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Grand Unified Flavour Physics

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Abstract. We probe the unification of down quarks and leptons in a supersymmetric SO(10) GUT. The large atmospheric neutrino mixing angle induces $b_R - s_R$ transitions, which can account for the sizeable CP phase ϕ_s measured in $B_s - \overline{B}_s$ mixing. Corrections to down-quark-lepton unification from higher-dimensional Yukawa terms translate neutrino mixing also into $s_R - d_R$ and $b_R - d_R$ currents. We find the flavour structure of Yukawa corrections to be strongly constrained by ε_K.

The embedding of the Standard Model into a larger symmetry group implies relations among Standard-Model parameters, which can be tested by experiment. In the framework of supersymmetric Grand Unified Theories, the unification of gauge couplings has passed such tests excellently. Flavour physics probes the unification of Yukawa couplings in low-energy observables. The unification of down (s)quarks and (s)leptons can transfer large mixing in the lepton sector into flavour-changing neutral currents among down squarks. In particular, the atmospheric neutrino mixing angle leaves imprints of Yukawa unification in B_s observables. For light fermions, the mass relations between down quarks and leptons need to be corrected. To this end, one introduces higher-dimensional Yukawa terms that a priori imply large mixing effects also in K and B_d physics observables. The precise knowledge of these observables allows to constrain the flavour structure of such Yukawa corrections. We show that corrections to down-quark-lepton unification must be basically aligned with the initial Yukawa couplings in flavour space.

GUT RELATIONS BETWEEN DOWN QUARKS AND LEPTONS

The unification of down quarks and leptons is a feature of SU(5) symmetry. Explicitly, the SU(2)-singlet down-type quarks d^c are combined with the lepton doublet $L = (\ell, \nu_\ell)$ in a $\overline{5}$ representation of SU(5),

$$\overline{5} = (d_1^c, d_2^c, d_3^c; \ell, \nu_\ell)\,, \qquad (1)$$

where $1,2,3$ are colour indices and a superscript c denotes charge conjugation. The quark doublet Q, as well as the quark and lepton singlets u^c and ℓ^c, are embedded into a 10. The Yukawa sector comprises fermion couplings to the Higgs representations $5_H \supset H_u$, $\overline{5}_H \supset H_d$, which contain the Higgs doublets H_u and H_d of a two-Higgs-doublet model,

$$W_Y = 10_i Y_5^{ij}\, 10_j\, 5_H + 10_i Y_{\overline{5}}^{ij}\, \overline{5}_j\, \overline{5}_H\,. \qquad (2)$$

The first term gives masses to up quarks, whereas the second term generates masses for both down quarks and leptons. Thereby the Standard-Model Yukawa couplings Y_d and Y_ℓ are unified at the scale of gauge coupling unification M_{GUT},

$$Y_d = Y_\ell^\top = Y_{\overline{5}}\,. \qquad (3)$$

By evolving this relation down to the electroweak scale M_Z and comparing with the measured down-quark and lepton masses, one learns that experimentally down-quark-lepton unification holds only for the bottom and tau couplings. Corrections for the mass relations between light fermions can be provided by adding terms of mass dimension five to the Yukawa sector [1],

$$W_Y^{d,\ell} = 10_i Y_{\bar{5}}^{ij}\, \bar{5}_j\, \bar{5}_H + 10_i \frac{24_H}{M_{\rm Pl}} Y_\sigma^{ij}\, \bar{5}_j\, \bar{5}_H\,. \tag{4}$$

If the adjoint Higgs representation 24_H acquires a vacuum expectation value proportional to $\mathrm{SU}(5)$ hypercharge, $\langle 24_H\rangle = \sigma\,\mathrm{diag}(2,2,2;-3,-3)$, relation (3) is modified below the GUT scale to

$$Y_d = Y_\ell^\top + 5\frac{\sigma}{M_{\rm Pl}} Y_\sigma\,. \tag{5}$$

Due to the suppression by $\sigma/M_{\rm Pl} = \mathcal{O}(10^{-3})$, the Yukawa corrections Y_σ affect only the light fermions, while preserving the successful bottom-tau Yukawa unification.

NEUTRINO MIXING IN DOWN-SQUARK INTERACTIONS

Yukawa corrections from dimension-five terms can be implemented straight into $\mathrm{SO}(10)$, where all fermions of one generation, plus a heavy right-handed neutrino N, are unified in one spinor representation $16 = 1 \oplus 10 \oplus \bar{5} = (N, (Q, u^c, \ell^c), (d^c, L))$. We examine a specific supersymmetric $\mathrm{SO}(10)$ model proposed by Chang, Masiero, and Murayama (CMM) [2]. The Yukawa sector is given by

$$W_{\rm CMM} = 16_i \widehat{Y}_{10}^{ii}\, 16_i\, 10_H + 16_i\, (V_q^* \widehat{Y}_{45} V_\ell)^{ij}\, 16_j \frac{45_H\, 10'_H}{M_{\rm Pl}} + 16_i \widehat{Y}_{16}^{ii}\, 16_i \frac{\overline{16}_H \overline{16}_H}{2M_{\rm Pl}}\,, \tag{6}$$

