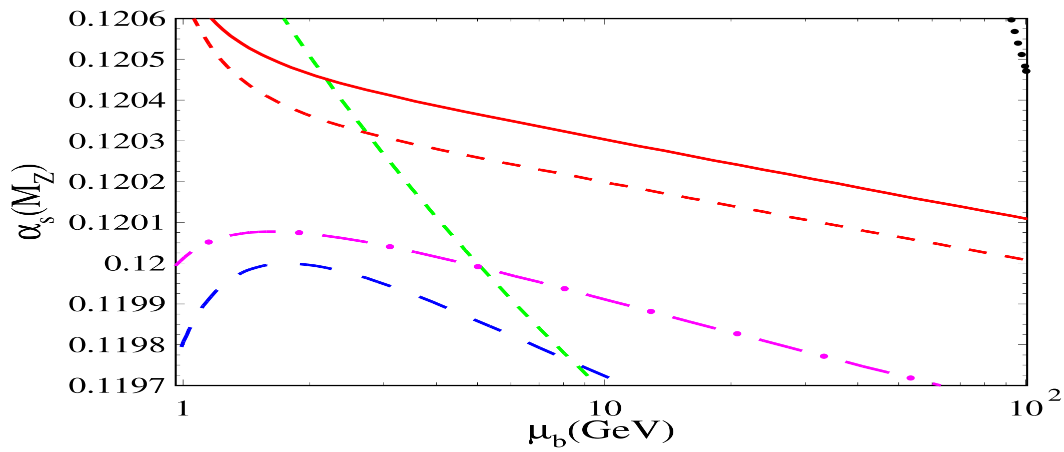
Running and decoupling of α_s

Matthias Steinhauser

Karlsruhe Institute of Technology

The β function governs the renormalization scale dependence of the strong coupling and is known to four-loop accuracy since almost 14 years [1][2]. There are different ways to solve the corresponding differential equation. The prefered method is the numerical solution with truncation of $\beta(\alpha_s)$ at the desired order. There are also several approximate (analytical) expressions, e.g., the one based on the iterative (perturbative) solution where the result for $\alpha_s(\mu)$ is given as an expansion in $1/L = 1/\ln(\mu^2/\Lambda^2)$ [3]. This formula should be used with care, in particular for small renormalization scales μ. If one considers, e.g., $\mu = M_\tau$ one observes a shift of $+0.004$ after including the four-loop corrections and negative shift of approximately the same order of magnitude at five-loop level. These numbers have to be compared with the current experimental precision which is cited as ± 0.005 in Ref. [4] (see also the other contributions on α_s from τ decays in these proceedings).

Next to the running itself also the decoupling of heavy quarks form the running of the strong coupling constant is a crucial ingredient of the precision determination of α_s. Every time a heavy quark threshold is crossed one has to apply the decoupling constants which relate α_s with n_f active quark flavours, usually denoted by $\alpha_s^{(n_f)}$, to the coupling with only $n_f - 1$ active quark flavours. The decoupling constants are obtained by matching n_f-flavour QCD to the effective theory with the number of quarks equal to $n_f - 1$. The theoretical framework for the calculation of the decoupling constants has been set up in Ref. [5] where formulae are given relating l-loop corrections to l-loop vacuum integrals.



As a consequence of the decoupling relations $\alpha_s(\mu)$ is not a continuous function of μ but has finite steps at the energy scale where the heavy quark is integrated out, $\mu_{\rm dec}$. This energy is not fixed by theory, should, however, be in the vicinity of the heavy quark mass. On general grounds the dependence on $\mu_{\rm dec}$ should become weaker if higher order perturbative corrections are included in the analysis. This is demonstrated in the figure above where $\alpha_s^{(5)}(M_Z)$ is computed using $\alpha_s^{(3)}(M_\tau)$ as a starting point. The decoupling of the charm quark is performed at the fixed scale $\mu_c = 3$ GeV and the decoupling scale of the the bottom quark μ_b is varied in the broad range between 1 GeV and 100 GeV. N-loop running goes along with $(N-1)$-loop decoupling. Results are shown for $N = 1$ (upper right dotted line), $N = 2$ (steep dashed line), $N = 3$ (lower dashed line) and $N = 4$ (dash-dotted line). One observes a dramatic reduction of the μ_b dependence with increasing N resulting in a quite flat four-loop result (Note that the scale on the ordinate only varies by 0.0009.).

For comparison we show in the figure two more curves which correspond to $N = 5$. They incorporate the four-loop decoupling relations [6][7]. For the unknown five-loop coefficient of the β function we have chosen $\beta_4 = 0$ (solid line) and $\beta_4 = 150$ (dashed line parallel to the solid one; the normalization corresponding to $\{\beta_0, \beta_1, \beta_2, \beta_3\} \approx \{1.92, 2.42, 2.83, 18.85\}$ has been chosen).

From the figure above it is possible to estimate an uncertainty on $\alpha_s^{(5)}(M_Z)$ as obtained from $\alpha_s^{(3)}(M_\tau)$ due to missing higher order corrections. If we restrict ourselves to a range of μ_b between 2 GeV and 10 GeV and take the difference between the three- and four-loop curve as an estimate for the uncertainty we obtain $\delta\alpha_s^{(5)}(M_Z) \approx 0.0002$. The difference between the four- and five-loop (dashed) curve would even lead to $\delta\alpha_s^{(5)}(M_Z) \approx 0.0003$. The variation of $\alpha_s^{(5)}(M_Z)$ due to the variation of μ_b leads to an additional uncertainty of $\delta\alpha_s^{(5)}(M_Z) \approx 0.0002$. A similar uncertainty is obtained from the variation of μ_c between 2 GeV and 5 GeV. (This can easily be checked with the program `RunDec` [8].) Thus a total uncertainty of ± 0.0004 (obtained by adding the three uncertainties in quadrature) should be assigned to $\alpha_s^{(5)}(M_Z)$. The uncertainties induced by the errors in the quark masses are much smaller.

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