{(1)}_{Tq_-,Y} = 0, \qquad \chi^{(1)}_{Tq_-,Y} = \bar{\chi}^{(1)}_{Tq_-,Y} = 0,$$
$$\Delta^{(1)}_{Tu-,Y} = \Delta^{(1)}_{T,Y}, \qquad \Delta^{(1)}_{Td-,Y} = \Delta^{(1)}_{T,Y}(\hat{t} \leftrightarrow \hat{u}), \qquad \bar{\Delta}^{(1)}_{Tq-,Y} = 0,$$
$$\Delta^{(1)}_{T,Y} = \frac{1}{36}\left(1 + 3\frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2}\right) L^2_{us} + \frac{1}{12}\left(1 + \frac{2}{3}\frac{\hat{u}}{\hat{t}}\right) L_{us} + \frac{67}{72} + \frac{\pi^2}{108}\left(1 + 9\frac{\hat{u}}{\hat{t}} + 3\frac{\hat{u}^2}{\hat{t}^2}\right)$$
$$-\frac{1}{36} L^2_Z + \frac{1}{2}\left(\frac{5}{6} - w_Z\right) L_Z - \frac{1}{2}\left(\frac{1}{\beta_Z} + w_Z \beta_Z\right) L_{\chi_Z}. \tag{56}$$

Longitudinal W, left-handed quarks, SU(2) component:

$$\gamma^{(1)}_{Lq-,SU(2)} = -3, \qquad \zeta^{(1)}_{Lq-,SU(2)} = \frac{21}{4} - \frac{3}{2} w_t, \qquad \xi^{(1)}_{Lq-,SU(2)} = L_Z, \qquad \boldsymbol{\chi}^{(1)}_{Lq_-,SU(2)} = \begin{pmatrix} -2L_{us} - 2L_{ts} & 4L_{ut} \\ \frac{3}{4} L_{ut} & 0 \end{pmatrix},$$
$$\chi^{(1)}_{Lu_-,SU(2)} = L_{ts} - 3L_{us}, \qquad \chi^{(1)}_{Ld_-,SU(2)} = L_{us} - 3L_{ts}, \qquad \bar{\chi}^{(1)}_{Lu_-,SU(2)} = 2L_{us}, \qquad \bar{\chi}^{(1)}_{Ld_-,SU(2)} = 2L_{ts},$$
$$\boldsymbol{\Delta}^{(1)}_{Lq-,SU(2)} = \begin{pmatrix} -\frac{1}{2}L_+ + \frac{575}{72} + \frac{5}{3}\pi^2 + \frac{19}{6} i\pi & L_- - 4L_{ut}L_Z - \frac{416}{3} - 82 i\pi \\ \frac{3}{16} L_- - \frac{1}{4} L_{ut} L_Z & -\frac{43}{24} - \frac{1}{3}\pi^2 \end{pmatrix} + \Delta^{(1)}_{L,SU(2)} \mathbb{1},$$
$$\Delta^{(1)}_{Lu-,SU(2)} = \frac{3}{4}\frac{\hat{s}}{\hat{t}}\left(L^2_{us} + \pi^2\right) - \frac{1}{4}\frac{\hat{s}}{\hat{u}}\left(L^2_{ts} + \pi^2\right) - L_{tu} L_Z + \frac{1471}{72} + \frac{2}{3}\pi^2 + \frac{71}{6} i\pi + \Delta^{(1)}_{L,SU(2)},$$
$$\Delta^{(1)}_{Ld-,SU(2)} = \frac{3}{4}\frac{\hat{s}}{\hat{u}}\left(L^2_{ts} + \pi^2\right) - \frac{1}{4}\frac{\hat{s}}{\hat{t}}\left(L^2_{us} + \pi^2\right) - L_{ut} L_Z - \frac{1025}{72} + \frac{2}{3}\pi^2 - \frac{26}{3} i\pi + \Delta^{(1)}_{L,SU(2)},$$
$$\bar{\Delta}^{(1)}_{Lu-,SU(2)} = -\frac{1}{2}\frac{\hat{s}}{\hat{t}}\left(L^2_{us} + \pi^2\right) + \frac{112}{9} - \pi^2 + \frac{26}{3} i\pi, \qquad \bar{\Delta}^{(1)}_{Ld-,SU(2)} = -\frac{1}{2}\frac{\hat{s}}{\hat{u}}\left(L^2_{ts} + \pi^2\right) - \frac{200}{9} - \pi^2 - \frac{71}{6} i\pi,$$
$$\Delta^{(1)}_{L,SU(2)} = -17 w_Z + \frac{27}{4} w^2_Z - w_H + \frac{1}{4} w^2_H + \frac{3}{2} w_t - \frac{3}{2} w^2_t - \frac{1}{4}\left(13 + 39 w_Z - \frac{117}{2} w^2_Z + \frac{27}{2} w^3_Z\right) L_Z$$
$$+ \frac{1}{2}\left(1 - 3w_H + \frac{5}{4} w^2_H - \frac{1}{4} w^3_H\right) L_H + \frac{3}{2} w_t L_t + \frac{3}{2}\left(1 - w^2_t\right) w_t L_{tW} - \frac{1}{4} L^2_Z + \frac{1}{4} L^2_{\chi_Z} + \frac{1}{2} L^2_{\chi_H}$$
$$+ \left[\frac{9}{2}\frac{1}{\beta_Z} + \left(\frac{1}{4} + \frac{63}{8} w_Z - \frac{27}{8} w^2_Z\right) w_Z \beta_Z\right] L_{\chi_Z} - \left[\frac{1}{2}\frac{1}{\beta_H} + \left(1 - \frac{3}{8} w_H + \frac{1}{8} w^2_H\right) w_H \beta_H\right] L_{\chi_H}. \tag{57}$$

