TTP11-03
TUM-HEP-792/11
February 2011

Numerical updates of lifetimes and mixing parameters of B mesons

Alexander Lenz

Physik Department
Technische Universität München
James-Franck-Str, D-85748 Garching, Germany

and

Ulrich Nierste

Institut für Theoretische Teilchenphysik
Karlsruhe Institute of Technology, Universität Karlsruhe
Engesserstraße 7, 76128 Karlsruhe, Germany

We update the Standard-Model predictions for several quantities related to $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ and $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing. The mass and width differences in the B_s system read $\Delta M_s^{\mathrm{SM}} = (17.3 \pm 2.6)\,\mathrm{ps}^{-1}$ and $\Delta\Gamma_s^{\mathrm{SM}} = (0.087 \pm 0.021)\,\mathrm{ps}^{-1}$, respectively. The CP asymmetries in flavour-specific decays are $a_{\mathrm{fs}}^{s,\mathrm{SM}} = (1.9{\pm}0.3)\cdot 10^{-5}$ and $a_{\mathrm{fs}}^{d,\mathrm{SM}} = -(4.1{\pm}0.6)\cdot 10^{-4}$. We further critically discuss the sensitivity of $\Delta\Gamma_d$ to new physics and the uncertainties in the relation between $\Delta\Gamma_s$ and the branching fraction of $\overset{(-)}{B}_s \to D_s^{(*)+} D_s^{(*)-}$. Then we present a numerical update of the average width Γ_s in the B_s system and correlate Γ_s with the B^+–B_d lifetime ratio. Finally we summarise the key results of our recent global analysis with the CKMfitter collaboration addressing new physics in $\mathrm{B}-\overline{\mathrm{B}}$ mixing. In an appropriately defined scenario parametrising new physics in $\mathrm{B}-\overline{\mathrm{B}}$ mixing by two complex parameters Δ_d and Δ_s the Standard-Model point $\Delta_d = \Delta_s = 1$ is disfavoured by 3.6 standard deviations.

PRESENTED AT

6th International Workshop on the CKM Unitarity Triangle
Warwick, United Kingdom, September 6-10, 2010

1 Introduction

On May 14, 2010, the DØ collaboration has reported evidence for an anomalous like-sign dimuon charge asymmetry $A_{\rm SL}$ in the decays of neutral B mesons [1]. The presented result corresponds to a data set composed of B_d and B_s mesons and quantifies CP violation in the $\rm B_d\!-\!\overline{B}_d$ and $\rm B_s\!-\!\overline{B}_s$ mixing amplitudes. Expressed in terms of the CP asymmetries $a_{\rm fs}^{d,s}$ in flavour-specific $B_{d,s}$ decays the measured quantity reads

$$A_{\rm SL} \;=\; (0.506 \pm 0.043) a_{\rm fs}^d + (0.494 \pm 0.043) a_{\rm fs}^s. \qquad (1)$$

The index SL refers to the use of semileptonic decays in the measurement. The DØ result $A_{\rm SL}^{\rm D\O} = -0.00957 \pm 0.00251 \pm 0.00146$ deviates from the Standard-Model (SM) prediction of Ref. [2], $A_{\rm SL} = \left(-0.23^{+0.05}_{-0.06}\right) \cdot 10^{-3}$, with a statistical significance of 3.2 standard deviations and the central value is off by a factor of 42. With the direct and indirect knowledge on $a_{\rm fs}^d$ from B factory data (which still leaves room for sizeable new physics contributions) any theoretical explanation of the DØ measurement necessarily requires large new physics contributions to the $\rm B_s\!-\!\overline{B}_s$ mixing amplitude. This contribution must be similar in magnitude to the SM contribution while having a very different phase.

$\rm B_q\!-\!\overline{B}_q$ mixing is described by two hermitian 2×2 matrices, the mass matrix M^q and the decay matrix Γ^q. By diagonalising $M^q - i\Gamma^q/2$ one determines the mass eigenstates $|B_q^H\rangle$ and $|B_q^L\rangle$ (with "H" and "L" denoting "heavy" and "light") as linear combinations of the flavour eigenstates $|B_q\rangle$ and $|\overline{B}_q\rangle$. The average mass of the two eigenstates is $M_{B_q} = M_{11}^q = M_{22}^q$ and the average width equals $\Gamma_q = \Gamma_{11}^q = \Gamma_{22}^q$. The off-diagonal elements M_{12}^q and Γ_{12}^q lead to $\rm B_q\!-\!\overline{B}_q$ mixing phenomena, namely a mass difference ΔM_q and a width difference $\Delta\Gamma_q$ between the eigenstates B_q^H and B_q^L and further to the CP asymmetry in flavour-specific decays, $a_{\rm fs}^q$. These quantities read

$$\Delta M_q \;=\; M_H^q - M_L^q \;\simeq\; 2|M_{12}^q|\,, \qquad \Delta\Gamma_q \;=\; \Gamma_L^q - \Gamma_H^q \;\simeq\; 2|\Gamma_{12}^q|\cos\phi_q\,,$$
$$a_{\rm fs}^q \;=\; \frac{|\Gamma_{12}^q|}{|M_{12}^q|}\sin\phi_q\,,$$

with the CP-violating phase

$$\phi_q \;\equiv\; \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right). \qquad (2)$$

For pedagogical introductions to $\rm B_q\!-\!\overline{B}_q$ mixing see Refs. [3]. In the SM the CP phases are small, $\phi_d \approx -4.3^\circ$ and $\phi_s \approx 0.22^\circ$, so that $\Delta\Gamma_q^{\rm SM} \simeq 2|\Gamma_{12}^q|$. New physics can affect magnitude and phase of M_{12}^q and ΔM_q and ϕ_q can deviate from their SM predictions substantially.

We present updates of the Standard-Model predictions for $\Delta\Gamma_q$ and the CP asymmetries in flavour-specific decays in Sec. 2. Sec 3 is devoted to Γ_s and in Sec. 4 we discuss the recent analysis of $\mathrm{B}-\overline{\mathrm{B}}$ mixing in Ref. [4], which shows evidence of new physics.

