Strong and Electromagnetic J/ψ and $\psi(2S)$ Decays into Pion and Kaon Pairs*

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A combined analysis of the electromagnetic pion and kaon form factors in the neighborhood of J/ψ and $\psi(2S)$ and of the strong decay amplitude of these resonances into kaons is presented. In the presence of a large relative phase between strong and electromagnetic resonance amplitudes the branching ratio, as measured in electron-positron annihilation, receives an additional contribution from the interference between resonance and continuum amplitude neglected in earlier papers. Our study is model independent and does not rely on the SU(3) symmetry assumptions used in earlier papers. We note that the large relative phase between strong and electromagnetic amplitudes observed in earlier analyses is model dependent and relies critically on the specific assumptions on SU(3) symmetry and breaking.

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I. INTRODUCTION

Exclusive decays of J/ψ and $\Psi(2S)$ into pseudoscalar meson pairs have attracted considerable attention both from the theoretical and the experimental side. In the case of $\pi^+\pi^-$ the branching ratio was used to determine the pion form factor at fairly high energies, which turned out to be significantly larger (see e. g. [1, 2]) than originally predicted in QCD. In the case of charged and neutral kaons both strong and electromagnetic interactions contribute with comparable strength and the branching ratio evidently depends critically on their relative phase. In Refs. [3–5] it has been argued this phase to be fairly large, close to 90° or 270° and interesting conclusions have been drawn for the similar interplay between strong and weak phases in exclusive B meson decays.

Recently new experimental results for the branching ratios and new measurements of the pion and kaon production cross section close to $\Psi(2S)$ became available. Furthermore there is the perspective of improved measurements of these quantities both through the radiative return [6–8] at B meson factories and from direct scanning at BES. A detailed analysis of strong and electromagnetic amplitudes which is less dependent on additional assumptions is therefore appropriate.

In the following we will demonstrate that a previously neglected interference term between continuum and resonance amplitude may seriously affect the analysis of the kaon modes. Furthermore, the claim of a large relative phase depends crucially on the assumption of an extremely small K_LK_S from factor, an assumption still to be validated.

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Our paper is organized as follows: In section II we present the formalism used to describe both resonance decays and continuum cross section in the neighborhood of the resonances. Section III is concerned with a model-independent extraction of resonance parameters and form factors based on the most recent data. Section IV contains a brief summary and our conclusions.

II. PSEUDOSCALAR PAIR PRODUCTION CLOSE TO RESONANCES

Various amplitudes can contribute to the production of hadronic final states close to a narrow resonance R: continuum production through the photon (A_{QED}, Fig. 1a), resonant production with electromagnetic decay (A^R_{QED}, Fig. 1b) and resonant production with hadronic decay (A^R_{QCD}, Fig. 1c). All contributions involving virtual photons include the vacuum polarization. The amplitudes for pseudoscalar meson (denoted P) pair production, considered in this paper, (relative to the Born amplitude $\bar{v}\gamma_\mu u(q_1-q_2)^\mu/s$) are thus given by

$$\begin{aligned}
A_{QED} &= \frac{e^2}{1-\Delta\alpha} F_P \\
A^R_{QED} &= \frac{e^2}{(1-\Delta\alpha)^2}\frac{f_R^2 M_R^4}{s-M_R^2+i\Gamma_R M_R}\frac{1}{s}F_P \\
A^R_{QCD} &= \frac{e}{1-\Delta\alpha}\;\frac{f_R M_R^2\,\mathcal{C}^R_P}{s-M_R^2+i\Gamma_R M_R}\;.
\end{aligned} \tag{1}$$

The s dependence of $\Delta\alpha \equiv \mathrm{Re}\Pi(s)$ and of the form factor F_P is implicitly understood. M_R and Γ_R denote mass and width of the narrow resonance. In the absence of dispersive contributions the coupling f_R of the virtual photon to the resonance is real, while the electromagnetic form factor F_P and the coupling $\mathcal{C}^R_P$ are complex. The contribution from the (lepton- and