Longitudinal W, left-handed quarks, Y component:

$$\gamma^{(1)}_{Lq-,Y} = -\frac{5}{9}, \qquad \zeta^{(1)}_{Lq-,Y} = \frac{13}{12}, \qquad \xi^{(1)}_{Lq-,Y} = \frac{5}{9} L_Z, \qquad \boldsymbol{\chi}^{(1)}_{Lq_-,Y} = -\frac{1}{3} L_{ut} \mathbb{1},$$
$$\chi^{(1)}_{Lq_-,Y} = -\frac{1}{3} L_{ut}, \qquad \bar{\chi}^{(1)}_{Lq_-,Y} = 0, \qquad \boldsymbol{\Delta}^{(1)}_{Lq-,Y} = \Delta^{(1)}_{L,Y} \mathbb{1}, \qquad \Delta^{(1)}_{Lq-,Y} = \Delta^{(1)}_{L,Y}, \qquad \bar{\Delta}^{(1)}_{Lq-,Y} = 0,$$
$$\Delta^{(1)}_{L,Y} = -\frac{1}{12} L_- + \frac{1}{3} L_{ut} L_Z - \frac{731}{72} - \frac{5}{27}\pi^2 - \frac{41}{6} i\pi + \frac{1}{2}\left(\frac{5}{6} - \frac{7}{2} w_Z + \frac{1}{2} w^2_Z\right) L_Z$$
$$-\frac{1}{36} L^2_Z - \frac{1}{4} L^2_{\chi_Z} - \frac{1}{4}\left[2\frac{1}{\beta_Z} + \left(5 - w_Z\right) w_Z \beta_Z\right] L_{\chi_Z} - \frac{1}{2} w_Z. \tag{58}$$

For the right-handed quarks the amplitudes are isospin singlet so that the $SU(2)$ and Y contributions factorize with respect to the Born amplitude:

$$\mathcal{A}^{(1)}_{Lq_+,i} = \left[\frac{\gamma^{(1)}_{Lq_+,i}}{2}\, L^2 + \left(\zeta^{(1)}_{Lq_+,i} + \xi^{(1)}_{Lq_+,i} + \chi^{(1)}_{Lq_+,i}\right) L + \Delta^{(1)}_{Lq_+,i}\right] \mathcal{A}^{(0)}_{Lq_+}, \qquad i = SU(2), Y. \tag{59}$$

