SFB/CPP-10-76
TTP10-34

Minimal Supersymmetric SU(5) and Gauge Coupling Unification at Three Loops

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Abstract

We consider the relations between the gauge couplings at the electroweak scale and the high scale where unification of the three gauge couplings is expected. Threshold corrections are incorporated both at the supersymmetric and at the grand unified scale and, where available three-loop running and two-loop decoupling are employed. We study the impact of the current experimental uncertainties of the coupling constants and the supersymmetric mass spectrum on the prediction of the super-heavy masses within the so-called minimal supersymmetric SU(5). As a main result of the three-loop analysis we confirm that minimal supersymmetric SU(5) cannot be ruled out by the current experimental data on proton decay rates.

PACS numbers: 11.25.Db 11.30.Pb 12.10.Kt 12.38.Bx

1 Introduction

An appealing hint in favour of supersymmetry (SUSY) is the apparent unification of gauge couplings at a scale of about 10^{16} GeV [1–3]. In particular, the gauge coupling unification is predicted, even under the assumption of a minimal particle content of the underlying Grand Unified Theory (GUT), the so-called minimal SUSY SU(5) model [4, 5]. An important feature of this model is that the requirement of minimality renders it the most predictive model from the currently known candidates for GUTs. However, immediately after the formulation of the minimal SUSY SU(5) it has been noticed that within SUSY GUTs new dimension-five operators cause a rapid proton decay [6, 7]. This aspect was intensively studied over the last thirty years with the extreme conclusions of Refs. [8, 9] that the minimal SUSY SU(5) model is ruled out by the combined constraints from proton decay and gauge coupling unification. However, in the following years several careful analyses have shown that the constraint on the coloured Higgs triplet mass from proton decay was actually too strong, and that minimal SUSY SU(5) is still a perfectly viable theory. The proton decay rate for the dominant channel $p \to K^+\overline{\nu}$ can be suppressed either by sfermion mixing [10] or by taking into account higher dimensional operators induced at the Planck scale [11–13]. Such operators can also lead to a successful down quark and charged lepton Yukawa coupling unification for the first two families, which is not possible in the renormalizable version of the minimal SUSY SU(5). Planck-scale-induced operators also change the boundary conditions for the gauge couplings at the GUT scale, since in their presence the colour octet and the isospin triplet Higgs bosons contained in the $\mathbf{24}_H$ representation do not necessarily have the same mass anymore, as it is the case in the minimal model. This fact allows for significantly weaker constraints from gauge coupling unification on the colour triplet Higgs boson than in the renormalizable model.

Recently, new experimental data for the relevant input parameters [14–16] and substantial progress on the theory side [17–20] became available. This encourages us to reanalyze minimal SUSY SU(5) focusing on the aspect of gauge coupling unification. We emphasize that in our analysis we restrict ourselves to the renormalizable version of minimal SUSY SU(5) although in this way the unification of the first and second generation Yukawa couplings can not be achieved. In our analysis we ignore this fact since it is not relevant here. The constraints on the mass of the coloured Higgs triplet that are derived can easily be translated to the non-renormalizable version of minimal SUSY SU(5) by comparison of, e.g., Eqs. (9) and (10) of Ref. [13] and Eq. (14) in our paper. In this sense, our results represent the “worst case scenario”.

More precisely, we review in this paper the constraints on the mass of the coloured Higgs triplet $M_{\mathrm{H_c}}$ and the grand unification scale[1] M_{G} within minimal SUSY SU(5) [4,5] taking into account the latest experimental data for the weak scale parameters and the most precise theoretical predictions currently available. We adopt the renormalization group method in the “bottom-up” approach to predict the values of the two parameters and take

[1]See Section 2 for the exact definition of $M_{\mathrm{H_c}}$ and M_{G}.

into account threshold corrections generated by the superpartners of the SM particles as well as those due to the super-heavy SUSY-GUT particles. In this way, we can derive the two SUSY-GUT parameters from the knowledge of the gauge coupling constants of the SM at the electroweak scale and the minimal supersymmetric SM (MSSM) mass spectrum. In addition, we implement the perturbativity restrictions for all gauge, Yukawa and Higgs self couplings, *i.e.*, we require that they are smaller than one up to the Planck scale.

Besides the minimal SU(5) we also briefly discuss the phenomenological consequences of the Missing Doublet Model (MDM) which has been designed in order to avoid unnatural doublet-triplet splitting in the Higgs fields of the **5** and $\bar{\mathbf{5}}$ representations.

The remainder of the paper is organized as follows: In the next Section we introduce our framework and describe the tools we have used for our analysis. In particular we specify the underlying GUT theory and describe in detail our procedure for the running and decoupling. In Section 3 the phenomenological consequences are discussed where we study in particular the constraints on the GUT masses of minimal SUSY SU(5). Finally, we present our conclusions in Section 4.

2 Theoretical framework

2.1 Minimal SU(5) and Missing Doublet Model

In Section 3 we discuss in detail the restrictions on "minimal SU(5)", however, mention also briefly the consequences for the so-called "Missing Doublet Model". For convenience of the reader we introduce in this Subsection some details on these models.

The superpotential of minimal SU(5) [4] is given by

$$\begin{aligned} \mathcal{W} &= M_1 \mathrm{Tr}(\Sigma^2) + \lambda_1 \mathrm{Tr}(\Sigma^3) + \lambda_2 \mathrm{\bar{H}\Sigma H} + \mathrm{M_2 \bar{H} H} \\ &\quad + \sqrt{2} Y_d^{ij} \Psi_i \phi_j \bar{H} + \frac{1}{4} Y_u^{ij} \Psi_i \Psi_j H \,, \end{aligned} \tag{1}$$

where Ψ_i and ϕ_i ($i = 1, 2, 3$ is a generation index) are matter multiplets in the **10**- and $\bar{\mathbf{5}}$-dimensional representation of SU(5), respectively, and the field H ($\bar{H}$) is realized in the **5** ($\bar{\mathbf{5}}$) representation. SU(5) is broken to SU(3)×SU(2)×U(1) if the adjoint Higgs boson $\Sigma \equiv \Phi^a T^a$ ($a = 1, \ldots, 24$) gets the vacuum expectation value $\langle\Sigma\rangle = V/(2\sqrt{30}) \times \mathrm{diag}(-2, -2, -2, 3, 3)$, with $V = -4\sqrt{30} M_1/(3\lambda_1)$. Choosing $\langle\bar{H}\rangle = \langle H\rangle \ll V$ and in addition imposing the (tree-level-)fine-tuning condition $M_2 = -\sqrt{3}\lambda_2 V/\sqrt{40}$ the isodoublets in H and $\bar{H}$ remain massless. Furthermore, one gets the following super-heavy mass spectrum:

$$M_X^2 = \frac{5}{12} g^2 V^2 \,, \quad M_{H_c}^2 = \frac{5}{24} \lambda_2^2 V^2 \,, \quad M_\Sigma^2 \equiv M_{(8,1)}^2 = M_{(1,3)}^2 = 25 M_{(1,1)}^2 = \frac{15}{32} \lambda_1^2 V^2 \,, \tag{2}$$

where the indices in round brackets refer to the SU(3) and SU(2) quantum numbers. Here M_Σ denotes the mass of the colour octet part of the adjoint Higgs boson Σ and

M_{H_c} stands for the mass of the colour triplets of H and $\bar{H}$, M_X is the mass of the gauge bosons and g is the gauge coupling. The equality $M^2_{(8,1)} = M^2_{(1,3)}$ only holds if one neglects operators that are suppressed by $1/M_{\rm Pl}$ as we do here. We emphasize again that taking into account such operators can considerably weaken the constraint on the colour triplet mass M_{H_c} that is derived later in this paper [11–13]. However, the altered constraint can easily be derived from our results.