2 Width differences and CP asymmetries

The calculation of Γ_{12}^q uses the heavy quark expansion (HQE), which is an operator product expansion exploiting the hierarchy $m_b \gg \Lambda_{\rm QCD}$. Γ_{12}^q is then predicted as a simultaneous expansion in the two parameters $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$. $\overset{(-)}{B}_q$ decays into final states which are common to B_q and $\overline{B}_q$ contribute to Γ_{12}^q and lead to a width difference between the two eigenstates of the $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ complex. Γ_{12}^s is dominated by the Cabibbo-favoured contribution with a $(c\overline{c})$ pair in these final states. Corrections of order $\Lambda_{\rm QCD}/m_b$ to Γ_{12}^s have been found in Ref. [5], the contributions of order $\alpha_s(m_b)$ have been calculated in Ref. [6] and were confirmed in Ref. [7]. The theory prediction of Γ_{12}^d to NLO in α_s and $1/m_b$ has been obtained in Refs. [7, 8]. In Ref. [2] these NLO results for Γ_{12}^q have been expressed in terms of a new operator basis, which avoids certain numerical cancellations and improves the NLO results by including some color-enhanced α_s/m_b corrections. Further Ref. [2] applies the all-order summation of $\alpha_s^n z \ln^n z$ terms (with $z = m_c^2/m_b^2$ and $n = 1, 2, \ldots$) of Ref [9] to Γ_{12}^q.

In the ratio $|\Gamma_{12}^q|/|M_{12}^{\rm SM,q}|$ hadronic uncertainties cancel to a large extent. In the B_d system on has [2]

$$\frac{|\Gamma_{12}^d|}{|M_{12}^{d,\rm SM}|} = \frac{2|\Gamma_{12}^d|}{|\Delta M_d^{\rm SM}|} = (54 \pm 10) \cdot 10^{-4} \tag{3}$$

In the absence of new physics we can identify $\Delta M_d^{\rm SM}$ with the experimental value $\Delta M_d^{\rm exp} = 0.507\,\mathrm{ps}^{-1}$ to find

$$\left.\frac{\Delta\Gamma_d}{\Gamma_d}\right|_{\rm SM} = (42 \pm 8) \cdot 10^{-4}. \tag{4}$$

Here also $\tau(B_d) = 1/\Gamma_d = (1.525 \pm 0.009)\,\mathrm{ps}$ has been used. In the presence of new physics Eq. (4) changes to

$$\frac{\Delta\Gamma_d}{\Gamma_d} = (45 \pm 10) \cdot 10^{-4} \cdot \cos\phi_d. \tag{5}$$

Different central values in Eqs. (4) and (5) occur, because we do not use $\Delta M_d^{\rm exp}$ in Eq. (5) and ΔM_d is about 7% larger than $\Delta M_d^{\rm exp}$ for our range of the relevant hadronic parameter, $f_{B_d}\sqrt{\mathcal{B}_{B_d}} = (174 \pm 13)$ MeV. Using the 3σ CL range in Tab. 11 of Ref. [4], $-30^\circ \leq \phi_d \leq -1^\circ$, one finds that

$$\frac{\Delta\Gamma_d}{\Gamma_d} = \left(45^{+10}_{-12}\right) \cdot 10^{-4} \tag{6}$$

is fulfilled in any model of new physics which leaves Γ_{12}^d unaffected. Can new physics enhance $\Delta\Gamma_d$ to an observable level? Γ_{12}^d is an inclusive quantity with the three doubly Cabibbo-suppressed contributions $\Gamma_{12}^{d,cc}$, $\Gamma_{12}^{d,uc}$ and $\Gamma_{12}^{d,uu}$. The first contribution stems from the interference of the decay $b \to c\overline{c}d$ of the $\overline{B}_d$ component with the decay $\overline{b} \to \overline{c}c\overline{d}$ of the B_d component of the mixed neutral meson state. This interference is possible, because both components can decay into the same flavourless $c\overline{c}d\overline{d}$ final state. $\Gamma_{12}^{d,uu}$ is the analogue involving $b \to u\overline{u}d$, while $\Gamma_{12}^{d,uc}$ arises from the interference of $b \to c\overline{u}d$ and $\overline{b} \to \overline{u}c\overline{d}$ decays or their charge-conjugate modes. It is difficult to find a model of new physics which can numerically compete with the dominant SM contribution $\Gamma_{12}^{d,cc}$ without violating other experimental constraints. For a recent discussion of the experimental aspects of $\Delta\Gamma_d$ see Ref. [10]. Formulae for the time evolution of B_d decays to flavourless states, which permit the extraction of $\Delta\Gamma_d$, can be found in Refs. [3, 11, 12]. We update ϕ_d and $a_{\rm fs}^d$ below in Eq. (11).

In the B_s system $\Delta\Gamma_s$ is found together with ϕ_s from an angular analysis of $B_s \to J/\psi\phi$ data. The calculated value of $|\Gamma_{12}^s|$ defines the physical "yellow band" in the $(\Delta\Gamma_s, \phi_s)$ plane. In any model of new physics $(\Delta\Gamma_s, \phi_s)$ must lie in this band, because new physics has a negligible impact on Γ_{12}^s which stems from Cabbibo-favoured $b \to c\overline{c}s$ decays. $|\Gamma_{12}^s|$ is proportional to the hadronic parameter $f_{B_s}^2 \mathcal{B}_{B_s}$. Updating our 2006 prediction in Ref. [2] to the 2010 world averages of the input parameters listed in Tabs. 6 and 7 of Ref. [4] we find

$$\begin{aligned} \Delta\Gamma_s^{\rm SM} &\simeq 2|\Gamma_{12}| = \\ &= \left(0.087 \pm 0.015|_{\widetilde{R}_2} \pm 0.012\big|_{f_{B_s}} \pm 0.007|_{\rm scale} \pm 0.007|_{\rm rest}\right) \,{\rm ps}^{-1} \qquad (7) \end{aligned}$$

The three largest sources of uncertainty stem from the matrix element of the operator $\widetilde{R}_2$ which occurs at order $1/m_b$, the decay constant f_{B_s} and the choice of the renormalisation scale (estimating higher-order corrections in α_s). Adding the errors in quadrature yields

$$\Delta\Gamma_s^{\rm SM} \simeq 2|\Gamma_{12}| = (0.087 \pm 0.021)\,{\rm ps}^{-1}. \qquad (8)$$