Longitudinal W, right-handed quarks, SU(2) component:

$$\begin{aligned}
\gamma^{(1)}_{Lq_+,SU(2)} &= -\frac{3}{2}, \qquad \zeta^{(1)}_{Lq_+,SU(2)} = 3 - \frac{3}{2}w_t, \qquad \chi^{(1)}_{Lq_+,SU(2)} = 0, \qquad \xi^{(1)}_{Lq_+,SU(2)} = \frac{1}{2}L_Z, \\
\Delta^{(1)}_{Lq_+,SU(2)} &= -17w_Z + \frac{27}{4}w_Z^2 - w_H + \frac{1}{4}w_H^2 + \frac{3}{2}w_t - \frac{3}{2}w_t^2 + \frac{5}{6} + \frac{1}{6}\pi^2 + \frac{3}{2}\left(1-w_t^2\right)w_t L_{tW} + \frac{3}{2}w_t L_t \\
&\quad -\frac{1}{4}\left(10+39w_Z - \frac{117}{2}w_Z^2 + \frac{27}{2}w_Z^3\right)L_Z + \frac{1}{2}\left(1-3w_H + \frac{5}{4}w_H^2 - \frac{1}{4}w_H^3\right)L_H + \frac{1}{4}L^2_{\chi_Z} + \frac{1}{2}L^2_{\chi_H} \\
&\quad +\left[\frac{9}{2}\frac{1}{\beta_Z} + \left(\frac{1}{4} + \frac{63}{8}w_Z - \frac{27}{8}w_Z^2\right)w_Z\beta_Z\right]L_{\chi_Z} - \left[\frac{1}{2}\frac{1}{\beta_H} + \left(1 - \frac{3}{8}w_H + \frac{1}{8}w_H^2\right)w_H\beta_H\right]L_{\chi_H}.
\end{aligned} \tag{60}$$

Longitudinal W, right-handed quarks, Y component:

$$\begin{aligned}
\gamma^{(1)}_{Lu_+,Y} &= -\frac{25}{18}, \qquad \zeta^{(1)}_{Lu_+,Y} = \frac{7}{3}, \qquad \chi^{(1)}_{Lu_+,Y} = -\frac{4}{3}L_{ut}, \qquad \xi^{(1)}_{Lu_+,Y} = \frac{25}{18}L_Z, \\
\Delta^{(1)}_{Lu_+,Y} &= -\frac{1}{3}L_- + \frac{4}{3}L_{ut}L_Z - \frac{209}{18} - \frac{25}{54}\pi^2 - \frac{1}{2}w_Z - \left(\frac{5}{6} - \frac{7}{4}w_Z + \frac{1}{4}w_Z^2\right)L_Z - \frac{4}{9}L_Z^2 \\
&\quad -\frac{1}{4}L^2_{\chi_Z} - \frac{1}{4}\left[\frac{2}{\beta_Z} + \left(5-w_Z\right)w_Z\beta_Z\right]L_{\chi_Z}, \\
\gamma^{(1)}_{Ld_+,Y} &= -\frac{13}{18}, \qquad \zeta^{(1)}_{Ld_+,Y} = \frac{4}{3}, \qquad \chi^{(1)}_{Ld_+,Y} = \frac{2}{3}L_{ut}, \qquad \xi^{(1)}_{u_+,L,Y} = \frac{13}{18}L_Z, \\
\Delta^{(1)}_{Ld_+,Y} &= \frac{1}{6}L_- - \frac{2}{3}L_{ut}L_Z - \frac{94}{9} + \frac{13}{54}\pi^2 - \frac{41}{6}i\pi - \frac{1}{2}w_Z + \left(\frac{1}{6} - \frac{7}{4}w_Z + \frac{1}{4}w_Z^2\right)L_Z - \frac{1}{9}L_Z^2 \\
&\quad -\frac{1}{4}L^2_{\chi_Z} - \frac{1}{4}\left[\frac{2}{\beta_Z} + \left(5-w_Z\right)w_Z\beta_Z\right]L_{\chi_Z}.
\end{aligned} \tag{61}$$

The NNLL two-loop amplitudes are obtained by using Eqs.(9,44,48):