The effects of the super-heavy particle masses on the MSSM gauge couplings can be parametrized with the help of decoupling coefficients. If we interpret the MSSM as the low-energy effective theory of the SUSY SU(5) model, we can define its three gauge coupling constants as functions of the unique SU(5) gauge coupling $\alpha^{\rm SU(5)}$ through

$$\alpha_i^{\rm MSSM}(\mu_{\rm GUT}) = \zeta_{\alpha_i}(\mu_{\rm GUT}, \alpha^{\rm SU(5)}, M_{H_c}, M_X, M_\Sigma)\, \alpha^{\rm SU(5)}(\mu_{\rm GUT})\,, \quad i = 1, 2, 3\,, \tag{3}$$

where $\zeta_{\alpha_i}(\mu_{\rm GUT}, \alpha^{\rm SU(5)}, M_{H_c}, M_X, M_\Sigma)$ denote the decoupling coefficients that can be obtained from Green's functions with external light particles computed in the full and effective theory. The scale $\mu_{\rm GUT}$ is an unphysical parameter, not fixed by theory. The dependence of physical observables on $\mu_{\rm GUT}$ thus provides an estimation of the theoretical uncertainties within fixed order perturbation theory.

For SUSY theories the most convenient regularization scheme is Dimensional Reduction (DRED) [21] which we also adopt in our calculation. As a consequence, the coupling constants appearing in Eq. (3) are renormalized minimally in the so-called $\overline{\rm DR}$ renormalization scheme. The one-loop formulas of the decoupling coefficients for a general gauge group have been known for a long time [22–25]. The specification to minimal SUSY SU(5) reads [26, 27]

$$\begin{aligned}
\zeta_{\alpha_1}(\mu) &= 1 + \frac{\alpha^{\rm SU(5)}(\mu)}{4\pi}\left(-\frac{2}{5}\, L_{\mu H_c} + 10\, L_{\mu X}\right)\,,\\
\zeta_{\alpha_2}(\mu) &= 1 + \frac{\alpha^{\rm SU(5)}(\mu)}{4\pi}\left(-2\, L_{\mu \Sigma} + 6\, L_{\mu X}\right)\,,\\
\zeta_{\alpha_3}(\mu) &= 1 + \frac{\alpha^{\rm SU(5)}(\mu)}{4\pi}\left(-L_{\mu H_c} - 3\, L_{\mu \Sigma} + 4\, L_{\mu X}\right)\,,
\end{aligned} \tag{4}$$

where $L_{\mu x} = \ln(\mu^2/M_x^2)$ and for simplicity we keep from the list of arguments of the coefficients ζ_{α_i} only the decoupling scale.

The Missing Doublet Model [28,29] is designed to avoid unnatural doublet-triplet splitting in the field H that is present in the minimal model. This is achieved at the cost of introducing additional Higgs fields Θ and $\bar{\Theta}$ in the large SU(5) representations **50** and $\overline{\mathbf{50}}$ that do not contain any isodoublets and thus only couple to the colour triplets in H and $\bar{H}$. To break SU(5) another Higgs field Σ in the **75** representation is used instead of the **24** as in the minimal model. The superpotential reads

$$\begin{aligned}
\mathcal{W} &= M_1{\rm Tr}(\Sigma^2) + \lambda_1{\rm Tr}(\Sigma^3) + \lambda_2{\rm H}\Sigma\Theta + \bar{\lambda}_2\bar{\rm H}\Sigma\bar{\Theta} + {\rm M}_2\bar{\Theta}\Theta\\
&\quad + \sqrt{2}Y_d^{ij}\Psi_i\phi_j\bar{H} + \frac{1}{4}Y_u^{ij}\Psi_i\Psi_j H\,.
\end{aligned} \tag{5}$$

After Σ develops a vacuum expectation value the spectrum of the theory can be parametrized by five mass parameters [27] $M_X, M_{H_c}, M_{H_{c'}}, M_\Sigma$ and M_2. The last is assumed to be of $\mathcal{O}(M_{\rm Pl})$ so that the representations **50** and $\overline{\mathbf{50}}$ do not contribute to the running above the unification scale and below $M_{\rm Pl}$. Otherwise, due to large group factors of these representations, the perturbativity requirement cannot be fulfilled. In this case the decoupling constants read [26][2]

$$\begin{aligned}
\zeta_{\alpha_1}(\mu) &= 1+\frac{\alpha^{\rm SU(5)}(\mu)}{4\pi}\left(-\frac{2}{5}\,L_{\mu H_c}-\frac{2}{5}\,L_{\mu H_{c'}}+10\,L_{\mu X}-20\,L_{\mu\Sigma}+10\ln\frac{64}{625}\right)\,,\\
\zeta_{\alpha_2}(\mu) &= 1+\frac{\alpha^{\rm SU(5)}(\mu)}{4\pi}\left(-22\,L_{\mu\Sigma}+6\,L_{\mu X}+6\ln\frac{4}{25}\right)\,,\\
\zeta_{\alpha_3}(\mu) &= 1+\frac{\alpha^{\rm SU(5)}(\mu)}{4\pi}\left(-L_{\mu H_c}-L_{\mu H_{c'}}-23\,L_{\mu\Sigma}+4\,L_{\mu X}+4\ln\frac{64}{78125}\right)\,. \qquad (6)
\end{aligned}$$

As we will see later, the presence of the large representation **75** in this model leads to huge theoretical uncertainties due to the variation of the unphysical scale $\mu_{\rm GUT}$.

2.2 Running and decoupling

It is well known that gauge coupling unification is highly sensitive to the super-heavy mass spectrum [30]. This property allows us to probe unification through precision measurements of low-energy parameters like the gauge couplings at the electroweak scale or the supersymmetric mass spectrum. A simple algebraic exercise taking into account the naive step-function approximation [31] based on one-loop RGEs provides analytical formulas for the determination of the GUT spectrum as a function of the three gauge couplings measured at the Z-boson mass scale. An estimate of three-loop as compared to two-loop running has been obtained in Ref. [32]. Note, however, that a consistent treatment requires the implementation of n-loop RGEs and $(n-1)$-loop threshold corrections. We have adopted this approach for $n = 1, 2$ and 3, whenever the required theoretical input was available, and have solved numerically the system of differential equations.

Crucial input for our analysis constitute the precise values of the gauge couplings at the electroweak scale. They are obtained from the weak mixing angle in the $\overline{\rm MS}$ scheme [15], the QED coupling constant at zero momentum transfer and its hadronic [33] contribution in order to obtain its counterpart at the Z-boson scale, and the strong coupling

[2] The occurrence of the last term in the round brackets of each equation is due to the use of relations between the super-heavy masses.

constant [16].[3] The corresponding central values and uncertainties read

$$\begin{aligned}
\sin^2\Theta^{\overline{\rm MS}} &= 0.23119 \pm 0.00014\,,\\
\alpha &= 1/137.036\,,\\
\Delta\alpha_{\rm had}^{(5)} &= 0.02761 \pm 0.00015\,,\\
\alpha_s(M_Z) &= 0.1184 \pm 0.0020\,.
\end{aligned}\tag{7}$$

Whereas $\sin^2\Theta^{\overline{\rm MS}}$ and $\alpha_s(M_Z)$ are already defined in the $\overline{\rm MS}$ scheme, $\Delta\alpha_{\rm had}^{(5)}$ constitute corrections to the on-shell value of α. In order to obtain the corresponding $\overline{\rm MS}$ result we add the leptonic [34] and top quark [35] contribution, $\Delta\alpha_{\rm lep}^{(5)} = 314.97686 \times 10^{-4}$ and $\Delta\alpha_{\rm top}^{(5)} = (-0.70 \pm 0.05) \times 10^{-4}$, and apply the transition formula to the $\overline{\rm MS}$ scheme [15]

$$\Delta\alpha^{(5),\overline{\rm MS}} - \Delta\alpha^{(5),{\rm OS}} = \frac{\alpha}{\pi}\left(\frac{100}{27} - \frac{1}{6} - \frac{7}{4}\ln\frac{M_Z^2}{M_W^2}\right) \approx 0.0072\,. \tag{8}$$