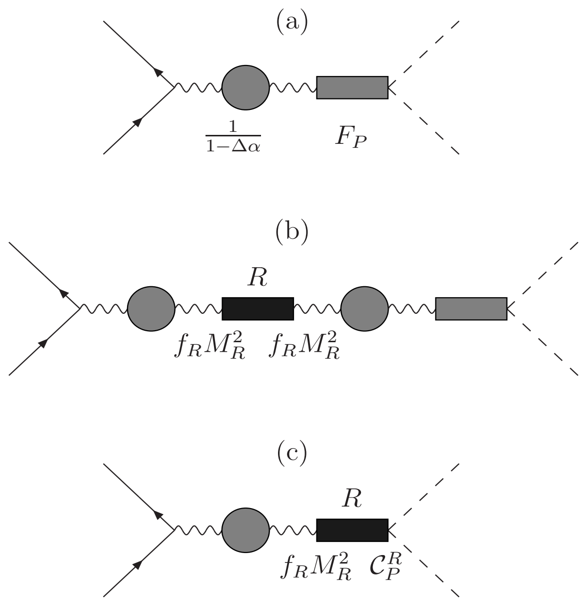

FIG. 1: (color online) The diagrams contributing to the cross section of the reaction $e^+e^- \to P\bar{P}$. The grey bubble stands for the vacuum polarization, the grey box for the 'bare' form factor, the black box for the Breit-Wigner amplitude.

hadron-induced) vacuum polarization is displayed explicitly, hence F_P denotes the "bare" form factor. (Note that in [2] the "dressed" form factor was used.) For the subsequent discussion it is convenient to express the combination $f_R^2 M_R^4/[s(1-\Delta\alpha)^2]$ by $3\sqrt{s}\ \Gamma_e^R/\alpha$ and $\mathcal{C}_P^R$ by $eF_P f_R M_R^2\ c_P^R/[s(1-\Delta\alpha)]$, where Γ_e^R is the electronic width of the narrow resonance, and $c_P^R = A_{QCD}^R/A_{QED}^R$.

The cross section for the reaction $e^+e^- \to P\bar{P}$ can then be written in the form

$$\begin{aligned}
&\sigma\left(e^+e^- \to P\bar{P}\right) = \\
&\quad \frac{\pi\alpha^2}{3s}|F_P|^2\ \beta^3 \\
&\quad \times\left|\frac{1}{1-\Delta\alpha} + \sum_R \frac{3\sqrt{s}}{\alpha}\frac{\Gamma_e^R(1+c_P^R)}{s-M_R^2+i\Gamma_R M_R}\right|^2 = \\
&\quad \frac{\pi\alpha^2}{3s}|F_P|^2\ \beta^3 \\
&\quad \times\left(\frac{1}{(1-\Delta\alpha)^2} + \sum_R\left\{\frac{9s}{\alpha^2}\frac{(\Gamma_e^R)^2}{(s-M_R^2)^2+\Gamma_R^2 M_R^2}\right.\right. \\
&\qquad \times\left[|1+c_P^R|^2 + \frac{2\alpha M_R}{3\sqrt{s}(1-\Delta\alpha)}\frac{\Gamma_R}{\Gamma_e^R}\mathrm{Im}(c_P^R)\right] \\
&\qquad \left.\left. + \frac{6\sqrt{s}\Gamma_e^R}{\alpha(1-\Delta\alpha)}\frac{\left(1+\mathrm{Re}(c_P^R)\right)\left(s-M_R^2\right)}{(s-M_R^2)^2+\Gamma_R^2 M_R^2}\right\}\right)\ ,
\end{aligned} \tag{2}$$

where $\beta = \sqrt{1-\frac{4m_P^2}{s}}$. For muons the factor β^3 is replaced by $4\beta(1+2m_\mu^2/s)$. The narrow resonances are well separated, thus in Eq.(2) their interferences are neglected. We also neglect here small contributions from the imaginary part of $\Delta\alpha$.

In the present discussion the narrow resonances J/ψ and $\psi(2S)$ are considered. For pion pairs the direct couplings c_P^R of the narrow resonances to the hadronic final states vanishes as a consequence of isospin conservation. For kaons, however, the direct coupling is important and the relative magnitude and the phase of QED versus hadronic amplitude can only be obtained from a study combining measurements on and off resonance. Such an investigation might, furthermore, even allow to disentangle the $I=0$ and $I=1$ amplitudes that contribute to the electromagnetic form factor. For the muon the form factor is set to 1. For baryons, which are not studied in this paper, one receives contributions from the electric and magnetic form factors (see e.g. [7] and references therein). Also for baryons there exist a direct coupling to the narrow resonances and both isospin zero and isospin one contribute to the form factors, hence the discussion is quite similar to the case of kaons.