$$\begin{aligned}
\mathcal{B}^{(2)}_{Tq_-} &= \frac{\gamma^{(1)}_{Tq_-}}{2}\mathcal{B}^{(1)}_{Tq_-}\, L^2, \\
\mathcal{A}^{(2)}_{Pq_\mp} &= \Bigg\{\frac{1}{8}\left(\gamma^{(1)}_{Pq_\mp}\right)^2 L^4 + \left[\frac{\gamma^{(1)}_{Pq_\mp}}{2}\left(\zeta^{(1)}_{Pq_\mp} + \xi^{(1)}_{Pq_\mp} + \chi^{(1)}_{Pq_\mp}\right) - \frac{1}{6}[\beta\gamma]^{(1)}_{Pq_\mp}\right]L^3 + \frac{1}{2}\Bigg[\gamma^{(2)}_{Pq_\mp} + \left(\zeta^{(1)}_{Pq_\mp} + \xi^{(1)}_{Pq_\mp}\right)^2 \\
&\quad + 2\left(\zeta^{(1)}_{Pq_\mp} + \xi^{(1)}_{Pq_\mp}\right)\chi^{(1)}_{Pq_\mp} + [\chi^2]^{(1)}_{Pq_\mp} - [\beta\zeta]^{(1)}_{Pq_\mp} - [\beta\chi]^{(1)}_{Pq_\mp} + \Delta^{Yuk}_{Pq_\mp} + \gamma^{(1)}_{Pq_\mp}\Delta^{(1)}_{Pq_\mp}\Bigg]L^2\Bigg\}\mathcal{A}^{(0)}_{Pq_\mp} \\
&\quad + \left\{\frac{\gamma^{(1)}_{Pq_\mp}}{2}\bar{\chi}^{(1)}_{Pq_\mp}L^3 + \frac{1}{2}\left[2\left(\zeta^{(1)}_{Pq_\mp} + \xi^{(1)}_{Pq_\mp}\right)\bar{\chi}^{(1)}_{Pq_\mp} + [\bar{\chi}^2]^{(1)}_{Pq_\mp} - [\beta\bar{\chi}]^{(1)}_{Pq_\mp} + \gamma^{(1)}_{Pq_\mp}\bar{\Delta}^{(1)}_{Pq_\mp}\right]L^2\right\}\bar{\mathcal{A}}^{(0)}_{Pq_\mp}
\end{aligned} \tag{62}$$

where

$$f^{(1)}_{Pq_\mp} = f^{(1)}_{Pq_\mp,SU(2)} + t^2_W f^{(1)}_{Pq_\mp,Y} - s^2_W f^{(1)}_{q,QED}, \qquad f=\gamma,\zeta,\xi,\chi,\Delta; \qquad \bar\chi^{(1)}_{Pq_-} = \bar\chi^{(1)}_{Pq_-,SU(2)}, \qquad \bar\chi^{(1)}_{Pq_+} = 0,$$

$$[\beta f]^{(1)}_{Pq_\mp} = \frac{19}{6} f^{(1)}_{Pq_\mp,SU(2)} - \frac{41}{6} t^4_W f^{(1)}_{Pq_\mp,Y} + \frac{80}{9} s^4_W f^{(1)}_{q,QED}, \quad f=\gamma,\zeta,\chi; \quad [\beta\bar\chi]^{(1)}_{Pq_-} = \frac{19}{6}\bar\chi^{(1)}_{Pq_-,SU(2)}, \quad [\beta\bar\chi]^{(1)}_{Pq_+} = 0,$$

$$\bar\Delta^{(1)}_{Lq_-} = \bar\Delta^{(1)}_{Lq_-,SU(2)}, \qquad \bar\Delta^{(1)}_{Tq_-} = \bar\Delta^{(1)}_{Lq_+} = 0, \qquad \Delta^{Yuk}_{Tq_-} = 0; \qquad \Delta^{Yuk}_{Lq_\pm} = -\frac{1}{4}w_t - 3w_t^2, \tag{63}$$

with $P = T, L$ and $q = u, d$. Note that the Yukawa contribution of Eq.(18) is partially contained already in $\zeta^{(1)}_{L,SU(2)}$ and the remaining piece is given by $\Delta^{Yuk}_{Lq_-}$. The square of the matrix $\boldsymbol{\chi}$ gives the following contribution:

$$[\chi^2]^{(1)}_{Pq_\mp} = [\chi^2]^{(1)}_{Pq_\mp,SU(2)} + 2\chi^{(1)}_{Pq_\mp,SU(2)}\left(t^2_W \chi^{(1)}_{Pq_\mp,Y} - s^2_W \chi^{(1)}_{q,QED}\right) + \left(t^2_W \chi^{(1)}_{Pq_\mp,Y} - s^2_W \chi^{(1)}_{q,QED}\right)^2$$