This leads to

$$\alpha^{\overline{\rm MS}}(M_Z) = \frac{\alpha}{1 - \Delta\alpha_{\rm lep}^{(5)} - \Delta\alpha_{\rm had}^{(5)} - \Delta\alpha_{\rm top}^{(5)} - 0.0072} = \frac{1}{127.960 \pm 0.021}\,. \tag{9}$$

In the quantities $\sin^2\Theta^{\overline{\rm MS}}$, $\alpha^{\overline{\rm MS}}(M_Z)$ and $\alpha_s(M_Z)$ the top quark is still (partly) decoupled. Thus, in a next step we compute the six-flavour SM quantities using the relations [15, 36]

$$\begin{aligned}
\alpha^{(6),\overline{\rm MS}} &= \alpha^{\overline{\rm MS}}\left\{1 + \frac{4}{9}\frac{\alpha^{\overline{\rm MS}}}{\pi}\left[\ln\frac{M_Z^2}{M_t^2}\left(1 + \frac{\alpha_s}{\pi} + \frac{\alpha^{\overline{\rm MS}}}{3\pi}\right) + \frac{15}{4}\left(\frac{\alpha_s}{\pi} + \frac{\alpha^{\overline{\rm MS}}}{3\pi}\right)\right]\right\}\,,\\
\sin^2\Theta^{(6),\overline{\rm MS}} &= \sin^2\Theta^{\overline{\rm MS}}\left\{1 + \frac{1}{6}\frac{\alpha^{\overline{\rm MS}}}{\pi}\left(\frac{1}{\sin^2\Theta^{\overline{\rm MS}}} - \frac{8}{3}\right)\left[\left(1 + \frac{\alpha_s}{\pi}\right)\ln\frac{M_t^2}{M_Z^2} - \frac{15}{4}\frac{\alpha_s}{\pi}\right]\right\}\,,
\end{aligned}\tag{10}$$

where all couplings are evaluated at the scale $\mu = M_Z$. We obtain[4]

$$\begin{aligned}
\alpha^{(6),\overline{\rm MS}}(M_Z) &= 1/(128.129 \pm 0.021)\,,\\
\sin^2\Theta^{(6),\overline{\rm MS}}(M_Z) &= 0.23138 \pm 0.00014\,,\\
\alpha_s^{(6)}(M_Z) &= 0.1173 \pm 0.0020\,.
\end{aligned}\tag{11}$$

[3]We adopt the central value from Ref. [16], however, use as our default choice for the uncertainty 0.0020 instead of 0.0007.

[4]Since we aim for gauge couplings at the electroweak scale with highest possible precision we use four-loop running and three-loop decoupling as implemented in `RunDec` [37] in order to obtain $\alpha_s^{(6)}$ from $\alpha_s(M_Z) \equiv \alpha_s^{(5)}(M_Z)$. At such high order in perturbation theory there is practically no dependence on the decoupling scale.

These quantities are related to the three gauge couplings via the equations

$$\begin{aligned}
\alpha_1 &= \frac{5}{3}\frac{\alpha^{(6),\overline{\rm MS}}}{\cos^2\Theta^{(6),\overline{\rm MS}}}\,,\\
\alpha_2 &= \frac{\alpha^{(6),\overline{\rm MS}}}{\sin^2\Theta^{(6),\overline{\rm MS}}}\,,\\
\alpha_3 &= \alpha_s^{(6)}\,,
\end{aligned}\tag{12}$$

which holds for any renormalization scale μ.

To the accuracy we are aiming at one has to worry about supersymmetric effects influencing the extraction of the couplings in Eq. (11) from experimental data. Due to the presence of the weak gauge bosons in the loop corrections the weak mixing angle receives the numerically largest contributions whereas the influence of supersymmetric particles on the electromagnetic and strong coupling can be neglected.

The procedure to incorporate the supersymmetric effects on the numerical value of $\sin^2\Theta^{(6),\overline{\rm MS}}(M_Z)$ is as follows: In a first step we transfer $\sin^2\Theta^{(6),\overline{\rm MS}}(M_Z)$ from Eq. (11) to the $\overline{\rm DR}$ scheme [38] and apply afterwards the supersymmetric one-loop corrections evaluated in Ref. [39] relating the weak mixing angle in the SM to the one in MSSM. In a next step we decouple the supersymmetric particles [40] and finally go back to the $\overline{\rm MS}$ scheme. As a result we obtain $\sin^2\Theta^{(6),\overline{\rm MS}}(M_Z)$ at the scale $\mu = M_Z$ including virtual MSSM contributions. Note that by construction these corrections are suppressed by the square of the supersymmetric mass scale. We anticipate that for typical supersymmetric benchmark scenarios the influence of supersymmetric corrections to $\sin^2\Theta^{(6),\overline{\rm MS}}(M_Z)$ can lead to shifts in $M_{\rm H_c}$ which are of the order of 10%. For the `mSUGRA` parameters in Eq. (16) the shift of $\sin^2\Theta^{(6),\overline{\rm MS}}(M_Z)$ amounts to about $1.4\cdot 10^{-5}$.

In addition to the input values for the gauge coupling constants, we also need the W- and Z-boson pole masses M_W and M_Z, the top-quark and tau-lepton pole masses M_t and M_τ and the running bottom-quark mass $m_b^{\overline{\rm MS}}$. For the convenience of the reader we also specify their numerical values [15, 41, 42]

$$\begin{aligned}
M_W &= 80.398\ \text{GeV}\,,\\
M_Z &= 91.1876\ \text{GeV}\,,\\
M_t &= 173.1\ \text{GeV}\,,\\
M_\tau &= 1.77684\ \text{GeV}\,,\\
m_b^{\overline{\rm MS}}(m_b^{\overline{\rm MS}}) &= 4.163\ \text{GeV}\,.
\end{aligned}\tag{13}$$

The corresponding uncertainties are not important for our analysis.

In the following we describe in detail the individual steps of the running analysis needed for the energy evolution of the gauge couplings from the scale $\mu = M_Z$ to the unification scale.

1. Running within the SM from $\mu = M_Z$ to the SUSY scale $\mu_{\rm SUSY}$.
   Starting from Eqs. (11) and (12) (and adding supersymmetric effects as discussed above) we use the three-loop beta function of QCD [43, 44] and the two-loop RGEs in the electroweak sector [45–47] in order to obtain the values of the gauge couplings at $\mu_{\rm SUSY} \approx 1$ TeV. We take into account the tau, bottom and top Yukawa couplings and thus solve a coupled system of six differential equations. Since the quartic Higgs coupling λ enters the equations of the Yukawa couplings starting from two-loop order only we neglect its contribution. At this point we want to stress that $\mu_{\rm SUSY}$ is not fixed but kept as a free parameter in our setup.

2. SUSY threshold corrections.
   For energies of about $\mu_{\rm SUSY} \approx 1$ TeV the SUSY particles become active and the proper matching between the SM and the MSSM has to be performed. We decouple all heavy non-SM particles simultaneously at the scale $\mu_{\rm SUSY}$ using the one-loop relations for α_1 and α_2 and the Yukawa couplings from Refs. [40] and [48], respectively. The SUSY-QCD decoupling effects for α_3 and m_b are known to two-loop order and have been computed in Refs. [18, 19]. The simultaneous decoupling might be problematic in case there is a huge splitting among the SUSY masses. In that case a step-by-step decoupling would be preferable (see, e.g., Ref. [49]), however, a two-loop calculation in that framework is still missing. Furthermore, the mass splitting in almost all benchmark scenarios currently discussed in the literature is rather small.

