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Two-loop parameter relations between dimensional regularization and dimensional reduction applied to SUSY-QCD

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Abstract

The two-loop relations between the running gluino-quark-squark coupling, the gluino and the quark mass defined in dimensional regularization (DREG) and dimensional reduction (DRED) in the framework of SUSY-QCD are presented. Furthermore, we verify with the help of these relations that the three-loop β-functions derived in the minimal subtraction scheme combined with DREG or DRED transform into each other. This result confirms the equivalence of the two schemes at the three-loop order, if applied to SUSY-QCD.

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1 Introduction

DRED has been introduced in Ref. [1] as a regularization scheme for supersymmetric gauge theories which maintains supersymmetry (SUSY) and at the same time retains the elegant features of DREG [2], especially the gauge invariance. The essential difference between DRED and DREG is that the continuation from 4 to D dimensions is made by *compactification*. After *dimensional reduction* to $D = 4 - 2\varepsilon$, it is only the D components of the gauge field that generate the actual gauge interactions. The remaining 2ε components behave under gauge transformations as a multiplet of scalar fields, usually called ε-scalars.

As pointed out by Siegel himself [3], there are potential problems with DRED. In Ref. [4] it has been shown that the variation δS of the action of a pure (no chiral matter) supersymmetric gauge theory is nonzero even with DRED. If δS gives a nonzero result when inserted in a Green's function this creates an apparent violation of supersymmetric Ward identities. Within DREG this happens at one-loop order. On the other hand, within DRED all explicit calculations up to two-loop order have found zero for such insertions [5, 6]. Recently, a mathematically consistent formulation of DRED [6] and rigorous methods to prove its supersymmetric properties [7] have been introduced.

Another way to verify the consistency of DRED with SUSY is to study the behaviour under the renormalisation of the ε-scalar-couplings (also called *evanescent* couplings) to matter and gauge fields. In a supersymmetric theory, they have to remain equal to the gauge coupling, if the renormalization scheme preserves SUSY. Explicit computations up to three-loop order within SUSY-QCD [8] confirmed this requirement for DRED in combination with the minimal subtraction scheme, *i.e.* the $\overline{\text{DR}}$ scheme. But, if DRED is applied to non-supersymmetric theories, like for example QCD, this equality is not preserved under the renormalization [9, 10]. However, even in softly broken supersymmetric theories like the Minimal Supersymmetric extension of the Standard Model (MSSM), one has to worry about the ε-scalars. In such theories, they will receive a loop-induced mass, which will also influence the renormalization of the genuine scalar masses. In order to decouple the ε-scalar masses from the β-functions of the genuine scalar masses, additional finite counterterms proportional to the ε-scalar masses have to be added to the renormalized scalar masses. This new renormalization scheme, usually known as the $\overline{\mathrm{DR}}'$ scheme, was introduced in Ref [11] to the one-loop order and extended through two-loops in Ref. [12]. The results presented in this letter are the same in the $\overline{\mathrm{DR}}$ and $\overline{\mathrm{DR}}'$ schemes, because we did not take into account dimensionful couplings.

As is well known, the equality of the Yukawa couplings of gauginos to matter multiplets and the gauge couplings, or the equality of the quartic scalar couplings, *e.g.* four-squark or four-slepton couplings, and the gauge couplings are not preserved under renormalization if DREG is employed. This is a direct manifestation of the fact that DREG breaks SUSY. It means that, if one demands that the renormalized couplings are the same at some renormalization scale, then they are different at another scale. This point becomes important if we want to relate a given theory at one scale to the same theory at another scale. This procedure is often known as the running analysis and it amounts to determine the fundamental parameters of the MSSM solving the system of the Renormalization Group Equations (RGEs) with two types of boundary conditions: i) universality conditions imposed at some very high energy scale like the unification scale; and ii) low-energy constraints obtained from experiment. The appropriate renormalization scheme at each step of the running analysis is not fixed *a priori*. In general the SM parameters and cross sections are mostly given in the $\overline{\mathrm{MS}}$ scheme [13], while the MSSM ones are usually given in the $\overline{\mathrm{DR}}$ scheme. Apart from the finite shifts of the running parameters associated with the change of renormalization scheme, also threshold corrections, which account for the non-decoupling of heavy particles in mass independent schemes have to be implemented. They are known at the one-loop order for the complete MSSM [14], and at the two-loop orders for the SUSY-QCD [15, 16].

Very recently, Refs. [17, 18] have shown that the QCD factorization theorem holds through one-loop order, if DRED is employed in computations of hadronic processes. They also provide translation rules from DRED to other regularization schemes through one-loop. However, it seems that the application of DRED to hadronic processes beyond one-loop becomes much more involved as compared to the standard procedure based on the DREG. It is thus advisable to use different regularization schemes for various parts of a practical computation. The consistency of such an approach is guaranteed by the fact that DRED and DREG are equivalent to all orders in perturbation theory if applied to a renormalisable theory [19]. This means that the two schemes are related by coupling constant redefinitions, under which the β-functions calculated in one scheme transform into those computed in the other one. In the framework of QCD, the translation rules for the change from DREG to DRED is known up to three loops for the strong coupling constant and the quark masses [10, 20]. In the MSSM, the one-loop relations are known for the gauge, Yukawa, quartic scalar couplings and for the coupling associated with the gaugino-chiral supermultiplet interactions, as well as for the gaugino masses [21]. The one-loop relation between the gauge coupling constant and the one associated with the interaction of the gluino and the quark-squark multiplet has also been verified by an on-shell computation in Ref. [22]. For the strong coupling constant even the two-loop conversion rule in SUSY-QCD is known [15].