$$[\chi^2]^{(1)}_{Tu_-,SU(2)} = 16L^2_{us} - 12L_{ts}L_{us} + 4L^2_{ts}, \qquad [\chi^2]^{(1)}_{Td_-,SU(2)} = 4L^2_{us} - 12L_{ts}L_{us} + 16L^2_{ts},$$

$$[\chi^2]^{(1)}_{Lu_-,SU(2)} = 9L^2_{us} - 2L_{ts}L_{us} + L^2_{ts}, \qquad [\chi^2]^{(1)}_{Ld_-,SU(2)} = L^2_{us} - 2L_{ts}L_{us} + 9L^2_{ts}, \qquad [\chi^2]^{(1)}_{Lq_+,SU(2)} = 0,$$

$$[\bar\chi^2]^{(1)}_{Pq_\mp} = [\bar\chi^2]^{(1)}_{Pq_\mp,SU(2)} + 2\bar\chi^{(1)}_{Pq_\mp,SU(2)}\left(t^2_W \chi^{(1)}_{Pq_\mp,Y} - s^2_W \chi^{(1)}_{q,QED}\right)$$

$$[\bar\chi^2]^{(1)}_{Tu_-,SU(2)} = -4L^2_{us} - 4L_{ts}L_{us}, \qquad [\bar\chi^2]^{(1)}_{Td_-,SU(2)} = -4L^2_{ts} - 4L_{ts}L_{us},$$

$$[\bar\chi^2]^{(1)}_{Lu_-,SU(2)} = -4L^2_{us} - 4L_{ts}L_{us}, \qquad [\bar\chi^2]^{(1)}_{Ld_-,SU(2)} = -4L^2_{ts} - 4L_{ts}L_{us}, \qquad [\bar\chi^2]^{(1)}_{Lq_+,SU(2)} = 0. \tag{64}$$

Finally, the pure two-loop quantities $\gamma^{(2)}_{Pq_\mp}$ are given by:

$$\gamma^{(2)}_{Pq_\pm} = \gamma^{(2)}_{Pq_\pm,SU(2)} + t^4_W \gamma^{(2)}_{Pq_\pm,Y} - s^4_W \gamma^{(2)}_{q,QED}, \qquad \gamma^{(2)}_{u,QED} = \frac{10400}{243}, \qquad \gamma^{(2)}_{d,QED} = \frac{8000}{243},$$

$$\gamma^{(2)}_{Tq_-,SU(2)} = -\frac{385}{9} + \frac{11}{3}\pi^2, \quad \gamma^{(2)}_{Tq_-,Y} = \frac{52}{81}, \qquad \gamma^{(2)}_{Lq_-,SU(2)} = -\frac{70}{3} + 2\pi^2, \quad \gamma^{(2)}_{Lq_-,Y} = \frac{520}{81},$$

$$\gamma^{(2)}_{Lq_+,SU(2)} = -\frac{35}{3} + \pi^2, \qquad \gamma^{(2)}_{Lu_+,Y} = \frac{1300}{81}, \qquad \gamma^{(2)}_{Ld_+,Y} = \frac{676}{81}. \tag{65}$$

Throughout this section the following notations have been used

$$L = \ln\frac{\hat s}{M_W^2} - i\pi, \qquad L_{ts} = \ln\frac{-\hat t}{\hat s} + i\pi, \qquad L_{us} = \ln\frac{-\hat u}{\hat s} + i\pi, \qquad L_{ut} = \ln\frac{\hat u}{\hat t}, \qquad L_{tu} = \ln\frac{\hat t}{\hat u},$$

$$L_\pm = \frac{-\hat s}{\hat t}\left(L^2_{us} + \pi^2\right) \pm \frac{-\hat s}{\hat u}\left(L^2_{ts} + \pi^2\right), \qquad L_{tW} = \ln\left(1 - \frac{M_W^2}{m_t^2}\right), \qquad L_\gamma = \ln\frac{\lambda^2}{M_Z^2},$$

$$L_i = \ln\frac{M_i^2}{M_W^2}, \qquad L_{\chi_i} = \ln\frac{1-\beta_i}{1+\beta_i}, \qquad \beta_i = -i\sqrt{4\frac{M_W^2}{M_i^2} - 1}, \qquad w_i = \frac{M_i^2}{M_W^2}, \qquad i = Z, H, t. \tag{66}$$