   As pointed out before, a fully consistent approach would require two-loop threshold corrections not only in the strong but also in the electroweak sector. They are not yet available, however, we also expect that their numerical impact is relatively small. Furthermore, consistency with the RG running would require mixed QCD-Yukawa corrections for α_3 which are not yet available. Since the effect of these kind of corrections on the (three-loop) running is numerically small we expect that also the impact on the decoupling is small.

   In our numerical analysis we generate the SUSY mass spectrum with the help of the program `SOFTSUSY` [50] and study the various SPS (Snowmass Points and Slopes) scenarios [51, 52].

   At this stage also the change of renormalization scheme from $\overline{\rm MS}$ to $\overline{\rm DR}$ has to be taken into account. We employed the one-loop conversion relations [38, 53] for all parameters except α_3 and m_b where two-loop relations [53, 54] have been used in order to be consistent with the decoupling at the SUSY scale.

3. Running within the MSSM from $\mu_{\rm SUSY}$ to the high-energy scale $\mu_{\rm GUT}$.
   We use the three-loop RGEs of the MSSM [17, 20] to evolve the gauge and Yukawa couplings from $\mu_{\rm SUSY}$ to some very high scale of the order of 10^{16} GeV, that we denote by $\mu_{\rm GUT}$, where we expect that SUSY-GUT particles become active.

4. SUSY-GUT threshold effects.

At the energy scale $\mu_{\rm GUT}$, threshold corrections induced by the non-degenerate SUSY-GUT spectrum have to be taken into account. Currently, only the one-loop corrections are available [22–24], which we include in our setup, although for consistency two-loop corrections would be necessary.

A suitable linear combination of the three one-loop equations for α_i in (3) and (4) leads to the following two relations

$$\begin{aligned} 4\pi\left(-\frac{1}{\alpha_1(\mu)}+3\,\frac{1}{\alpha_2(\mu)}-2\,\frac{1}{\alpha_3(\mu)}\right) &= -\frac{12}{5}L_{\mu H_c}\,, \\ 4\pi\left(5\,\frac{1}{\alpha_1(\mu)}-3\,\frac{1}{\alpha_2(\mu)}-2\,\frac{1}{\alpha_3(\mu)}\right) &= -24\left(L_{\mu X}+\frac{1}{2}L_{\mu\Sigma}\right)\,, \end{aligned} \tag{14}$$

where $\alpha^{\rm SU(5)}$ has been eliminated. These equations allow for the prediction of the coloured triplet Higgs boson mass $M_{\rm H_c}$ from the knowledge of the MSSM gauge couplings at the energy scale $\mu = \mu_{\rm GUT}$. It is furthermore common to define a new mass parameter $M_G = \sqrt[3]{M_X^2 M_\Sigma}$, the so-called grand unified mass scale, that can also be determined from the knowledge of the MSSM gauge couplings at $\mu_{\rm GUT}$. These observations makes it quite easy to test the minimal SUSY SU(5) model once the required experimental data are available in combination with a high-order analysis.

For the Missing Doublet Model the above relations read

$$\begin{aligned} 4\pi\left(-\frac{1}{\alpha_1(\mu)}+3\,\frac{1}{\alpha_2(\mu)}-2\,\frac{1}{\alpha_3(\mu)}\right) &= -\frac{12}{5}\left(L_{\mu H_c}+L_{\mu H_{c'}}\right)+12\ln\frac{64}{3125}\,, \\ 4\pi\left(5\,\frac{1}{\alpha_1(\mu)}-3\,\frac{1}{\alpha_2(\mu)}-2\,\frac{1}{\alpha_3(\mu)}\right) &= -24\left(L_{\mu X}+\frac{1}{2}L_{\mu\Sigma}\right)-12\ln\frac{262144}{1953125}\,, \end{aligned} \tag{15}$$

5. Running from $\mu_{\rm GUT}$ to the Planck scale $M_{\rm Pl}$.
   The last sequence of our approach consists in the running within the SUSY-SU(5) model. We implemented the three-loop RGEs for the gauge [55], and the one-loop formulas for the Yukawa and Higgs self couplings [56] and impose that they can be described within perturbation theory up to the Planck scale.

In Tab. 1 we summarize for the individual steps of the running-decoupling procedure to which loop order the perturbative corrections are currently available and implemented in our setup. The numbers in parenthesis indicate the loop order which would be necessary in order to perform a fully consistent analysis with three-loop running and two-loop decoupling. The phenomenologically most important ingredient which is still lacking are the two-loop decoupling relations at $\mu_{\rm GUT}$ as we will show in the next Section.

	run	dec	run	dec	run
	$M_Z \to \mu_{\rm SUSY}$	$\alpha_i \to \alpha_i^{\rm (MSSM)}$	$\mu_{\rm SUSY} \to \mu_{\rm GUT}$	$\alpha_i^{\rm (MSSM)} \to \alpha^{\rm (GUT)}$	$\mu_{\rm GUT} \to M_{\rm pl}$
EW	2(3)	1(2)	3	1(2)	$3^\star$
QCD	3	2	3		

Table 1: Loop corrections available for the individual steps of the running-decoupling procedure. The number is brackets indicate the loop-order needed for a consistent analysis. ($\star$ The running of the gauge and Yukawa coupling is performed to three- and one-loop accuracy, respectively.)

3 Phenomenological constraints on the SUSY GUT mass spectrum

The running-decoupling prescription described in Section 2.2 introduces two decoupling scales, $\mu_{\rm SUSY}$ and $\mu_{\rm GUT}$, for the supersymmetric and GUT particles, respectively. These scales are not determined by theory. However, on general grounds one expects that the GUT parameters become insensitive to the precise choice of these scales when going to higher orders. In Fig. 1(a) this is demonstrated for $M_{\rm H_c}$ considering the dependence on $\mu_{\rm SUSY}$ where the latter is varied between 100 GeV and 10 TeV. For illustration we adopt the `mSUGRA` scenario for the SUSY breaking mechanism with

$$\begin{aligned} m_0 &= m_{1/2} = -A_0 = 1000 \text{ GeV}\,, \\ \tan\beta &= 3\,, \\ \mu &> 0\,, \end{aligned} \tag{16}$$

and generate with the help of `SOFTSUSY` [50] the supersymmetric mass spectrum. This results in squark masses which are of the order to 2 TeV and thus in the upper range of what can be measured at the CERN LHC. The dotted, dashed and solid lines in Fig. 1(a) correspond to the one-, two- and three-loop running analysis, respectively, where the decoupling is performed at one order lower as required by consistency. One finds a quite sizeable dependence of $M_{\rm H_c}$ at one-loop order varying by almost two orders of magnitude. A significant reduction is observed after inclusion of the two-loop effects leading to a variation of $M_{\rm H_c}$ by only a factor two to three. Finally, after incorporating three-loop running and two-loop matching corrections the variation of $M_{\rm H_c}$ on $\mu_{\rm SUSY}$ is about $5 \cdot 10^{14}$ GeV in the considered range for the matching scale. It is furthermore remarkable that for $\mu_{\rm SUSY}$ around 1000 GeV, which is close to the average of the supersymmetric masses, the two-loop corrections show a maximum and the three-loop corrections are practically zero. In particular, they are significantly smaller than for $\mu_{\rm SUSY} = M_Z$ which has often been used as decoupling scale for the supersymmetric particles.

In Fig. 1(b) the dependence of $M_{\rm H_c}$ on $\mu_{\rm GUT}$ is studied. A consistent three-loop analysis can not be performed since the two-loop GUT matching relation is not yet available.