It is the purpose of this letter to extend the translation "dictionary" between the two schemes in the framework of SUSY-CQD to two-loop order. More precisely, we give in Section 2 the differences between the running gluino-quark-squark coupling and the running quark and gluino masses computed in the $\overline{\text{MS}}$ and the $\overline{\text{DR}}$ schemes at the two-loops. As a by-product result we reconfirm the two-loop conversion relation derived in [15]. In Section 3 we explicitly verify that the three-loop $\overline{\text{DR}}$ β-functions and the fermion mass anomalous dimensions can be obtained from the $\overline{\text{MS}}$ results just converting all running parameters (couplings and masses) according to the two-loop results derived before. In Appendix A we discuss the one-loop renormalization of the four-squark couplings within the $\overline{\text{MS}}$ scheme.

2 Two-loop conversion rules from DRED to DREG

In this letter, we restrict the discussion to the translation rules for the running parameters of the SUSY-QCD. Thus we just need the $SU(3)$ part of the MSSM Lagrangian. However, we give here the results valid for a general supersymmetric theory based on an $SU(N)$ gauge group, with one gauge supermultiplet in the adjoint representation (A), comprising the gluon and gluino, and N_f sets of matter multiplets in the fundamental representation (F), containing the fermions and their superpartners.[1]

2.1 Running coupling constants

In order to compute the relations between running parameters defined in two different renormalization schemes, one has to relate them to physical observables which cannot depend on the choice of scheme. For example, the relationship between the strong coupling constant defined in the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ schemes can be obtained from the S-matrix amplitude of a physical process involving the gauge coupling computed in the two schemes. However, beyond one-loop the computation of the physical amplitudes becomes very much involved. We applied this method only for the computation of the two-loop *effective* charges of the gluon-quark-quark and gluino-quark-squark couplings in the $\overline{\text{DR}}$ scheme, in order to prove the equality of the corresponding couplings at this order in perturbation theory. We considered the simplifying case of a supersymmetric theory, *i.e.* massless gluino and equal-mass quarks and squarks and required the external particles to be on-shell. For the computation of the resulting two-loop on-shell integrals we used existing automated programs [23].The effective charges computed for on-shell gluons and gluinos are not infrared safe, but the infrared divergences of the two charges are equal. This can be understood from the fact that they are proportional to the corresponding one-loop *effective* charges, which have been shown to be equal [22], and the proportionality factors are universal quantities equal for gluon and gluinos in a supersymmetric theory. We found that the two *effective* charges are equal, which implies that the couplings themselves are also equal in the $\overline{\text{DR}}$ scheme through two-loops. The equality of the two couplings in the $\overline{\text{DR}}$ scheme has been confirmed even at the three-loop order in Ref. [8]. This result proves on the one hand the supersymmetric character of the $\overline{\text{DR}}$ scheme, and on the other hand it allows us to derive the relation between the two

[1]We work with Dirac fermions and complex scalar fields.

couplings valid in the $\overline{\rm MS}$ scheme, as we discuss below.
For the computation of the translation relations between the $\overline{\rm MS}$ and $\overline{\rm DR}$ schemes we employed a simpler computation method [10]. Starting from the observation that the ratio of the charge renormalization constants calculated using DREG or DRED is momentum and mass independent, one can derive them avoiding the use of the on-shell kinematics. Instead, one introduces *physical* renormalization constants, which are computed choosing a convenient kinematics for which the "large-momentum" or the "hard-mass" procedures can be applied, and retains the divergent as well as the finite pieces of the renormalization constants. Up to three loops this procedure is quite well established (for a detail description of the method see Ref. [10]) and automated programs exist to perform such calculations [24–26].

Considering the *physical* charge of the gluon-quark-quark coupling at two-loop order we reconfirm the result derived in [15]. For completeness we reproduce it here

$$\alpha_s^{\overline{\rm MS}} = \alpha_s^{\overline{\rm DR}}\left[1-\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}\frac{C_A}{3}+\left(\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}\right)^2\left(-\frac{11}{9}C_A^2+2T_FN_fC_F\right)\right], \qquad (1)$$

where $\alpha_s^{\overline{\rm MS}}=(g_s^{\overline{\rm MS}})^2/(4\pi)$ and $\alpha_s^{\overline{\rm DR}}=(g_s^{\overline{\rm DR}})^2/(4\pi)$ denote the strong coupling constant in the $\overline{\rm MS}$ and $\overline{\rm DR}$ scheme, respectively. We choose the usual normalization for the Dynkin index T_F of the fundamental representation $Tr(T^aT^b)=T_F\delta^{ab}=\frac{1}{2}\delta^{ab}$. Accordingly, the quadratic Casimir invariant for the fundamental representation is given by $C_F=T_FN_A/d(F)$, where $N_A=N^2-1$ is the number of generators and $d(F)=N$ is the dimension of the fundamental representation. The Casimir invariant for the adjoint representation reads $C_A=N$.

