

## Higgs boson production at LHC to NNLO accuracy and finite top quark mass effects

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In this contribution we consider the production of a Standard Model Higgs boson to next-to-next-to-leading order accuracy taking into account a finite top quark mass. This result improves the one based on the effective theory and is important for precise predictions of Higgs boson production both at Tevatron and LHC.

*35th International Conference of High Energy Physics*

*July 22-28, 2010*

*Paris, France*

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<sup>†</sup>This work was supported by the DFG through the SFB/TR 9 “Computational Particle Physics”. Preprint Nos.: SFB/PPP-10-83, TTP10-37

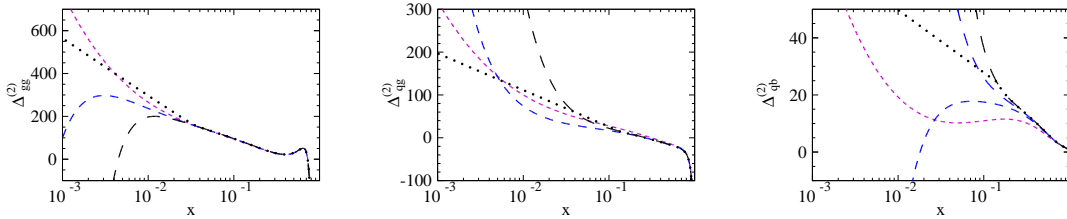
## 1. Introduction

Calculations of higher orders in perturbation theory for the production of a Standard Model Higgs boson have a long history. Already 20 years ago the next-to-leading (NLO) QCD corrections became available [1] and about ten years ago also the NNLO corrections have been computed [2] although only within the framework of an effective theory where it has been assumed that the top quark mass  $M_t$  is much heavier than all other mass scales involved in the process — even the center-of-mass energy which can be much larger than  $M_t$ . It is thus very important to perform the NNLO calculation within full QCD in order to obtain reliable predictions up to NNLO accuracy. Recently two independent groups have completed this big enterprise [3, 4, 5, 6, 7]. In this contribution we briefly describe the calculation and results obtained in Refs. [3, 6].<sup>1</sup>

## 2. Outline of the calculation

Our result for the partonic cross section is based on the proper combination of two ingredients: the evaluation in the limit of large center-of-mass energy and the asymptotic expansion of the total cross section in inverse powers of  $M_t$ . The leading contribution of the former, which is a constant at NLO and a logarithm at NNLO, has been computed in Ref. [8]. Due to an involved asymptotic expansion the latter is technically more challenging and requires a significant amount of computer resources in order to obtain several terms in the expansion.

In Ref. [6] we have decided to consider the forward scattering amplitudes and evaluate those imaginary parts which involve a cut of the Higgs boson line. In a first step we generate the diagrams and apply subsequently the asymptotic expansion in the limit<sup>2</sup>  $M_t^2 \gg \hat{s}, M_H^2$ , implemented in two independent programs. This procedure factorizes the original triple-scale forward scattering functions into massive vacuum integrals (with a single scale  $M_t$ ) up to three loops and four-point one- and two-loop integrals dependent on  $\hat{s}$  and  $M_H$ . As a result one obtains an expansion in  $1/M_t$  which is valid for  $x = M_H^2/\hat{s} > x_{th} = M_H^2/(4M_t^2)$ . We match this result to a function  $3C_1 + ax$  (NLO) or  $-9C_2 \ln x + b$  (NNLO), where coefficients  $C_1$  and  $C_2$  are tabulated in Ref. [8] and  $a, b$  and the matching point  $x_m$  is chosen to provide the most “natural” smooth behaviour of the function. We found



**Figure 1:** Partonic NNLO cross sections for the  $gg$ ,  $qg$  and  $q\bar{q}$  channels (from left to right) as function of  $x$  for  $M_H = 130$  GeV. Lines with longer dashes include higher order terms in  $\rho$ . The dotted lines correspond to the matched result (see Refs. [6, 9]).

<sup>1</sup>In Refs. [5, 7] an expansion in  $x = M_H^2/\hat{s}$  has been performed whereas in Ref. [6] the full  $x$ -dependence is kept. Apart from that the results of [5, 7] and [6] agree.

<sup>2</sup> $\hat{s}$  is the partonic center-of-mass energy.

that an  $x_m$  such that the function and its first derivative match smoothly is a good choice at the NLO; at the NNLO, matching at  $x_m = x_{th}/4$  produces reasonable results for  $110 \text{ GeV} \leq M_H \leq 300 \text{ GeV}$  and is consistent with the region of  $x$  where higher  $\mathcal{O}(\rho^n)$  ( $\rho = M_H^2/M_t^2$ ) corrections demonstrate good convergence. By varying the constants and interpolating function shapes we have checked that the dependence of the hadronic cross section on the exact details of the matching procedure is quite small and that only the asymptotics near  $x \rightarrow 0$  are important. In the next Section we discuss the resulting partonic and hadronic cross sections.