The uncertainty of the theory prediction has reduced from 41% in 2006 to 24% in 2010 because of an impressive progress in the lattice calculations of $f_{B_s}\sqrt{\mathcal{B}_{B_s}} = 212 \pm 14$ MeV (average from [4] using [13]). The corresponding value used in Ref. [2] was $f_{B_s}\sqrt{\mathcal{B}_{B_s}} = 221 \pm 46$ MeV. The central value has decreased due to the smaller $f_{B_s}\sqrt{\mathcal{B}_{B_s}}$ and slightly smaller values of m_b, V_{cb} and α_s. Eq. (8) implies

$$\frac{\Delta\Gamma_s^{\rm SM}}{\Gamma_s} \simeq \frac{2|\Gamma_{12}|}{\Gamma_s} = 0.133 \pm 0.032. \qquad (9)$$

Here $\Gamma_s = \Gamma_d$ has been used, deviations from this relation are discussed in the next section. If one assumes that there is no new physics in the mixing amplitude, one can determine $\Delta\Gamma_s^{\rm SM}/\Gamma_s$ in a more precise way:*

$$\frac{\Delta\Gamma_s^{\rm SM}}{\Gamma_s} = \frac{\Delta\Gamma_s^{\rm SM}}{\Delta M_s^{\rm SM}} \cdot \Delta M_s^{\rm Exp.} \cdot \tau_{B_d}^{\rm Exp} = 0.137 \pm 0.027. \tag{10}$$

The CP phases read

$$\phi_s^{\rm SM} = 0.22^\circ \pm 0.06^\circ, \qquad \phi_d^{\rm SM} = -4.3^\circ \pm 1.4^\circ. \tag{11}$$

The CP asymmetries in flavour-specific decays read

$$a_{\rm fs}^{s,\rm SM} = (1.9 \pm 0.3) \cdot 10^{-5}, \qquad a_{\rm fs}^{d,\rm SM} = -(4.1 \pm 0.6) \cdot 10^{-4}. \tag{12}$$

In Eqs. (11) and (12) the result $|V_{ub}| = (3.56^{+0.15}_{-0.20}) \cdot 10^{-3}$ of the SM fit in Ref. [4] has been used. If one uses instead the experimental value $|V_{ub}| = (3.92 \pm 0.46) \cdot 10^{-3}$, one finds slightly larger values, e.g. $a_{\rm fs}^{s,\rm SM} = (2.1 \pm 0.4) \cdot 10^{-5}$ and $a_{\rm fs}^{d,\rm SM} = -(4.5 \pm 0.8) \cdot 10^{-4}$. Experiments address different linear combinations of these two CP asymmetries: The DØ collaboration has measured the dimuon asymmetry A_{SL} defined in Eq. (1), while LHCb will measure the difference of these asymmetries. The corresponding Standard-Model predictions read

$$A_{SL}^{\rm SM} = -(2.0 \pm 0.3) \cdot 10^{-4}, \tag{13}$$

$$a_{\rm fs}^{s,\rm SM} - a_{\rm fs}^{d,\rm SM} = (4.3 \pm 0.7) \cdot 10^{-4}. \tag{14}$$

The central values in Eqs. (3)-(14) correspond to a renormalisation scheme using $\overline{\rm MS}$ quark masses and $\overline{z} = \overline{m}_c^2(\overline{m}_b)/\overline{m}_b^2(\overline{m}_b)$. For completeness we also update the NLO prediction [18] for $\Delta M_s^{\rm SM}$:

$$\Delta M_s^{\rm SM} = (17.3 \pm 2.6)\,{\rm ps}^{-1}. \tag{15}$$

The quoted central value corresponds to $f_{B_s} = 231$ MeV and $\mathcal{B}_{B_s} = 0.841$ (see Ref. [4]). Often $\Delta M_s^{\rm SM}$ is expressed in terms of the scheme-independent parameter $\widehat{\mathcal{B}}_{B_s}$ and $\mathcal{B}_{B_s} = 0.841$ translates to $\widehat{\mathcal{B}}_{B_s} = 1.281$.

We finally discuss the width difference between the CP eigenstates defined as

$$|B_{s,\rm CP\pm}\rangle = \frac{|B_s\rangle \mp |\overline{B}_s\rangle}{\sqrt{2}}.$$

The width of the CP-even state exceeds that of the CP-odd state by

$$\Delta\Gamma_{\rm CP} = 2|\Gamma_{12}^s| \tag{16}$$

*In Eq. (3.27) of Ref. [2] we have erroneously used $\tau_{B_s}^{\rm exp}$ instead of $\tau_{B_d}^{\rm exp}$ in the calculation. The quoted number should read $\Delta\Gamma_s \tau_{B_d} = 0.135 \pm 0.026$.

and is unaffected by new physics in M_{12}^s [11]. Aleksan et al. have shown that in the simultaneous limits $m_c \to \infty$, $m_b - 2m_c \to 0$ and an infinite number of colours, $N_c \to \infty$, $\Delta\Gamma_{\rm CP}$ is exhausted by decays into just four final states [14]:

$$2\,B(\overset{(-)}{B}_s \to D_s^{(*)+} D_s^{(*)-}) = \frac{\Delta\Gamma_{\rm CP}}{\Gamma_s}\left[1+\mathcal{O}\left(\frac{\Delta\Gamma}{\Gamma_s}\right)\right]. \qquad (17)$$

On the experimental side we have the Belle measurement [15]

$$\frac{\Delta\Gamma_{\rm CP}}{\Gamma_s} = 0.147^{+0.036}_{-0.030}\Big|_{\rm stat}\,{}^{+0.044}_{-0.042}\Big|_{\rm syst}$$

and the DØ result [16]

$$\frac{\Delta\Gamma_{\rm CP}}{\Gamma_s} = 0.072 \pm 0.030.$$

While it is flattering that the Belle central value exactly conicides with our 2006 prediction for $\Delta\Gamma_{\rm CP}/\Gamma_s$ in Ref. [2], the data are not accurate enough to assess the accuracy of the limits $m_c \to \infty$, $m_b - 2m_c \to 0$ and $N_c \to \infty$ adopted in Ref. [14]. However, the calculated $1/m_b$ corrections are of order 20% [5, 2] and in the NLO result of Refs. [6, 7, 2] the $1/N_c$ terms are non-negligible. Also the deviation of the hadronic "bag" factor $B \simeq 0.85$ from 1 is an $1/N_c$ effect. Therefore one cannot rule out large corrections to Eq. (17), possibly of order 100%. On the experimental side one may look for multi-body $c\bar{c}s\bar{s}$ final states which are absent in the limit $N_c \to \infty$. One ingredient of Eq. (17) is the prediction that $B_{s,{\rm CP}-}$ does not contribute to $\overset{(-)}{B}_s \to D_s^{(*)+} D_s^{(*)-}$. This can be checked by studying the lifetime in these modes, which should then be equal to $1/\Gamma_L^s$ as measured in the CP-even component of $\overset{(-)}{B}_s \to J/\psi\phi$ [11].