Similarly, one can determine the conversion rules for the coupling constant $\hat{\alpha}_s=(\hat{g}_s)^2/(4\pi)$ of the Yukawa interaction of the gluino and the quark-squark multiplet

$$\mathcal{L}_{\tilde{g}q\tilde{q}}=-\sqrt{2}\hat{g}_sT^a_{ij}\left[\bar{q}_{L,i}\tilde{g}^a\tilde{q}_{L,j}-\bar{q}_{R,i}\tilde{g}^a\tilde{q}_{R,j}+h.c.\right]. \qquad (2)$$

Here $\tilde{g}, q$ and $\tilde{q}$ denote as usual the gluino, quark and squark fields, L and R subscripts stand for the left- and right-handed components of the quark and squark fields, and a and i,j are color indices of the adjoint and fundamental representations, respectively.
Let us remark that we performed the calculation for a general covariant gauge and used the cancellation of the gauge parameter in the final results as an internal check. For the derivation of the two-loop formulae given above, also the one-loop relation between the gauge parameter defined in the $\overline{\rm DR}$ and $\overline{\rm MS}$ schemes is necessary. In order to properly take into account the Majorana character of the gluino, the rules given in [27] are applied with the help of a specially written PERL program [28].

So, for the two-loop conversion rule of the gluino-quark-squark coupling, we obtain

$$\begin{aligned}\hat{\alpha}_s^{\overline{\rm MS}} &= \alpha_s^{\overline{\rm DR}}\left[1+\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}(C_A-C_F)\right.\\ &+ \left.\left(\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}\right)^2\left(\frac{23}{6}C_A^2-\frac{137}{12}C_AC_F+\frac{25}{4}C_F^2+2T_FN_f(C_F-C_A)\right)\right], \qquad (3)\end{aligned}$$

and together with Eq. (1) we get the relationship between $\hat{\alpha}_s^{\overline{\rm MS}}$ and $\alpha_s^{\overline{\rm MS}}$

$$\begin{aligned}\hat{\alpha}_s^{\overline{\rm MS}} &= \alpha_s^{\overline{\rm MS}}\Bigg[+\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\left(\frac{4}{3}C_A - C_F\right)\\ &+ \left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2\left(\frac{107}{18}C_A^2 - \frac{145}{12}C_AC_F + \frac{25}{4}C_F^2 - 2T_FN_fC_A\right)\Bigg]. \end{aligned}\tag{4}$$

As a consistency check, we will show in Section 3 that the three-loop β-functions of α_s and $\hat{\alpha}_s$ computed in the $\overline{\rm MS}$ scheme can be converted into the $\overline{\rm DR}$ β-function [8, 29] only by means of the finite shifts of the running couplings.

2.2 Running fermion masses

The particle masses are other fundamental parameters of the MSSM, that acquired a lot of attention both theoretically and phenomenologically. In this letter, we provide the two-loop translation relations for the fermion masses. They are functions only of the coupling constants and colour factors. The relations between the running masses defined in $\overline{\rm MS}$ and $\overline{\rm DR}$ can be obtained using the same requirement as for the coupling constants, that physical observables have to be renormalization scheme independent.

In practice, we have employed the easier method of *physical* renormalization schemes as discussed above. So, the running quark mass defined in the $\overline{\rm MS}$ scheme can be translated into the running mass in the $\overline{\rm DR}$ scheme through

$$m_q^{\overline{\rm MS}} = m_q^{\overline{\rm DR}}\left[1+\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}C_F + \left(\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}\right)^2\left(\frac{7}{12}C_AC_F + \frac{7}{4}C_F^2 - \frac{1}{2}C_FT_FN_f\right)\right]. \tag{5}$$

For the running gluino mass we get the following conversion relation

$$m_{\tilde{g}}^{\overline{\rm MS}} = m_{\tilde{g}}^{\overline{\rm DR}}\left[1+\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}C_A + \left(\frac{\alpha_s^{\overline{\rm DR}}}{4\pi}\right)^2\left(\frac{23}{6}C_A^2 - 4C_AT_FN_f + \frac{1}{2}C_FTN_f\right)\right]. \tag{6}$$

Again, one can verify the correctness of these relations by showing that the three-loop mass anomalous dimensions computed in the $\overline{\rm MS}$ scheme can be translated into the $\overline{\rm DR}$ ones, by employing only the mass and coupling redefinitions given above. This point will be discussed in detail in the next section.

Let us point out that the relations between the running masses defined in different renormalization schemes are free of the renormalon problems which affects the pole masses. It is thus advisable to use these relations in high precision calculations of the supersymmetric mass spectrum.

3 Three-loop renormalization group functions in DREG

The renormalization group functions provide the scale variation of the parameters of a quantum field theory. They have been extensively studied and an impressive theoretical accuracy has been achieved. In the $\overline{\text{MS}}$ scheme, the anomalous dimensions of all SM parameters are known up to two-loop level [30,31], while for QCD even the four-loop order results are available [32–35]. For a more general theory containing gauge, Yukawa and quartic scalar interactions, the gauge β-function is known through three-loops [36] both in the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ scheme. In the case of the MSSM, the three-loop anomalous dimensions for dimensionless as well as dimensionful couplings were derived in the $\overline{\text{DR}}$ scheme in Refs. [29, 37, 38]. The three-loop anomalous dimensions for the dimensionless couplings of SUSY-QCD were re-confirmed in Ref. [8].

