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Virtual three-loop corrections to Higgs boson production in gluon fusion for finite top quark mass

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Abstract

In this letter we present the three-loop virtual corrections to the Higgs boson production in the gluon fusion channel where finite top quark mass effects are taken into account. We perform an asymptotic expansion and manage to evaluate five terms in the expansion parameter M_H^2/M_t^2. A good convergence is observed almost until $M_H \approx 2M_t$.

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1 Introduction

Among the main tasks of the CERN Large Hadron Collider (LHC) will be the uncovering of the mechanism which provides particles with their masses. A crucial role in this respect is assigned to the Higgs boson whose discovery is awaited with great eagerness.

At LHC the Standard Model Higgs boson is mainly produced in the so-called gluon fusion process where two gluons couple via a closed quark loop to the Higgs boson. The leading order (LO) process for this channel has been evaluated in Refs. [1–4] and already almost 15 years ago also the next-to-leading order (NLO) corrections became available [5, 6]. To this order the production cross section could be evaluated without any assumption on the hierarchy between the mass of the quark in the loop, the Higgs boson mass, and the partonic center-of-mass energy.

On the contrary, at next-to-next-to-leading order (NNLO) only quantum corrections involving the top quark Yukawa coupling are available. It has been performed under the assumption that the top quark is much heavier than the Higgs boson. In this limit it is suggestive to construct an effective theory where the top quark is integrated out. The coefficient function of the corresponding effective operator has been computed to NNLO in Refs. [7, 8] (see also Ref. [9]) and the production cross section has been evaluated in Refs. [10–13]. Let us mention that recently the virtual contributions to the NNNLO corrections have been completed [7, 14].

In this letter we provide the first results beyond the effective-theory approach where three building blocks are required to NNLO: virtual three-loop corrections to the $2 \to 1$ process, two-loop corrections to the $2 \to 2$ process where next to the Higgs boson a parton is radiated off, and one-loop corrections with radiation of two additional partons. We present results for the three-loop virtual corrections to the process $gg \to H$ including finite top quark mass effects. Our results constitute a building block for the NNLO corrections beyond the heavy top quark limit.

Let us mention that there are also results beyond the fixed-order approximation. In particular, in Ref. [15] large logarithms in connection with soft gluon radiation have been resummed. A step further has been taken in Ref. [16], where certain π^2 terms have been resummed leading to a perturbative series which is significantly better behaved as compared to the unresummed approach. In Ref. [17] the limit of high partonic center-of-mass energies has been considered for the gluon-fusion process and an approximation for the NNLO cross section has been derived which goes beyond the $M_t \to \infty$ result. Electroweak corrections have been considered in Refs. [18,19]. For recent numerical predictions of Higgs boson production in gluon fusion both at the Tevatron and the LHC we refer to Ref. [20].

The remainder of the paper is organized as follows: in the next section we briefly describe details of our calculation and the Section 3 contains our results and conclusions.

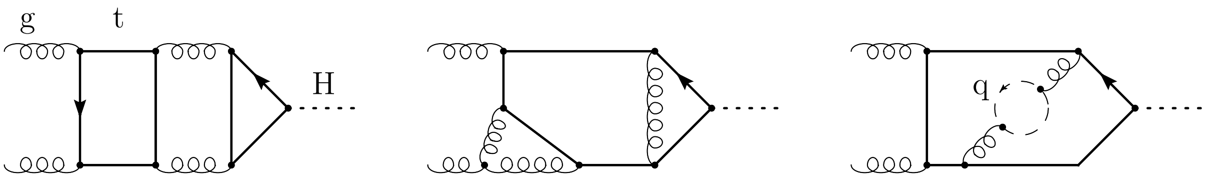

Figure 1: Sample diagrams contributing to the NNLO virtual corrections to $gg \to h$.

2 Calculation

The existing NNLO calculations to the Higgs boson production have been performed in the framework of an effective theory where the top quark has been integrated out. In this way effective vertices[1] are generated between the Higgs boson and two, three or four gluons. The number of loops to be considered for the calculation of the cross section is effectively reduced by one, leading to virtual two-loop $2 \to 1$, one-loop $2 \to 2$ and tree-level $2 \to 3$ corrections.