3 Average B_s width

We define

$$\tau_{B_s} \equiv \frac{1}{\Gamma_{B_s}}. \qquad (18)$$

Any decay $\overset{(-)}{B}_s \to f$ obeys a two-exponential law, $\Gamma[\overset{(-)}{B}_s \to f, t] \to A_f \exp[-\Gamma_L^s t] + B_f \exp[-\Gamma_H^s t]$. In a flavour-specific decay like $B_s \to X\ell^+\nu_\ell$ or $B_s \to D_s^-\pi^+$ the two coefficients are equal, $A_f = B_f$. Fitting the decay to a single exponential, one determines τ_{B_s} up to a calculable correction of order $\Delta\Gamma^2/\Gamma_s^2$ [17, 11]. If the experimental selection efficiencies vary over the decay length, this method can lead to a bias towards Γ_L^s or Γ_H^s, therefore it is recommended to use the correct two-exponential formulae in the experimental analyses with simultaneous fits to Γ_s and $\Delta\Gamma_s$.

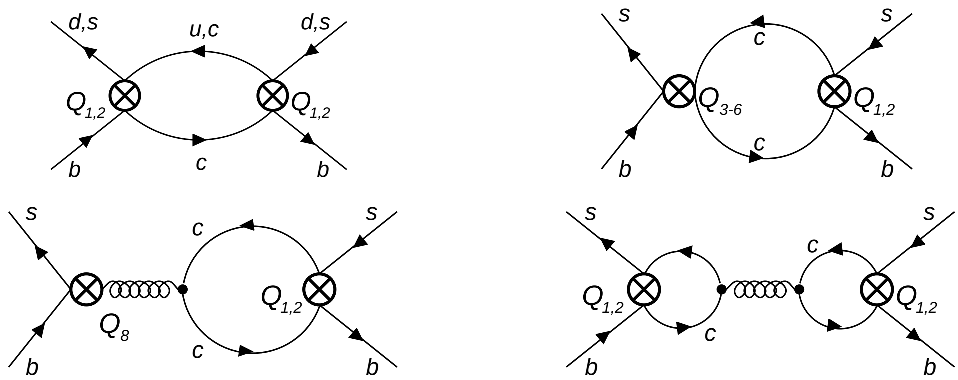

Figure 1: *CKM-favoured weak annihilation diagrams: The upper left diagram contributes nearly equally to τ_{B_s} and τ_{B_d}, the other diagrams involve small penguin coefficients C_{3-6} or are suppressed by α_s.*

We next discuss the ratio $\tau_{B_s}/\tau_{B_d} = \Gamma_d/\Gamma_s$: The deviation of τ_{B_s}/τ_{B_d} from 1 stems from the weak annihilation (WA) diagrams of Fig. 1. The upper left diagram [19, 20, 5] involves the large Wilson coefficients $C_{1,2}$, but this contribution almost cancels from τ_{B_s}/τ_{B_d} up to terms of order $z = m_c^2/m_b^2$ and $1 - f_{B_d}^2 M_{B_d}/f_{B_s}^2 M_{B_s}$. The other diagrams [21] essentially only contribute to τ_{B_s}, but involve small coefficients or a factor of $\alpha_s(m_b)$. Since moreover WA diagrams are individually small, $|\tau_{B_s}/\tau_{B_d} - 1|$ has been estimated to be of order 0.01 or smaller by several authors [19, 20, 5, 21]. (There is also a contribution to τ_{B_s}/τ_{B_d} from $\mathrm{SU(3)_F}$ violation in the kinetic-energy and chromomagnetic operators, which is of order 10^{-3} and negligible [5].) The theory prediction involves four hadronic parameters [20], B_1, B_2, ϵ_1 and ϵ_2, which are multiplied by the B_s meson decay constant f_{B_s}, and further the ratio $f_{B_s}/f_{B_d} = 1.209 \pm 0.007 \pm 0.023$ [4, 22].

We find

$$\frac{\tau_{B_s}}{\tau_{B_d}} - 1 \;=\; 10^{-3} \cdot \left(\frac{f_{B_s}}{231\ \text{MeV}}\right)^2 \Big[(0.77 \pm 0.10) B_1 - (1.00 \pm 0.13) B_2 + (36 \pm 5)\epsilon_1 - (51 \pm 7)\epsilon_2 \Big]. \tag{19}$$

Using the results of the quenched lattice-QCD calculation for $B_{1,2}$ and $\epsilon_{1,2}$ of Ref. [24], $(B_1, B_2, \epsilon_1, \epsilon_2) = (1.10 \pm 0.20,\ 0.79 \pm 0.10,\ -0.02 \pm 0.02,\ 0.03 \pm 0.01)$, we find $-4 \cdot 10^{-3} \leq \frac{\tau_{B_s}}{\tau_{B_d}} - 1 \leq 0$. The lower end of this interval exceeds the HFAG value of $\tau_{B_s}/\tau_{B_d} - 1 = -0.027 \pm 0.015$ [23] by 1.5 standard deviations.