In this section, we discuss the results for the three-loop β-function of the gauge and gluino-quark-squark couplings and the three-loop mass anomalous dimensions of the quark and gluino masses in the framework of SUSY-QCD with $\overline{\text{MS}}$ as renormalization scheme. For such a calculation one can exploit that the divergent part of a logarithmically divergent integral is independent of the masses and external momenta. Thus the latter can be chosen in a convenient way: we set to zero all masses and one of the external momenta in the three-point functions paying attention to not introduce spurious infrared divergences. The resulting three-loop integrals can be evaluated with the help of existing programs [24, 25]. At the three-loop order in perturbation theory, the use of γ_5 requires special care. We adopted here the prescription introduced in Ref [8].

Apart from the technical difficulties, related to the genuine three-loop calculation, one has to bare in mind that the couplings of the gluino-quark-squark and four-squark interactions are different from the gauge coupling even at the one-loop order, if the $\overline{\text{MS}}$ scheme is employed. Since the four-squark couplings occur in the two-loop β-function of $\hat{\alpha}_s^{\overline{\text{MS}}}$, one needs their one-loop renormalization constants for the derivation of the three-loop β-function of $\hat{\alpha}_s^{\overline{\text{MS}}}$. In addition, for the conversion of this result into the $\overline{\text{DR}}$ scheme the one-loop translation rules from $\overline{\text{MS}}$ to $\overline{\text{DR}}$ of the four-squark couplings are needed. They have been known for quite some time for a general renormalizable theory with scalars, fermions, and gauge fields at one- and two-loop order [30, 39]. In SUSY-QCD the tree-level four-squark interaction is given by

$$\mathcal{L}_{\tilde{q}\tilde{q}\tilde{q}\tilde{q}} = -\sum_{A,B} \frac{1}{2} g_s^2 T^a_{ij} T^a_{kl} (\tilde{q}^{A,*}_{L,i} \tilde{q}^A_{L,j} - \tilde{q}^{A,*}_{R,i} \tilde{q}^A_{R,j})(\tilde{q}^{B,*}_{L,k} \tilde{q}^B_{L,l} - \tilde{q}^{B,*}_{R,k} \tilde{q}^B_{R,l}) \tag{7}$$

with A, B flavour indices, and a and i, j, k, l colour indices. At the tree-level, the four-squark couplings are equal to the gauge coupling. After renormalization in the $\overline{\text{MS}}$ scheme, one has to distinguish four types of quartic scalar couplings: i) the coupling of squarks with the same chirality and flavour $g_L^A\,, g_R^A\,,$ ii) the coupling of squarks with different chiralities but the same flavour $g_{LR}^A\,, g_{RL}^A\,,$ iii) the coupling of squarks with the same chirality but of different flavours $g_L^{AB}\,, g_R^{AB}\,,$ iv) the coupling of squarks with different chiralities and flavours $g_{LR}^{AB}\,, g_{RL}^{AB}$. Another subtlety which occurs beyond tree-level is that the group colour factors do not factorize, so that one has to keep track of various colour tensors in the computation of the one-loop renormalization constants. We introduce the following tensors for the quartic squark couplings

$$\begin{aligned}
(S_L^A)_{ij;kl} &= \frac{(g_L^A)^2}{4\pi}(T^a_{ij}T^a_{kl} + T^a_{il}T^a_{kj}) = \frac{(g_R^A)^2}{4\pi}(T^a_{ij}T^a_{kl} + T^a_{il}T^a_{kj})\,,\\
(S_{LR}^A)_{ij;kl} &= -\frac{(g_{LR}^A)^2}{4\pi}(T^a_{ij}T^a_{kl}) = -\frac{(g_{RL}^A)^2}{4\pi}(T^a_{ij}T^a_{kl})\,,\\
(S_L^{AB})_{ij;kl} &= \frac{(g_L^{AB})^2}{4\pi}(T^a_{ij}T^a_{kl}) = \frac{(g_R^{AB})^2}{4\pi}(T^a_{ij}T^a_{kl})\,,\\
(S_{LR}^{AB})_{ij;kl} &= -\frac{(g_{LR}^{AB})^2}{4\pi}(T^a_{ij}T^a_{kl}) = -\frac{(g_{RL}^{AB})^2}{4\pi}(T^a_{ij}T^a_{kl})\,,
\end{aligned} \tag{8}$$

and the associated coupling constants

$$\alpha_L^A = \frac{(g_L^A)^2}{4\pi}\,, \quad \alpha_{LR}^A = \frac{(g_{LR}^A)^2}{4\pi}\,, \quad \alpha_L^{AB} = \frac{(g_L^{AB})^2}{4\pi}\,, \quad \alpha_{LR}^{AB} = \frac{(g_{LR}^{AB})^2}{4\pi}\,. \tag{9}$$

We provide in Appendix A the one-loop $\overline{\rm MS}$ β-function for the coupling tensors retaining the complete colour structure dependence. The calculation in the $\overline{\rm DR}$ scheme is significantly simpler since the colour tensors factorize. The resulting β-functions of scalar couplings are equal to the gauge β-function as required by SUSY.

