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EDM constraints on flavored CP-violating phases

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Abstract

The CP-violating phenomenology of the MSSM with Minimal Flavor Violation (MFV) in the lepton sector is revisited. To this end, the most general parametrizations of the slepton soft-breaking terms are constructed assuming a seesaw mechanism of type I. After a critical reassessment of how the CP-symmetry is broken within the MFV framework, all possible CP-violating phases are introduced. From the strong hierarchy of their contributions to the Electric Dipole Moments (EDMs), these phases are split into three classes: flavor-blind, flavor-diagonal and flavor off-diagonal. In particular, the phases from the neutrino sector belong to the last class; they start to contribute only at the second order in the mass-insertion approximation and have thus a negligible effect. It is then shown that to each class of phases corresponds a unique largely dominant term in the MFV expansion. Numerically, for a realistic range of MSSM and neutrino parameters, such that $B(\mu \to e\gamma)$ does not exceed its experimental bound, the three types of phases are found to be allowed by the current bound on the electron EDM, though the next generation of experiments should constrain tightly the flavor-blind phase. Finally, we relax the MFV hypothesis and show how in the general MSSM, the MFV operator basis can be used to judge of the naturality of the slepton soft-breaking terms.

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1 Introduction

In the past few years, the Minimal Flavor Violation (MFV) hypothesis has emerged as an appealing framework, allowing to reconcile the ever more constraining low-energy data with the possibility of New Physics below the TeV scale. Indeed, when the New Physics particles are so light, the absence of significant deviations with respect to the Standard Model (SM) in K and B observables or in Lepton Flavor Violating (LFV) processes like $\mu \to e\gamma$ yields very tight bounds on the absolute value and CP-violating phase of the flavor-violating couplings. At this stage, one could simply forbid New Physics to induce any flavor transitions, i.e. force it to be flavor-blind. However, this seems far too constraining given that flavor transitions do occur in the SM. By contrast, the MFV hypothesis permits non-trivial flavor structures for New Physics, but severely limits them by naturally relating them to those of the SM.

However, there is still some latitude in the precise implementation of MFV, especially regarding the presence of new CP-violating phases. Indeed, most approaches try to limit the CP-violating phases in the flavor sector to those already present in the SM. For instance, in the quark sector, only the CKM phase is allowed. However, this has several conceptual drawbacks. First, MFV can be neatly formulated as a symmetry principle, but this procedure leaves the CP-violating phases completely free. Second, most New Physics models do contain CP-violating phases beyond those of the SM quark and lepton sectors, so this restriction appears unnatural, and may even be difficult to maintain if the flavor-blind and flavored sectors can communicate. Finally, as will be discussed in the present work, the appearance of new CP-violating phases is actually deeply rooted in the formulation of MFV itself; it could only be naturally avoided by imposing the CP-symmetry on the whole theory, including on the SM.

Lifting all restrictions on the CP-violating phases, there is a serious danger of failing to account for the tight bounds on the Electric Dipole Moments (EDMs), which are flavor-diagonal CP-violating observables. Indeed, for flavor transitions, one can rely on various hierarchies (like those in the CKM matrix or in the neutrino masses) to suppress the effects of the new phases, but no such mechanism is available for flavor-diagonal observables. To be more specific, the dimension-six effective electromagnetic operator which will concern us in the following,

$$H_{eff} = e\frac{\mathcal{C}^{IJ}}{\Lambda^2}\bar{\psi}_R^I \sigma_{\mu\nu}\psi_L^J F^{\mu\nu} H_d \; , \tag{1}$$

induces the flavor-violating transitions between fermion species $\psi^I \to \psi^J\gamma$ when $\mathcal{C}^{IJ} \neq 0$, as well as an EDM for the fermion ψ^I when $\mathrm{Im}\,\mathcal{C}^{II} \neq 0$. While the couplings $\mathcal{C}^{IJ}$ naturally exhibit a hierarchical structure for $I \neq J$, inherited from those in the quark and lepton masses and mixings, the flavor-diagonal $\mathcal{C}^{II}$ are only constrained to be $\mathcal{O}(1)$ complex numbers by MFV, and can easily induce unacceptably large EDMs. However, before being drawn to the conclusion that MFV must be supplemented by some sort of mechanism protecting it from large flavor-diagonal CP-violating effects, it is worth analyzing the situation in a precise theoretical setting, to see whether Eq. (1) gives a fair account or misses important effects.

This is the purpose of the present paper. We will work within the MSSM, with neutrino masses generated through a seesaw mechanism of type I, and investigate whether the constraints from the leptonic EDMs rule out or allow for the most general CP-violating phases in the slepton sector, as introduced by MFV. For this program, in the next section, we will start by deriving the MFV expansions for the slepton mass terms and trilinear couplings and identify all the CP-violating phases.

Then, we will analyze in detail the rationale behind these additional phases, and classify them according to their impact on the EDMs. In the third section, we will present the results of our numerical analyses, showing that all types of MFV phases are allowed, though the planned improvements in the search for the electron EDM could change that picture. We will then compare the situation in the MSSM with the model-independent approach, and show how one can adapt Eq. (1) so as to reflect the fact that flavored phases are compatible with experimental bounds on the leptonic EDMs. In the fourth section, we will move to the general MSSM, and show how the operator basis constructed in the context of MFV offers a new perspective on the size of the slepton mass insertions. Finally, our results are summarized in the conclusion.

2 The MFV expansion in the slepton sector

The gauge sector of the MSSM exhibits the $U(3)^5$ flavor symmetry [1], one $U(3)$ factor for each of the quark and lepton superfields

$$G_F = U(3)^5 = G_q \times G_\ell \text{ with } G_q \equiv U(3)_Q \times U(3)_U \times U(3)_D \;,\; G_\ell \equiv U(3)_L \times U(3)_E \;. \tag{2}$$

This flavor symmetry is broken by the Yukawa couplings in the superpotential and by the squark and slepton soft-breaking terms. However, only the former have been measured precisely through the quark and lepton masses and mixings. The latter are only constrained to induce limited flavor-mixings when squarks and sleptons masses are below the TeV scale so as not to violate the many bounds coming from FCNC and other low-energy observables.

If the mechanism responsible for the breakings of G_F by the superpotential couplings is also behind those due to the soft-breaking terms, both are necessarily related. The MFV hypothesis is a model-independent description of such a relationship under the condition of minimality: if the flavor symmetry is broken just such as to generate the known quark and lepton masses and mixings, it is then possible to reconstruct the flavor-dependent soft-breaking couplings [2, 3].

Technically, the minimality statement translates as the existence of only a limited number of spurion fields breaking G_F. In the quark sector, the spurions are taken aligned with the Yukawa couplings

$$\mathbf{Y}_u \sim (\bar{3}, 3, 1)_{G_q} \;,\;\; \mathbf{Y}_d \sim (\bar{3}, 1, 3)_{G_q} \;. \tag{3}$$

This immediately makes the MSSM superpotential invariant under G_q. Then, the soft-breaking terms are also made formally invariant by writing them entirely in terms of the Yukawa spurions. For example, the left-squark mass term takes the form [2, 3]

$$\mathbf{m}_Q^2 = m_0^2(a_1 \mathbf{1} + a_2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + a_3 \mathbf{Y}_d^\dagger \mathbf{Y}_d + ...) \;. \tag{4}$$

To get the corresponding MFV expansions in the slepton sector is the purpose of the present section.

2.1 Seesaw spurions

To account for neutrino masses without introducing unnaturally small neutrino Yukawa couplings, the MSSM is supplemented with a seesaw mechanism of type I [4], i.e. three heavy right-handed neutrino superfields N are added to its particle content. Though this formally extends the flavor symmetry group to $G_F \times U(3)_N$, the N fields never occur at low-energy and only the spurion combinations which are singlets under $U(3)_N$ are needed [5]. Expanding in the inverse right-handed neutrino mass matrix $\mathbf{M}_R$, these are

$$\mathbf{Y}_e \sim (\bar{3},3)_{G_\ell}\,,\ \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \sim (8,1)_{G_\ell}\,,\ \mathbf{Y}_\nu^T(\mathbf{M}_R^{-1})\mathbf{Y}_\nu \sim (\bar{6},1)_{G_\ell}\,,\ \mathbf{Y}_\nu^\dagger(\mathbf{M}_R^{-1})^*(\mathbf{M}_R^{-1})\mathbf{Y}_\nu \sim (8,1)_{G_\ell}\,,\ ... \tag{5}$$

The $\mathbf{M}_R^{-1}$ term corresponds to the Majorana mass term for left-handed neutrinos. We work in the basis where the charged lepton Yukawa coupling is diagonal, hence

$$\mathbf{\Upsilon}_\nu \equiv v_u \mathbf{Y}_\nu^T(\mathbf{M}_R^{-1})\mathbf{Y}_\nu = \frac{1}{v_u}U^*\mathbf{m}_\nu U^\dagger\ ,\ \ \mathbf{Y}_e = \frac{\mathbf{m}_e}{v_d}\ , \tag{6}$$

with $\mathbf{m}_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$, $\mathbf{m}_\nu = \mathrm{diag}(m_{\nu1}, m_{\nu2}, m_{\nu3})$, and $v_{u,d} = \langle 0|H_{u,d}^0|0\rangle$ the two Higgs boson vacuum expectation values. The mixing matrix U is related to the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix [6] as $U = U_{\mathrm{PMNS}}.\,\mathrm{diag}(1, e^{i\alpha/2}, e^{i\beta/2})$, with U_{PMNS} involving one CP-violating Dirac phase γ and where α and β are the two Majorana phases.

Provided $\mathbf{M}_R$ is sufficiently large, the neutrino Yukawa couplings can be of $\mathcal{O}(1)$ and so it is the leading spurion $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ which will concern us in the following. As is well-known, it is impossible to fix $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ unambiguously from neutrino mixing parameters alone. The arbitrariness can be collected into an unknown complex orthogonal matrix $\mathbf{R}$ [7]

$$\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu = \frac{1}{v_u^2}U(\mathbf{m}_\nu^{1/2})\,\mathbf{R}^\dagger\,\mathbf{M}_R\,\mathbf{R}\,(\mathbf{m}_\nu^{1/2})U^\dagger\ . \tag{7}$$

As discussed in Ref. [8], the right-handed neutrinos are, to a good approximation, degenerate when MFV is enforced at the seesaw scale. Taking $\mathbf{M}_R = M_R\mathbf{1}$, the above spurion simplifies to [9]

$$\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu = \frac{M_R}{v_u^2}U(\mathbf{m}_\nu^{1/2})\,e^{2i\mathbf{\Phi}}\,(\mathbf{m}_\nu^{1/2})U^\dagger\ , \tag{8}$$

with the matrix $\mathbf{\Phi}^{ij} = \varepsilon^{ijk}\phi_k$ involving three (real) parameters $\phi_{1,2,3}$, on which little is known. They affect the size of the CP-conserving entries in $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ and induce CP-violating imaginary parts, since

$$e^{2i\mathbf{\Phi}} = e^{i\mathbf{\Phi}}.e^{i\mathbf{\Phi}} = \mathbf{1} - \frac{2\sinh^2 r}{r^2}\mathbf{\Phi}.\mathbf{\Phi} + i\frac{\sinh 2r}{r}\mathbf{\Phi},\ \ r \equiv \sqrt{\phi_1^2+\phi_2^2+\phi_3^2}\ . \tag{9}$$

Note that if $\phi_i = 0$, $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ becomes independent of the Majorana phases α and β. If the Dirac phase γ also vanishes, one simply has $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu = (M_R/v_u)\mathbf{\Upsilon}_\nu$.