In contrast to this approach, our starting point is the full QCD with six active flavours. Some sample diagrams contributing to the virtual corrections are shown in Fig. 1; altogether 657 three-loop diagrams have to be considered which in the sum lead to the structure

$$X(\rho)(q_1 \cdot q_2)g_{\mu\nu} + Y(\rho)q_{1\nu}q_{2\mu} + \dots\,, \tag{1}$$

where $\rho = M_H^2/M_t^2$ and q_1 and q_2 are the incoming momenta of the two gluons with polarization vectors $\varepsilon^\mu(q_1)$ and $\varepsilon^\nu(q_2)$. In our calculation we construct projectors on X and Y and check the condition $X = -Y$ which follows from gauge invariance. The ellipses in Eq. (1) represent further structures which do not contribute to the physical cross section. They would receive contributions from vertex corrections with external ghosts which we do not consider in this paper.

The virtual contribution to the partonic cross section can be cast in the form

$$\hat{\sigma}_{ggh}^{\mathrm{virt}} = \hat{\sigma}_{\mathrm{LO}}\left(1 + \frac{\alpha_s}{\pi}\,\delta^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\delta^{(2)} + \dots\right)\,, \tag{2}$$

where the LO cross section is given by

$$\hat{\sigma}_{\mathrm{LO}} = \frac{G_F\;\alpha_s^2}{288\sqrt{2}\pi}\frac{f_0(\rho,\epsilon)}{(1-\epsilon)}\,\delta(1-x)\,, \tag{3}$$

with $x = M_H^2/\hat{s}$ and dimension of space $D = 4 - 2\epsilon$. $\sqrt{\hat{s}}$ is the partonic center-of-mass

[1]The effective coupling has even been computed to four- and five-loop order in Refs. [7] and [21, 22], respectively.

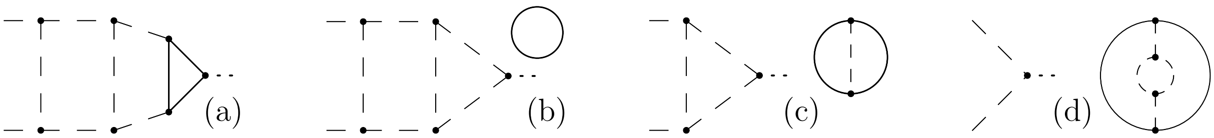

Figure 2: Factorized regions appearing from the asymptotic expansion of the double-scale integral (a). Solid lines carry the mass M_t, dotted lines mass M_H, and the dashed lines are massless. Cases (b),(c), and (d) correspond to one, two, and three loop momenta at the scale M_t, respectively, with the remaining loops at the scale M_H.

energy and $f_0(\rho,\epsilon)$ reads

$$f_0(\rho,0) = \frac{36}{\rho^2}\left|1+\left(1-\frac{4}{\rho}\right)\arcsin^2\left(\frac{\sqrt{\rho}}{2}\right)\right|^2, \qquad (\rho \le 4), \tag{4}$$

$$\begin{aligned} f_0(\rho,\epsilon) &= \left[1+\frac{7+7\epsilon}{60}\rho+\frac{1543+2486\epsilon+943\epsilon^2}{100800}\rho^2+\frac{226+461\epsilon+296\epsilon^2+61\epsilon^3}{100800}\rho^3\right.\\ &\left.+\ \frac{55354+130873\epsilon+109848\epsilon^2+39533\epsilon^3+5204\epsilon^4}{155232000}\rho^4+\ldots\right]\frac{\Gamma^2(1+\epsilon)}{(M_t^2/\mu^2)^{2\epsilon}}, \end{aligned} \tag{5}$$

where an expansion through $\mathcal{O}(\rho^4)$ and $\mathcal{O}(\epsilon^4)$ has been performed in the second line. In Section 3 we will present results for $\delta^{(1)}$ and $\delta^{(2)}$.