The translation rules for the four-squark couplings can be obtained from the finite pieces of the charge renormalization functions computed in the two schemes. We did the calculation for vanishing external momenta and regularized the infrared divergences giving a common mass to all particles [40]. To one-loop order they read:

$$\begin{aligned}
(S_\lambda^{\delta,\overline{\rm MS}})_{ij;kl} &= (S_\lambda^{\delta,\overline{\rm DR}})_{ij;kl} - \frac{\alpha_s^{\overline{\rm DR}}}{4\pi}\Big(\{T^a,T^b\}_{ij}\{T^a,T^b\}_{kl} + \{T^a,T^b\}_{il}\{T^a,T^b\}_{kj}\Big)\,, \ \delta = A\,, \lambda = L\,,\\
(S_\lambda^{\delta,\overline{\rm MS}})_{ij;kl} &= (S_\lambda^{\delta,\overline{\rm DR}})_{ij;kl} - \frac{\alpha_s^{\overline{\rm DR}}}{4\pi}\{T^a,T^b\}_{ij}\{T^a,T^b\}_{kl} \quad \text{otherwise}\,.
\end{aligned} \tag{10}$$

Here $\{T^a,T^b\}$ denotes the anti-commutator of the group generators.
The one-loop translation rules from $\overline{\rm MS}$ to $\overline{\rm DR}$ of the quartic scalar couplings are known for the case of identical flavour scalars [21]. These relations coincide with those of $(S_L^A)_{ij;kl}$ couplings in SUSY-QCD.

3.1 Three-loop β-functions in DREG

The β-functions for the gauge and the gluino-quark-squark couplings are defined through

$$\beta_{\alpha_s}^{\overline{\rm MS}} = \mu^2\frac{\rm d}{{\rm d}\mu^2}\frac{\alpha_s^{\overline{\rm MS}}}{\pi}\,, \quad \text{and} \quad \beta_{\hat\alpha_s}^{\overline{\rm MS}} = \mu^2\frac{\rm d}{{\rm d}\mu^2}\frac{\hat\alpha_s^{\overline{\rm MS}}}{\pi}\,. \tag{11}$$

Writing

$$\beta_\alpha^{\overline{\rm MS}} = \sum_{i=1}^{3}\beta_\alpha^{\overline{\rm MS},(i)}\,, \quad \alpha = \alpha_s\,,\hat\alpha_s\,, \tag{12}$$

where (i) stands for the loop order, we find for the gauge β-function

$$\begin{aligned}
\beta_{\alpha_s}^{\overline{\rm MS},(1)} &= \left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2 4(-3C_A + 2N_fT_F)\,,\\
\beta_{\alpha_s}^{\overline{\rm MS},(2)} &= \left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^3 4(-6C_A^2 + 8C_AN_fT_F + 12C_FN_fT_F) - \left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}16N_fT_F(C_A+C_F)\,,\\
\beta_{\alpha_s}^{\overline{\rm MS},(3)} &= \left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^4 4\Big[-19C_A^3 + 2\left(12C_A^2 + 25C_AC_F - 10C_F^2\right)N_FT_F - 4\left(C_A + 5C_F\right)T_F^2N_F^2\Big]\,.
\end{aligned} \tag{13}$$

In the expression for $\beta^{\overline{\rm MS},(3)}_{\alpha_s}$ as well as in all the other three-loop formulae quoted in this letter, we identify all couplings with $\alpha_s^{\overline{\rm MS}}$. The inaccuracy induced in this way is of the four-loop order, so that the simplified formulae are enough to perform consistency checks of the two-loop translation relations given in the previous section. In practice, we derived the formulae distinguishing between the various couplings, but the results are too long to be presented here. The three-loop results with complete dependence on different couplings can be obtained in electronic form from the author.