The interference term between continuum and any of the resonances consists of two parts: one proportional to $\mathrm{Im}(c_P^R)$ and the second one proportional to $\left(1+\mathrm{Re}(c_P^R)\right)$. The term $\sim \mathrm{Im}(c_P^R)$ contributes both to the integrated resonant cross section and to its peak value. These quantities thus are not, as it is often assumed (see e.g. [3–5]), proportional to $\Gamma(R\to P\bar{P})$ which is defined without the interference with the continuum:

$$\Gamma(R\to P\bar{P}) = \Gamma^{QED}\times|1+c_P^R|^2\ , \tag{3}$$

where Γ^{QED} represents the decay rate induced through A_{QED}^R alone. The correct formula, to be used to calculate the branching ratios of J/ψ and $\psi(2S)$ decays, which are extracted from the integrated cross section, thus reads

$$\begin{aligned}
\Gamma(R\to P\bar{P}) &= \Gamma^{QED}\times\left[|1+c_P^R|^2\right. \\
&\quad \left. + \frac{2\alpha M_R}{3\sqrt{s}\,(1-\Delta\alpha)}\frac{\Gamma_R}{\Gamma_e^R}\mathrm{Im}(c_P^R)\right]\ .
\end{aligned} \tag{4}$$

The additional term correctly takes into account the interference effects between continuum and resonance.

From Eq.(2) it is evident that electromagnetic and hadronic decay amplitudes, A_{QED}^R and A_{QCD}^R, can only be disentangled, if both the even part (contributing to the peak value, the branching ratio and the far off resonance behaviour) and the odd part (in the neighborhood of the resonance) are measured.

At present the complete analysis can be performed only for the $\psi(2S)$ decaying to charged kaons where an off peak measurement of the cross section is available [9]. For the $\psi(2S)$ decay to neutral kaons as well as for the J/ψ decays off peak data are not available and the only information about the coupling c_P^R comes from the branching ratios. We will return to this point in the next Section.

III. ANALYSIS OF RESONANCE PARAMETERS AND FORM FACTORS

In the following we shall present an analysis of the resonance parameters valid for J/ψ and $\psi(2S)$ decays into $\pi^+\pi^-$, K^+K^- and $K^0\bar{K}^0$. Masses, decay rates, branching ratios, form factors and values of $\Delta\alpha$ are listed in Table I.

	J/ψ	$\psi(2S)$
M [MeV]	3096.916±0.011	3686.09±0.04
Γ_{ee} [keV]	5.55±0.14±0.02	2.38±0.04
$\mathcal{B}(e^+e^-)$[%]	5.94±0.06	0.752±0.017
$\mathcal{B}(K^+K^-)[10^{-5}]$	23.7±3.1	6.3±0.7
$\mathcal{B}(K^0_SK^0_L)[10^{-5}]$	14.6±2.6	5.4±0.5
$\mathcal{B}(\pi^+\pi^-)[10^{-5}]$	14.7±2.3	0.9±0.5[13]
$\sigma(K^+K^-)$[pb]	-	5.7±0.8 [9]
$\sigma(K^0_SK^0_L)$[pb]	-	< 0.74 (90% C.L.) [12]
$\sigma(\pi^+\pi^-)$[pb]	-	9.0±2.2 [9]
$\Delta\alpha$	0.02117 [8, 11]	0.02219[8, 11]

TABLE I: Resonance parameters, vacuum polarization and cross sections used in this paper. If not stated differently the values are taken from [10]. The cross sections are measured at $\sqrt{s} = 3.671$ GeV .

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Let us first discus the decays to $\pi^+\pi^-$. Isospin symmetry forbids the direct hadronic decay, whence $A^R_{QCD} = 0$. In this case the form factor can be directly derived from the branching ratios listed in Table I:

$$|F_\pi|^2 = \frac{4\mathcal{B}(R \to \pi^+\pi^-)}{\beta_\pi^3 \mathcal{B}(R \to e^+e^-)} \ . \tag{5}$$

For some time it has been argued [1, 2] that the result $|F_\pi(\sqrt{s} = M_{J/\psi})|^2 = (10.0 \pm 1.6) \cdot 10^{-3}$, as derived from this consideration, is surprisingly large, when confronted [2] with predictions based on the asymptotic pion wave function and derived in perturbative QCD. However, an independent and direct measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ in the neighborhood of $\psi(2S)$ [9] is consistent with this relatively large form factor. Thus we shall adopt $A^R_{QCD}(\pi^+\pi^-) = 0$ in this paper, although an independent measurement of the pion form factor in the J/ψ region would be highly desirable. The results for $|F_\pi(\sqrt{s} = M_{J/\psi})|^2$ and $|F_\pi(\sqrt{s} = 3.671\text{GeV})|^2$ are listed in Table II. Our value for the pion form factor $|F_\pi|^2 = (5.92 \pm 1.46) \cdot 10^{-3}$, as obtained from the off-resonance cross section measurement [9] and listed in Table I, is about 6% higher than the result quoted in [9] as a consequence of the interference between the resonance and the continuum neglected in [9].