where the three terms generate masses for up quarks, down quarks and leptons, and right-handed neutrinos, respectively. Under the assumption that the up-quark and neutrino Yukawa couplings Y_{10} and Y_{16} are simultaneously diagonal, flavour mixing is fully contained in the down-quark-lepton coupling Y_{45}. In the fermion mass eigenbasis, the mixing matrices V_q and V_ℓ are identified with $V_{\rm CKM}$ and $V_{\rm PMNS}$ up to phases. Due to down-quark-lepton unification, $V_{\rm PMNS}$ rotates both leptons and right-handed down quarks. The large atmospheric neutrino mixing angle $\theta_{23} \simeq 45^\circ$ thereby translates into significant $b_R - s_R$ transitions.

As in $\mathrm{SU}(5)$, corrections to Yukawa unification for light fermions arise from the $\mathrm{SU}(5)$-breaking vacuum expectation value $\langle 24_H\rangle$, since the 45_H of $\mathrm{SO}(10)$ contains the 24_H of $\mathrm{SU}(5)$. They generate additional rotations of down quarks with respect to leptons, parametrized by a mixing angle θ. Up to phases, the rotation matrix of right-handed down quarks is thus given by

$$R_d = (UV_\ell)^\top, \qquad U = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{7}$$

As a consequence of this additional rotation, the atmospheric neutrino mixing angle also appears in $s_R - d_R$ and $b_R - d_R$ transitions. In a supersymmetric framework, these effects are visible in the couplings of down squarks if the mass spectrum of scalar superpartners is not degenerate. We assume flavour-blind supersymmetry breaking at the Planck scale, which corresponds to universal soft masses m_0^2 and trilinear couplings A_0 at $M_{\rm Pl}$. Effects of the large top Yukawa coupling in the SO(10) renormalization group evolution induce a mass splitting $\Delta_{\tilde{d}} = \mathcal{O}(0.5)$ in the down-squark soft mass matrix (here in the up-quark mass eigenbasis),

$$M_{\rm Pl}:\ (M_{\tilde{d}}^2)^U = m_0^2 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \overset{\rm RGE}{\longrightarrow} \quad M_Z:\ (M_{\tilde{d}}^2)^U = m_{\tilde{d}}^2 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\Delta_{\tilde{d}} \end{pmatrix}. \qquad (8)$$

In the super-CKM basis, where the scalar superpartners are simultaneously rotated with the fermions, the mass matrix of singlet down squarks reads

$$\begin{aligned}(M_{\tilde{d}}^2)^{\rm sCKM} &= R_d^\dagger \, (M_{\tilde{d}}^2)^U \, R_d \\ &= m_{\tilde{d}}^2 \cdot \begin{pmatrix} 1-\sin^2\theta\,\Delta_{\tilde{d}}/2 & \sin(2\theta)\,e^{-i\phi_K}\,\Delta_{\tilde{d}}/4 & \sin\theta\,e^{-i\phi_{B_d}}\,\Delta_{\tilde{d}}/2 \\ \sin(2\theta)\,e^{i\phi_K}\,\Delta_{\tilde{d}}/4 & 1-\cos^2\theta\,\Delta_{\tilde{d}}/2 & -\cos\theta\,e^{-i\phi_{B_s}}\,\Delta_{\tilde{d}}/2 \\ \sin\theta\,e^{i\phi_{B_d}}\,\Delta_{\tilde{d}}/2 & -\cos\theta\,e^{i\phi_{B_s}}\,\Delta_{\tilde{d}}/2 & 1-\Delta_{\tilde{d}}/2 \end{pmatrix}, \qquad (9)\end{aligned}$$

assuming tri-bi-maximal lepton mixing. The off-diagonal elements induce effects of atmospheric neutrino mixing in K, B_d, and B_s physics observables. The phases ϕ_K, ϕ_{B_d}, and $\phi_{B_s} = \phi_{B_d} - \phi_K$ are new sources of CP violation. We will concentrate on the characteristic signatures from down-quark-squark gluino box diagrams in meson mixing.