The three-loop β-function of the gluino-quark-squark coupling reads

$$
\begin{aligned}
\beta^{\overline{\rm MS},(1)}_{\hat\alpha_s} &= -\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}12(C_A+C_F) + \left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2 4(3C_F+2T_FN_F)\,,\\
\beta^{\overline{\rm MS},(2)}_{\hat\alpha_s} &= \left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^3 4\left[4C_A^2-12C_AC_F+2C_F(C_F-7N_FT_F)\right]\\
&+\left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2 4\Bigg[\left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)\frac{6C_A^2+9C_AC_F+21C_F^2-2C_AT_FN_F+34C_FT_FN_F}{2}\\
&-4\sum_{\delta=A,AB}\frac{(T^aT^b)_{ji}(S_L^\delta)_{ij;kl}(T^bT^a)_{lk}}{N_AT_F} - 4\sum_{\delta=A,AB}\frac{(T^aT^b)_{ji}(S_{LR}^\delta)_{ij;kl}(T^aT^b)_{lk}}{N_AT_F}\Bigg]\\
&+\left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)4\Bigg[\left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2\left(-16C_A^2+\frac{11}{4}C_AC_F-6C_F^2+7C_AN_FT_F+4C_FN_FT_F\right)\\
&+\frac{C_F}{2}\sum_{\delta=A,AB}\sum_{\lambda=L,LR}\frac{(S_\lambda^\delta)_{ij;kl}(S_\lambda^\delta)_{ji;lk}}{N_AT_F}\Bigg]\\
&= \left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^3 4\left[4C_A^2-12C_AC_F+2C_F(C_F-7N_FT_F)\right]\\
&+\left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2 4\Bigg[\left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)\frac{6C_A^2+9C_AC_F+21C_F^2-2C_AT_FN_F+34C_FT_FN_F}{2}\\
&-\left(\frac{\alpha_L^A}{4\pi}\right)(C_AT_F+16D_3(F)T_F+C_A^2-4C_AC_F+4C_F^2)-\left(\frac{\alpha_{LR}^A}{4\pi}\right)(C_A-16D_3(F))T_F\\
&-\left(\frac{\alpha_L^{AB}}{4\pi}\right)(C_A+16D_3(F))T_FN_Q-\left(\frac{\alpha_{LR}^{AB}}{4\pi}\right)(C_A-16D_3(F))T_FN_Q\Bigg]\\
&+\left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)4\Bigg[\left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2\left(-16C_A^2+\frac{11}{4}C_AC_F-6C_F^2+7C_AN_FT_F+4C_FN_FT_F\right)\\
&+\left(\frac{\alpha_L^A}{4\pi}\right)^2C_F\frac{2T_F-C_A+2C_F}{4}+\left(\frac{\alpha_{LR}^A}{4\pi}\right)^2\frac{C_FT_F}{2}\\
&+\left(\frac{\alpha_L^{AB}}{4\pi}\right)^2\frac{C_FN_QT_F}{2}+\left(\frac{\alpha_{LR}^{AB}}{4\pi}\right)^2\frac{C_FN_QT_F}{2}\Bigg]\,,
\end{aligned}
$$

$$\beta^{\overline{\rm MS},(3)}_{\hat{\alpha}_s} = \left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^4 4\Big[-\frac{188}{3}C_A^3 + \frac{167}{3}C_A^2 C_F + \frac{145}{3}C_A^2 N_F T_F + \frac{17}{4}C_F^2 C_A$$
$$+ \frac{107}{2}C_F C_A N_F T_F - 8 C_A N_F^2 T_F^2 - \frac{81}{4}C_F^3 - \frac{49}{2}C_F^2 N_F T_F - 20 C_F N_F^2 T_F^2$$
$$- 32 D_4(FA) - 128 D_4(FF) N_F T_F \Big] , \tag{14}$$

where $N_Q = N_F - 1$ counts the number of quark/squark flavours B different from the external quark/squark flavour A. The additional colour factors occurring in the above results are defined as

$$D_3(F) = \frac{d_F^{abc} d_F^{abc}}{N_A} = \frac{N^2-4}{16N} , \qquad D_4(FA) = \frac{d_F^{abcd} d_A^{abcd}}{N_A} = \frac{N(N^2+6)}{48} ,$$
$$D_4(FF) = \frac{d_F^{abcd} d_F^{abcd}}{N_A} = \frac{18 - 6N^2 + N^4}{96 N^2} , \tag{15}$$

where $d_F^{abc}, d_F^{abcd}, d_A^{abcd}$ are the fully symmetric rank three and four tensors of SU(N), as defined in Ref. [41].
$\beta^{\overline{\rm MS},(2)}_{\hat{\alpha}_s}$ is given first as a function of the quartic scalar coupling tensors. Implementing their explicit expressions (8) one gets the RHS of the second equality sign. The explicit dependence on the coupling tensors is needed for the computation of the three-loop result $\beta^{\overline{\rm MS},(3)}_{\hat{\alpha}_s}$ as a function of the different types of couplings. As can be understood from the above formulae, when the quartic scalar coupling tensors occurring in the two-loop diagrams are renormalized, their one-loop renormalization functions are contracted with three different colour structures. We did the renormalization at the diagram level, employing the appropriate colour projectors. For the derivation of the three-loop results with all couplings set to be equal to $\alpha_s^{\overline{\rm MS}}$, one can avoid the introduction of coupling tensors for the four-squark interaction. In this case the colour structures of the tree-level couplings are preserved under the renormalization to the one-loop order and so, their renormalization can be done as usual. However, for the conversion of the three-loop $\overline{\rm MS}$ results to the $\overline{\rm DR}$ scheme the introduction of the coupling tensors is unavoidable, because the colour structures do not factorize in the second equation of the translation relations (10).
Furthermore, it is a straightforward calculation to show that employing the conversion rules given in Eqs. (1), (3), (10) into the $\overline{\rm MS}$ three-loop β-functions (13) and (14), one obtains the $\overline{\rm DR}$ β-function computed in Refs. [8, 29]. Since the couplings α_s and $\hat{\alpha}_s$ occur already at the one-loop order, their two-loop translation rules are necessary to convert the three-loop β-functions from $\overline{\rm MS}$ to $\overline{\rm DR}$. This is a strong consistency check for the translation rules we discussed in the previous section.