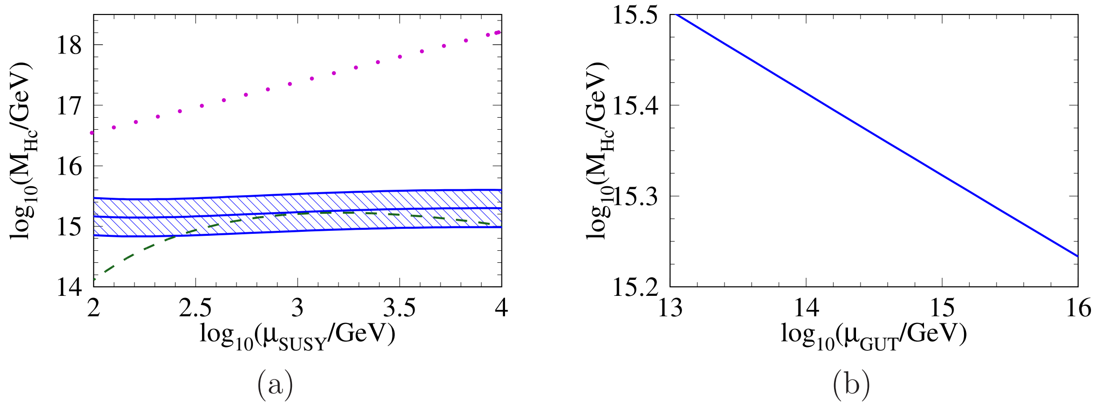


Figure 1: Dependence of $M_{\mathrm{H_c}}$ on μ_{SUSY} (a) and μ_{GUT} (b) including successively higher orders. Dotted, dashed and solid lines correspond to the one-, two- and three-loop running analysis, respectively.

Nevertheless it is tempting to combine the three-loop running with the one-loop matching effects which is represented by the solid line. One observes that $M_{\mathrm{H_c}}$ varies by about $1.5 \cdot 10^{15}$ GeV. This is of the same order of magnitude as the three-loop effect at the SUSY scale if the matching is performed at M_Z (see Fig. 1(a)).

In the following we discuss the dependence of $M_{\mathrm{H_c}}$ and M_{G} on various parameters entering our analysis. We start with varying the supersymmetric mass spectrum by considering different SPS scenarios [51,52] and use Eq. (14) in order to extract in each case both $M_{\mathrm{H_c}}$ and M_{G}. The decoupling scales are fixed to $\mu_{\mathrm{SUSY}} = 1000$ GeV and $\mu_{\mathrm{GUT}} = 10^{16}$ GeV, respectively, which ensures, according to the previous discussion, that the three-loop effect is rather small. In Fig. 2 the results are shown in the $M_{\mathrm{H_c}} - M_{\mathrm{G}}$ plane where the lines indicate the slopes in case of SPS1a, SPS2, SPS3, SPS7, SPS8 and SPS9. Most scenarios lead to $M_{\mathrm{H_c}}$ masses between $0.25 \cdot 10^{14}$ GeV and about $1 \cdot 10^{15}$ GeV: SPS9 (anomaly-mediated SUSY breaking) gives the smallest and SPS2 the largest value of $M_{\mathrm{H_c}}$.

A further illustration of the dependence of the GUT parameters on the SUSY spectrum can be found in Fig. 3 where we adopt the parameters of Eq. (16) and vary $m_{1/2}$. In this way we alter the SUSY spectrum which enters the prediction of $M_{\mathrm{H_c}}$ and M_{G} via the decoupling procedure at $\mu = \mu_{\mathrm{SUSY}}$. The solid and dashed lines correspond to $M_{\mathrm{H_c}}$ and M_{G}, respectively, which show a substantial variation. On the other hand, m_0, $\tan\beta$ and A_0 have only a minor influence on the GUT masses and thus we refrain from explicitly showing the dependence.

In the following we study the effects of the uncertainties of the input values α_i (c.f. Eq. (12)) on $M_{\mathrm{H_c}}$ and M_{G}. We fix the SUSY spectrum as before (see Eq. (16)) and set in a first step $\mu_{\mathrm{SUSY}} = M_Z$ which has often been common practice in similar analyses (see, e.g., Ref. [9]). Taking into account correlated errors and performing a χ^2 analysis leads to ellipses in the $M_{\mathrm{H_c}} - M_{\mathrm{G}}$ plane. Let us mention that we can reproduce the results

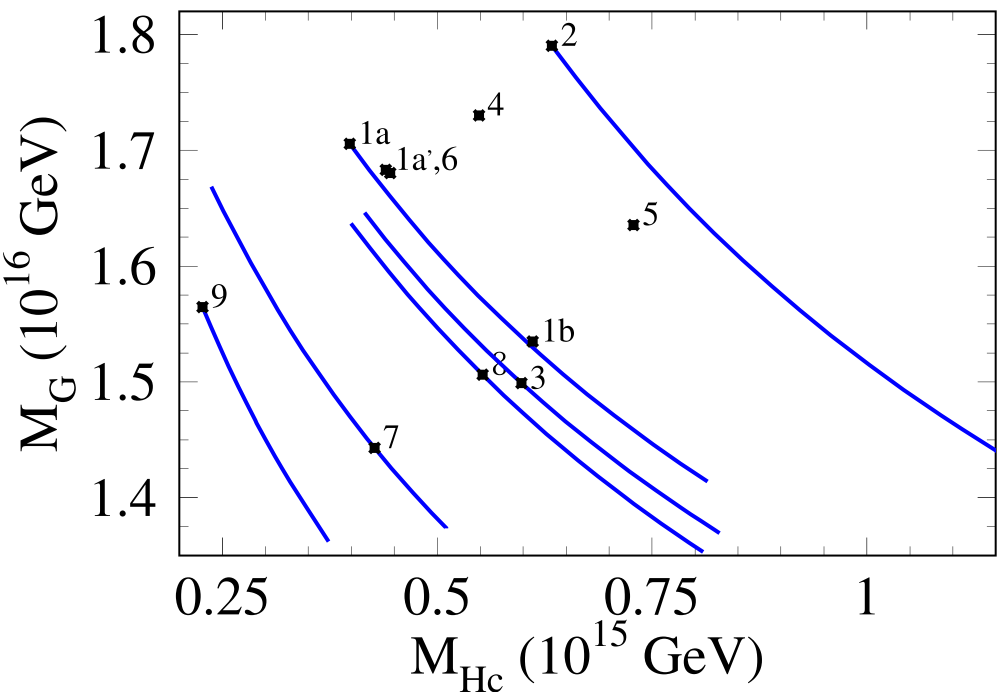


Figure 2: Dependence of $M_{\mathrm{H_c}}$ and M_{G} on the various SPS scenarios.

of Ref. [9] after adopting their parameters and restricting ourselves to the perturbative input used in that publication.

In Fig. 4(a) we show our results for the two- (dashed lines) and three-loop (continuous lines) analyses. In each case the two concentric ellipses correspond to 68% and 90% confidence level, respectively, where only parametric uncertainties from Eq. (11) have been taken into account. A significant shift to higher masses of about an order of magnitude is observed for $M_{\mathrm{H_c}}$; M_{G} increases by about $2 \cdot 10^{15}$ GeV. This demonstrates the importance of the two-loop matching and three-loop running corrections. As has been discussed in the context of Fig. 1 they are essential in order to remove the dependence on μ_{SUSY}. In fact, adopting in Fig. 4 $\mu_{\mathrm{SUSY}} = 1$ TeV would lead to ellipses for the two- and three-loop analysis which were almost on top of each other and which would coincide with the three-loop ellipses (solid lines) in Fig. 4.