2.2 Reparametrizations and algebraic reductions

The MSSM is made formally invariant under G_ℓ by writing the soft-breaking terms $\mathbf{m}_L^2$, $\mathbf{m}_E^2$, and $\mathbf{A}_e$ in terms of the two spurions $\mathbf{Y}_e$ and $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$. Let us first consider a generic operator $\mathbf{Q}$ transforming as an octet under $U(3)_L$. In full generality, it can be parametrized as an infinite series of products of powers of

$$\mathbf{A} \equiv \mathbf{Y}_e^\dagger\mathbf{Y}_e,\ \mathbf{B} \equiv \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\ , \tag{10}$$

also transforming as octets under $U(3)_L$, as

$$\mathbf{Q} = \sum_{i,j,k,...=0,1,2,...} z_{ijk...}\mathbf{A}^i\mathbf{B}^j\mathbf{A}^k...\ , \tag{11}$$

for some appropriate coefficients $z_{ijk\ldots}$, a priori all complex. This series can be partially resummed using the Cayley-Hamilton identity for a generic 3×3 matrix $\mathbf{X}$

$$\mathbf{X}^3-\langle\mathbf{X}\rangle\mathbf{X}^2+\tfrac{1}{2}\mathbf{X}(\langle\mathbf{X}\rangle^2-\langle\mathbf{X}^2\rangle)-\det\mathbf{X}=0\;, \tag{12}$$

where $\langle\mathbf{X}\rangle$ denotes the trace of $\mathbf{X}$, as well as identities derived from it (see e.g. Ref. [10]), leaving the operator $\mathbf{Q}$ with a finite number of terms:

$$\begin{aligned}\mathbf{Q}=&\;y_1\mathbf{1}+y_2\mathbf{A}+y_3\mathbf{B}+y_4\mathbf{A}^2+y_5\mathbf{B}^2+y_6\mathbf{AB}+y_7\mathbf{BA}+y_8\mathbf{ABA}+y_9\mathbf{BA}^2+y_{10}\mathbf{BAB}.\\&+y_{11}\mathbf{AB}^2+y_{12}\mathbf{ABA}^2+y_{13}\mathbf{A}^2\mathbf{B}^2+y_{14}\mathbf{B}^2\mathbf{A}^2+y_{15}\mathbf{B}^2\mathbf{AB}+y_{16}\mathbf{AB}^2\mathbf{A}^2+y_{17}\mathbf{B}^2\mathbf{A}^2\mathbf{B}\;.\end{aligned} \tag{13}$$

The only non-trivial reduction is that for the term $\mathbf{B}^2\mathbf{ABA}^2$, related to the leptonic Jarlskog invariant [11], and can be done starting with $\mathbf{X}=[\mathbf{A},\mathbf{B}]$ in Eq. (12).

Still using only the Cayley-Hamilton identities, and thanks to the hermiticity of $\mathbf{Y}_e^\dagger\mathbf{Y}_e$ and $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$, we can manipulate the expansion (13) to write $\mathbf{Q}$ entirely in terms of hermitian operators as

$$\begin{aligned}\mathbf{Q}=&\;x_1\mathbf{1}+x_2\mathbf{A}+x_3\mathbf{B}+x_4\mathbf{A}^2+x_5\mathbf{B}^2+x_6i[\mathbf{A},\mathbf{B}]+x_7\{\mathbf{A},\mathbf{B}\}+c_8\mathbf{ABA}\\&+x_9i[\mathbf{B},\mathbf{A}^2]+x_{10}\mathbf{BAB}+x_{11}i[\mathbf{A},\mathbf{B}^2]+x_{12}\mathbf{BA}^2\mathbf{B}+x_{13}i[\mathbf{A}^2,\mathbf{B}^2]\\&+x_{14}i(\mathbf{ABA}^2-\mathbf{A}^2\mathbf{BA})+x_{15}i(\mathbf{B}^2\mathbf{AB}-\mathbf{BAB}^2)\\&+x_{16}i(\mathbf{AB}^2\mathbf{A}^2-\mathbf{A}^2\mathbf{B}^2\mathbf{A})+x_{17}i(\mathbf{B}^2\mathbf{A}^2\mathbf{B}-\mathbf{BA}^2\mathbf{B}^2)\;.\end{aligned} \tag{14}$$

For the last four operators, the corresponding "+" hermitian combinations are fully reducible. For example, the combination $\mathbf{ABA}^2+\mathbf{A}^2\mathbf{BA}$ is entirely absorbed into lower-order terms using

$$\begin{aligned}\mathbf{A}^2\mathbf{BA}+\mathbf{ABA}^2=&\;\mathbf{ABA}\langle\mathbf{A}\rangle+\mathbf{A}^2\langle\mathbf{BA}\rangle-\tfrac{1}{6}\mathbf{B}(\langle\mathbf{A}\rangle^3+2\langle\mathbf{A}^3\rangle-3\langle\mathbf{A}\rangle\langle\mathbf{A}^2\rangle)+\mathbf{A}\left\langle\mathbf{BA}^2\right\rangle\\&-\mathbf{A}\langle\mathbf{BA}\rangle\langle\mathbf{A}\rangle+\tfrac{1}{2}\langle\mathbf{BA}\rangle(\langle\mathbf{A}\rangle^2-\left\langle\mathbf{A}^2\right\rangle)+\left\langle\mathbf{A}^2\mathbf{BA}\right\rangle-\langle\mathbf{A}\rangle\left\langle\mathbf{BA}^2\right\rangle\;,\end{aligned} \tag{15}$$

which is derived from Eq. (12).

The most general expansions for $\mathbf{m}_L^2$, $\mathbf{m}_E^2$ and $\mathbf{A}_e$ are directly obtained from the expansion of $\mathbf{Q}$ as

$$\mathbf{m}_L^2=m_0^2\mathbf{Q},\;\;\mathbf{m}_E^2=m_0^2(\mathbf{1}+\mathbf{Y}_e\mathbf{Q}\mathbf{Y}_e^\dagger),\;\;\mathbf{A}_e=A_0\mathbf{Y}_e\mathbf{Q}\;, \tag{16}$$

where it is understood that the MFV coefficients for $\mathbf{m}_L^2$ and $\mathbf{m}_E^2$ are real, since these mass terms are hermitian, while those for $\mathbf{A}_e$ can be complex. The mass parameters m_0 and A_0 set the supersymmetry breaking scale. These expansions are the most general parametrizations of the slepton soft-breaking terms in presence of a type I seesaw mechanism. In particular, if the neutrino sector communicates to the slepton sector through RGE effects [12], they correspond to the most general form one could attain starting with universal slepton soft-breaking terms, up to arbitrary high orders. Finally, if one allows for $U(1)_L$ and $U(1)_E$-breaking terms, operators involving contractions of $\mathbf{Q}$ with the $SU(3)_{L,E}$ Levi-Civita tensors can be constructed for $\mathbf{A}_e$. These are a priori smaller and will not be included here [13].

It should be also remarked that all these developments can be immediately applied to the quark sector by substituting $\mathbf{A}\equiv\mathbf{Y}_e^\dagger\mathbf{Y}_e\to\mathbf{Y}_d^\dagger\mathbf{Y}_d$ and $\mathbf{B}\equiv\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\to\mathbf{Y}_u^\dagger\mathbf{Y}_u$ in Eqs. (11), (13) or (14). The corresponding expansions for $\mathbf{m}_Q^2$, $\mathbf{m}_U^2$, $\mathbf{m}_D^2$, $\mathbf{A}_u$ and $\mathbf{A}_d$ have been derived in Ref. [10].

2.3 MFV expansions and numerical reductions

The expansions (16) are fully general and do not correspond to MFV yet. Indeed, any matrix can be expanded in the basis (14), which is thus more a reparametrization than an expansion. However, projecting an arbitrary matrix, the coefficients x_i in general span several orders of magnitudes because the spurion operators in $\mathbf{Q}$ are nearly aligned [10, 14].

In a more realistic framework, one would expect the coefficients $z_{ijk...}$ of Eq. (11) to be at most $\mathcal{O}(1)$ complex numbers. Then, the coefficients y_i of Eq. (13) and x_i of Eq. (14) are also of $\mathcal{O}(1)$, since the Cayley-Hamilton identities never generate large numerical coefficients (all the traces of $\mathbf{A}$, $\mathbf{B}$ and combinations are $\mathcal{O}(1)$ or smaller). This constraint of naturalness on the size of the expansion coefficients is the essence of the MFV hypothesis. The MFV expansions for the slepton soft-breaking terms differ from a mere reparametrization only in the initial constraints imposed on the coefficients.

Still, once the expansion coefficients are assumed to be at most of $\mathcal{O}(1)$, an additional reduction of the number of terms is possible. It stems from the large mass hierarchy between the charged leptons and, contrary to the Cayley-Hamilton reduction, is only approximate. Following Ref. [10], this hierarchy is accounted for through the identification $(\mathbf{Y}_e^\dagger \mathbf{Y}_e)^2 \approx y_\tau^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e$. Of course, no similar identity exists for the neutrino spurion. Getting rid of all the terms involving $\mathbf{A}^2$, the MFV series reduces to

$$\begin{aligned}\mathbf{Q} = c_1\mathbf{1} + c_2\mathbf{A} + c_3\mathbf{B} + c_4\mathbf{B}^2 + c_5\{\mathbf{A},\mathbf{B}\} + c_6\mathbf{BAB}\\ + c_7 i[\mathbf{A},\mathbf{B}] + c_8 i[\mathbf{A},\mathbf{B}^2] + c_9 i(\mathbf{B}^2\mathbf{AB} - \mathbf{BAB}^2)\ ,\end{aligned} \tag{17}$$

where the nine coefficients c_i are $\mathcal{O}(1)$ complex numbers, or if $\mathbf{Q}$ is hermitian, nine $\mathcal{O}(1)$ real numbers. Plugging these expressions into Eq. (16), the final MFV expansions for the slepton soft-breaking terms are (remember $\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e$ and $\mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$):

$$\begin{aligned}\mathbf{m}_L^2 = m_0^2(a_1\mathbf{1} + a_2\mathbf{A} + a_3\mathbf{B} + a_4\mathbf{B}^2 + a_5\{\mathbf{A},\mathbf{B}\} + a_6\mathbf{BAB}\\ + ib_1[\mathbf{A},\mathbf{B}] + ib_2[\mathbf{A},\mathbf{B}^2] + ib_3(\mathbf{B}^2\mathbf{AB} - \mathbf{BAB}^2))\ ,\end{aligned} \tag{18a}$$

$$\begin{aligned}\mathbf{m}_E^2 = m_0^2(a_7\mathbf{1} + \mathbf{Y}_e(a_8\mathbf{1} + a_9\mathbf{B} + a_{10}\mathbf{B}^2 + a_{11}\{\mathbf{A},\mathbf{B}\} + a_{12}\mathbf{BAB}\\ + ib_4[\mathbf{A},\mathbf{B}] + ib_5[\mathbf{A},\mathbf{B}^2] + ib_6(\mathbf{B}^2\mathbf{AB} - \mathbf{BAB}^2))\mathbf{Y}_e^\dagger)\ ,\end{aligned} \tag{18b}$$

$$\begin{aligned}\mathbf{A}_e = A_0\mathbf{Y}_e(c_1\mathbf{1} + c_2\mathbf{A} + c_3\mathbf{B} + c_4\mathbf{B}^2 + c_5\{\mathbf{A},\mathbf{B}\} + c_6\mathbf{BAB}\\ + d_1 i[\mathbf{A},\mathbf{B}] + d_2 i[\mathbf{A},\mathbf{B}^2] + d_3 i(\mathbf{B}^2\mathbf{AB} - \mathbf{BAB}^2))\ .\end{aligned} \tag{18c}$$

Altogether, the 18 real a_i and b_i, and the nine complex c_i and d_i MFV coefficients sum up to 36 free real parameters, in addition to the dimensionful SUSY-breaking scale parameters m_0 and A_0. These expansions span the whole space of the complex (hermitian) matrices for complex (real) coefficients, but let us stress once more that expanding an arbitrary matrix in those bases generates huge coefficients, in contradiction with the MFV hypothesis [10, 14].