Appendix B

In this appendix we present the result for the one and two-loop corrections to the partonic cross section in NNLL approximation. The differential cross sections are obtained from the amplitudes given in the previous appendix through the relations:

$$\frac{d\,\hat\sigma_{Tq_-}}{dc_\theta}=\frac{N_c}{32\pi\hat s}\overline{\sum_{SPIN}}\ \overline{\sum_{\kappa=\pm1}}\,\left|A_{Tq_-}\right|^2=\frac{\hat s\,s_\theta^2}{256\pi}\left[\left|\mathcal{A}_{Tq_-}\right|^2(1+c_\theta^2)-\mathrm{Re}\Big(\mathcal{A}_{Tq_-}\mathcal{B}^*_{Tq_-}\Big)\frac{\hat s}{2}s_\theta^2\,c_\theta+\left|\mathcal{B}_{Tq_-}\right|^2\frac{\hat s^2}{16}\,s_\theta^4\right],$$
$$\frac{d\,\hat\sigma_{Lq_\mp}}{dc_\theta}=\frac{N_c}{32\pi\hat s}\overline{\sum_{SPIN}}\left|A_{Lq_\mp}\right|^2=\frac{\hat s\,s_\theta^2}{128\pi}\left|\mathcal{A}_{Lq_\mp}\right|^2,\qquad\qquad s_\theta=\sin\theta,\quad c_\theta=\cos\theta. \tag{67}$$

The perturbative series Eq.(24) for the cross section takes the form

$$\frac{d\,\hat\sigma_{Pq_\mp}}{dc_\theta}=\left[1+\left(\frac{\alpha}{4\pi}\right)\delta^{(1)}_{Pq_\mp}+\left(\frac{\alpha}{4\pi}\right)^2\delta^{(2)}_{Pq_\mp}+\ldots\right]\frac{d\,\hat\sigma^{(0)}_{Pq_\mp}}{dc_\theta}.\qquad\qquad \alpha=\frac{\alpha_e}{s_W^2},$$
$$\frac{d\,\hat\sigma^{(0)}_{Tq_-}}{dc_\theta}=N_c\frac{\alpha^2\pi}{16}\,\hat s\,s_\theta^2\,(1+c_\theta^2)\,\left|\mathcal{A}^{(0)}_{Tq_-}\right|^2,\qquad\qquad \frac{d\,\hat\sigma^{(0)}_{Lq_\mp}}{dc_\theta}=N_c\frac{\alpha^2\pi}{8}\,\hat s\,s_\theta^2\,\left|\mathcal{A}^{(0)}_{Lq_\mp}\right|^2. \tag{68}$$

We expand the corrections terms $\delta^{(n)}_{Pq_\mp}$ in powers of the large logarithm $\mathcal{L}=\ln(\hat s/M_W^2)$

$$\delta^{(1)}_{Pq_\mp}=a^{(1)}_{Pq_\mp}\,\mathcal{L}^2+b^{(1)}_{Pq_\mp}\,\mathcal{L}+c^{(1)}_{Pq_\mp},\qquad\qquad \delta^{(2)}_{Pq_\mp}=a^{(2)}_{Pq_\mp}\,\mathcal{L}^4+b^{(2)}_{Pq_\mp}\,\mathcal{L}^3+c^{(2)}_{Pq_\mp}\,\mathcal{L}^2. \tag{69}$$

Numerically for the one-loop coefficients we get

$$a^{(1)}_{Tu_-}=-4.85\,,\qquad\qquad b^{(1)}_{Tu_-}=\Big(-6.77+4\frac{\hat u}{\hat t}\Big)l_u+2.77\,l_t+4.86\,,$$
$$c^{(1)}_{Tu_-}=\Big(-2.48+1.55\frac{\hat u}{\hat t}-2.48\frac{\hat u^2}{\hat t^2}\Big)l_u^2+3\,l_t^2-4\frac{s}{t}\,l_u\,l_t+\Big(5.25-4.97\frac{\hat u}{\hat t}\Big)l_u-0.70\,l_t-3.24$$
$$+\frac{(\hat t-\hat u)\,\hat u}{\hat u^2+\hat t^2}\left[\Big(4.03\frac{\hat u}{\hat t}-2.48\frac{\hat u^2}{\hat t^2}\Big)l_u^2-3\frac{\hat t}{\hat u}\,l_t^2+\Big(4.55-4.97\frac{\hat u}{\hat t}\Big)l_u-2.48\right];$$