Let us now move to the case of kaons. As stated in the Introduction, a completely independent determination of the three amplitudes A_{QED}, A^R_{QED} and A^R_{QCD}, without further assumptions can only be obtained if both the even and the odd parts of the cross section are measured in the neighborhood of the resonance. In the first step we will discuss the decay $\psi(2S) \to K^+K^-$. The resonant symmetric part, which is determined by the branching ratio, receives a contribution from $|1 + c^R_P|$ and a contribution proportional to $\mathrm{Im}(c^R_P)$, which is often ignored. Its numerical relevance will be discussed below. For the ratio $\mathcal{B}(K^+K^-)/\mathcal{B}(e^+e^-)$ we find

$$R_+ \equiv \frac{4\mathcal{B}(K^+K^-)}{\beta^3_{K^+}\mathcal{B}(e^+e^-)} = |F_{K^+}|^2 \left[|1 + c_+|^2 + r\mathrm{Im}c_+\right] \ , \tag{6}$$

where for simplicity we use notation $c_+ = c^{\psi(2S)}_{K^+}$. Moreover

$$\beta^3_{K^+} = \left(1 - \frac{4m^2_{K^+}}{M^2_{\psi(2S)}}\right)^{3/2} = 0.894$$
$$R_+ = (3.75 \pm 0.43) \cdot 10^{-2}$$
$$r = \frac{2\alpha}{3(1-\Delta\alpha)\mathcal{B}(e^+e^-)} = 0.663 \pm 0.015 \ . \tag{7}$$

Anticipating a value of c_+ with a modulus around 3 and a sizable imaginary part, it is clear that the interference term $r\mathrm{Im}c_+$ cannot be neglected.

In order to disentangle the form factor from the modulus and the phase of $|c_+|$, a measurement of $|F_{K^+}|$ far away from resonance and a second measurement closer to the peak would be required. This is illustrated by evaluating Eq.(2) off resonance, ($|E - M_R| \gg \Gamma_R$). In this case Eq.(2) can be cast into the form

$$S_+ - R_+\frac{\gamma^2}{4} = |F_{K^+}|^2 \left[1 + \gamma(1 + \mathrm{Re}c_+)\right], \tag{8}$$

with

$$S_+ = \sigma(e^+e^- \to K^+K^-)(1-\Delta\alpha)^2 / \left(\frac{\pi\alpha^2}{3s}\beta^3_{K^+}\right) \tag{9}$$

and

$$\gamma = \frac{\Gamma_e}{E - M_R}\frac{3(1-\Delta\alpha)}{\alpha}. \tag{10}$$

	J/ψ	$\psi(2S)$
$|F_\pi|^2[10^{-3}]$ Eq.(5)	10.0±1.6	4.8±2.73
$|F_\pi|^2[10^{-3}]$ Eq.(2)	-	5.92±1.46
$|F_{K^+}|^2[10^{-3}]$Eq.(12)	-	4.0±1.3
$|F_{K^0}|^2[10^{-3}]$Eq.(17)	-	< 0.81

TABLE II: Form factors obtained from our analysis.

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The cross section $\sigma(e^+e^- \to K^+K^-)$, as defined in Eq.(2), still contains all vacuum polarization corrections. When using experimental data one has to remember that it is often corrected for the leptonic part of the vacuum polarization.

Given $|F_{K^+}|$ from one measurement far enough from the resonance to suppress the interference ($\gamma \to 0$), and performing another one with $E - M_R$ such that $\gamma \sim -1$ Eq. (8) will lead to a model-independent determination of $|F_{K^+}|^2$ and $\mathrm{Re}c_+$. Eq. (6) can then be used to determine $\mathrm{Im}c_+$ up to a twofold ambiguity. A measurement above the resonance would suffer from contributions from the radiative return to the resonance, thus it is easier to make the measurement below.