3.2 Three-loop fermion mass anomalous dimensions in DREG

In this section we provide the fermion (quark and gluino) mass anomalous dimensions within the $\overline{\rm MS}$ scheme through three-loops. They are derived from the renormalization constants of the fermion masses, which can be calculated by decomposing the fermion self-energy into its vector and scalar parts and then computing the counterterms for the wave functions and masses. We define the fermion (quark or gluino) mass anomalous dimensions as

$$\gamma_A = \frac{\mu^2}{m_A}\frac{\mathrm{dm_A}}{\mathrm{d}\mu^2}\,, \quad A = q, \tilde{g}\,. \tag{16}$$

Writing their expansion in the perturbation theory like

$$\gamma_A = \sum_{i=1}^{3} \gamma_A^{\overline{\mathrm{MS}},(i)}\,, \tag{17}$$

we have for the quark mass anomalous dimension

$$\begin{aligned}
\gamma_q^{\overline{\mathrm{MS}},(1)} &= \left(\frac{\hat{\alpha}_s^{\overline{\mathrm{MS}}}}{4\pi}\right) C_F - \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right) 3C_F\,,\\
\gamma_q^{\overline{\mathrm{MS}},(2)} &= \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right)^2 \left(-\frac{29}{2}C_A C_F - \frac{3}{2}C_F^2 + 7C_F N_F T_F\right)\\
&+ \left(\frac{\hat{\alpha}_s^{\overline{\mathrm{MS}}}}{4\pi}\right)^2 (-2C_A C_F + C_F^2 - C_F N_F T_F) + \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right)\left(\frac{\hat{\alpha}_s^{\overline{\mathrm{MS}}}}{4\pi}\right)\frac{11}{2}(C_A C_F + C_F^2)\,,\\
\gamma_q^{\overline{\mathrm{MS}},(3)} &= \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right)^3 \Bigg[-\frac{115}{3}C_A^2 C_F + \frac{43}{4}C_A C_F^2 - \frac{59}{4}C_F^3 + 6C_F N_F^2 T_F^2\\
&+ (14C_A C_F + 47C_F^2 + 48C_A C_F\zeta(3) - 48C_F^2\zeta(3))N_F T_F\Bigg]\,,
\end{aligned} \tag{18}$$

where ζ denotes the Riemann's zeta function with $\zeta(3) = 1.20206$. For the gluino mass anomalous dimension we obtain

$$\begin{aligned}
\gamma_{\tilde{g}}^{\overline{\mathrm{MS}},(1)} &= \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right)(-3)C_A + \left(\frac{\hat{\alpha}_s^{\overline{\mathrm{MS}}}}{4\pi}\right) 2N_F T_F\,,\\
\gamma_{\tilde{g}}^{\overline{\mathrm{MS}},(2)} &= \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right)^2 C_A(-16C_A + 7N_F T_F) - \left(\frac{\hat{\alpha}_s^{\overline{\mathrm{MS}}}}{4\pi}\right)^2 (4C_A - C_F)N_F T_F\\
&+ \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right)\left(\frac{\hat{\alpha}_s^{\overline{\mathrm{MS}}}}{4\pi}\right)(5C_A + 17C_F)N_F T_F\,,\\
\gamma_{\tilde{g}}^{\overline{\mathrm{MS}},(3)} &= \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\right)^3 \Bigg[-\frac{310}{3}C_A^3 + (103C_A^2 + \frac{347}{2}C_A C_F - \frac{83}{2}C_F^2)N_F T_F\\
&+ (-24C_A - 74C_F)N_F^2 T_F^2\Bigg]\,.
\end{aligned} \tag{19}$$

It is an easy exercise to verify that the three-loop $\overline{\mathrm{MS}}$ mass anomalous dimensions given above differ from the ones computed in $\overline{\mathrm{DR}}$ scheme [8,37] only by the finite shifts for coupling constants and masses discussed in Section 2. Let us point out that, for the conversion of the three-loop mass anomalous dimensions the two-loop relations for masses and couplings are needed. So, this provide us with another important consistency check of Eqs. (1, 3, 5, 6).

4 Conclusions

In this letter we present the two-loop translation rules between $\overline{\rm DR}$ and $\overline{\rm MS}$ scheme for the running gluino-quark-squark coupling and for the gluino and quark masses. We also confirm the two-loop relation for the gauge coupling given in Ref. [15]. Furthermore, we prove that the three-loop β-function of the gauge and gluino-quark-squark couplings and the anomalous dimensions of the quark and gluino masses calculated in the $\overline{\rm MS}$ scheme can be converted into the known $\overline{\rm DR}$ results, by means of these two-loop parameter redefinitions. This is a powerful consistency check of our two-loop results.
As a by-product of our calculation, we derive the one-loop RGEs for the four-squark coupling in the $\overline{\rm MS}$ scheme and their conversion rules to the $\overline{\rm DR}$ scheme.