We can improve our prediction by using the information of the lifetime difference between B^+ and B_d meson, which involves the same hadronic parameters, up to $\mathrm{SU(3)_F}$ corrections. Adding $1/m_b$ corrections to the prediction in Ref. [9] and using

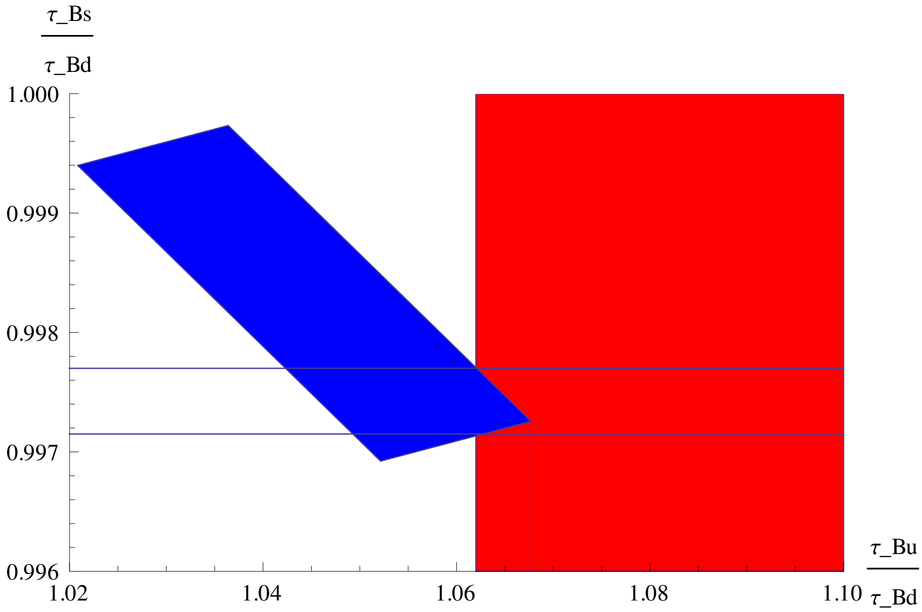

Figure 2: *Allowed range for τ_{B_s}/τ_{B_d} and τ_{B^+}/τ_{B_d}. The predictions for τ_{B^+}/τ_{B_d} and τ_{B_s}/τ_{B_d} (blue region) derived with the decade-old hadronic parameters of Ref. [24] barely overlaps with the experimental 3σ region (red region to the right). This shows the importance of a modern lattice calculation of these parameters.*

the input parameters of Ref. [4] we obtain

$$\begin{aligned}\frac{\tau_{B^+}}{\tau_{B_d}} - 1 \;&=\; 0.0324 \left(\frac{f_B}{200\,\text{MeV}}\right)^2 \Big[(1.0 \pm 0.2)\, B_1 \;+\; (0.1 \pm 0.1)\, B_2 \\ &\qquad -\; (17.8 \pm 0.9)\, \epsilon_1 \;+\; (3.9 \pm 0.2)\, \epsilon_2 - 0.26\Big]. \qquad (20)\end{aligned}$$

The last term are the $1/m_b$ corrections calculated in the vacuum saturation approximation. With the hadronic parameters of Ref. [24] quoted above we correlate τ_{B_s}/τ_{B_d} with τ_{B^+}/τ_{B_d} in Fig. 2. We notice that the experimental range $\tau_{B^+}/\tau_{B_d} = 1.081 \pm 0.006$ [23] prefers a larger deviation of τ_{B_s}/τ_{B_d} from 1. Our analysis omits effects of $\text{SU}(3)_\text{F}$ violation other than $1 - f_{B_d}^2 M_{B_d}/f_{B_s}^2 M_{B_s}$; $\text{SU}(3)_\text{F}$ violation in $B_1, B_2, \epsilon_1, \epsilon_2$ may lower τ_{B_s}/τ_{B_d} a bit further [25]. Most importantly, we observe a tension between theory and experiment in τ_{B^+}/τ_{B_d}. This calls for a new effort in the calculation of B_1, B_2, ϵ_1, and ϵ_2 on the lattice. We further note that with the 2001 values of these hadronic parameters the theory prediction for τ_{B_s}/τ_{B_d} is in better agreement with experiment than that of τ_{B^+}/τ_{B_d}.

4 New physics in $\mathrm{B}-\overline{\mathrm{B}}$ mixing

New physics in B–$\overline{\mathrm{B}}$ mixing can be parametrised in terms of the complex parameters Δ_d and Δ_s defined as [2]

$$\Delta_q \equiv \frac{M_{12}^q}{M_{12}^{\mathrm{SM},q}}, \qquad \Delta_q \equiv |\Delta_q| e^{i\phi_q^\Delta}. \tag{21}$$

The average of the DØ [1] and CDF [26] measurements of A_{SL},

$$A_{\mathrm{SL}} = -0.0085 \pm 0.0028, \tag{22}$$

is 2.9σ away from the SM prediction. Since A_{SL} involves both $\mathrm{B_s}$–$\overline{\mathrm{B}}_{\mathrm{s}}$ mixing and $\mathrm{B_d}$–$\overline{\mathrm{B}}_{\mathrm{d}}$ mixing, a combined analysis of Δ_d and Δ_s is desirable. For the SM prediction of the two key observables related to $\mathrm{B_d}$–$\overline{\mathrm{B}}_{\mathrm{d}}$ mixing, ΔM_d and $A_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_S)$, one needs the information on the apex $(\overline{\rho}, \overline{\eta})$ of the unitarity triangle (UT). $\overline{\rho}$ and $\overline{\eta}$ in turn depend on Δ_d and, through $\Delta M_d/\Delta M_s$, even on $|\Delta_s|$. These interdependences require a joint fit to the CKM elements and Δ_d and Δ_s. Further, theoretical assumptions on the nature of new physics are needed: A plausible framework is the hypothesis that observable effects of new physics are confined to $\mathrm{B}-\overline{\mathrm{B}}$ and $\mathrm{K}-\overline{\mathrm{K}}$ mixing, which are typically more sensitive to new physics than the other quantities entering the global fit of the UT.