2.4 CP-violation under the MFV hypothesis

There are two possible sources of CP-violation in the expansions (18). First, the spurion $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ involves several CP-violating parameters: the Dirac phase γ, the two Majorana phases α and β, and the three parameters ϕ_i. The second source are the MFV coefficients themselves. The b_i are all purely CP-violating, while the c_i and d_i can have a CP-violating component. The goal of the present section is to characterize these flavored phases –i.e., coming from the slepton masses and trilinear terms of Eq. (18)–, by organizing them into three classes: flavor-blind, flavor diagonal and flavor off-diagonal. Before this, we first discuss in detail why CP-violating phases have to be allowed for the MFV coefficients.

Necessity for CP-violating coefficients. The MFV coefficients are not required by the $U(3)^5$ symmetry to be CP-conserving, but it is actually a matter of consistency to allow them to violate CP. Looking back at the reduction from the general expansion (11) down to (13) or (14), traces of combinations of the spurions are absorbed into the coefficients through the use of Cayley-Hamilton identities. These traces can be complex when the spurion $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ involves CP-violating phases, so the coefficients have to be allowed to be complex. In fact, independently of Cayley-Hamilton identities, MFV coefficients are always understood to include (potentially complex) traces of products of the spurions, up to any order, so even the $z_{ijk...}$ of Eq. (11) should not be taken real. The same requirement arises if the MFV expansion is set at a different scale: the necessity for complex coefficients appears when the radiative corrections to the soft-breaking terms are projected back on the standard expansions using the Cayley-Hamilton identities [10]. In this case, one also sees that it would be inconsistent to partially account for CP-violating phases, for example by allowing complex MFV coefficients for $\mathbf{A}_e$ but not $\mathbf{m}_{L,E}^2$, because they are linked through the RGE [15].

Phenomenologically, it would nevertheless be useful to have at hand a well-defined CP-limit for the MFV coefficients, but this is actually ill-defined: it depends on the operator basis chosen. Naively, one may think that taking all the MFV coefficients real corresponds to the CP-limit, but this is obviously not correct since, looking at Eq. (13) and (14),

$$\mathbf{Q}_{\mathrm{Im}\,y_i\to 0} \not\Leftrightarrow \mathbf{Q}_{\mathrm{Im}\,x_i\to 0}\ . \tag{19}$$

So long as the spurion is complex, there is no reason to prefer one limit over the other. On the other hand, if the CP-limit is enforced also for the spurion $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$, the CP-limit for $\mathbf{Q}$ is obtained setting $\mathrm{Im}\,y_i\to 0$ or $\mathrm{Im}\,x_{1-8}\to 0$, $\mathrm{Re}\,x_{9-17}\to 0$, since then $\mathbf{Q}$ becomes real. These two limits are further equivalent when $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ is complex, because Eq. (13) and (14) essentially differ by some i factors and some real rearrangements, but this needs not be the case. For example, the CP-limit is obtained from Eq. (11) and (13) by setting $\mathrm{Im}\,z_i\to 0$ and $\mathrm{Im}\,y_i\to 0$ when $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ is real. However, when $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ is complex, these same limits are no longer equivalent,

$$\mathbf{Q}_{\mathrm{Im}\,z_i\to 0} \not\Leftrightarrow \mathbf{Q}_{\mathrm{Im}\,y_i\to 0}\ , \tag{20}$$

because in going from (11) to (13), some y_i have absorbed complex traces. Therefore, we arrive at the conclusion that the CP-limit for the MFV coefficient is basis-dependent, and thus again that as a matter of principle, it makes no sense to impose a CP-limit on the MFV coefficients while allowing the spurions to be CP-violating.

In the present case, the spurions $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ and $\mathbf{Y}_e^\dagger\mathbf{Y}_e$ are both hermitian, hence all the complex traces can be reduced to the Jarlskog invariant using the Cayley-Hamilton identities

$$J \equiv \mathrm{Im}\langle(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu(\mathbf{Y}_e^\dagger\mathbf{Y}_e)^2\rangle = \frac{i}{2}\det[\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu,\mathbf{Y}_e^\dagger\mathbf{Y}_e] \neq 0\ . \tag{21}$$

This invariant is very small, and thus the ambiguities in the imaginary parts of the MFV coefficients discussed before are also very small. The potentially large CP-violating effects from the ϕ_i's cannot enhance J much because these parameters affect both the real and imaginary parts of the entries of

$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, see Eq. (9), and the perturbativity bound $|\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu| \lesssim 1$ fails much before J could reach even the percent level. However, the crucial point is that even if J is very small, its non-zero value implies that the CP-symmetry cannot be defined separately for the MFV coefficients and for the spurions. It can therefore not be used to enforce naturally some reality conditions on the MFV coefficients. Without any acting symmetry, enforcing such a condition brings instead a fine-tuning problem. This goes against the very principle of MFV which is to restore naturality in the flavor sector.

Of course, this fine-tuning of the CP-violating phases could have a dynamical explanation, in which case we proved above that it is numerically stable under a change of MFV operator basis and under the RG evolution. For instance, in the quark sector, it was shown in Ref. [10] that imposing MFV at the GUT scale and running it down, the MFV coefficients exhibit a quasi fixed-point behavior, with their imaginary parts running towards negligible values. However, this peculiar behavior can be traced back to the fast evolution of the flavor-blind parameters –essentially evolving like the gluino mass–, and MFV coefficients in the slepton sector are not expected to exhibit such a behavior. Therefore, lacking a dynamical mechanism able to enforce a fine-tuning of the slepton MFV coefficients, we will allow them to be $\mathcal{O}(1)$ complex numbers. Ultimately, it is the comparison with experiment which will tell us if this setting is viable or not.

Finally, it should be mentioned that neutrino spurions are not always hermitian. When the right-handed neutrinos are not degenerate, their successive decouplings at different scales give rise to non-hermitian spurions [16]. In such a case, fine-tuning the MFV coefficients to CP-conserving values is not longer numerically stable, because there can be complex traces much larger than the Jarlskog invariant. Any MFV implementation must then necessarily involve complex coefficients. These effects will be left for subsequent studies, and we will concentrate here on the case of hermitian spurions only.

Flavor-blind phases and phase-convention dependences. For the slepton mass terms $\mathbf{m}_L^2$ and $\mathbf{m}_E^2$, MFV only introduces relative phases between coefficients because of the hermiticity constraint; specifically, all the a_i and b_i are taken real. For the trilinear term $\mathbf{A}_e$, all the c_i and d_i coefficients can be complex. Their phases thus have to be defined relative to those in the gauge sector of the MSSM. Since the latter are flavor-blind (i.e., not related to $U(3)^5$ breakings), it appears that MFV introduces a relative flavor-blind phase in $\mathbf{A}_e$. In other words, parts of the $\mathbf{A}_e$ phases can be moved into the flavor-blind parameters of the MSSM (gaugino masses, μ term,...) by changing the phase conventions (see e.g. Ref. [17]).[1]

At this stage, there are two possible points of view. One can simply accept that all the phases in $\mathbf{A}_e$ are free and well-defined once those in the rest of the MSSM are fixed. The c_i and d_i are then allowed to take any $\mathcal{O}(1)$ complex values, provided they are compatible with EDM constraints. The second point of view is to consider only relative phases between MFV coefficients, and to assume that the flavor-blind relative phase between the MFV expansion and the rest of the MSSM is fixed by some other, unknown mechanism. Indeed, it is well-known that the flavor-blind phases of the gaugino masses or the μ term can easily lead to much too large EDMs if sparticles are not very heavy (the so-called MSSM CP problem [18]). MFV does not constrain these phases, it is up to some other mechanism to restrict their sizes.[2] This unknown mechanism could then also constrain the overall flavor-blind phase of $\mathbf{A}_e$. In the present work, we will not take this second point of view, because

[1] Strictly speaking, the flavor-blind phase is nevertheless "flavored" since $\mathbf{A}_e$ has no purely flavor-blind component. Indeed, without the spurion $\mathbf{Y}_e$, $\mathbf{A}_e$ is forbidden in MFV.

[2] Alternatively, cancellations between the MSSM contributions to the EDM could be at play, see e.g. Refs. [19].

we want to quantify the size of the effects induced by natural phases in $\mathbf{A}_e$ (in the MFV sense), but this possibility should be kept in mind. Further, the phase of the other relevant parameters for the EDMs, μ, M_1 and M_2, will be set to zero since they are beyond the reach of MFV.

Three types of CP-violating phases for leptonic EDMs. Given the above provisions, we distinguish three types of CP-violating phases according to their effects on leptonic EDMs.

The first type is the *flavor-blind* one. Looking back at the general expansion (11), it seems natural to identify it as the phase of z_0, leaving to the physics behind the MFV expansion the task to generate the relative phases of all the other coefficients $z_{ijk...}$, whose operators explicitly break the flavor symmetry. This identification does not immediately permit to pinpoint the flavor-blind phase in the expansion (18), because of the systematic use of Cayley-Hamilton identities; the initial constraint on z_0 is only passed to c_1 when the spurions are sufficiently suppressed. This is however the case in a large portion of the parameter space, as we will see in the next section. We therefore call $\arg c_1$ (or, with a small abuse of language, $\operatorname{Im} c_1$) the flavor-blind phase. When present, the contribution of this phase to the EDMs dominates.

All the other CP-violating phases can be split into two classes according to the order in the Mass-Insertion Approximation (MIA) at which they start to contribute to the leptonic EDMs. This is the place where having written the MFV series entirely in terms of hermitian operators becomes important. Indeed, the diagonal entries of these operators are automatically real and their contributions to the EDMs are relegated to the second order in the MIA. In other words, the EDMs are effectively shielded from direct effects from the CP-violating phases occurring in $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$.[3] This situation is similar to the flavor off-diagonal CP-violation scenarios discussed e.g. in Ref. [17], even though $\mathbf{A}_e$ itself is not hermitian here.

The parameters $\operatorname{Im} c_i$ and $\operatorname{Im} d_3$ induce leading-order MIA effects, and are thus called *flavor-diagonal phases*. On the other hand, b_i, $d_{1,2}$ and all the $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ spurion phases contribute only at the second order in the MIA, and are called *flavor off-diagonal phases*. Note that $d_{1,2}$ do not contribute to the leading order in the MIA because the diagonal entries of $[\mathbf{A}, \mathbf{B}]$ and $[\mathbf{A}, \mathbf{B}^2]$ always vanish (this can be traced back to the fact that those of $\mathbf{AB}$ and $\mathbf{AB}^2$ are purely real). On the other hand, d_3 does have non-vanishing diagonal entries but, being of third order in $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, they are too suppressed to play any role.

3 MFV phases and leptonic observables

All the CP-violating phases occurring in the MFV expansions induce corrections to the leptonic EDMs. The goal of the present section is to estimate the order of magnitude of these effects for each type of phases, i.e. flavor-blind, flavor diagonal and flavor off-diagonal, and to see whether they are compatible with the experimental bounds. Also, we want to characterize the contribution of each type of phases by finding which of the MFV operators are relevant, and which are always negligible.

Instead of a full scan over the MSSM parameter space, our strategy will be to first identify a reasonable range of parameters, i.e. such that LFV transitions (essentially $\mu \to e\gamma$) satisfy their experimental bounds, and then to compute the EDMs over this range. In this way, the loose correlation between these two types of leptonic observables is fully exploited, and the effect of their different dependences on MFV coefficients and neutrino parameters can be probed most thoroughly.

[3] This holds also for $\mathbf{A}_e$, which is the product of hermitian operators with $\mathbf{Y}_e$, real and diagonal in the basis (6).

LFV transitions:		Electric dipole moments:	
$B(\mu \to e\gamma) < 1.2 \times 10^{-11}$	[28]	$\lvert d_e \rvert < 1.6\ 10^{-27}\ e\,\mathrm{cm}$	[31]
$B(\tau \to e\gamma) < 1.1 \times 10^{-7}$	[29]	$\lvert d_\mu \rvert < 1.8\ 10^{-19}\ e\,\mathrm{cm}$	[32]
$B(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$	[30]	$d_\tau \in [-2.2, 4.5] \times 10^{-17}\ e\,\mathrm{cm}$	[33]

Table 1: Current experimental bounds on LFV transitions $\ell^I \to \ell^J\gamma$ and leptonic EDMs. The electron, muon and tau EDMs are derived from the bound on the thallium EDM, from the $(g-2)_\mu$ experiment, and from the $e^+e^- \to \tau^+\tau^-$ data, respectively.