As expected, the uncertainty of α_s induces the largest contributions to the uncertainties of $M_{\mathrm{H_c}}$ and M_{G}. In particular, it essentially determines the semimajor axis of the ellipses. Thus it is tempting to assign a more optimistic uncertainty of $\delta\alpha_s = 0.0010$ and redo the previous analysis. The results, shown in Fig. 4(b), underline even more the importance of the three-loop analysis. Whereas for $\delta\alpha_s = 0.0020$ the parametric uncertainty (*i.e.* the size of the ellipses) and the shift due to higher perturbative corrections are of the same order of magnitude, in Fig. 4(b) the latter is about twice as big as the former. Let us,

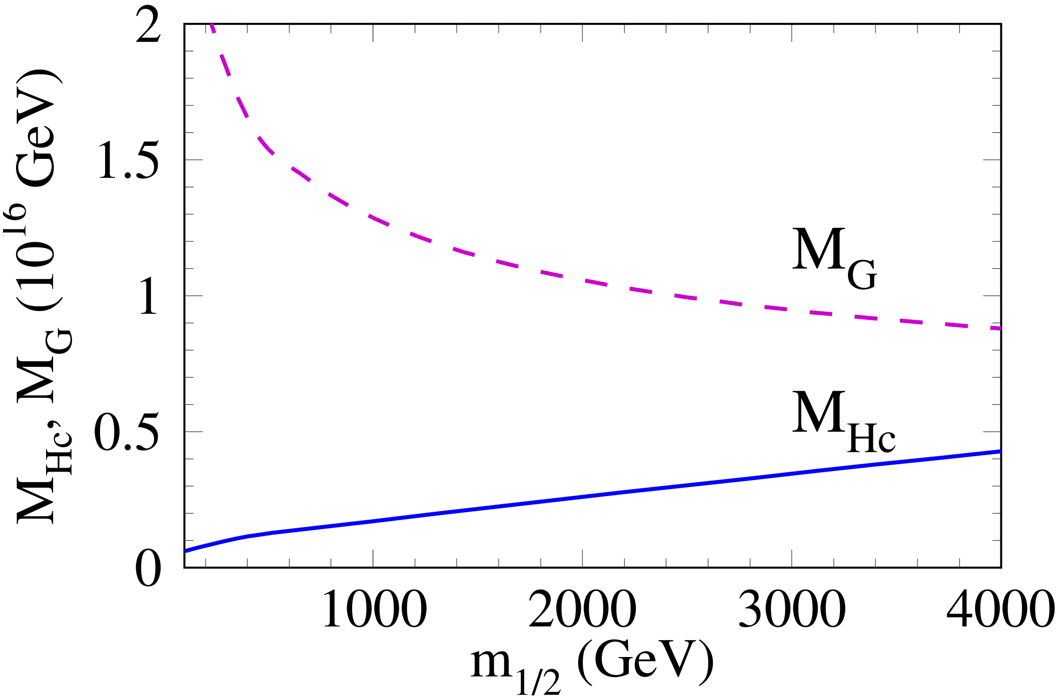
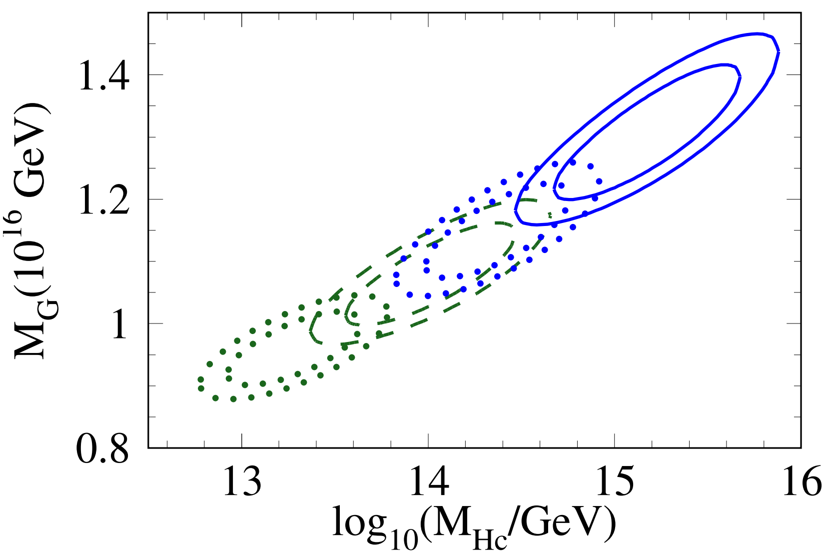


Figure 3: Dependence of $M_{\mathrm{H_c}}$ (solid) and M_{G} (dashed) on $m_{1/2}$.

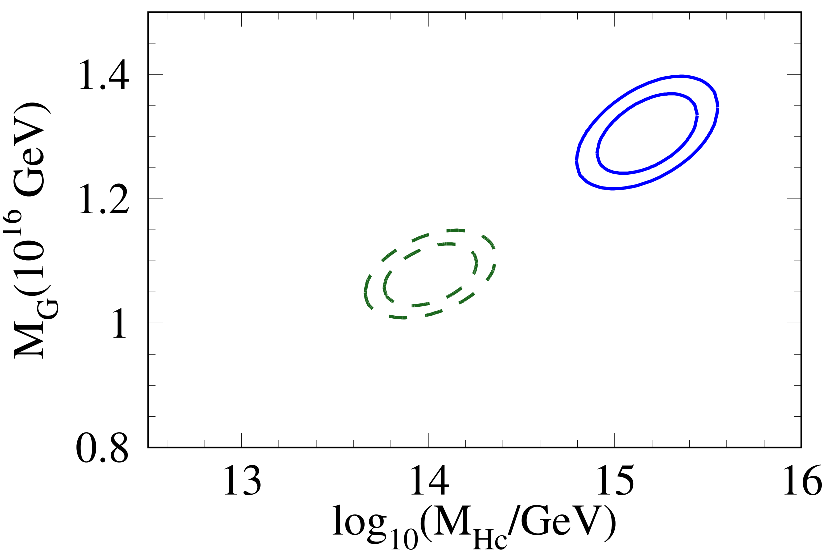
however, stress once again that choosing μ_{SUSY} close to the supersymmetric mass scales leads to small three-loop effects since the two-loop ellipses are essentially shifted on top of the three-loop ones.

Recently there have been a few extractions of α_s based on higher order perturbative corrections with uncertainties slightly above 1%, which, however, obtain central values for α_s close to 0.113 (see, e.g., Ref. [57]). Since these results are significantly lower than the value given in Eq. (7) it is interesting to show in Fig. 4(a) also the corresponding 68% and 90% confidence level ellipses (for the two- and three-loop analysises) as dotted lines adopting $\alpha_s(M_Z) = 0.1135 \pm 0.0014$ [58]. One observes a big shift in the GUT masses, in the case of $M_{\mathrm{H_c}}$ the central value is about one order of magnitude lower than for the α_s value of Eq. (7).

In Fig. 5 we visualize the running (and decoupling) of the gauge couplings where the parameters of Eq. (16) together with $\mu_{\mathrm{SUSY}} = 1000$ GeV and $\mu_{\mathrm{GUT}} = 10^{16}$ GeV have been adopted. In addition we have chosen $M_\Sigma = 1{\cdot}10^{15}$ GeV which leads to $M_{\mathrm{H_c}} = 1.7{\cdot}10^{15}$ GeV and $M_X = 4.6{\cdot}10^{16}$ GeV. One can clearly see the discontinuities at the matching scales and the change of the slopes when passing them. In panel (b) the region around $\mu = 10^{16}$ GeV is enlarged which allows for a closer look at the unification region. The bands indicate 1σ uncertainties of α_i at the electroweak scale (cf. Eq. (11)). In panel (b) we furthermore perform the decoupling of the super-heavy masses for two different values of μ_{GUT}. One