The CLEO determination [9] of the cross section 15 MeV below the $\psi(2S)$ resonance leads to $\gamma = -0.063 \pm 0.001$, $S_+ = (3.90 \pm 0.52) \cdot 10^{-3}$ and $S_+ - R_+\gamma^2/4 = (3.86 \pm 0.52) \cdot 10^{-3}$. In this case the γ^2 term is negligible and the term linear in γ is a small correction. In contrast, for a value of $|E - M_R|$ say around 2 MeV and correspondingly $\gamma \sim -0.5$, the interference term is large and the measurement is sensitive to the product $\gamma\mathrm{Re}c_+$. Let us now evaluate $|c_+|$ and $|F_{K^+}|^2$ as a function of ϕ_+ ($c_+ = |c_+|e^{i\phi_+}$) for the present set of measurements. Combining Eqs. (8) and (6) one finds

$$\frac{R_+}{S_+ - R_+\frac{\gamma^2}{4}} = \frac{1 + 2|c_+|\cos(\phi_+) + |c_+|^2 + r|c_+|\sin(\phi_+)}{1 + \gamma(1 + |c_+|\cos(\phi_+))} \tag{11}$$

and the values of $|c_+|$ can be determined as a function of its phase ϕ_+ (vertically dashed region in the upper plot of Fig.2).

As a consequence of the smallness of γ, the form factor $|F_{K^+}|^2$ as extracted from Eq.(8) is only moderately dependent on ϕ_+ (Fig.2). Nevertheless, the experimentally allowed interference term might not be negligible as assumed in [9]. Thus the error on $|F_{K^+}|^2$ as extracted from $\sigma(e^+e^- \to K^+K^-)$ has to be enlarged as compared to [9]. For an unknown phase ϕ_+ the experimentally allowed region of $|F_{K^+}|$ is given by (see Fig. 2)

$$0.052 < |F_{K^+}| < 0.073 \tag{12}$$

as compared to the value obtained in [9]

$$0.059 < |F_{K^+}| < 0.067\ , \tag{13}$$

which corresponds to the special case $\gamma(1 + \mathrm{Re}c_+) = 0$. As stated above, if a second measurement of $\sigma(e^+e^- \to K^+K^-)$ closer to the $\psi(2S)$ would be available a model independent determination of $\mathrm{Re}c_+$, $|F_{K^+}|^2$ with a twofold solution for $\mathrm{Im}c_+$ would be feasible. This is illustrated in Fig. 3 where the analysis based on two fictitious measurements

$$\sigma(e^+e^- \to K^+K^-, E = M_{\psi(2S)} - 20\mathrm{MeV}) = (5.55 \pm 0.28)pb \tag{14}$$

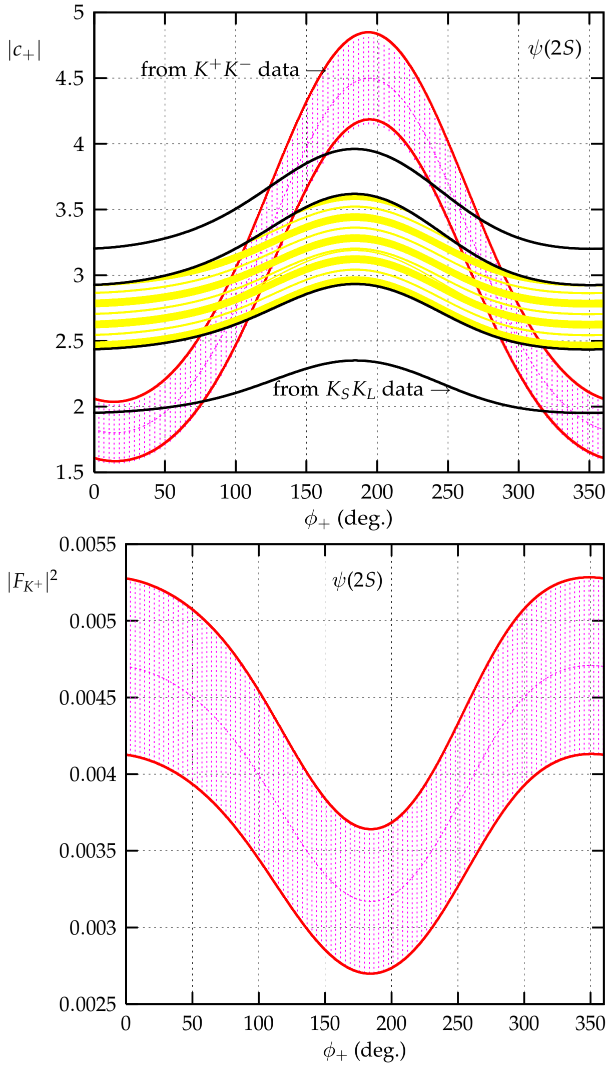

FIG. 2: (color online) The experimentally allowed regions of $|c_+|$, $|F_{K^+}|^2$ as functions of ϕ_+. To obtain the vertically dashed regions we used K^+K^- data. The narrow horizontal band was obtained using $A_{QED}(K^0) = 0$, while the wider horizontal region was obtained using neutral kaon data and in addition $|F_{K^+}|^2(\phi_+)$ (see text for details).