First, in the next subsection, we give the relevant expressions for the one-loop contributions to the EDM and LFV transitions in the MSSM, together with the current experimental bounds on these observables. Also, the various experimental inputs, as well as the ranges of variation we allow for the unknown seesaw and MSSM parameters, are described.

3.1 Preliminaries: Electromagnetic operator and parameter input values

The supersymmetric contributions can be collected into a single dimension-five effective operator

$$H_{eff} = e\mathcal{M}^{IJ}\bar{\psi}_L^I\sigma_{\mu\nu}\psi_R^J F^{\mu\nu} + h.c. \tag{22}$$

The flavor diagonal parts $\mathcal{M}^{II}$ are directly related to the EDMs and Magnetic Dipole Moments (MDMs), $d_I/e = 2\,\mathrm{Im}\,\mathcal{M}^{II}$ and $a_I = 4m_{e^I}\,\mathrm{Re}\,\mathcal{M}^{II}$, respectively, while the $\ell^I \to \ell^J\gamma$ transitions arise from the off-diagonal parts $\mathcal{M}^{IJ}$.

For our purposes, it is sufficient to consider only the neutralino and chargino one-loop contributions. We thus neglect Barr-Zee type contributions [20] as well as contributions arising from the $\tan\beta$-enhanced non-holomorphic corrections to the Yukawa couplings [21]. We take the results of Ref. [22], with LFV transitions to first order in the MIA,

$$\begin{aligned} B\left(\ell^I \to \ell^J\gamma\right) = &\frac{3\alpha}{2\pi}\tan^4\theta_W \times B\left(\ell^I \to \ell^J\bar{\nu}^J\nu^I\right) \times \frac{M_W^4\,M_1^2\tan^2\beta}{|\mu|^2} \\ &\times \left(\left|\delta_{LL}^{JI}\mathcal{F}_1 + \delta_{LR}^{JI}\frac{m_R m_L}{\mu m_e^I\tan\beta}\mathcal{F}_2\right|^2 + \left|\delta_{RR}^{JI}\mathcal{F}_3 + \delta_{RL}^{JI}\frac{m_R m_L}{\mu^* m_e^I\tan\beta}\mathcal{F}_2\right|^2\right)\,, \end{aligned} \tag{23}$$

and the lepton EDMs and MDMs up to second order,

$$\begin{aligned} \frac{a_I}{m_e^I} + 2i\frac{d_I}{e} = \frac{\alpha M_1}{4\pi|\mu|^2\cos^2\theta_W}\Big[&m_e^I\left(\mu\tan\beta\mathcal{F}_4 - A_e^{I*}\mathcal{F}_5\right) + m_R m_L\left(\delta_{LR}^{IK}\delta_{RR}^{KI}\mathcal{F}_7 + \delta_{LL}^{IK}\delta_{LR}^{KI}\mathcal{F}_8\right) \\ &+m_{e^K}\left(\delta_{LL}^{IK}\delta_{RR}^{KI}\left(\mu\tan\beta - A_e^{K*}\right) + \delta_{LR}^{IK}\delta_{LR}^{KI}\left(\mu^*\tan\beta - A_e^K\right)\right)\mathcal{F}_6 + ...\Big]\,. \end{aligned} \tag{24}$$

In these expressions, m_e^I is the ℓ^I mass, m_L and m_R are the average left and right slepton masses used in defining the mass-insertions δ_{LL}, δ_{RR} and δ_{LR}, while A_e^I are defined as the diagonal entries of $(\mathbf{Y}_e)^{-1}\mathbf{A}_e$. The loop functions $\mathcal{F}_i$, all of mass-dimension M^{-2}, can be found in Ref. [22]. In the actual computation, we use also the full one-loop results of Ref. [23]. We checked, as stated in Ref. [22], that the MIA is excellent over most of the parameter space and for our purpose gives a sufficiently good approximation even for relatively light sparticles.

The experimental situation is shown in Table 1. Several of these measurements are expected to be improved in the near future, with sensitivities reaching 10^{-29} $e\,$cm for d_e, 10^{-24} $e\,$cm for d_μ, $10^{-13} - 10^{-14}$ for $B(\mu \to e\gamma)$ at the MEG experiment [24], and 2×10^{-9} for $B(\tau \to (e,\mu)\gamma)$ at B factories (for reviews, see e.g. Refs. [25, 26]).

We now describe the input values for the various parameters needed to estimate the EDMs and LFV transitions. All the parameters entering $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ and $\mathbf{Y}_e$ are fixed at the electroweak scale. If one were to impose MFV at the high-energy scale M_R or above, RGE effects would play an important role [12]. However, the MFV expansions are RGE invariant, so all the RGE effects can be described through running MFV coefficients. In the squark sector, it was shown in Ref. [10, 27] that the coefficients at the low scale are at most $\mathcal{O}(1)$ if they were of $\mathcal{O}(1)$ at the GUT scale, and the same should hold also for the slepton sector (though the converse is not necessarily true). Therefore, in the present work, to avoid prescribing anything about the dynamics at the high-scale, we impose MFV at the electroweak scale and allow for a rather generous range of variation for the MFV coefficients (see Eq. (29) below).

First, neutrino mixing parameters are taken from the best-fit of Ref. [34]

$$\begin{gathered}\Delta m_{21}^2 = \Delta m_\odot^2 = 7.65^{+0.23}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2,\ |\Delta m_{31}^2| = \Delta m_{atm}^2 = 2.4^{+0.12}_{-0.11} \times 10^{-3}\,\mathrm{eV}^2\ , \\ \sin^2\theta_\odot = 0.304^{+0.022}_{-0.016},\ \sin^2\theta_{atm} = 0.50^{+0.07}_{-0.06},\ \sin^2\theta_{13} \leq 0.056\ . \end{gathered} \tag{25}$$

The neutrino mass-scale $m_\nu \equiv m_{\nu 1}$ is unknown, but should not exceed about 1 eV if the cosmological bound $\sum_i m_i \lesssim 1$ eV holds [35]. For M_R we enforce the perturbativity condition $|\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu| \lesssim 1$, which translates as $M_R \lesssim 10^{13}$ GeV for $m_\nu \simeq 1\,$eV, and goes up to a few 10^{14} GeV when $m_\nu \simeq 0$. The ϕ_i parameters affect both the phase and norm of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, and we allow them to vary between $\pm 1/2$ so that they never upset the perturbativity bound (larger values are in any case disfavored by bounds on LFV transitions, see next section). Finally, the other CP-violating parameters are varied throughout their allowed ranges. In summary, we take

$$\begin{array}{llll} \gamma, \alpha, \beta & \in [-\pi,\, +\pi]\ , & m_\nu\ (\mathrm{eV}) & \in [0,\, 0.1]\ , \\ \sin^2\theta_{13} & \in [0,\, 0.056]\ , & M_R\ (\mathrm{GeV}) & \in [10^9,\, 10^{14}]\ , \\ \phi_1, \phi_2, \phi_3 & \in [-1/2,\, +1/2]\ , & & \end{array} \tag{26}$$

while the solar and atmospheric mass-differences and angles are fixed to their central values in Eq. (25).

For the MSSM parameters, we vary $\tan\beta \equiv v_u/v_d$ between 10 and 50, but fix the gaugino masses and the μ term to $\mu = \pm 400$ GeV, $M_1 = 200$ GeV and $M_2 = 400$ GeV at the electroweak scale. Since our goal is to analyze the consequences of the phases in the slepton sector, we take them real at that scale (so that RGE effects do not regenerate their phases from the complex trilinear couplings [36]). With these parameters, the tree-level chargino and neutralino masses are

$$m_{\chi_1^\pm} \approx 354\ \mathrm{GeV},\ \ m_{\chi_2^\pm} \approx 456\ \mathrm{GeV}, \tag{27a}$$

$$m_{\chi_1^0} \approx 198\ \mathrm{GeV},\ \ m_{\chi_2^0} \approx 354\ \mathrm{GeV},\ \ m_{\chi_3^0} \approx 407\ \mathrm{GeV},\ \ m_{\chi_4^0} \approx 455\ \mathrm{GeV}\ . \tag{27b}$$

The slepton masses follow from the soft-breaking terms Eq. (18). We fix

$$m_0 = 600\ \mathrm{GeV},\ \ A_0 = 400\ \mathrm{GeV}\ , \tag{28}$$

and scan over

$$a_i, b_i, \mathrm{Re}\, c_i, \mathrm{Re}\, d_i, \mathrm{Im}\, c_i, \mathrm{Im}\, d_i \in \pm\ [0.1, 8]\ , \tag{29}$$

discarding points which lead to slepton or sneutrino masses below 100 GeV.

Constraints on the MFV coefficients:	Constraints on the $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ spurion:		
	$m_\nu = 0$	$m_\nu = 0.1\,eV$	$m_\nu = 0.1\,eV$
	$\phi_i = 0$	$\phi_i = 0$	$\phi_i \neq 0$
All possible complex coefficients	Red	Dark red	Light red
No flavor-blind phase ($\mathrm{Im}\, c_1 = 0$)	Blue	Dark blue	Light blue
No flavor-diagonal phases ($\mathrm{Im}\, c_i = 0$)	Green	Dark green	Light green

Table 2: Scenarios for the CP-violating phases coming from the spurion $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ and from the MFV coefficients. It is understood that ϕ_i denotes collectively all the nine CP-violating phase entering $\mathbf{Y}_\nu$, i.e. the Dirac phase γ, the two Majorana phases α and β, and the three ϕ_i parameters are varied within the ranges (26). The colors correspond to those of the 90% contours in Fig.1. The situation with $m_\nu = 0$, $\phi_i \neq 0$ is essentially identical to $m_\nu = 0$, $\phi_i = 0$. Finally, it is understood that the normal spectrum is supposed for neutrino masses.

3.2 Leptonic EDMs versus LFV transitions

The numerical discussion is organized into several scenarios. The two spurions $\mathbf{Y}_e^\dagger\mathbf{Y}_e$ and $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ vary according to different parameters: the former is tuned entirely by $\tan\beta$, while the latter is linear in M_R, see Eq. (8). Hence, to span a large range of possibilities, we consider separately

$$M_R = 10^9, 10^{11}, 10^{13}\,\mathrm{GeV}, \quad \tan\beta = 10, 50\ . \tag{30}$$

Given the MSSM mass spectrum specified in Eqs. (27–29), larger values for M_R are disfavored by the current bounds on LFV transitions, as will be detailed below. For such values, the neutrino Yukawa couplings $\mathbf{Y}_\nu$ are relatively small, powers of $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ become very suppressed, and the MFV expansions simplify to (remember $\mathbf{A} \equiv \mathbf{Y}_e^\dagger\mathbf{Y}_e$ and $\mathbf{B} \equiv \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$)

$$\mathbf{m}_L^2 = m_0^2(a_1\mathbf{1} + a_2\mathbf{A} + a_3\mathbf{B} + a_5\{\mathbf{A},\mathbf{B}\} + ib_1[\mathbf{A},\mathbf{B}])\ , \tag{31a}$$

$$\mathbf{m}_E^2 = m_0^2(a_7\mathbf{1} + \mathbf{Y}_e(a_8\mathbf{1} + a_9\mathbf{B} + a_{11}\{\mathbf{A},\mathbf{B}\} + ib_4[\mathbf{A},\mathbf{B}])\mathbf{Y}_e^\dagger)\ , \tag{31b}$$

$$\mathbf{A}_e = A_0\mathbf{Y}_e(c_1\mathbf{1} + c_2\mathbf{A} + c_3\mathbf{B} + c_5\{\mathbf{A},\mathbf{B}\} + d_1 i[\mathbf{A},\mathbf{B}])\ . \tag{31c}$$

Further, numerically, the operator a_5 and a_{11} are at most of 30% relative to the leading contributions and could be neglected in a first approximation. Note that since $\mathbf{Y}_e^\dagger\mathbf{Y}_e$ is diagonal, $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ should not be discarded everywhere since it is the only source of flavor transitions, no matter how small is M_R. Similarly, at low $\tan\beta$, the suppressed $\mathbf{Y}_e^\dagger\mathbf{Y}_e$ is needed to account for CP-violation in the $\mathbf{m}_L^2$ and $\mathbf{m}_E^2$ sectors (see the b_1 and b_4 operators).