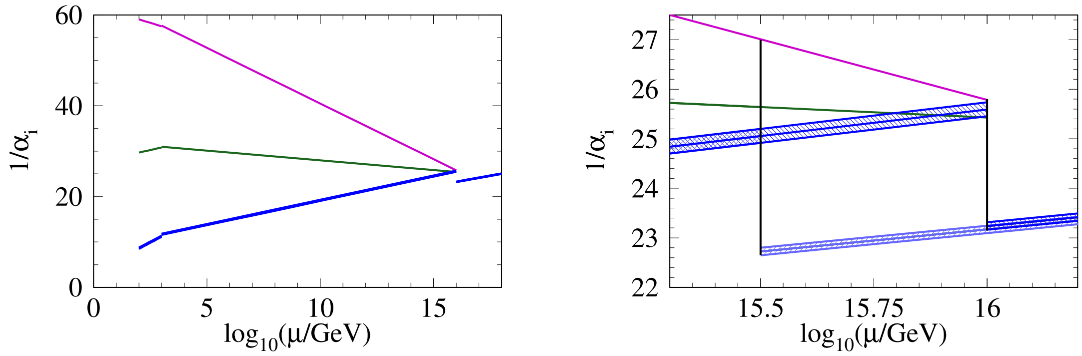


(a)



(b)

Figure 4: Ellipses in the $M_{\mathrm{H_c}} - M_{\mathrm{G}}$ plane obtained from the uncertainties of the gauge couplings at the electroweak scale. In (a) the input parameters of Eq. (7) have been used whereas in (b) $\delta\alpha_s = 0.0010$ has been chosen. Dashed and solid lines correspond to the two- and three-loop analysis, respectively. The dotted lines in (a) have been obtained for $\alpha_s(M_Z) = 0.1135 \pm 0.0014$ where the lower (upper) ellipse corresponds to the two- (three-) loop analysis.



(a) (b)

Figure 5: Running of the gauge couplings from the electroweak to the Planck scale. The discontinuity for $\mu = \mu_{\rm SUSY}$ and $\mu = \mu_{\rm GUT}$ are clearly visible. In panel (b) an enlargement of (a) for the region around $\mu = \mu_{\rm GUT}$ is shown where for the decoupling the two values $\mu_{\rm GUT} = 10^{15.5}$ GeV $\approx 3.2 \cdot 10^{15}$ GeV and $\mu_{\rm GUT} = 10^{16}$ GeV have been chosen.

observes quite different threshold corrections leading to a nice agreement of $\alpha^{\rm SU(5)}$ above 10^{16} GeV. Fig. 5 stresses again that the uncertainty of α_s is the most important one for the constraints that one can set on GUT models from low-energy data. Furthermore, it illustrates the size of the GUT threshold corrections and emphasizes the importance of the two-loop corrections for the corresponding decoupling constants.

Finally we discuss the phenomenological consequences of our analysis in a "top-down" approach where we specify the parameters at the high scale and examine the effect on the gauge couplings for $\mu = M_Z$. For minimal SUSY SU(5) we choose the following parameters

$$
\begin{aligned}
M_{\rm H_c} &= 3.67 \cdot 10^{14}\ {\rm GeV}\,, \\
M_{\Sigma} &= 2 \cdot 10^{16}\ {\rm GeV}\,, \\
M_X &= 1.58 \cdot 10^{16}\ {\rm GeV}\,, \\
\alpha^{\rm SU(5)}(10^{17}\ {\rm GeV}) &= 0.03986\,,
\end{aligned}
\tag{17}
$$

which guarantee the agreement of $\alpha^{(6),\overline{\rm MS}}(M_Z)$, $\sin^2\Theta^{(6),\overline{\rm MS}}(M_Z)$ and $\alpha_s^{(6),\overline{\rm MS}}(M_Z)$ with their experimental counterparts for $\mu_{\rm GUT} = 10^{16}$ GeV. In Fig. 6(a) we fix $\mu_{\rm SUSY} = 500$ GeV and the SUSY spectrum according to the SPS1a benchmark scenario which allows for a comparison with the MDM (cf. Fig. 6(b)). We vary $\mu_{\rm GUT}$ by three orders of magnitude where the dotted, dashed and solid line correspond to the one-, two- and three-loop analysis of the described procedure, respectively, and the bands reflect the uncertainties as given in Eq. (11). For this parameter choice we observe, as expected from the above discussions, a sizeable two-loop effect but only a mild change after including the three-loop corrections. Furthermore, the overall variation is small for all three gauge couplings. Note that the one-loop curve for α_s is independent of $\mu_{\rm GUT}$ since the one-loop coefficients of the β functions in SUSY QCD and minimal SUSY SU(5) are identical.

In Fig. 6(b) we perform the same analysis within the MDM adopting SPS1a,[5] $\mu_{\rm SUSY} =$ 500 GeV and the parameters[6]

$$\begin{aligned} M_{\rm H_c} &= 6\cdot 10^{18}\ \mathrm{GeV}\,, \\ M_{\rm H_c'} &= 1\cdot 10^{16}\ \mathrm{GeV}\,, \\ M_{\Sigma} &= 2\cdot 10^{15}\ \mathrm{GeV}\,, \\ M_{X} &= 3\cdot 10^{16}\ \mathrm{GeV}\,, \\ \alpha^{\rm SU(5)}(10^{17}\ \mathrm{GeV}) &= 0.1504\,. \end{aligned} \tag{18}$$

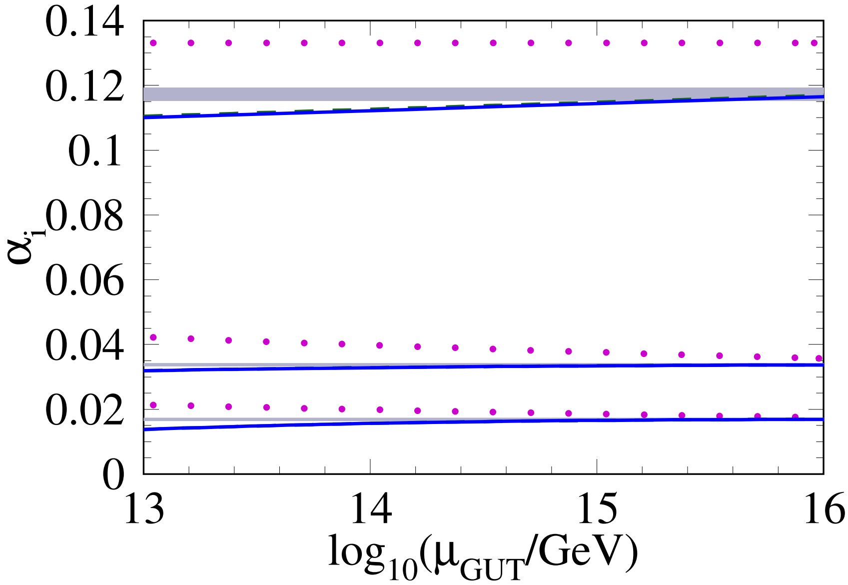
One observes a much stronger variation of the predicted values of α_i, in particular in the case of the strong coupling which varies between 0.06 and 0.13 in the considered range of $\mu_{\rm GUT}$. This demonstrates the importance of the two-loop threshold corrections which are expected to significantly reduce the dependence on $\mu_{\rm GUT}$.

At this point a discussion about the additional constraint on the Higgs triplet mass M_{H_c} that can be derived from the non-observation of the proton decay is in order. The latest upper bound on the proton decay rate for the channel $p \to K^+\bar{\nu}$ [14] is $\Gamma_{\rm exp} = 4.35\times 10^{-34}/\mathrm{y}$. In order to translate it into a lower bound for the the Higgs triplet mass, one needs an additional assumption about the Yukawa couplings that enter the expression of the decay rate $\Gamma(p\to K^+\bar{\nu})$. As pointed out in Ref. [11] this is because down quark and lepton Yukawa couplings fail to unify within the minimal renormalizable SUSY SU(5) model and so a completely consistent treatment is not possible. Therefore one could either choose[7] (i) $Y_{ql} = Y_{ud} = Y_d$ or (ii) $Y_{ql} = Y_{ud} = Y_e$, which leads to completely different phenomenological consequences. We are aware that both cases are equally justified once higher dimensional operators are included. Since these operators further weaken the bounds presented below, we refrain to include them into the analysis in this paragraph. For the case (i) and sparticle masses around 1 TeV the lower bound for the Higgs triplet mass can be read off from Fig. 2 of Ref. [11] and amounts to $M_{H_c} \geq 1.05\times 10^{17}$GeV whereas for the second choice it becomes $M_{H_c} \geq 5.25\times 10^{15}$GeV. From our phenomenological analysis presented above it turns out that within the minimal SUSY SU(5) model the upper bound for M_{H_c} is of about 10^{16}GeV. So, the substantial increase of about one order of magnitude for the upper bound on M_{H_c} induced by the three-loop order running analysis attenuates the tension between the theoretical predictions made under the assumption (i) and the experimental data. The choice (ii) for the Yukawa couplings clearly shows that the minimal SUSY SU(5) model cannot be ruled out by the current experimental data on proton decay rates. More experimental information about the SUSY mass spectrum and proton decay rates is required in order to be able to draw a firm conclusion.