and

$$\sigma(e^+e^- \to K^+K^-, E = M_{\psi(2S)} - 2\mathrm{MeV}) = (7.68 \pm 0.38)pb \tag{15}$$

in combination with an improved determination of $\mathcal{B}(\psi(2S) \to K^+K^-) = (6.3 \pm 0.35) \cdot 10^{-5}$ (we assume that the central remains unchanged and the error is reduced by a factor 2 as compared to [10]). With a luminosity of BES III [14] of $2.6 \cdot 10^3 \mathrm{pb}^{-1}$/month one expects about 1000 K^+K^- events in 2 days of running, thus the proposed measurements are certainly possible. As it is clear from Fig. 3 a remarkable accuracy for $|c_+|$, $|F_{K^+}|^2$ and ϕ_+ can be expected. Two solutions are allowed for $|c_+|$ and ϕ_+ corresponding to two values of Imc_+. This ambiguity can only be resolved with information coming from neutral kaon production.

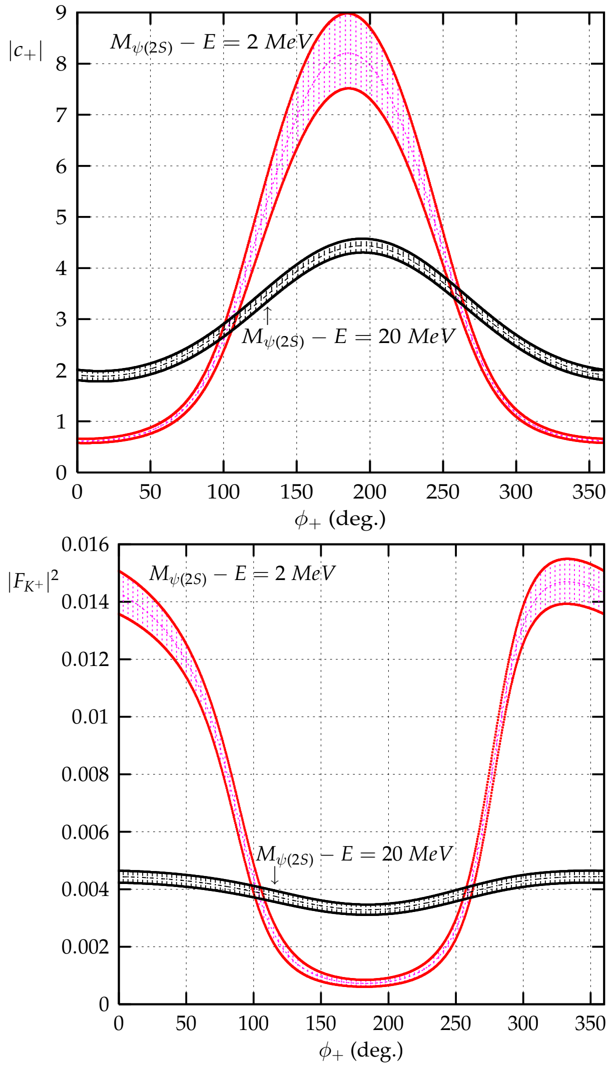

FIG. 3: (color online) Allowed regions of $|c_+|$, $|F_{K^+}|^2$ and ϕ_+ in the fictitious experimental situation described in the text.