To analyze the impact of the different types of CP-violating phases on the EDMs, we distinguish nine scenarios, each specified by a set of conditions imposed on the MFV coefficients on one hand, and on the parameters entering the spurion $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ on the other, as summarized in Table 2. For each of these nine scenarios, the results of scanning over the parameters according to flat distributions (and subject to the corresponding constraints) are shown in Fig. 1 as 90% contours in the $B\,(\mu \to e\gamma) - d_e$ plane (normalized to their respective experimental bound). We do not give any statistical interpretation to these contours: one can understand the 90% limit as a simple procedure to remove peculiar, fine-tuned situations in parameter space. Finally, we also discard points for which a_μ is not acceptable. Given the current discrepancy between theory and experiment of about $(30 \pm 10) \times 10^{-10}$ (see Ref. [26] and references therein), we keep points for which

$$0 \leq a_\mu^{\mathrm{SUSY}} \leq 40 \times 10^{-10}\ . \tag{32}$$

Apart from fixing the sign of μ, given our mass spectrum, only less than 5% of the points have to be thrown away, and this only for large $\tan\beta$.

3.2.1 LFV transitions

As visible in Fig. 1, the LFV transitions are insensitive to the constraints imposed on the CP-violating part of the MFV coefficients, in stark contrast to the EDMs, as will be discussed below. This can be understood from the fact that the amplitudes for the LFV processes are always dominated by the δ_{LL}^{JI} term in Eq. (23). Given the scaling of the operators in Eq. (31), we can write Eq. (23) as:

$$B\left(\ell^I \to \ell^J \gamma\right) \sim \frac{M_W^4\, M_1^2 \tan^2\beta}{|\mu|^2} \left|\delta_{LL}^{JI}\mathcal{F}_1\right|^2 \ , \ \ \delta_{LL} \approx \frac{a_3}{a_1}\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \frac{a_5}{a_1}\{\mathbf{Y}_e^\dagger \mathbf{Y}_e, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\} \ , \tag{33}$$

with, for the $\mu \to e\gamma$ transition, δ_{LL}^{12} fully dominated by the a_3 operator (see Fig. 2A)

$$\delta_{LL}^{12} \approx \frac{a_3}{a_1}(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^{12} = \frac{a_3}{a_1}\frac{M_R}{v_u^2}(U(\mathbf{m}_\nu^{1/2})\, e^{2i\mathbf{\Phi}}\, (\mathbf{m}_\nu^{1/2})U^\dagger)^{12} \ . \tag{34}$$

In these expressions, the operator b_1 is absent: it is competitive only for the imaginary parts of the off-diagonal entries of δ_{LL}^{JI}, not for their norms. The loop function $\mathcal{F}_1$ has a strong non-polynomial dependence on the masses, hence also on a_1 since $m_L^2 \approx m_0^2 a_1$ when $M_R \lesssim 10^{13}$ GeV and $\tan\beta$ is not too large. Nevertheless, this dependence is monotonic, and Eq. (34) is sufficient to understand the behavior of $B\left(\mu \to e\gamma\right)$ as the MFV and neutrino parameters are varied.

Sparticle mass dependences: The LFV decay rates are quadratic in M_R, and increasing this parameter beyond 10^{13} GeV would violate the experimental bound for most values of a_3 and a_1. This maximal value for M_R follows from the choice we made for the MSSM mass spectrum, Eqs. (27) and (28). For instance, taking m_0 larger so as to make sleptons and sneutrinos heavier would shift the center of the contours towards the lower left corner, suppressing both EDMs and LFV transitions, and thus allowing for slightly larger M_R. Still, note that with $m_0 = 600$ GeV and a_1 between 0.1 and 8, we are already scanning over a large range of slepton masses as m_L varies between about 100 GeV and 2 TeV. Further, the contours in Fig. 1 span several orders of magnitude for $B\left(\mu \to e\gamma\right)$, but the corresponding d_e values are much more concentrated. Therefore, changing m_0 affects more $B\left(\mu \to e\gamma\right)$ than d_e, and $m_0 = 600$ GeV appears as a reasonably small value still allowing to probe relatively large M_R without violating the current $B\left(\mu \to e\gamma\right)$ bound.

Neutrino spurion parameters: When the lightest neutrino is massless (in the normal spectrum), the $\mu \to e\gamma$ transition is essentially independent of the CP-violating phases in the spurion $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ (given their range of variation specified in (26)). When m_ν increases, either $B\left(\mu \to e\gamma\right)$ is suppressed if $\phi_i = 0$ or increased if the ϕ_i are varied in their ranges. This is easy to understand from the behavior of the off-diagonal entries of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. When $\phi_i = 0$, they are tuned entirely by the neutrino mass-differences $m_{\nu 3} - m_\nu$ or $m_{\nu 2} - m_\nu$, which decrease with increasing m_ν keeping $\Delta m_\odot^2$ and Δm_{atm}^2 fixed to their experimental values. On the contrary, when $\phi_i \neq 0$, off-diagonal entries receive a contribution linear in m_ν and ϕ_i (see Eq. (8)),

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^{12} = \frac{M_R}{\sqrt{2}v_u^2}\left(s_\odot c_\odot \Delta m_{21} + 2im_\nu\left(\phi_1 + c_\odot\phi_2 + s_\odot\phi_3\right) + \ldots\right) \ , \tag{35}$$

which dominates for $m_\nu > \Delta m_{21}$, given the ranges (26) for the ϕ_i parameters.

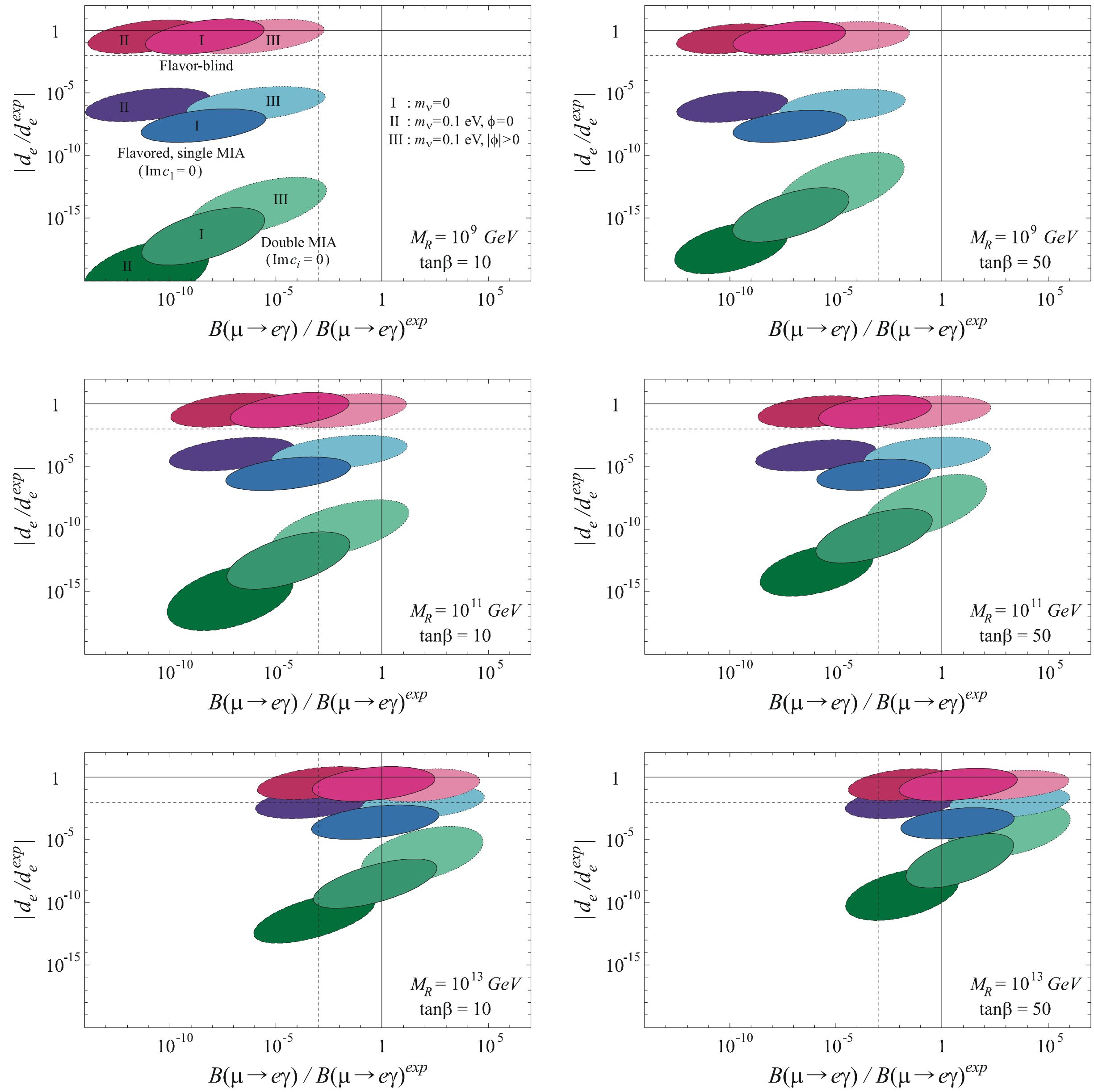

Figure 1: Contours at 90% in the $B(\mu \to e\gamma) - d_e$ plane (normalized to their experimental bounds) corresponding to scanning over the parameters as given in Eqs. (26) and (29), but subjected to the constraints of the scenarios in Table 2 and Eq. (30). The contours for $m_\nu = 0$ are drawn for real spurions only. Those for complex spurions mostly overlap and extend them by at most one order of magnitude towards larger $B(\mu \to e\gamma)$ without any shift in d_e. Finally, the vertical and horizontal dashed lines show the expected sensitivities of the next generation of experiments searching for $B(\mu \to e\gamma)$ and d_e, respectively.