### 3. Partonic and hadronic results

We introduce the following notation for the partonic cross section

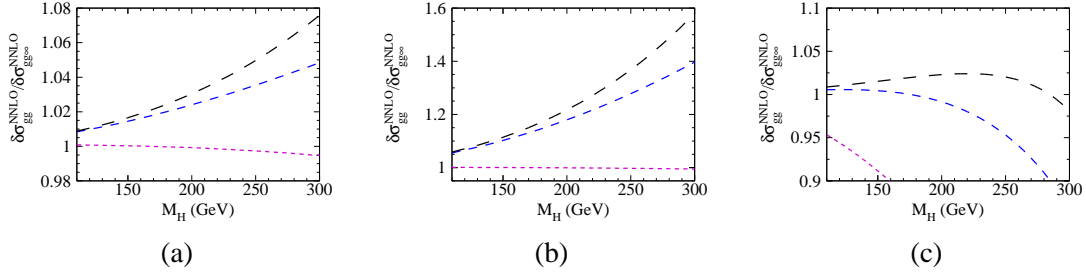
$$\hat{\sigma}_{ij \rightarrow H+X} = \hat{A}_{\text{LO}} \left( \Delta_{ij}^{(0)} + \frac{\alpha_s}{\pi} \Delta_{ij}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Delta_{ij}^{(2)} + \dots \right), \quad \hat{A}_{\text{LO}} = \frac{G_F \alpha_s^2}{288\sqrt{2}\pi} f_0(\rho, 0), \quad (3.1)$$

where  $ij$  denote one of the possible initial states:  $gg, qg, \bar{q}g, q\bar{q}, qq, \text{ or } qq'$ , where  $q$  and  $q'$  stand for (different) massless quark flavours. At NNLO the Higgs boson in the final state may be accompanied by zero, one or two gluons or light quarks. In general, the quantities  $\Delta_{ij}^{(k)}$  depend on  $x$  and  $\rho$ . Leading order mass dependence is then described by the function  $f_0(\rho, 0)$  which can be found in Ref. [3]. In Fig. 1 we show the partonic NNLO cross sections for the numerically most important contributions  $gg, qg$  and  $q\bar{q}$  as function of  $x$ . Our final result obtained from the above matching procedure is represented by the dotted lines.

The hadronic cross section is obtained by the convolution of the partonic cross section  $\hat{\sigma}_{ij \rightarrow H+X}$  with the corresponding parton distribution functions (PDFs). In the following we present results for  $pp$  collisions at the LHC peak energy  $\sqrt{s} = 14 \text{ TeV}$ . We use the parton distribution function (PDF) set MSTW2008 [10] and the  $\alpha_s$  evolution at LO, NLO and NNLO when computing predictions to the cross section at the corresponding order.

To discuss the numerical effect of our calculation we decompose the prediction of the total cross section into its LO, NLO and NNLO contributions  $\sigma_{pp' \rightarrow H+X}(s) = \sigma^{\text{LO}} + \delta\sigma^{\text{NLO}} + \delta\sigma^{\text{NNLO}}$  and denote the heavy top quark approximation with an additional subscript  $\infty$ . In the following we present the numerically most important contribution from the  $gg$  channel. In Figs. 2(a)–(c) we show the NNLO contribution to the hadronic cross section,  $\delta\sigma^{\text{NNLO}}$ , normalized to the infinite top quark mass result where in each case the three lines correspond to the inclusion of terms of order  $\rho^0$  (short dashes),  $\rho^1$  and  $\rho^2$  (long dashes). The difference among the three plots is that in (a), the exact LO top quark mass dependence is factored out as in Eq. (3.1), while in (b) the partonic cross sections both in numerator and denominator are strictly expanded in  $\rho$ . Finally, in (c) we expand  $\hat{A}_{\text{LO}}$  in the numerator but keep it exact in the denominator.

For the fully expanded option (b) one observes for  $M_H = 300 \text{ GeV}$  corrections up to 40% originating from the linear  $\rho$  term which further increase to almost 60% after including the  $\rho^2$  term. However, when the exact leading-order top quark mass dependence is factored out (case (a)), the corrections amount to at most 8%. Considering the fact that the NNLO terms contribute about 10% of the total NNLO cross section we conclude that the top quark mass suppressed terms at NNLO alter the prediction by less than 1%. This justifies the use of the heavy top mass approximation for the evaluation of the NNLO hadronic cross section. The latter conclusion is also obtained



**Figure 2:** (a), (b) and (c): Ratio of the NNLO hadronic cross section ( $gg$  contribution) including successive higher orders in  $1/M_t$  normalized to the infinite top quark mass result. In (a) the exact LO mass dependence is factorized both in the numerator and denominator. In (b) numerator and denominator are expanded in  $\rho$ , and in (c) only the numerator is expanded.

from Fig. 2(c). It is interesting to remark that the slight deviation of the  $\rho^0$ -curve in (a) is an effect of the matching procedure which is not present in the denominator of the ordinate axis. Furthermore, panel (c) indicates that the infinite-top quark mass result (with factored exact LO result) approximates the exact result (including finite top quark mass effects) to a few percent level. Let us also stress that the matched result obtained from the different approximation in  $\rho$  only leads to slightly different hadronic contributions. The difference in the three curves in Fig. 2 essentially comes from the top quark mass corrections to the  $\delta$ -function part of the partonic cross section.

*Acknowledgments:* I would like to thank A. Pak and M. Rogal for a fruitful collaboration on the subjects presented in this contribution.

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