A One-loop β-functions of the four-squark couplings in $\overline{\rm MS}$

As mentioned before, the four-squark couplings behave as tensors in colour space. Their one-loop β-functions in the $\overline{\rm MS}$ scheme read

$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}\frac{(S_\lambda^{\delta,\overline{\rm MS}})_{ij;kl}}{\pi} = \frac{1}{4\pi^2}(\Lambda_\lambda^\delta)_{ij;kl} - 32\left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2 (H_\lambda^\delta)_{ij;kl} + 4\left(\frac{\hat\alpha_s^{\overline{\rm MS}}}{4\pi}\right)\frac{(S_\lambda^\delta)_{ij;kl}}{\pi}C_F$$
$$+6\left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)^2 (G_\lambda^\delta)_{ij;kl} - 6\left(\frac{\alpha_s^{\overline{\rm MS}}}{4\pi}\right)\frac{(S_\lambda^\delta)_{ij;kl}}{\pi}C_F + \frac{1}{4\pi^2}(\Omega_\lambda^\delta)_{ij;kl}\,,$$
$$\text{with}\quad \delta = A, AB\,,\quad \text{and}\quad \lambda = L, LR\,. \tag{20}$$

The new colour tensors are defined as follows

$$\begin{aligned}
(\Lambda_L^A)_{ij;kl} &= (S_L^A)_{ij;mn}(S_L^A)_{nm;kl} + (S_L^A)_{il;mn}(S_L^A)_{nm;kj} + \frac{1}{2}(S_L^A)_{im;kn}(S_L^A)_{mj;nl}\,,\\
(H_L^A)_{ij;kl} &= (T^aT^b)_{ij}(T^bT^a)_{kl} + (T^aT^b)_{kj}(T^bT^a)_{il}\,,\\
(G_L^A)_{ij;kl} &= \{T^a,T^b\}_{ij}\{T^a,T^b\}_{kl} + \{T^a,T^b\}_{il}\{T^a,T^b\}_{kj}\\
(\Omega_L^A)_{ij;kl} &= (S_{LR}^A)_{ij;mn}(S_{LR}^A)_{nm;kl} + (S_{LR}^A)_{il;mn}(S_{LR}^A)_{nm;kj}\\
&\quad + \sum_{B\neq A}\left[(S_L^{AB})_{ij;mn}(S_L^{AB})_{nm;kl} + (S_L^{AB})_{il;mn}(S_L^{AB})_{nm;kj}\right]\\
&\quad + \sum_{B\neq A}\left[(S_{LR}^{AB})_{ij;mn}(S_{LR}^{AB})_{nm;kl} + (S_{LR}^{AB})_{il;mn}(S_{LR}^{AB})_{nm;kj}\right],
\end{aligned}$$

$$\begin{aligned}
(\Lambda_{LR}^A)_{ij;kl} &= (S_{LR}^A)_{im;nl}(S_{LR}^A)_{mj;kn} + (S_{LR}^A)_{im;kn}(S_{LR}^A)_{mj;nl}\,,\\
(H_{LR}^A)_{ij;kl} &= (T^aT^b)_{ij}(T^aT^b)_{kl}\,,\\
(G_{LR}^A)_{ij;kl} &= \{T^a,T^b\}_{ij}\{T^a,T^b\}_{kl}\,,\\
(\Omega_{LR}^A)_{ij;kl} &= 2(S_L^A)_{ij;mn}(S_{LR}^A)_{nm;kl} + 2\sum_{B\neq A}(S_L^{AB})_{ij;mn}(S_{LR}^{AB})_{nm;kl}\,,
\end{aligned}$$

$$
\begin{aligned}
(\Lambda_L^{AB})_{ij;kl} &= (S_L^{AB})_{im;nl}(S_L^{AB})_{mj;kn} + (S_L^{AB})_{im;kn}(S_L^{AB})_{mj;nl}\,,\\
(H_L^{AB})_{ij;kl} &= (T^aT^b)_{ij}(T^bT^a)_{kl}\,,\\
(G_L^{AB})_{ij;kl} &= G_{LR}^{A})_{ij;kl}\,,\\
(\Omega_L^{AB})_{ij;kl} &= \sum_C (S_L^{AC})_{ij;mn}(S_L^{BC})_{nm;kl} + \sum_C (S_{LR}^{AC})_{ij;mn}(S_{LR}^{BC})_{nm;kl}\,,
\end{aligned}
$$

$$
\begin{aligned}
(\Lambda_{LR}^{AB})_{ij;kl} &= (S_{LR}^{AB})_{im;nl}(S_{LR}^{AB})_{mj;kn} + (S_{LR}^{AB})_{im;kn}(S_{LR}^{AB})_{mj;nl}\,,\\
(H_{LR}^{AB})_{ij;kl} &= (H_{LR}^{A})_{ij;kl}\,,\\
(G_{LR}^{AB})_{ij;kl} &= (G_{LR}^{A})_{ij;kl}\,,\\
(\Omega_{LR}^{AB})_{ij;kl} &= \sum_C (S_L^{AC})_{ij;mn}(S_{LR}^{BC})_{nm;kl} + \sum_C (S_{LR}^{AC})_{ij;mn}(S_L^{BC})_{nm;kl}\,.
\end{aligned}
\tag{21}
$$

As can be easily verified, even if we identify the four types of quartic scalar interactions their one-loop β-functions remain different. If in addition, one sets them equal to the gauge coupling and to the gluino-squark-quark coupling equal, *i.e.* if the $\overline{\rm DR}$ scheme constraints are fulfilled, then the colour structures factorize. The resulting one-loop β-functions for the scalar couplings are identical with the one-loop $\overline{\rm DR}$ gauge β-function, as required by SUSY.

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