Note that the quantities $\delta^{(i)}$ only depend on ρ, since the Feynman diagrams have to be evaluated for on-shell external partons. It is thus tempting to evaluate them in the limit $M_H \ll 2M_t$ which is expected to show good convergence properties even up to $M_H \approx 2M_t$ [23]. In Ref. [24] the NNLO corrections to the decay of a Higgs boson into gluons have been considered. The optical theorem in combination with the asymptotic expansion was used in order to evaluate three expansion terms in M_H^2/M_t^2 where rapid convergence has been observed for $M_H \approx M_t$. The asymptotic expansion [25] in the limit $M_H \ll 2M_t$ leads to one-, two- and three-loop vacuum integrals where the scale is given by the top-quark mass and to one- and two-loop vertex diagrams with massless internal lines and external momentum at the scale M_H. In Fig. 2 we exemplify the asymptotic expansion in diagrammatic form for a typical contribution.

For our calculation we have used two independent set-ups. In the first one all Feynman diagrams are generated with `QGRAF` [26]. The various diagram topologies are identified and transformed to `FORM` [27] notation with the help of `q2e` and `exp` [28,29]. The program `exp` is also used in order to apply the asymptotic expansion (see, e.g., Ref. [25]) in the various mass hierarchies. The actual evaluation of the integrals is performed with the packages `MATAD` [30], which is used for the vacuum integrals, and `FIRE` [31], employed to reduce the massless three-point functions to master integrals. The latter can, e.g., be found in Ref. [32].

The second set-up also relies on `QGRAF` for the generation of the Feynman diagrams. Afterwards the asymptotic expansion is done with a Perl program, and, two- and three-loop integrals are reduced by an independent implementation of the Laporta algorithm [33, 34].

We have performed the evaluation of the vertex corrections up to order ρ^2 for general QCD gauge parameter and checked that it drops out in the sum of all bare three-loop diagrams which serves as a welcome check of our calculation.

In the sum of all three-loop diagrams we observe poles up to order $1/\epsilon^4$ which is due to a mixture of ultra-violet and infra-red singularities. As usual, the ultra-violet poles are treated via renormalization. In our case we have to renormalize the gluon wave function, top quark mass and strong coupling constant to two-loop order. The remaining infra-red poles are only cancelled after including the corrections from the real radiation and mass factorization.

In the following section we present our results expressed in terms of $\alpha_s^{(5)}$, the strong coupling constant in the $\overline{\mathrm{MS}}$ scheme defined in five-flavour QCD, and the on-shell top quark mass. Since our two-loop result contains poles up to $\mathcal{O}(1/\epsilon^2)$, the one-loop top quark mass counterterm is needed up to $\mathcal{O}(\epsilon^2)$ which can be found in Ref. [35]. The two-loop counterterm for α_s can be found, e.g., in Ref. [36] and the one for the gluon wave function in Ref. [7].

The transition from $\alpha_s^{(6)}$ to $\alpha_s^{(5)}$ is performed with the help of the formulae derived in Ref. [7]. Since there are poles in the NLO expression, higher order terms in ϵ are necessary for the decoupling relation. The explicit result can be found in Eq. (12) of Ref. [37].

3 Results

At the three-loop order we were able to evaluate the first five terms in the expansion around $\rho = 0$. The NLO and NNLO corrections to the partonic cross section are given by (adopting common $\overline{\mathrm{MS}}$ conventions)

$$\begin{aligned}\delta^{(1)} &= -\frac{3}{\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{23}{6} - 3L_{\mu H}\right) + \frac{11}{2} + \frac{21}{2}\zeta(2) - \frac{3}{2}L_{\mu H}^2 + \frac{34}{135}\rho + \frac{3553}{113400}\rho^2 \\ &\quad + \frac{917641}{190512000}\rho^3 + \frac{208588843}{251475840000}\rho^4 + \mathcal{O}(\rho^5)\,, \end{aligned} \tag{6}$$