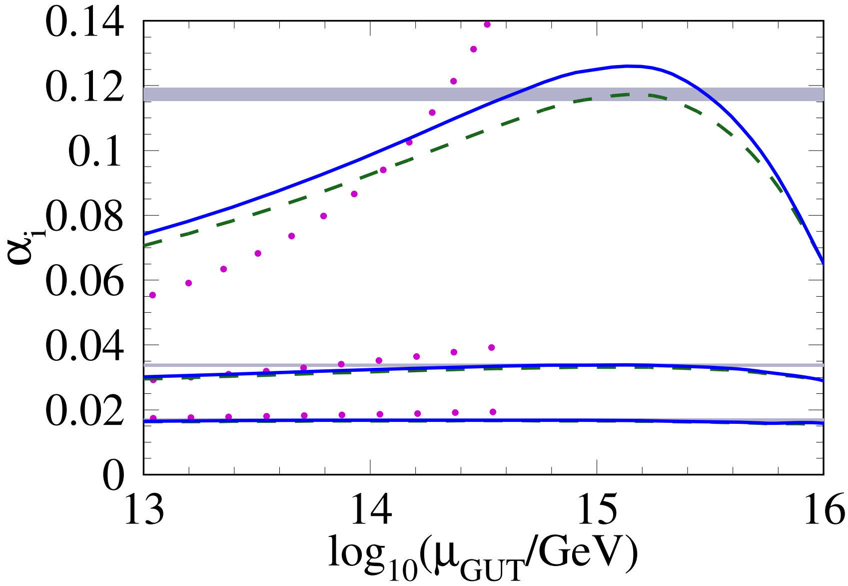
[5] Adopting the parameters from Eq. (16) leads to a non-perturbative values of the gauge coupling at the Planck scale.

[6] The relatively large value of $\alpha^{\rm SU(5)}(10^{17}\ \mathrm{GeV})$ is required due to the large Casimir constants entering the β function above the GUT scale.

[7] Y_{ql} is the Yukawa coupling of the quark and lepton doublet to the Higgs colour triplet. Y_{ud} is the corresponding coupling for the up and down quark singlet.



(a)



(b)

Figure 6: Gauge couplings at the weak scale as obtained by a top-down approach within minimal SUSY SU(5) (a) and the MDM (b) as a function of $\mu_{\rm GUT}$ (see text). In (b) the one-loop curves are only shown up to $\mu_{\rm GUT} \approx 4 \cdot 10^{14}$ GeV since beyond this scale α_3 becomes quite large.

On the other hand, it is commonly recognized that SUSY SU(5) in its minimal version cannot provide the underlying model for a consistent GUT because of the failure of Yukawa unification. As mentioned in the Introduction, one of the most appealing solutions of this problem is the inclusion of $M_{\rm Pl}$ scale effects using the effective theory approach, *i.e.*, including higher dimensional operators. Explicitly, this translates into [13]

$$M_{(8,1)} \neq M_{(1,3)} \quad \text{and} \quad M_{H_c} = M_{H_c}^{(0)} \left(\frac{M_{(1,3)}}{M_{(8,1)}} \right)^{\frac{5}{2}} , \tag{19}$$

where $M_{H_c}^{(0)}$ denotes the mass of the Higgs triplet in the minimal renormalizable model. In this case, a suitable choice of the Yukawa matrices can significantly reduce the proton decay amplitudes, so that values for M_{H_c} of the order $\mathcal{O}(10^{15}\text{GeV})$ even for larger values of $\tan\beta \simeq 15$ are allowed. So the prediction of the non-renormalizable version of the SUSY SU(5) model is well under the experimental upper bound. However, the inclusion of new undetermined parameters together with the higher dimensional operators makes the tests of such models much more complicated.

At this point we try to compare with the findings of the analysis [59] based on the non-renormalizable version of minimal SU(5) where the bound $M_{\rm H_c} > 3.7 \times 10^{17}$ GeV is given. This is the most stringent bound[8] that can be derived from current proton decay experimental data. In order to be consistent with our upper bound on $M_{\rm H_c}^{(0)}$ (cf. Fig. 4) of about 10^{16} GeV one requires $M_{(1,3)}/M_{(8,1)} \gtrsim 4.2$ which is a quite moderate value.

4 Conclusions

Already in the early days of the Standard Model there have been studies of theories which predict the unification of the coupling constants at high energies. Since it is not possible to reach such energies in collider experiments it is necessary to establish relations between the couplings at the electroweak scale, where precise measurements are available, and the corresponding quantities at the unification scale. Next to the beta functions covering the running also threshold corrections at the SUSY and GUT scale constitute crucial input.

We have considered the so-called minimal supersymmetric SU(5) GUT theory and have studied the gauge coupling unification applying one-, two- and three-loop running. The main effect of the three-loop running (accompanied by two-loop decoupling relations) is the stabilization w.r.t. the variation of the decoupling scales. In general sizable three-loop effects are observed if the decoupling scale for the supersymmetric particles is not chosen in the vicinity of the masses of the supersymmetric particles. In particular, for $\mu_{\rm SUSY} = M_Z$, which is the canonical choice often adopted in the literature, one observes an increase of the coloured Higgs triplet mass by about an order of magnitude. Already in previous studies it has been shown that the non-renormalizable version of minimal SUSY

[8] See Ref. [59] for the exact definition of this bound.

SU(5) cannot be excluded by the experimental bound on the proton decay rate [10,11,13]. Our results attenuate the tension between the theoretical predictions of $M_{\mathrm{H_c}}$ and the experimental results even more.

Our analysis includes all the state-of-the-art theoretical input. It could be improved by including the two-loop GUT threshold corrections which are not yet available. This induces an uncertainty of about 0.3 on $\log_{10}(M_{\mathrm{H_c}}/\mathrm{GeV})$ which constitutes the major theory-uncertainty. We observe that the effects of the two-loop GUT threshold corrections become particularly important in the Missing Doublet Model where also larger effects can be observed. As far as the parametric uncertainty is concerned we mention the dependence on the supersymmetric mass spectrum and the uncertainty on α_s. In both cases a shift of $M_{\mathrm{H_c}}$ of a few times 10^{15} GeV is observed.

The most popular extensions of the minimal SUSY SU(5) model, the Missing Doublet Model and the non-renormalizable version of the SUSY SU(5) model comprising Planck-scale operators, can not be excluded using only the currently available theoretical and experimental data. They either contain additional free parameters as compared to the minimal model or are affected by large theoretical uncertainties, so that no firm conclusion can be drawn for them.

Acknowledgements

We would like to thank Mikołaj Misiak and Robert Harlander for carefully reading the manuscript and many useful comments. We are grateful to Thomas Teubner and Daisuke Nomura for communications regarding $\Delta\alpha^{(5)}_{\mathrm{had}}$. This work was supported by the DFG through SFB/TR 9 “Computational Particle Physics”, the Graduiertenkolleg “Hochenergiephysik und Teilchenastrophysik”, the “Studienstiftung des Deutschen Volkes” and the “Landesgraduiertenförderung” of the state of Baden-Württemberg.

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