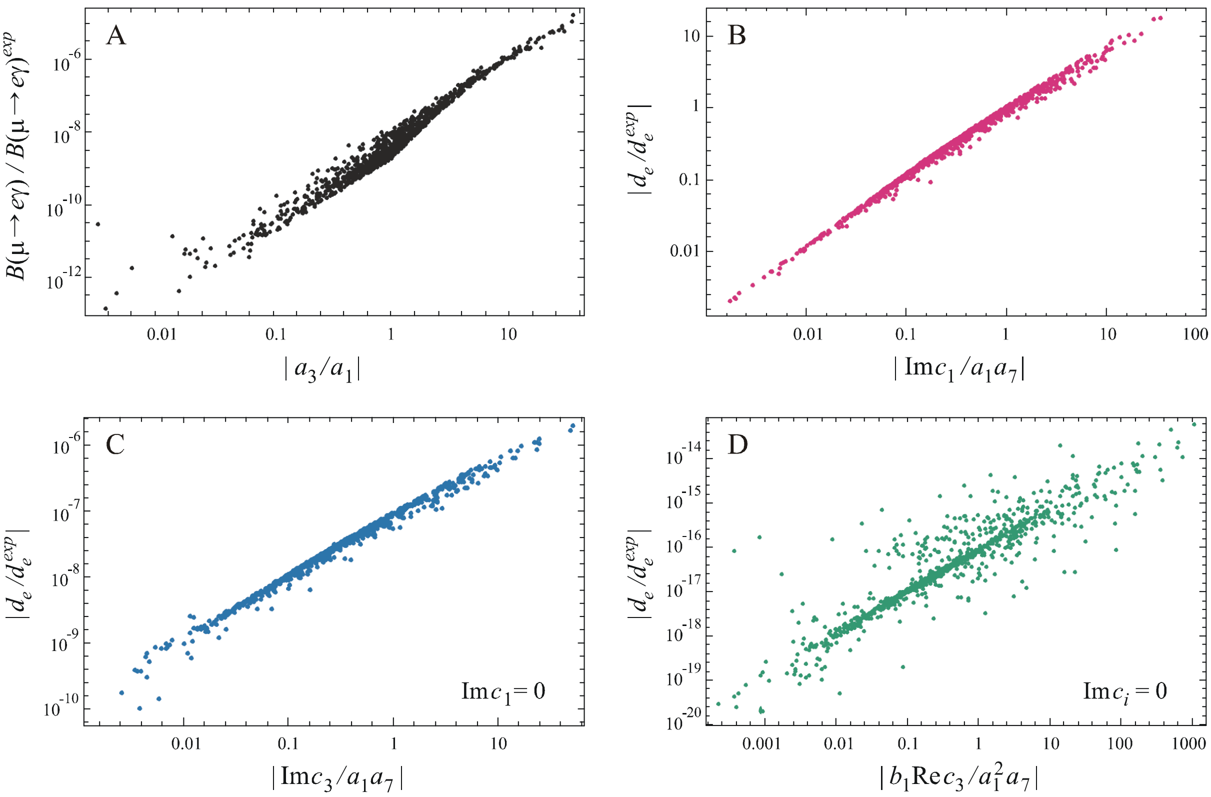

Figure 2: Illustration of the hierarchical dominance of a single phase per scenario for $M_R = 10^9$ GeV, $\tan\beta = 10$, and $m_\nu = 0$. Colors refer to Fig. 1. A. Dominance of the $\delta_{LL}^{12} \approx a_3/a_1$ contribution for the $B\,(\mu \to e\gamma)$ transition. The small spread of the points is due to the dependences of the loop functions on the masses, especially strong for light sleptons ($\approx$ small a_1). B. Dominance of the flavor-blind phase $\mathrm{Im}\,c_1$ for d_e. Again, the small spread of the points is due to the loop function. C. Dominance of the subleading flavor-diagonal phase $\mathrm{Im}\,c_3$. D. Approximate dominance of the flavor off-diagonal phase from b_1. Here, the spread of points is also due to additional operators whose contributions can be competitive, especially for light sparticles (small a_1 and/or a_7) or when b_1 and/or $\mathrm{Re}\,c_3$ are small. The effect of taking $\tan\beta$ larger affects C and D, taking M_R larger affects D, while taking m_ν larger and including fixed CP-violating phases in the spurion does not affect the dominance, but shifts the points to smaller or larger values, as shown in Fig. 1.

Correlations and tanβ: As long as there is only the a_3 operator, the three LFV transitions $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are clearly correlated among themselves. The ratios of their width can be predicted in terms of the spurion parameters only, i.e. in terms of the relevant off-diagonal entries of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, and are thus identical to those already extensively studied in the context of the supersymmetric seesaw. For example, these ratios show a strong sensitivity to the ϕ_i parameters and a further dependence on the Majorana phases [7, 9, 37, 38]. Note though that this sensitivity to neutrino parameters is smaller for the $\tau \to \mu\gamma$ mode, tuned by the larger Δm^2_{atm} in the limit $m_\nu = 0$.

At large $\tan\beta$, the situation changes slightly because the operator a_5 becomes competitive for $\tau \to \mu\gamma$ and $\tau \to e\gamma$. Though the ratio of the widths of these two modes still depends only on the spurion parameters, their ratios to $B(\mu \to e\gamma)$ are function also of the MFV parameters a_5 and a_3. However, the operator a_5 never becomes dominant, and thus never changes the order of magnitude of the $\tau \to \mu\gamma$ and $\tau \to e\gamma$ modes.

Altogether, the predictions for the $\tau \to \mu\gamma$ or $\tau \to e\gamma$ modes within MFV are always at least two orders of magnitude farther away from their respective experimental limits than $\mu \to e\gamma$. Therefore, in the MFV setting, $\mu \to e\gamma$ currently gives the best constraints, or opportunity for discovery. In Fig. 1 we have drawn the reach of the MEG experiment, which will probe this mode down to the $10^{-13} - 10^{-14}$ range, and will essentially exclude $M_R \gtrsim 10^{13}$ GeV with our range of sparticle masses.

3.2.2 Leptonic EDMs

Let us now turn to the lepton EDMs, starting from Eq. (24). As will be explored in detail below, only three contributions are actually relevant:

$$\frac{d_I}{e} \approx \frac{-m_{eI}\,\alpha M_1 A_0}{8\pi\cos^2\theta_W m_0^4}\left(\frac{\operatorname{Im} c_1}{a_1 a_7} + \frac{\operatorname{Im} c_2 \mathbf{Y}_e^\dagger \mathbf{Y}_e + \operatorname{Im} c_3 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu}{a_1 a_7} - \frac{b_1 \operatorname{Re} c_3}{a_1^2 a_7}[\mathbf{Y}_e^\dagger \mathbf{Y}_e, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + ...\right)^{II} . \tag{36}$$

The first two terms come from the $-A_e^{I*}$ piece in Eq. (24), i.e. the diagonal entries of the trilinear term $\mathbf{A}_e$, while the last originates from the double mass-insertion $\delta_{LL}^{IK}\delta_{LR}^{KI}$, i.e. from $\mathbf{m}_L^2$ and $\mathbf{A}_e$. Contrary to the case of LFV transitions, the loop functions $\mathcal{F}_5$ and $\mathcal{F}_8$ are much flatter for our range of masses, and can be approximated by $|\mu^2|/m_L^2 m_R^2 \approx |\mu^2|/(m_0^4 a_1 a_7)$.

To show the main dependences on the neutrino parameters, let us write d_e in the $m_\nu = 0$ limit, keeping $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ real:

$$\frac{d_e}{e} \approx \frac{-m_e\,\alpha M_1}{8\pi\cos^2\theta_W}\frac{A_0}{m_0^4}\left(\frac{\operatorname{Im} c_1}{a_1 a_7} + \frac{M_R \Delta m_{21} s_\odot^2}{v_u^2}\frac{\operatorname{Im} c_3}{a_1 a_7} - \frac{M_R^2 c_\odot^2 s_\odot^2\,(\Delta m_{21})^2\, m_\tau^2}{2 v_d^2 v_u^4}\frac{b_1 \operatorname{Re} c_3}{a_1^2 a_7} + ...\right) . \tag{37}$$

The term $\operatorname{Im} c_2$ does not contribute: being quadratic in the lepton mass, it is relevant only for d_μ and d_τ. The striking feature of d_e within MFV is the very strong hierarchy between these three terms. Each of them corresponds to one scenario for the MFV coefficients given in Table 2, and their strong hierarchy is obvious in Fig. 1. Analytically, Eq. (37) shows that the contribution from the flavor-blind $\operatorname{Im} c_1$ is much larger than the one from the flavor-diagonal phase $\operatorname{Im} c_3$, linear in Δm_{21}, which is itself much larger than the double MIA contribution of the flavor off-diagonal phase b_1, quadratic in Δm_{21}, but this hierarchy gets mitigated as M_R increases. Also, only the third term shows a strong, quadratic sensitivity to $\tan\beta$. All this is immediately apparent in Fig. 1.

Let us look more closely at each scenario.

The dominant flavor-blind phase: When c_1 is complex, it completely dominates, as seen in Fig. 1 and Fig. 2B. This scenario corresponds essentially to what has been analyzed in Ref. [39] (see also Ref. [40]). Contrary to the LFV transitions, which always scale at least quadratically with M_R, this dominant contribution is independent of M_R. Further, it is clearly independent of the neutrino parameters. The important point is that even for the relatively light MSSM mass-spectrum specified in Eqs. (27–29), d_e can easily satisfy its experimental upper bound. For example, with $\mathrm{Im}\, c_1 = 1$, one immediately reads from Fig. 2B that the current bound on d_e imposes $a_1 a_7 \gtrsim 1$, which corresponds to $m_L^2 \approx m_R^2 \gtrsim 600$ GeV when $a_1 = a_7$. On the contrary, the expected two order-of-magnitude improvement in the measurement of d_e would rule out this scenario, except for prohibitively high slepton masses, or unnaturally small $\mathrm{Im}\, c_1$. The same conclusion was reached in Ref. [39]. The lower bound $d_e \gtrsim 10^{-28}\, e\,\mathrm{cm}$ they found is comparable (though slightly larger) than the one we can extract from Fig. 1.

At moderate $\tan\beta$, only c_1 contributes and the relative sizes of d_e, d_μ and d_τ are simply ruled by the ratios of the lepton masses. For our specific MSSM mass spectrum, this translates as

$$\frac{d_\mu}{d_\mu^{\exp}} \approx \frac{m_\mu}{m_e}\frac{d_e}{d_\mu^{\exp}} \lesssim 10^{-5}, \quad \frac{d_\tau}{d_\tau^{\exp}} \approx \frac{m_\tau}{m_e}\frac{d_e}{d_\tau^{\exp}} \lesssim 10^{-6} \; . \tag{38}$$

This means that while d_e is already around its current experimental bound, future experiments aiming at d_μ and d_τ should gain respectively five and six orders of magnitude in sensitivity to be just barely competitive. This conclusion is unchanged at large $\tan\beta$, because even though the MFV operator c_2 contributes for d_τ, it always affects the scaling (38) by less than an order of magnitude.

The subdominant flavor diagonal phases: If the flavor-blind phase is turned off by setting $\mathrm{Im}\, c_1 = 0$, the electron EDM is generated entirely by $\mathrm{Im}\, c_3$, see Fig. 2C. The dependence on the neutrino parameters is now apparent in Fig. 1: while d_e is essentially independent of the ϕ_i's no matter m_ν, it increases with m_ν as can be understood looking at the diagonal entries of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. In any case, except for very large M_R, d_e is rather far from its experimental bound and the additional CP-violating phases in the trilinear sector brought in by MFV are completely free. When M_R becomes very large, except for some contrived set of spurion parameters, the LFV transitions fail to pass their experimental bounds well before d_e.

In this scenario, the EDMs no longer scale as the lepton masses. Even if there were only the $\mathrm{Im}\, c_3$ contribution, the ratios of EDMs would depend on the neutrino parameters. For example, d_μ and d_τ would be enhanced by $\Delta m_{atm}/\Delta m_{sol}$ when $m_\nu = 0$. But in addition to $\mathrm{Im}\, c_3$, which always dominates for d_e, the $\mathrm{Im}\, c_2$ contribution is competitive for d_μ when $M_R \lesssim 10^{11}$, and dominates d_τ for $M_R \lesssim 10^{13}$ GeV, even at low $\tan\beta$. Further, for $\tan\beta = 50$, the tau Yukawa coupling is of $\mathcal{O}(1)$, $m_\tau^2/v_d^2 \approx 1/4$, bringing d_τ very close to the values attainable with the flavor-blind phase. This makes d_τ particularly interesting to test the presence of new CP-violating phases in the slepton sector, both flavor-blind and flavor diagonal. However, the bound (38) cannot be evaded, and d_τ should remain beyond the experimental reach for the near future.