$$\delta^{(2)} = \sum_{i\geq 0} \delta_i^{(2)} \rho^i\,, \tag{7}$$

with

$$\begin{aligned}
\delta_0^{(2)} &= \frac{9}{2\epsilon^4} + \frac{1}{\epsilon^3}\left(\frac{253}{16} + 9L_{\mu H}\right) + \frac{1}{\epsilon^2}\left(-\frac{69}{8} - \frac{243}{8}\zeta(2) + \frac{115}{8}L_{\mu H} + 9L_{\mu H}^2\right) \\
&+\frac{1}{\epsilon}\Bigg[-\frac{236}{9} - \frac{621}{16}\zeta(2) - \frac{159}{8}\zeta(3) - \frac{33}{2}L_{Ht} + \frac{23}{4}L_{\mu H}^2 + 6L_{\mu H}^3 \\
&+ L_{\mu H}\left(-\frac{943}{24} - \frac{243}{4}\zeta(2)\right)\Bigg] - \frac{125}{216} + \frac{1547}{16}\zeta(2) + \frac{1161}{8}\zeta(4) - \frac{381}{8}\zeta(3) \\
&-\frac{163}{8}L_{Ht} - \frac{33}{4}L_{Ht}^2 + 3L_{\mu H}^4 + \frac{23}{24}L_{\mu H}^3 + L_{\mu H}^2\left(-\frac{943}{24} - \frac{243}{4}\zeta(2)\right) \\
&+L_{\mu H}\left(-\frac{1151}{72} - \frac{69}{4}\zeta(2) - \frac{159}{4}\zeta(3) - 33L_{Ht}\right) ,
\end{aligned} \tag{8}$$

$$\begin{aligned}
\delta_1^{(2)} &= -\frac{34}{45\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{124997}{32400} - \frac{34}{45}L_{Ht} - \frac{68}{45}L_{\mu H}\right) - \frac{464570749}{10368000} - \frac{457253}{129600}L_{Ht} \\
&+ \frac{211}{90}\zeta(2) + \frac{7}{45}\zeta(2)\ln 2 + \frac{1909181}{55296}\zeta(3) - \frac{17}{45}L_{Ht}^2 - \frac{68}{45}L_{\mu H}^2 \\
&+ \left(-\frac{101537}{16200} - \frac{68}{45}L_{Ht}\right)L_{\mu H} ,
\end{aligned} \tag{9}$$

$$\begin{aligned}
\delta_2^{(2)} &= -\frac{3553}{37800\epsilon^2} - \frac{1}{\epsilon}\left(\frac{19652233}{38102400} + \frac{3553}{37800}L_{Ht} + \frac{3553}{18900}L_{\mu H}\right) - \frac{39974688999319}{4096770048000} \\
&+ \frac{887}{3024}\zeta(2) + \frac{857}{37800}\zeta(2)\ln 2 + \frac{267179777}{35389440}\zeta(3) - \frac{24507239}{50803200}L_{Ht} - \frac{3553}{75600}L_{Ht}^2 \\
&- \frac{3553}{18900}L_{\mu H}^2 - L_{\mu H}\left(\frac{3244007}{3810240} + \frac{3553}{18900}L_{Ht}\right) ,
\end{aligned} \tag{10}$$

$$\begin{aligned}
\delta_3^{(2)} &= -\frac{917641}{63504000\epsilon^2} - \frac{1}{\epsilon}\left(\frac{13727463943}{160030080000} + \frac{917641}{63504000}L_{Ht} + \frac{917641}{31752000}L_{\mu H}\right) \\
&- \frac{120540849644832968 71}{275302947225600000} + \frac{287809}{6350400}\zeta(2) + \frac{17881}{4536000}\zeta(2)\ln 2 \\
&+ \frac{5756378217151}{158544691200}\zeta(3) - \frac{5713528199}{71124480000}L_{Ht} - \frac{917641}{127008000}L_{Ht}^2 - \frac{917641}{31752000}L_{\mu H}^2 \\
&- L_{\mu H}\left(\frac{359730029}{2500470000} + \frac{917641}{31752000}L_{Ht}\right) ,
\end{aligned} \tag{11}$$