Here we briefly report on our analysis with the CKMfitter collaboration, restricting ourselves to the first of three studied scenarios, and refer to Ref. [4] for details. This scenario treats Δ_d and Δ_s as independent quantities. The fit results are shown in Fig. 3. The tension on the SM seen in $\Delta_d \neq 1$ is driven by several BaBar and Belle measurements [27] of $B(B^+ \to \tau^+\nu_\tau)$, which is proportional to $|V_{ub}|^2 f_B^2$. The different determinations of $|V_{ub}| \propto \sqrt{\overline{\rho}^2 + \overline{\eta}^2}$ are consistent with each other, but the corresponding best-fit region in the $(\overline{\rho}, \overline{\eta})$ plane is not spiked by the ray stemming from $\beta \simeq 21^\circ$ measured through $A_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_S)$. In the presence of new physics in $\mathrm{B_d}-\overline{\mathrm{B}}_{\mathrm{d}}$ mixing the latter quantity determines $\sin(2\beta + \phi_d)$ and $\phi_d < 0$ (favoured in Fig. 3) alleviates the tension. Negative values of ϕ_d further help to accomodate the large negative result for A_{SL}. Not only A_{SL} but also both the DØ and CDF measurements of ϕ_s^Δ through $B_s \to J/\psi\phi$ favour $\phi_s^\Delta < 0$. Our best-fit value is $\phi_s^\Delta = (-52^{+32}_{-25})^\circ$ at 95% CL. This is consistent with the 2010 results found from $B_s \to J/\psi\phi$, $\phi_s^{\Delta,CDF} = (-29^{+44}_{-49})^\circ$ and $\phi_s^{\Delta,DØ} = (-44^{+59}_{-51})^\circ$ at 95% CL each. If we remove A_{SL} from the fit and predict it instead from the other observables through the global fit, we find $A_{\mathrm{SL}} = \left(-4.2^{+2.9}_{-2.7}\right) \cdot 10^{-3}$ at 95% CL, which is just $1.5\,\sigma$ away from the experimental number in Eq. (22). The new-physics scenario with $\Delta_{d,s} \neq 1$ gives an excellent fit, with the SM point $\Delta_d = \Delta_s = 1$ excluded at the level of 3.6 standard deviations [4]. We remark that this result is only marginally influenced by

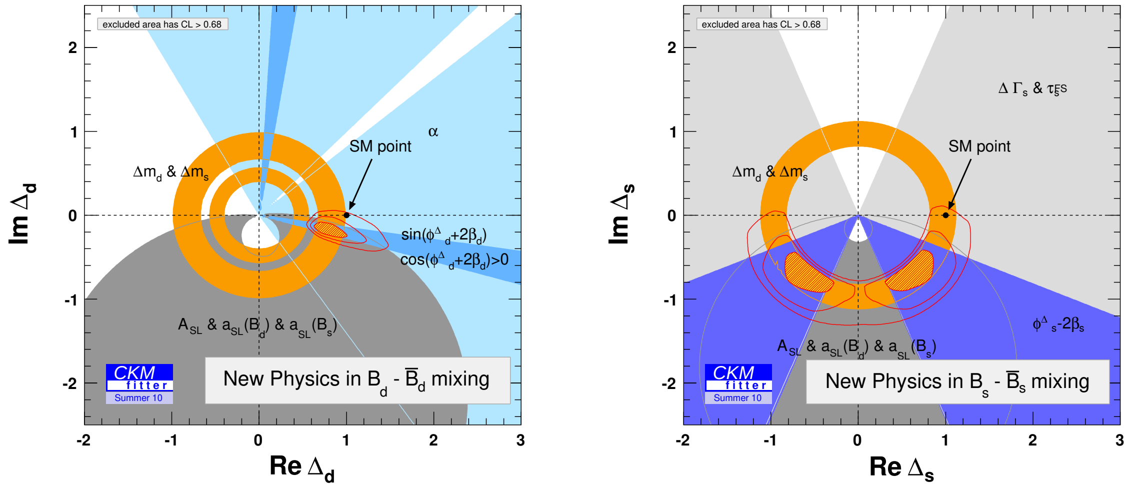

Figure 3: *Allowed regions for Δ_d and Δ_s [4].*

uncertainties of lattice calculations. Using the fit results of [4] for scenario I we can also quote a number for the quantity

$$S_{J/\psi\phi} \quad = \quad \sin\left(-2\beta_s + \Phi_s^{\Delta}\right) \quad = \quad -0.78^{+0.19}_{-0.12} \tag{23}$$

used in theoretical papers. We have assumed the absence of new physics in the tree-dominated decay $B_s \to J/\psi\phi$. This value differs sizeably from the Standard-Model expectation of

$$S^{\rm SM}_{J/\psi\phi} \quad = \quad -0.036 \pm 0.002\ . \tag{24}$$

In the literature numerous extensions of the Standard Model are discussed to explain this difference, see e.g. [4, 28] and references therein.

In conclusion we have updated several theory predictions related to B meson lifetimes and $\mathrm{B}-\overline{\mathrm{B}}$ mixing quantities. We have further briefly discussed the essential results of our study with the CKMFitter group [4], which has found evidence of physics beyond the SM in $\mathrm{B}-\overline{\mathrm{B}}$ mixing.

ACKNOWLEDGEMENTS

UN thanks the conference organisers for their invitation and financial support. The presented work is supported by BMBF grant 05H09VKF.

Appendix: Theory errors

In this appendix we give a detailed list of the different sources of the theoretical error for observables in the B_s mixing system. We compare this numbers with the corresponding ones from Ref. [2], but slightly proceed in a different way: In accordance with Ref. [4] we use the $\overline{\rm MS}$ scheme for m_b, while in [2] we were using in addition the pole scheme and our numbers and errors were averages of these two quark mass schemes. The numerical values and uncertainties of the input parameters are taken from Table 6 and Table 7 of Reference [4]. Contrary to Ref. [4] we do not use the Rfit method for the statistical analysis in these proceedings, instead we simply add different uncertainties in quadrature.

$\overline{m}_c(\overline{m}_c) = (1.286 \pm 0.042)$ GeV and $\overline{m}_b(\overline{m}_b) = (4.248 \pm 0.051)$ GeV imply

$$\overline{z} = \frac{\overline{m}_c^2(\overline{m}_b)}{\overline{m}_b^2(\overline{m}_b)} = 0.0474 \pm 0.0033 \, . \tag{25}$$

For the CKM angle γ we do not use the direct measurement given in Table 6 of Ref. [4], but instead the direct bounds on $\alpha^{\rm exp}$ and $\beta^{\rm exp}$ from the same table:

$$\gamma \;=\; \pi - \alpha^{\rm exp} - \beta^{\rm exp} \;=\; 1.220 \pm 0.077 \;=\; 69.9^\circ \pm 4.4^\circ . \tag{26}$$

In this way we find a precise value for γ from which new physics in B–$\overline{\rm B}$ mixing drops out. New physics can only change the value in Eq. (26) through novel electroweak $b \to d$ penguin effects, which stay within the quoted error in all plausible models. We calculate the CKM elements using V_{us}, V_{cb}, $|V_{ub}/V_{cb}|$ and γ as inputs.