$$a^{(1)}_{Td_-}=-5.00\,,\qquad\qquad b^{(1)}_{Td_-}=\Big(-7.38+4\frac{\hat t}{\hat u}\Big)l_t+3.38\,l_u+5.40\,,$$
$$c^{(1)}_{Td_-}=\Big(-2.48+1.55\frac{\hat t}{\hat u}-2.48\frac{\hat t^2}{\hat u^2}\Big)l_t^2+3\,l_u^2-4\frac{s}{u}\,l_u\,l_t+\Big(5.40-4.97\frac{\hat t}{\hat u}\Big)l_t-0.85\,l_u-3.37$$
$$+\frac{(\hat u-\hat t)\,\hat t}{\hat u^2+\hat t^2}\left[\Big(4.03\frac{\hat t}{\hat u}-2.48\frac{\hat t^2}{\hat u^2}\Big)l_t^2-3\frac{\hat u}{\hat t}\,l_u^2+\Big(4.55-4.97\frac{\hat t}{\hat u}\Big)l_t-2.48\right];$$

$$a^{(1)}_{Lu_-}=-2.50\,,\qquad\qquad b^{(1)}_{Lu_-}=-4.97\,l_u+0.97\,l_t-2.81\,,$$
$$c^{(1)}_{Lu_-}=1.55\frac{s}{t}\,l_u^2-0.55\frac{s}{u}\,l_t^2+0.24\,l_u-0.24\,l_t+70.47\,;$$

$$a^{(1)}_{Ld_-}=-2.65\,,\qquad\qquad b^{(1)}_{Ld_-}=-5.18\,l_t+1.18\,l_u-2.27\,,$$
$$c^{(1)}_{Ld_-}=1.45\frac{s}{u}\,l_t^2-0.45\frac{s}{t}\,l_u^2+0.30\,l_t-0.30\,l_u+1.01\,;$$

$$
\begin{aligned}
a^{(1)}_{Lu_+} &= -1.25\,, & b^{(1)}_{Lu_+} &= -0.43\, l_t + 0.43\, l_u - 6.69\,, \\
c^{(1)}_{Lu_+} &= 0.20\frac{s}{t}\, l_u^2 - 0.20\frac{s}{u}\, l_t^2 - 0.11\, l_u + 0.11\, l_t + 46.23\,;
\end{aligned}
$$

$$
\begin{aligned}
a^{(1)}_{Ld_+} &= -1.20\,, & b^{(1)}_{Ld_+} &= 0.22\, l_t - 0.22\, l_u - 6.85\,, \\
c^{(1)}_{Ld_+} &= 0.10\frac{s}{u}\, l_t^2 - 0.10\frac{s}{t}\, l_u^2 - 0.05\, l_t + 0.05\, l_u + 46.31\,.
\end{aligned}
\tag{70}
$$

The two-loop coefficients read

$$
\begin{aligned}
a^{(2)}_{Tu_-} &= 11.76\,, \qquad b^{(2)}_{Tu_-} = \left(32.82 - 19.40\frac{\hat{u}}{\hat{t}}\right) l_u - 13.42\, l_t - 17.34\,, \\
c^{(2)}_{Tu_-} &= \left(34.95 - 22.59\frac{\hat{u}}{\hat{t}} + 16.04\frac{\hat{u}^2}{\hat{t}^2}\right) l_u^2 - 10.72\, l_t^2 + \left(-34.13 - 20.33\frac{\hat{u}}{\hat{t}}\right) l_u\, l_t \\
&\quad + \left(-44.44 + 37.21\frac{\hat{u}}{\hat{t}}\right) l_u + 9.24\, l_t - 28.46 + 39.48\frac{\hat{u}^2}{\hat{t}^2} \\
&\quad + \frac{(\hat{t}-\hat{u})\,\hat{u}}{\hat{u}^2+\hat{t}^2}\left[\left(-19.56\frac{\hat{u}}{\hat{t}} + 12.04\frac{\hat{u}^2}{\hat{t}^2}\right) l_u^2 + 14.55\frac{\hat{t}}{\hat{u}}\, l_t^2 + \left(-22.06 + 24.08\frac{\hat{u}}{\hat{t}}\right) l_u + 12.04\right];
\end{aligned}
$$