For the moment, with only one off-resonance measurement available, one has to rely on the measurements of the $\psi(2S)$ decay into $K^0\bar{K}^0$, limited information on the neutral kaon form factor F_{K^0}, and isospin symmetry relating the hadronic amplitudes $A^R_{QCD}(K^+K^-) = A^R_{QCD}(K^0\bar{K}^0)$. The determination of $A^R_{QCD}(K^0\bar{K}^0)$ makes use of the upper limit $\sigma(e^+e^- \to K^0_S K^0_L) < 0.74$pb at 90% C.L., obtained [12] 15 MeV below the resonance. In [12] this measurement was translated into the limit $|F_{K^0}|^2 < 0.53 \cdot 10^{-3}$, neglecting the interference term in Eq.(2), i.e. by setting $\gamma = 0$. Based on the smallness of $|F_{K^0}|$ one might then set $A^R_{QED} = 0$ and derive the hadronic amplitude A^R_{QCD} from the branching ratio into $K^0\bar{K}^0$. Using isospin invariance this would lead to $|c_+ F_{K^+}| = 0.179 \pm 0.009$. The result for $|c_+|$ is shown in Fig. 2 as a narrow waved band.

A more refined and conservative analysis, which is sensitive to the interference term and makes use only of the fore mentioned upper limit on the cross section, leads in the first step to

$$| \frac{1}{c_0} | = | \frac{A^R_{QED}}{A^R_{QCD}} | < 0.187 \pm 0.008 \; . \tag{16}$$

The uncertainty comes from the error on R_0. The limit does depend on the phase ϕ_0 and we present above only the absolute limit ($\max_{\phi_0}(|1/c_0|)$). Using the analog of Eq.(6), this translates into

$$|F_{K^0}| < 0.0282 \pm 0.0003 \tag{17}$$

to be compared with the limit obtained by CLEO [12]

$$|F_{K^0}| < 0.023 \; , \tag{18}$$

where the interference with the resonance has been neglected. This analysis also implies

$$|F_{K^0} \cdot c_0| < 0.174 \pm 0.009 \pm 0.024 \tag{19}$$

where the first error is due to the error on R_0 and the second originates from the unknown strength of the interference between A^R_{QED} and A^R_{QCD} (when combined the errors should added linearly). In the last step the hadronic amplitudes of charged and neutral modes are identified

$$|F_{K^0} \cdot c_0| = |F_{K^+} \cdot c_+| \; . \tag{20}$$

Using $|F_{K^+}|$ as determined from Eq.(8) (see also Fig.2) one obtains experimentally allowed region of $|c_+|$ and ϕ_+ shown in Fig.2 as wide horizontal band. The intersection of the vertically dashed region (obtained from charged kaons only) and the wider horizontal band characterizes the experimentally allowed $|c_+|$ and ϕ_+ values. This more conservative analysis, which does not set $F_{K^0} = 0$ and uses only the upper limit in Eq. (17), thus leads to $|c_+| = 2.94 \pm 0.99$ and no relevant constraint on ϕ_+.

Although the same formulae are applicable to J/ψ decays the situation is markedly different as far as the analysis is concerned. At present, there are no off-resonance measurements of the cross section, hence only Eq.(6) (and its analog for neutral kaons) can be exploited. Furthermore, the quantity $r(J/\psi) = 0.0835 \pm 0.0008$, which characterizes the contribution from the resonance–continuum interference to the branching ratio is significantly smaller than $r(\psi(2S)) = 0.663$, such that the term

$r\mathrm{Im}c$ is significantly less important. Previous analyses of the J/ψ branching ratios have systematically neglected the $r\mathrm{Im}c$-term (see e.g. [3–5]). Furthermore the authors made simplifying assumptions about the π^+, K^+ and K^0 form factors. Let us discuss these now in detail:

Isospin symmetry implies $A^R_{QCD}(K^+) = A^R_{QCD}(K^0)$ and, furthermore, $A^R_{QCD}(\pi^+) = 0$, assumptions also used in the present analysis. The result for the pion form factor has been discussed above and is listed in Table II. The ratio $|F_\pi|^2(\psi(2S))/|F_\pi|^2(J/\psi) \simeq 0.5$ is well compatible with $F_\pi \sim 1/Q^2$ expected for the asymptotic high energy region. More specific, model dependent, assumptions have been used in [3–5].