The error budget for $\Delta M_s^{\rm SM}$ is as follows:

$\Delta M_s^{\rm SM}$	this work	Ref. [2]
Central Value	$17.3\,{\rm ps}^{-1}$	$19.3\,{\rm ps}^{-1}$
$\delta(f_{B_s})$	13.2%	33.4%
$\delta(V_{cb})$	3.4%	4.9%
$\delta(\mathcal{B}_{B_s})$	2.9%	7.1%
$\delta(m_t)$	1.1%	1.8%
$\delta(\alpha_s)$	0.4%	2.0%
$\delta(\gamma)$	0.3%	1.0%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.2%	0.5%
$\delta(\overline{m}_b)$	0.1%	$---$
$\sum \delta$	14.0%	34.6%

For the mass difference we observe a considerable reduction of the overall error from 35% in 2006 to 14% today. This is mainly driven by the progress in the lattice determination of $f_{B_s}^2 \mathcal{B}_{B_s}$. The situation for $\Delta\Gamma_s^{\rm SM}$ is similar:

$\Delta\Gamma_s^{\rm SM}$	this work	Ref. [2]
Central Value	$0.087\,{\rm ps}^{-1}$	$0.096\,{\rm ps}^{-1}$
$\delta(\mathcal{B}_{\tilde{R}_2})$	17.2%	15.7%
$\delta(f_{B_s})$	13.2%	33.4%
$\delta(\mu)$	7.8%	13.7%
$\delta(\widetilde{\mathcal{B}}_{S,B_s})$	4.8%	3.1%
$\delta(\mathcal{B}_{R_0})$	3.4%	3.0%
$\delta(V_{cb})$	3.4%	4.9%
$\delta(\mathcal{B}_{B_s})$	2.7%	6.6%
$\delta(\mathcal{B}_{\tilde{R}_1})$	1.9%	$---$
$\delta(\bar{z})$	1.5%	1.9%
$\delta(m_s)$	1.0%	1.0%
$\delta(\mathcal{B}_{R_1})$	0.8%	$---$
$\delta(\mathcal{B}_{\tilde{R}_3})$	0.5%	$----$
$\delta(\alpha_s)$	0.4%	0.1%
$\delta(\gamma)$	0.3%	1.0%
$\delta(\mathcal{B}_{R_3})$	0.2%	$---$
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.2%	0.5%
$\delta(m_b)$	0.1%	1.0%
$\sum\delta$	24.5%	40.5%

For the decay rate difference we also find a strong reduction of the overall error from 40.5% in 2006 to 24.5%. This is again due to our more precise knowledge of the decay constant and the bag parameter $\mathcal{B}_{B_s}$. However, in the $\overline{\rm MS}$-scheme for the quark masses also the renormalisation-scale dependence is reduced. It is interesting to note that now the dominant uncertainty stems from the value of the matrix element of the power-suppressed operator $\tilde{R}_2$ parametrised by $\mathcal{B}_{\tilde{R}_2}$.

Next we discuss the ratio of $\Delta\Gamma_s^{\rm SM}/\Delta M_s^{\rm SM}$:

$\Delta\Gamma_s^{\rm SM}/\Delta M_s^{\rm SM}$	this work	Ref. [2]
Central Value	$50.4 \cdot 10^{-4}$	$49.7 \cdot 10^{-4}$
$\delta(\mathcal{B}_{R_2})$	17.2%	15.7%
$\delta(\mu)$	7.8%	9.1%
$\delta(\widetilde{\mathcal{B}}_{S,B_s})$	4.8%	3.1%
$\delta(\mathcal{B}_{R_0})$	3.4%	3.0%
$\delta(\mathcal{B}_{\tilde{R}_1})$	1.9%	$---$
$\delta(\bar{z})$	1.5%	1.9%
$\delta(m_b)$	1.4%	1.0%
$\delta(m_t)$	1.1%	1.8%
$\delta(m_s)$	1.0%	0.1%
$\delta(\alpha_s)$	0.8%	0.1%
$\delta(\mathcal{B}_{R_1})$	0.8%	$---$
$\delta(\mathcal{B}_{\tilde{R}_3})$	0.5%	$----$
$\delta(\mathcal{B}_{R_3})$	0.2%	$---$
$\delta(\mathcal{B}_{B_s})$	0.1%	0.5%
$\delta(\gamma)$	0.0%	0.1%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.0%	0.1%
$\delta(V_{cb})$	0.0%	0.0%
$\sum \delta$	20.1%	18.9%

For $\Delta\Gamma_s/\Delta M_s$ we do not find any improvement. The decay constant cancels out in this ratio and therefore we do not profit from the progress in lattice simulations. Also the CKM dependence cancels to a large extent. Our last error budget concerns the

CP asymmetry in flavour-specific B_s decays:

$a_{\rm fs}^{s,\rm SM}$	This work	Ref. [2]
Central Value	$2.11 \cdot 10^{-5}$	$2.06 \cdot 10^{-5}$
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	11.6%	19.5%
$\delta(\mu)$	8.9%	12.7%
$\delta(\bar{z})$	7.9%	9.3%
$\delta(\gamma)$	3.1%	11.3%
$\delta(\mathcal{B}_{\tilde{R}_3})$	2.8%	2.5%
$\delta(m_s)$	2.0%	3.7%
$\delta(\alpha_s)$	1.8%	0.7%
$\delta(\mathcal{B}_{R_3})$	1.2%	1.1%
$\delta(m_t)$	1.1%	1.8%
$\delta(\widetilde{\mathcal{B}}_{S,B_s})$	0.6%	0.4%
$\delta(\mathcal{B}_{R_0})$	0.3%	$---$
$\delta(\mathcal{B}_{\tilde{R}_1})$	0.2%	$---$
$\delta(\mathcal{B}_{B_s})$	0.2%	0.6%
$\delta(m_s)$	0.1%	0.1%
$\delta(\mathcal{B}_{R_2})$	0.1%	$---$
$\delta(\mathcal{B}_{R_1})$	0.0%	$---$
$\delta(V_{cb})$	0.0%	0.0%
$\sum \delta$	17.3%	27.9%

Finally we also observe a large improvement for the CP asymmetry $a_{\rm fs}^s$. The overall error went down from 27.9% to 17.3%. In $a_{\rm fs}^s$ also the decay constant cancels, but in contrast to $\Delta\Gamma_s/\Delta M_s$ we now have a strong dependence on the CKM elements. Here we benefit from more precise values of the CKM elements. The central value for $a_{\rm fs}^s$ and the error are different from the one quoted in Eq. (12), because in the table above we have computed the CKM elements from the experimental range for $|V_{ub}|$ centered around $|V_{ub}| = 3.92 \cdot 10^{-3}$ (see Tab. 6 of Ref. [4]). In Eq. (12) the CKM elements are calculated instead from the more precise result $|V_{ub}| = (3.56^{+0.15}_{-0.20}) \cdot 10^{-3}$ of the global CKM fit.