The tiny flavor off-diagonal phases: When all the flavor-diagonal phases are turned off by setting $\mathrm{Im}\, c_i = 0$, the EDMs are generated by second order effects in the mass insertions. As explained in Section 2, this is the order at which we would expect to see a strong sensitivity to the CP-violating phase in the $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ spurion (the Dirac and Majorana phases and ϕ_i parameters), but these turn out to be subleading. The dominant CP-violating effect when $\mathrm{Im}\, c_i = 0$ comes instead from the CP-violating b_1 parameters taken together with $\mathrm{Re}\, c_3$. Of course, this $b_1\, \mathrm{Re}\, c_3$ contribution shows a strong dependence on the ϕ_i when $m_\nu \neq 0$ but this comes entirely from the sensitivity of the off-diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ on the ϕ_i, as can be seen in Eq. (35), and thus is not a CP-violating effect.[4]

It should be stressed that the dominance of the $b_1 \operatorname{Re} c_3$ contribution is not as strong as in the previous two scenarios, see Fig. 2D. Other effects compete at the double MIA order, especially when $M_R \gtrsim 10^{11}$ GeV or when $\tan\beta$ is large. For example, in the latter case, the double MIA contribution to d_e and d_μ comes from the $\delta_{LL}^{I3}\delta_{LR}^{3I}$ term in Eq. (24), which involves the third generation. As a result, the presence of additional operators in the large $\tan\beta$ limit blurs the dominance of $b_1 \operatorname{Re} c_3$. On the other hand, d_τ is still dominated by $b_1 \operatorname{Re} c_3$ since $\delta_{LR}^{33} = 0$. This shows that though the dominance is not complete, the overall picture can still be grasped with the help of the $b_1 \operatorname{Re} c_3$ contribution alone. In particular, whether these competing effects are turned on or off does not visibly change the contours in Fig. 1.

In view of the previous comment, it is clear that the ratios of d_e, d_μ and d_τ do not scale like the lepton masses. When all of them are dominated by the single $b_1 \operatorname{Re} c_3$ operator combination, which requires low M_R and $\tan\beta$, their ratios can be predicted entirely in terms of the neutrino parameters. Still, being quadratic in the off-diagonal entries of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, these ratios are even more sensitive to these parameters than the ratios of LFV transitions discussed before, and span several orders of magnitude when $m_\nu > 0$ and the ϕ_i vary in the ranges (26). When M_R or $\tan\beta$ are larger, additional operators enter and ratios of EDMs are no longer fixed in terms of neutrino parameters.

Overall, the EDMs are very suppressed in the absence of CP-violating phases in the c_i operators, no matter the CP-violating phases in the spurions, see Fig. 1. Only in the limit of very large M_R is this suppression compensated, but this is forbidden as LFV transitions violate their experimental bounds much before the EDMs become even reasonably close to their present limits.

3.3 Comparison with the model-independent approach

When MFV is used to directly parametrize the New Physics operator responsible for $\ell^I \to \ell^J \gamma$ transitions, EDMs and MDMs, one obtains [3, 5]

$$\begin{aligned} H_{eff} &= e\frac{1}{\Lambda^2}\frac{\alpha}{4\pi}\left(\mathbf{Y}_e^{II}\mathcal{C}^{IJ}\right)\bar{\psi}_R^I \sigma_{\mu\nu}\psi_L^J F^{\mu\nu} H_d + h.c.\ , \\ \mathcal{C} &= h_1\mathbf{1} + h_2\mathbf{A} + h_3\mathbf{B} + h_4\mathbf{B}^2 + h_5\{\mathbf{A},\mathbf{B}\} + h_6\mathbf{BAB} \\ &\quad + g_1 i[\mathbf{A},\mathbf{B}] + g_2 i[\mathbf{A},\mathbf{B}^2] + g_3 i(\mathbf{B}^2\mathbf{AB} - \mathbf{BAB}^2)\ , \end{aligned} \tag{39}$$

where $\mathcal{C}$ transforms as $\mathbf{Q}$ in Eq. (17) and, as before, $\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e$ and $\mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ with $\mathbf{Y}_e$ diagonal in our basis. This expression is manifestly invariant under the flavor group $U(3)^5$. As explained in Section 2, the $U(3)^5$ symmetry does not force the coefficients to be real, hence we allow all the h_i and g_i to be complex. The classification of the CP-violating phases performed in Sec. 2.4 still holds: $\operatorname{Im} h_1$ is flavor-blind, $\operatorname{Im} h_{2-6}$ and $\operatorname{Im} g_3$ are flavor-diagonal, and $\operatorname{Im} g_{1,2}$ as well as all the CP-violating phases in $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ are flavor off-diagonal since all the operators are hermitian. Note that the denomination 'flavor-blind' is to be understood as in the MSSM: strictly speaking, both the c_1 operator in Eq. (18) and the h_1 operator in Eq. (39) are forbidden when $\mathbf{Y}_e$ is absent since the whole $\mathbf{A}_e$ and H_{eff} would be forbidden.

Though the MFV parametrization of the effective operator in Eq. (39) is intended to be as model-independent as possible, we have introduced the loop factor and gauge coupling explicitly (compare

[4]We saw in Sec. 2 that $\operatorname{Im} c_1$ is never exactly zero, but is at least as large as the Jarlskog invariant (21). We checked that the contributions from $\operatorname{Im} c_1 \sim J$ are always at least one order of magnitude smaller than those from $b_1 \operatorname{Re} c_3$.

with Eq. (1)). Indeed, LFV and EDMs arise at the loop level in the MSSM, so it makes sense to factor out these effects in the following comparison. From Eq. (39), the leptonic observables are (neglecting terms suppressed by m_{e^J}/m_{e^I})

$$\frac{B\left(\ell^I \to \ell^J\gamma\right)}{B\left(\ell^I \to \ell^J\bar{\nu}^J\nu^I\right)} = \frac{\alpha\sin^4\theta_W}{24\pi}\frac{M_W^4}{\Lambda^4}|\mathcal{C}^{IJ}|^2, \;\; a_I = \frac{\alpha m_{e^I}^2}{\pi\Lambda^2}\,\mathrm{Re}\,\mathcal{C}^{II}, \;\; \frac{d_I}{e} = \frac{\alpha m_{e^I}}{2\pi\Lambda^2}\,\mathrm{Im}\,\mathcal{C}^{II}\;. \tag{40}$$

The similarity with Eqs. (23) and (24) is manifest in the limit where all sparticles are degenerate (remember that the loop functions scale as $\mathcal{F}_i \sim 1/M_{\mathrm{SUSY}}^2$). Let us compare the two MFV implementations in detail.

- The LFV transitions behave essentially as in the MSSM case studied before. Indeed, the operator h_3 dominates, hence LFV transitions scale as the off-diagonal entries of $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$, exactly like in Eq. (33). Correlations between LFV transitions are thus similar to those for the supersymmetric case [5, 41, 42]. The order of magnitude is slightly smaller in the effective theory when $\Lambda \sim M_{\mathrm{SUSY}}$, because of the different numerical factors and lack of the $\tan^2\beta$ enhancement.

- For the EDMs, the flavor blind phase $\mathrm{Im}\,h_1$ dominates. Comparing Eq. (39) with (24), $\mathrm{Im}\,h_1$ is the effective coupling describing both the effects of the flavor-blind phase of $\mathbf{A}_e$ and of all the phases of the flavor-blind parameters, including μ. As a result, the matching with the MSSM is slightly ambiguous. Indeed, taken at face-value, Eq. (40) implies

$$\frac{\alpha m_{e^I}}{2\pi\Lambda^2} \lesssim 1.6\times 10^{-27}\;e\,\mathrm{cm} \Rightarrow \Lambda \gtrsim 2.7\ \mathrm{TeV}\;, \tag{41}$$

  when $\mathrm{Im}\,h_1 \sim \mathcal{O}(1)$. Compared to the MSSM, this is an intermediate situation. An arbitrary phase for μ pushes Λ well above 5 TeV, essentially because of the $\tan\beta$ enhancement. On the contrary, the contribution of the flavor-blind phase of $\mathbf{A}_e$ is easily suppressed by more than a factor of ten, once the precise mass dependences are taken into account, pulling Λ down below 800 GeV (see Fig. 2B).

- Concerning the flavor-diagonal phases, these are dominated by $\mathrm{Im}\,h_2$ and $\mathrm{Im}\,h_3$, which are analogous to the $\mathrm{Im}\,c_2$ and $\mathrm{Im}\,c_3$ contributions in the MSSM, respectively. For d_e, only $\mathrm{Im}\,h_3$ contributes, bringing in a suppression factor $M_R\Delta m_{21}/v_u^2$ (see Eq. (37)). Interestingly, the model-independent formalism imposes a (loose) correlation between LFV transitions and d_e, since they are all due to the same MFV operator h_3. In the MSSM, this correlation is completely absent because the LFV transitions are tuned by the slepton mass term $\mathbf{m}_L^2$, while EDM by the trilinear coupling $\mathbf{A}_e$. For d_μ and d_τ, both $\mathrm{Im}\,h_2$ and $\mathrm{Im}\,h_3$ can contribute, exactly like in the MSSM. Finally, as for the flavor-blind phase $\mathrm{Im}\,h_1$, significant numerical factors affect these contributions and prevent a precise comparison of the scale Λ with the SUSY scale.

- The effects of the off-diagonal CP-violating phases are absent, since there is no such thing as a double mass-insertion in the effective operator formalism. Therefore, when $\mathrm{Im}\,h_i = 0$, only the very suppressed contribution of the flavor-diagonal phase of the g_3 operator generates the EDMs. Though the suppression is less strong in the MSSM, the situation is similar to the scenario with $\mathrm{Im}\,c_i = 0$, see Fig. 1. In other words, the effective operator formalism correctly predicts that in the absence of flavor blind or diagonal phases, the EDMs are far too suppressed to be seen in the near future.

Overall, the effective operator formalism catches the MSSM features quite satisfactorily, provided one allows for some latitude in the numerical coefficients. The loop factor and the gauge couplings obviously have to be added by hand, but an additional factor of about ten should also be allowed when extracting model-independent bounds on the scale Λ. This near correspondence between the effective formalism and the MSSM holds even if the latter has much more degrees of freedom, including more MFV coefficients, because numerically, only a few operators are dominant in both cases, and they have the same spurion structures. Still, some correlations between LFV transitions and EDMs are absent in the MSSM, first because the EDMs depend on both the left and right slepton masses while LFV transitions care only about those of the left sleptons (see the occurrence of a_7 in Eq. (24)), and then simply because in the MSSM, different MFV coefficients enter in δ_{LL} and δ_{LR}.

4 Beyond MFV: leptonic observables in the general MSSM

In the general MSSM, the bounds on LFV processes and lepton EDMs are usually expressed as limits on the real and imaginary parts of the slepton mass insertions, i.e. on the properly normalized off-diagonal elements of $\mathbf{m}_L^2$, $\mathbf{m}_E^2$, and $\mathbf{A}_e$. As shown e.g. in Ref. [22], such limits can be quite tight, with many mass insertions required to be extremely small. However, this does not tell anything yet about how natural those small values are. To judge of their naturality, one should relate them to the flavor-breakings observed in the SM. Indeed, it is only if a specific flavor-breaking in the slepton sector is required to be significantly smaller than the known flavor-breakings in the lepton sector that one can speak of a flavor puzzle.[5]

For this purpose, the basis of MFV operators we constructed in Section 2 is optimal. As explained there, the spurion expansions are merely reparametrizations as long as nothing is imposed on their coefficients [10, 14]. The essence of MFV, on the other hand, is to require the size of those coefficients to be at most of $\mathcal{O}(1)$. Relaxing this constraint, we are back to the full MSSM. Therefore, it is interesting to translate the current bounds on LFV processes and EDMs into bounds on the coefficients of the expansions (18). If a coefficient is required to be much smaller than one, we can say that it is fine-tuned. Otherwise, the structures of the soft-breaking terms $\mathbf{m}_L^2$, $\mathbf{m}_E^2$, and $\mathbf{A}_e$, though maybe peculiar, are no less natural (or unnatural) than those of the SM quark and lepton masses and mixings.