$$
\begin{aligned}
a^{(2)}_{Td_-} &= 12.52\,, \qquad b^{(2)}_{Td_-} = \left(36.94 - 20.01\frac{\hat{t}}{\hat{u}}\right) l_t - 16.93\, l_u - 20.89\,, \\
c^{(2)}_{Td_-} &= \left(39.69 - 25.29\frac{\hat{t}}{\hat{u}} + 16.42\frac{\hat{t}^2}{\hat{u}^2}\right) l_t^2 - 9.28\, l_u^2 + \left(-41.00 - 18.48\frac{\hat{t}}{\hat{u}}\right) l_u\, l_t \\
&\quad + \left(-53.63 + 40.13\frac{\hat{t}}{\hat{u}}\right) l_t + 15.58\, l_u - 23.56 + 39.48\frac{\hat{t}^2}{\hat{u}^2} \\
&\quad + \frac{(\hat{u}-\hat{t})\,\hat{t}}{\hat{u}^2+\hat{t}^2}\left[\left(-20.18\frac{\hat{t}}{\hat{u}} + 12.42\frac{\hat{t}^2}{\hat{u}^2}\right) l_t^2 + 15.01\frac{\hat{u}}{\hat{t}}\, l_u^2 + \left(-22.77 + 24.85\frac{\hat{t}}{\hat{u}}\right) l_t + 12.42\right];
\end{aligned}
$$

$$
\begin{aligned}
a^{(2)}_{Lu_-} &= 3.12\,, \qquad b^{(2)}_{Lu_-} = 12.42\, l_u - 2.42\, l_t + 10.54\,, \\
c^{(2)}_{Lu_-} &= \left(-3.87\frac{s}{t} + 12.34\right) l_u^2 + \left(1.37\frac{s}{u} + 0.47\right) l_t^2 - 0.81\, l_u\, l_t + 23.93\, l_u - 6.34\, l_t - 274.80\,;
\end{aligned}
$$

$$
\begin{aligned}
a^{(2)}_{Ld_-} &= 3.52\,, \qquad b^{(2)}_{Ld_-} = 13.76\, l_t - 3.14\, l_u + 9.44\,, \\
c^{(2)}_{Ld_-} &= \left(-3.85\frac{s}{u} + 13.44\right) l_t^2 + \left(1.19\frac{s}{t} + 0.70\right) l_u^2 - 2.14\, l_t\, l_u + 21.33\, l_t - 5.91\, l_u - 101.71\,;
\end{aligned}
$$

$$
\begin{aligned}
a^{(2)}_{Lu_+} &= 0.78\,, \qquad b^{(2)}_{Lu_+} = 0.54\, l_t - 0.54\, l_u + 10.11\,, \\
c^{(2)}_{Lu_+} &= \left(0.34 + 0.25\frac{\hat{u}}{\hat{t}}\right) l_u^2 + \left(-0.16 - 0.25\frac{\hat{t}}{\hat{u}}\right) l_t^2 - 0.19\, l_u\, l_t - 2.30\, l_u + 2.30\, l_t - 88.00\,;
\end{aligned}
$$

$$
\begin{aligned}
a^{(2)}_{Ld_+} &= 0.72\,, \qquad b^{(2)}_{Ld_+} = -0.26\, l_t + 0.26\, l_u + 10.03\,, \\
c^{(2)}_{Ld_+} &= \left(0.14 + 0.12\frac{\hat{t}}{\hat{u}}\right) l_t^2 + \left(-0.10 - 0.12\frac{\hat{u}}{\hat{t}}\right) l_u^2 - 0.05\, l_u\, l_t - 1.19\, l_t + 1.19\, l_u - 85.17\,.
\end{aligned}
\tag{71}
$$

Here $l_u = \ln(-\hat{u}/\hat{s})$ and $l_t = \ln(-\hat{t}/\hat{s})$.

Appendix C

In Ref. [21] the contribution of the imaginary part of the anomalous dimension matrix $\chi_T^{(1)}$ (given in Eq. (55) above) has been missed in the numerical estimates. This contribution changes the NNLL two-loop correction in the transverse boson production cross section. It results in an additional term

$$4\pi^2 \frac{x_-^2 - x_+^2}{x_+^2}$$

in the coefficient of the quadratic logarithm in Eqs. (14, 33) of Ref. [21].

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