For $\theta = 0$, corresponding to vanishing Yukawa corrections, imprints of the atmospheric neutrino mixing angle are confined to B_s observables. In $B_s - \overline{B}_s$ mixing, both the mass difference ΔM_s and the CP-violating phase ϕ_s receive significant contributions. The SUSY spectrum in the CMM model is determined by five input parameters at M_Z, which are the gluino mass $m_{\tilde{g}}$, the universal down-squark mass $m_{\tilde{d}}$, the ratio of down-quark trilinear and Yukawa couplings $a_d = (A_d)^{11}/(Y_d)^{11}$, the phase of the higgsino mass parameter $\arg(\mu)$, and the ratio of the Higgs-doublet vacuum expectation values $\tan\beta$. The renormalization group links these inputs via the universality conditions at $M_{\rm Pl}$ to the entire SUSY spectrum at low scales. In Fig. 1 left, we show the effects of large atmospheric neutrino mixing on ΔM_s for two typical sets of input parameters distinguished by $m_{\tilde{g}}$. For gluino masses $m_{\tilde{g}} \simeq 400\,\text{GeV}$, the CMM phase is constrained from ΔM_s to $1.2 \lesssim |2\phi_{B_s}| \lesssim 2.4$. Comparing with Fig. 1 right, one observes that within this parameter region the CP phase in $B_s - \overline{B}_s$ mixing amounts to $\phi_s = -0.5$ (contrarily to the tiny $\phi_s = -0.04$ in the SM). The CMM model can thus account for the measured phase $\phi_s^{\rm exp} = -0.77^{+0.29}_{-0.37}$ or $-2.36^{+0.37}_{-0.29}$ [3], while fulfilling the constraints from ΔM_s.

CONSTRAINTS ON YUKAWA CORRECTIONS FROM ε_K

The magnitude of neutrino mixing effects in $K - \overline{K}$ and $B_d - \overline{B}_d$ mixing from Yukawa corrections Y_σ is parametrized by the rotation angle θ, which a priori can be sizeable. The CP-violating observable ε_K in the kaon system, however, sets strong constraints on θ. ε_K is very well measured, theoretically rather clean, and extremely sensitive to new-physics contributions. One derives the upper bound $\theta^{\rm max} \simeq 1^\circ$ if

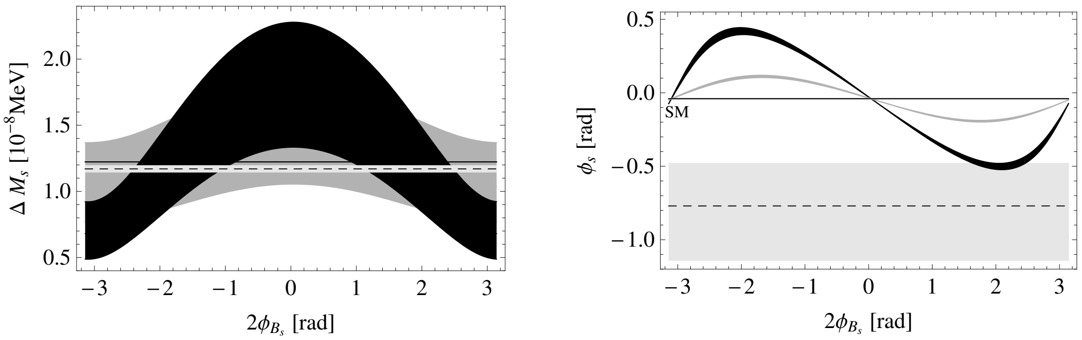

Figure 1. Effects of large atmospheric neutrino mixing in $B_s - \overline{B}_s$ mixing for fixed inputs $m_{\tilde{d}} = 2\,\text{TeV}$, $a_d/m_{\tilde{d}} = 1.8$, $\arg(\mu) = 0$, $\tan\beta = 5$, and $m_{\tilde{g}} = 400\,\text{GeV}$ (black curve), $700\,\text{GeV}$ (gray curve). Left: ΔM_s, gray band: experimental three-sigma range. Right: $\phi_s = \arg(-M_{12}/\Gamma_{12})$, gray band: experimental one-sigma range. Standard-Model values are indicated by a black line.

the phase ϕ_K is different from $\phi_K = 0, \pi/2$ [4]. Excluding any miraculous cancellations, this bound allows to specify the flavour structure of Yukawa corrections:

$$\boxed{\text{The Yukawa corrections } Y_\sigma \text{ have to be aligned with } Y_d \text{ and } Y_\ell\text{, see Eq. (5).}}$$

In other words, corrections to down-quark-lepton unification for light fermions cannot introduce new flavour structures with respect to the initial unified Yukawa couplings if the constraint from ε_K holds. Effects of large neutrino mixing in less sensitive K and B_d observables are consequently negligibly tiny. In the case of vanishing effects in ε_K, θ can still be restrained from $B - \overline{B}$ mixing observables to $\theta^{\max} \simeq 20^\circ$ [4].

This finding is of general interest for GUT model building: Once down-quark-lepton Yukawa unification of light fermions is corrected, ε_K strongly constrains the penetration of large atmospheric neutrino mixing into $s_R - d_R$ and $b_R - d_R$ currents. In the CMM model, effects are induced by the large down-squark mass splitting $\Delta_{\tilde{d}}$ due to $\text{SO}(10)$ running. There might be other sources that break the squark mass universality, like down-squark couplings to heavy right-handed neutrinos in supersymmetric $\text{SU}(5)$ models [5]. Whenever such a mass splitting occurs, the flavour structure of corrections to down-quark-lepton Yukawa unification is restricted to be aligned with the initial couplings.

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