The bounds on the coefficients are collected in Table 3, along with their origin. We assume the specific MSSM mass-spectrum of Eqs. (27) and (28), and take $M_R = 10^{12}$ GeV, $\tan\beta = 20$, and $m_\nu = 0$, values for which we expect rather tight bounds given Fig. 1. Further, in a way similar to how mass-insertions are defined, we normalize the $\mathbf{m}_L^2$ coefficients with respect to a_1, those of $\mathbf{m}_E^2$ with respect to a_7, and those of $\mathbf{A}_e$ with respect to the product $a_1 \times a_7$. Only one coefficient is turned on at a time, in addition to a_1 and a_7 which are allowed to vary between 0.1 and 8. This means that contributions requiring simultaneously two spurion operators are absent. This is sufficient for our purpose, given the suppression of the spurions when M_R and $\tan\beta$ are not too large.

The striking feature of Table 3 is how loose the current experimental bounds are when translated on the coefficients. As could have been expected from the strong hierarchies between contributions discussed in the previous section, only a_3/a_1 and $\operatorname{Im} c_1/(a_1 a_7)$ are actually constrained to be of $\mathcal{O}(1-10)$, while all the others can take huge values. One should also realize that in using the coefficients of the expansions (18) as measures of leptonic flavor-violation, one is performing a rather radical change of basis. Indeed, let us recall that a fully generic 3×3 matrix projected on these MFV

[5]Of course, one remains with the known lepton masses and mixings, whose peculiar structures are by themselves a flavor puzzle.

$\mathbf{m}_L^2$		$\mathbf{m}_E^2$		$\mathrm{Re}\,\mathbf{A}_e$		$\mathrm{Im}\,\mathbf{A}_e$	
$a_2 \lesssim 10^3$	[Masses]	$a_8 \lesssim 10^3$	[Masses]	$\mathrm{Re}\,c_1 \lesssim 10^2$	[Stab.]	$\mathrm{Im}\,c_1 \lesssim 2$	[d_e]
$a_3 \lesssim 10$	[$\mu \to e\gamma$]	$a_9 \lesssim 10^6$	[$\tau \to \mu\gamma$]	$\mathrm{Re}\,c_2 \lesssim 10^3$	[Stab.]	$\mathrm{Im}\,c_2 \lesssim 10^3$	[Stab.]
$a_4 \lesssim 10^4$	[$\mu \to e\gamma$]	$_{10} \lesssim 10^9$	[$\tau \to \mu\gamma$]	$\mathrm{Re}\,c_3 \lesssim 10^3$	[$\mu \to e\gamma$]	$\mathrm{Im}\,c_3 \lesssim 10^3$	[$\mu \to e\gamma$]
$a_5 \lesssim 10^3$	[$\tau \to \mu\gamma$]	$a_{11} \lesssim 10^7$	[$\tau \to \mu\gamma$]	$\mathrm{Re}\,c_4 \lesssim 10^6$	[$\mu \to e\gamma$]	$\mathrm{Im}\,c_4 \lesssim 10^6$	[$\mu \to e\gamma$]
$a_6 \lesssim 10^4$	[$\mu \to e\gamma$]	$a_{12} \lesssim 10^{11}$	[$\tau \to \mu\gamma$]	$\mathrm{Re}\,c_5 \lesssim 10^5$	[$\tau \to \mu\gamma$]	$\mathrm{Im}\,c_5 \lesssim 10^5$	[$\tau \to \mu\gamma$]
$b_1 \lesssim 10^3$	[$\tau \to \mu\gamma$]	$b_4 \lesssim 10^7$	[$\tau \to \mu\gamma$]	$\mathrm{Re}\,c_6 \lesssim 10^7$	[$\mu \to e\gamma$]	$\mathrm{Im}\,c_6 \lesssim 10^7$	[$\mu \to e\gamma$]
$b_2 \lesssim 10^6$	[$\tau \to \mu\gamma$]	$b_5 \lesssim 10^{10}$	[$\tau \to \mu\gamma$]	$\mathrm{Re}\,d_1 \lesssim 10^5$	[$\tau \to \mu\gamma$]	$\mathrm{Im}\,d_1 \lesssim 10^5$	[$\tau \to \mu\gamma$]
$b_3 \lesssim 10^8$	[$\mu \to e\gamma$]	$b_6 \lesssim 10^{13}$	[$\tau \to \mu\gamma$]	$\mathrm{Re}\,d_2 \lesssim 10^8$	[$\tau \to \mu\gamma$]	$\mathrm{Im}\,d_2 \lesssim 10^8$	[$\tau \to \mu\gamma$]
				$\mathrm{Re}\,d_3 \lesssim 10^{10}$	[$\mu \to e\gamma$]	$\mathrm{Im}\,d_3 \lesssim 10^{10}$	[$\mu \to e\gamma$]

Table 3: Bounds on the slepton soft-breaking terms of the general MSSM, expressed in terms of the coefficients of the expansions (18). The $\mathbf{m}_L^2$, $\mathbf{m}_E^2$, and $\mathbf{A}_e$ coefficients are normalized to a_1, a_7 and $a_1 a_7$, respectively. To establish those bounds, the MSSM spectrum (27, 28) is assumed, as well as $M_R = 10^{12}$ GeV, $\tan\beta = 20$, $m_\nu = 0$. Finally, the bounds on $a_{2,8}$ come from requiring sleptons to be lighter that 4 TeV, while those on $\mathrm{Re}\,c_{1,2}$ and $\mathrm{Im}\,c_2$ from the vacuum stability bounds, approximately enforced as $|c_{1,2}|^2 \lesssim 3(a_1 + a_7)$.

basis generates coefficients spanning several, sometimes even tens of orders of magnitude, simply because the SM flavor structures used as building blocks are very special.

Another interesting aspect of Table 3 is the dominance of the bounds from the $\tau \to \mu\gamma$ mode for a majority of coefficients, especially in the $\mathbf{m}_E^2$ sector. On the contrary, d_e is at present competitive only for the flavor-blind phase $\mathrm{Im}\,c_1$ while d_μ and d_τ are simply absent. Further, for several entries, the experimental bounds are actually unable to say anything, and we quote in Table 3 the order of magnitude of the bounds one gets restricting all slepton masses to be below 4 TeV, as well as from the stability of the potential (which we approximately impose as $|c_{1,2}|^2 \lesssim 3(a_1 + a_7)$) [43]. This dominance of the bounds on LFV transitions over those on EDMs can be understood as follows. First, by turning on $\mathrm{Im}\,c_3$, one actually turns on a whole set of mass insertions. When both $\mathrm{Im}\,c_3$ and a_3 are of $\mathcal{O}(1)$, the δ_{LR} mass-insertion contributions to LFV modes are subleading, but since we turn on only one operator at a time, they now dominate. Second, it is clear from Fig. 1 that $\mathrm{Im}\,c_3$ generates quite small EDMs, far from their current experimental bounds. Therefore, what Table 3 actually shows is that when $\mathrm{Im}\,c_3$ increases by orders of magnitude, one reaches the bound on $\mu \to e\gamma$ before that on d_e.

Overall, this analysis clearly demonstrates that in the lepton sector, the current experimental sensitivity is still far from the level required to actually probe non-natural flavor structures, at least in the MFV sense. In other words, even if the bounds on the mass insertions show peculiar patterns, they should not be interpreted as posing a fine-tuning problem yet, because they can all be understood naturally in terms of the patterns observed in the lepton masses and mixings.

5 Conclusion

In the present paper, we have constructed the most general expansions for the slepton soft-breaking terms $\mathbf{m}_L^2$, $\mathbf{m}_E^2$, and $\mathbf{A}_e$ within the MSSM, assuming Minimal Flavor Violation and a seesaw mechanism of type I. All the possible CP-violating phases were introduced, classified, and their impact on leptonic EDMs analyzed. Our main results may be summarized as follows:

1. The occurrence of complex phases in MFV has been thoroughly examined. We have proved that in principle, the MFV coefficients can never be assumed to respect CP when CP-violating phases are present in the Yukawa couplings. Indeed, these coefficients are always defined up to some phases proportional to the Jarlskog invariant (21). On one hand, this result implies that limiting the CP-violating sources within MFV to those present already in the SM cannot be based on a symmetry principle, and is thus instead a kind of fine-tuning. On the other hand, it shows that if one can live with this fine-tuning or if one has a mechanism able to enforce it, it is a numerically stable situation thanks to the smallness of the Jarlskog invariant. We have argued that ultimately, it is by comparing with experiment that one can decide whether such a fine-tuning is present or not, and thus that for now, MFV coefficients should be taken complex. This conclusion is fully general: it applies to both the lepton and quark sectors, and whether MFV is imposed on the MSSM or used model-independently to parametrize generic New Physics operators [3, 5, 41, 44].

2. We have characterized the CP-violating phases by ordering them into three classes: flavor-blind, flavor-diagonal and flavor off-diagonal; according to their hierarchical, decreasing impacts on the EDMs (see Fig. 1). Interestingly, because the MFV operator basis we constructed is hermitian, all the CP-violating phases coming from the spurion (i.e., from $\mathbf{Y}_\nu$) are of the flavor off-diagonal type: they have a negligible impact on the EDMs because they start to contribute only at the second order in the mass-insertion approximation. Also, to a good approximation, only one MFV operator is relevant for each type of phases contributing to d_e, while d_μ and d_τ can receive additional contributions, especially at large $\tan\beta$. Even if the ratios of EDMs scale as the lepton masses only in the presence of the flavor-blind phase, no scenario can enhance d_μ or d_τ sufficiently to make them more promising than d_e for finding a New Physics signal in the near future. Finally, though we concentrated exclusively on the slepton sector, our classification of the CP-violating phases can be immediately applied to the quark sector, for which the MFV expansions can also be written entirely in terms of hermitian operators.

3. For a realistic range of MSSM and neutrino parameters, such that $B(\mu \to e\gamma)$ satisfies its experimental bound, the three types of phases are allowed by the current bound on d_e. The next generation of experiments searching for d_e should cover most of the range of values induced by the flavor-blind phase of $\mathbf{A}_e$, but will probably not probe the flavor-diagonal or off-diagonal phases yet. Still, without a signal, the flavor blind phase of $\mathbf{A}_e$ would pose the same problem as the other flavor blind phases, i.e. those of μ, M_1 or M_2, whose sizes already have to be tightly constrained to pass the current bound on d_e when sparticles are not extremely heavy. A similar conclusion was reached recently in Ref. [39], including also the flavor-blind phases of the squark sector, though in a slightly more constrained scenario than MFV.

4. When the general MSSM soft-breaking terms are projected on the MFV spurion basis, the coefficients can a priori span several orders of magnitude, instead of $\mathcal{O}(1)$ values within MFV [10]. We have argued that translating the experimental limits into bounds on these coefficients permits to unambiguously appreciate the amount of fine-tuning involved in the flavor sector, something the usual mass-insertions do not immediately permit. Indeed, it is respectively to the SM flavor-breaking that those of the MSSM should be compared. We have performed such an analysis in the slepton sector, and found that current experimental bounds are far from the MFV level, except for two operators whose coefficients are already constrained to be of $\mathcal{O}(1)$ by $\mu \to e\gamma$ and d_e. Also, the $\tau \to \mu\gamma$ mode emerges as the most promising to improve the various limits

in this framework. It would be necessary to extend this analysis to the quark sector to fully appreciate the level at which MFV is currently tested, and on which observables to concentrate in order to gain the most information.

In conclusion, the perspectives for a richer CP-violating phenomenology within the MFV framework are very promising. In particular, the study of the quark sector initiated in Ref. [39] should be extended into a full-fledged MFV analysis, with potentially interesting results for K and B physics phenomenology [10, 44–47]. Also, in the MSSM without R-parity, the rather small value for M_R implied by $B(\mu \to e\gamma)$ means that the bounds on the proton lifetime are quite easy to satisfy with the help of MFV alone [13, 48]. This setting then predicts significant CP-violating and baryon number violating couplings, whose phenomenological implications have not yet been fully explored. Finally, leptogenesis within the MFV framework has led to interesting recent developments [8, 38, 49], and the impacts of the additional CP-violating phases are yet to be studied in that context.

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