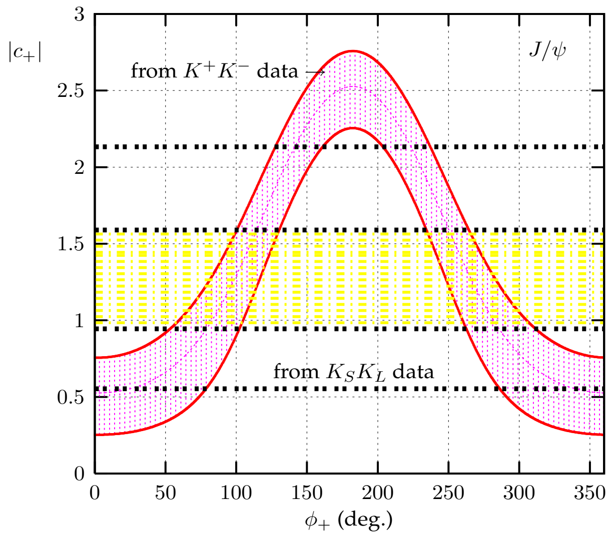

FIG. 4: (color online) The experimentally allowed region of $|c_+|$ as functions of ϕ_+. To obtain the vertically dashed regions we used K^+K^- data. The narrow horizontal band was obtained using $A_{QED}(K^0) = 0$, while the wider horizontal region was obtained using neutral kaon data and in addition $|F_{K^+}|^2(\phi_+) = (8.03 \pm 2.61) \cdot 10^{-3}$ (see text for details).

The analysis for charged and neutral kaons depends critically on the assumptions on the kaon form factors, which have not yet been measured close to J/ψ. In the first step we combine the measured branching ratio $\mathcal{B}(J/\psi \to K^+K^-)$ with an assumption on the charged form factor $|F_{K^+}|^2 = (8.0 \pm 2.6) \cdot 10^{-3}$. The latter is obtained from scaling law $|F_{K^+}|$ proportional to $1/Q^2$ and using its measured value at the $\psi(2S)$ resonance as input. This value is also close to the pion form factor as expected from $SU(3)$ symmetry. Since the values of the hadronic and QED amplitudes are of comparable size the result for $c_+ = A^{J/\psi}_{QCD}/A^{J/\psi}_{QED}$ depends crucially on their relative phase (vertically dashed region in Fig. 4). In the second step the branching ratio into neutral kaon is used to determine $A^{J/\psi}_{QCD}$. In principle $\mathcal{B}(J/\psi \to K^0\bar{K}^0)$ receives contributions both from $A^{J/\psi}_{QCD}$ and $A^{J/\psi}_{QED}(K^0_SK^0_L)$. Adopting, however, as an extreme case $A^{J/\psi}_{QED}(K^0_SK^0_L) = 0$ the modulus of $A^{J/\psi}_{QCD}(K^0_SK^0_L) = A^{J/\psi}_{QCD}(K^+K^-)$ is fixed, and correspondingly $|c_+F_{K^+}|$. Together with the assumption on $|F_{K^+}|$ this leads to $|c_+| = 1.27 \pm 0.32$ and the relative phase of $53^\circ < \phi_+ < 130^\circ$ or $234^\circ < \phi_+ < 311^\circ$. Although this observation is consistent with the observation of Rosner [3] of ϕ_+ around 90° or 270° (see also [4]), our uncertainty on the phase is significantly larger, a consequence of the larger errors both on the branching ratio and the charged kaon form factor. However, even the results discussed above depend critically on the assumption of very small $|F_{K^0}|$. Although the neutral kaon form factor is indeed expected to be significantly smaller than $|F_{K^+}|$, a more conservative analysis should be based on the upper limit on $|F_{K^0}|$ only, as derived from $|F_{K^0}|$ determined close to $\psi(2S)$, and the scaling law $|F_{K^0}| \sim 1/Q^2$, whence $|F_{K^0}|^2 < 1.63 \cdot 10^{-3}$ in the neighborhood of J/ψ. This implies $0.55 < |c_+| < 2.13$ and excludes only a small region between 161° and 204° for the phase ϕ_+. As it is clear from this discussion, a significant improvement on the charged and neutral kaon form factor measurement in the neighborhood of J/ψ is required to disentangle hadronic and electromagnetic amplitudes. Let us also stress that two measurements combined, one several MeV, the second more then 20 MeV below the resonance, would allow to determine all the amplitudes up to a two-fold ambiguity.

IV. SUMMARY

Recent experimental results for J/ψ and $\Psi(2S)$ decays into pairs of pseudoscalar mesons have been used to extract the hadronic decay amplitude and the electromagnetic form factors at the corresponding energies. A previously neglected interference term leads to significant shifts of the parameters. It is demonstrated that the previous results, observing a large relative phase between strong and electromagnetic amplitude, depend sensitively on specific assumptions about the neutral kaon form factor which are not tested by experiment.It is, furthermore, shown how the combination of two cross section measurements close to the resonance with the corresponding branching ratio would lead to a model independent determination (up to a twofold ambiguity) of strong amplitude, form factors and their relative phase.

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