References

[1] Talk by G. Borissov at *Joint Experimental-Theoretical Physics Seminar*, Fermilab, Batavia, USA, May 14, 2010. V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82** (2010) 032001 [arXiv:1005.2757 [hep-ex]]. V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **105** (2010) 081801 [arXiv:1007.0395 [hep-ex]].

[2] A. Lenz and U. Nierste, JHEP **0706**, 072 (2007), arxiv:hep-ph/0612167.

[3] K. Anikeev *et al.*, *B physics at the Tevatron: Run II and beyond*, [hep-ph/0201071]. U. Nierste, arXiv:hep-ph/0406300. Y. Nir, arXiv:hep-ph/0510413. U. Nierste, arXiv:0904.1869 [hep-ph].

[4] A. Lenz, U. Nierste, and J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker, S. Monteil, V. Niess, S. T'Jampens [CKMfitter Group]. arXiv:1008.1593 [hep-ph], to appear in Phys. Rev. D.

[5] M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. D **54**, 4419 (1996), arxiv:hep-ph/9605259;

[6] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459**, 631 (1999), arxiv:hep-ph/9808385;

[7] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP **0308**, 031 (2003), arxiv:hep-ph/0308029.

[8] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B **576** (2003) 173 [arXiv:hep-ph/0307344].

[9] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Nucl. Phys. B **639**, 389 (2002), arxiv:hep-ph/0202106.

[10] T. Gershon, J. Phys. G **38** (2011) 015007 [arXiv:1007.5135 [hep-ph]].

[11] I. Dunietz, R. Fleischer and U. Nierste, Phys. Rev. D **63**, 114015 (2001), arxiv:hep-ph/0012219.

[12] U. Nierste, arXiv:hep-ph/0406300.

[13] S. Aoki et al. [JLQCD Collaboration], Phys. Rev. Lett. **91**, 212001 (2003), arxiv:hep-ph/0307039. B. Blossier *et al.* [ETM Collaboration], PoS **LATTICE2009**, 151 (2009) [arXiv:0911.3757 [hep-lat]]. M. Wingate, C. T. H. Davies, A. Gray, G. P. Lepage and J. Shigemitsu, Phys. Rev. Lett. **92**, 162001 (2004), arxiv:hep-ph/0311130. C. Bernard et al. [Fermilab Lattice, MILC and HPQCD Collaborations], PoS LAT2008 , 278 (2008), arxiv:0904.1895 [hep-lat]. E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D **80** (2009) 014503, arxiv:0902.1815 [hep-lat]. E. Dalgic et al., Phys. Rev. D **76**, 011501 (2007), arxiv:hep-lat/0610104. C. Albertus et al. [RBC and UKQCD Collaborations], PoS LAT2007 , 376 (2007).

[14] R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. B **316** (1993) 567.

[15] S. Esen *et al.* [Belle Collaboration], Phys. Rev. Lett. **105** (2010) 201802 [arXiv:1005.5177 [hep-ex]].

[16] [DØ Collaboration], DØ Note 6093-CONF (2010).

[17] K. Hartkorn and H. G. Moser, Eur. Phys. J. C **8** (1999) 381.

[18] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347** (1990) 491.

[19] I.I. Bigi, B. Blok, M. Shifman, N. Uraltsev and A. Vainshtein, in *B decays*, ed. S. Stone, 2nd edition, *World Scientific*, Singapore, 1994, 132.
I.I. Bigi, hep-ph/9508408.

[20] M. Neubert and C. T. Sachrajda, Nucl. Phys. B **483** (1997) 339 [arXiv:hep-ph/9603202].

[21] Y. Y. Keum and U. Nierste, Phys. Rev. D **57** (1998) 4282 [arXiv:hep-ph/9710512].

[22] A. Ali Khan et al. [CP-PACS Collaboration], Phys. Rev. D **64**, 054504 (2001), arxiv:hep-lat/0103020. C. Bernard et al. [MILC Collaboration], Phys. Rev. D **66**, 094501 (2002), arxiv:hep-lat/0206016. B. Blossier *et al.* [ETM Collaboration], PoS **LATTICE2009**, 151 (2009) [arXiv:0911.3757 [hep-lat]]. M. Wingate, C. T. H. Davies, A. Gray, G. P. Lepage and J. Shigemitsu, Phys. Rev. Lett. **92**, 162001 (2004), arxiv:hep-ph/0311130.

[23] Heavy Flavor Averaging Group (HFAG),
http://www.slac.stanford.edu/xorg/hfag/osc/end_2009.

[24] D. Becirevic, arXiv:hep-ph/0110124.

[25] D. Becirevic, S. Fajfer and J. F. Kamenik, Phys. Lett. B **671** (2009) 66 [arXiv:0804.1750 [hep-ph]].

[26] T. Aaltonen *et al.* [CDF collaboration], Public Note 9015 (2007).

[27] B. Aubert *et al.* (BaBar collaboration), Phys. Rev. D **77**, 011107(R) (2008), arxiv:0708.2260 [hep-ex]. I. Adachi *et al.* (Belle collaboration), arxiv:0809.3834 [hep-ex]. B. Aubert *et al.* (BaBar collaboration), Phys. Rev. D **81**, 051101 (2010), arxiv:0809.4027 [hep-ex]. K. Hara *et al.* (Belle collaboration), arxiv:1006.4201 [hep-ex]. P. d. A. Sanchez *et al.* (BaBar collaboration), arxiv:1008.0104 [hep-ex].

[28] J. Girrbach, S. Jager, M. Knopf *et al.*, [arXiv:1101.6047 [hep-ph]]; Y. Kaburaki, K. Konya, J. Kubo *et al.*, [arXiv:1012.2435 [hep-ph]]; J. Kubo, A. Lenz, Phys. Rev. **D82** (2010) 075001. [arXiv:1007.0680 [hep-ph]]; O. Eberhardt, A. Lenz, J. Rohrwild, Phys. Rev. **D82** (2010) 095006. [arXiv:1005.3505 [hep-ph]].