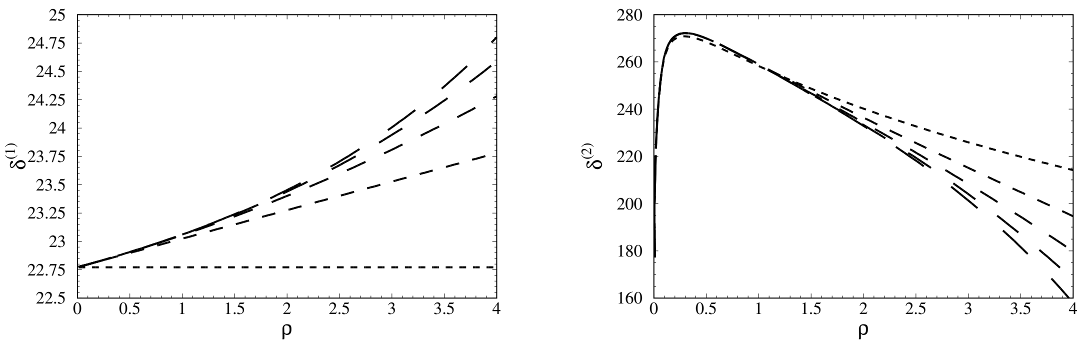

Figure 3: Finite part of $\delta^{(1)}$ (left) and $\delta^{(2)}$ (right) as a function of ρ. The longer-dashed lines include successively higher orders in ρ.

$$\begin{aligned}
\delta_4^{(2)} \quad = \quad & -\frac{208588843}{83825280000\epsilon^2} - \frac{1}{\epsilon}\left(\frac{36471674738759}{2323636761600000} + \frac{208588843}{83825280000}L_{Ht}\right. \\
& \left. + \frac{208588843}{41912640000}L_{\mu H}\right) - \frac{7493811653667964109815870}{2198569336543641600000000} + \frac{65703703}{8382528000}\zeta(2) \\
& + \frac{31270501}{41912640000}\zeta(2)\ln 2 + \frac{89834770435139}{3170893824000}\zeta(3) - \frac{45644737075181}{3098182348800000}L_{Ht} \\
& - \frac{208588843}{167650560000}L_{Ht}^2 - \frac{208588843}{41912640000}L_{\mu H}^2 - L_{\mu H}\left(\frac{1933157007779}{72613648800000}\right. \\
& \left. + \frac{208588843}{41912640000}L_{Ht}\right) , \qquad (12)
\end{aligned}$$

where $L_{\mu H} = \ln(\mu^2/M_H^2)$, $L_{Ht} = \ln(M_H^2/M_t^2)$ and $\zeta(n)$ is the Riemann's zeta function. Furthermore, $SU(3)$ colour factors have been applied and the number of massless quark flavours is set to $n_l = 5$. The analytic expression for the generic values of N_c and n_l can be found in [38].

We have checked that the expansion of $\delta^{(1)}$ to order ρ agrees with [39]. At the NNLO the leading term in the inverse top quark mass expansion agrees with the results of Refs. [7,10].

Although the final result is divergent, it is instructive to look at the finite parts of $\delta^{(1)}$ and $\delta^{(2)}$. In Fig. 3 the corresponding two- and three-loop expressions are shown for $\mu = M_H$ in the range between $\rho = 0$ and $\rho = 4$ corresponding to $M_H = 2M_t$. The longer-dashed lines include successively higher orders in ρ up to order ρ^4. One observes good convergence up to $\rho \approx 3$ which corresponds to $M_H \approx 1.7M_t$.

To conclude, we have presented the virtual corrections to the partonic cross section $gg \to H$ including finite top quark mass effects. Our calculation confirms the results obtained in the framework of the effective theory and provides four more expansion terms in M_H^2/M_t^2. We observe a rapid convergence almost up to $M_H \approx 2M_t$. The results presented in this letter constitute a building block for the NNLO corrections to the Higgs boson production in the gluon fusion channel beyond the heavy top quark limit.

When this paper was in preparation, we had a chance to learn about the parallel publication [40] and establish the full agreement of the results.

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