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# Supersymmetric renormalisation of the CKM matrix and new constraints on the squark mass matrices

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**Abstract**

We compute the finite renormalisation of the Cabibbo-Kobayashi-Maskawa (CKM) matrix induced by gluino–squark diagrams in the MSSM with non-minimal sources of flavour violation. Subsequently we derive bounds on the flavour–off–diagonal elements of the squark mass matrices by requiring that the radiative corrections to the CKM elements do not exceed the experimental values. Our constraints on the associated dimensionless quantities $\delta_{ij}^{d\,LR}$, $j > i$, are stronger than the bounds from flavour-changing neutral current (FCNC) processes if gluino and squarks are heavier than 500 GeV. Our bound on $|\delta_{12}^{u\,LR}|$ is stronger than the FCNC bound from $D$–$\overline{D}$ mixing for superpartner masses above 900 GeV. We further find a useful bound on $|\delta_{13}^{u\,LR}|$, for which no FCNC constraint is known. Our results imply that it is still possible to generate all observed flavour violation from the soft supersymmetry-breaking terms without conflicting with present-day data on FCNC processes. We suggest that a flavour symmetry renders the Yukawa sector flavour–diagonal and the trilinear supersymmetry-breaking terms are the spurion fields breaking this flavour symmetry. We further derive the dominant supersymmetric radiative corrections to the couplings of charged Higgs bosons and charginos to quarks and squarks.

## 1 Introduction

The generic Minimal Supersymmetric Standard Model (MSSM) contains a plethora of new sources of flavour violation, which reside in the supersymmetry–breaking sector. The origin of these flavour–violating terms is easily understood: In the Standard Model (SM) the Yukawa matrices are diagonalised by unitary rotations in flavour space and the resulting basis defines the quark mass eigenstates. If the same rotations are carried out on the squark fields of the MSSM, one obtains the super–CKM basis in which no tree–level FCNC couplings are present. However, neither the $3 \times 3$ mass matrices $\tilde{M}_{qLL}^2$ and $\tilde{M}_{qRR}^2$, $q = u, d$, of left–handed and right–handed squarks nor the trilinear Higgs–squark–squark couplings are necessarily diagonal in this basis. The trilinear $\overline{Q} H_d \mathbf{A}^d d_R$ and $\overline{Q} H_u \mathbf{A}^u u_R$ terms induce mixing between left–handed and right–handed squarks after the Higgs doublets $H_d$ and $H_u$ acquire their vacuum expectation values (vevs) $v_d$ and $v_u$, respectively. In the conventions of Ref. [1] the full $6 \times 6$ mass matrix for the down–squarks reads

$$M_{\tilde{d}}^2 = \begin{pmatrix} \left(M_{1L}^{\tilde{d}}\right)^2 & \Delta_{12}^{\tilde{d}LL} & \Delta_{13}^{\tilde{d}LL} & \Delta_{11}^{\tilde{d}LR} & \Delta_{12}^{\tilde{d}LR} & \Delta_{13}^{\tilde{d}LR} \\ \Delta_{12}^{\tilde{d}LL^*} & \left(M_{2L}^{\tilde{d}}\right)^2 & \Delta_{23}^{\tilde{d}LL} & \Delta_{12}^{\tilde{d}RL^*} & \Delta_{22}^{\tilde{d}LR} & \Delta_{23}^{\tilde{d}LR} \\ \Delta_{13}^{\tilde{d}LL^*} & \Delta_{23}^{\tilde{d}LL^*} & \left(M_{3L}^{\tilde{d}}\right)^2 & \Delta_{13}^{\tilde{d}RL^*} & \Delta_{23}^{RL*} & \Delta_{33}^{\tilde{d}LR} \\ \Delta_{11}^{\tilde{d}LR^*} & \Delta_{12}^{\tilde{d}RL} & \Delta_{13}^{\tilde{d}RL} & \left(M_{1R}^{\tilde{d}}\right)^2 & \Delta_{12}^{\tilde{d}RR} & \Delta_{13}^{\tilde{d}RR} \\ \Delta_{12}^{\tilde{d}LR^*} & \Delta_{22}^{\tilde{d}LR*} & \Delta_{23}^{\tilde{d}RL} & \Delta_{12}^{\tilde{d}RR^*} & \left(M_{2R}^{\tilde{d}}\right)^2 & \Delta_{23}^{\tilde{d}RR} \\ \Delta_{13}^{\tilde{d}LR^*} & \Delta_{23}^{\tilde{d}LR^*} & \Delta_{33}^{\tilde{d}LR^*} & \Delta_{13}^{\tilde{d}RR^*} & \Delta_{23}^{\tilde{d}RR^*} & \left(M_{3R}^{\tilde{d}}\right)^2 \end{pmatrix} \tag{1}$$

and the up–type squark mass matrix is defined in an analogous way with $\tilde{d}$ replaced by $\tilde{u}$. Here the $\Delta_{ij}^{\tilde{q}LR}$, $i,j = 1,\dots 3$, are related to the trilinear terms as

$$\Delta_{ij}^{\tilde{d}LR} \;=\; A_{ij}^{d} v_d \;=\; A_{ij}^{d} v\cos\beta, \quad \Delta_{ij}^{\tilde{u}LR} \;=\; A_{ij}^{u} v_u \;=\; A_{ij}^{u} v\sin\beta \qquad \text{for } j > i. \tag{2}$$

We normalise the Higgs vevs as $v = \sqrt{v_u^2 + v_d^2} \simeq 174\,\text{GeV}$ and define $\tan\beta = v_u/v_d$ as usual. The complete squark mass matrix is given in the appendix, where we also elaborate on the relationship between weak bases and the super–CKM basis. The diagonalisation of $M_{\tilde{q}}^2$ involves a rotation of the squark fields in flavour space which leads to various flavour–changing neutral couplings. In particular the gluino now couples to quarks and squarks of different generations and FCNC processes occur through strong gluino–squark loops, which easily dominate over the highly CKM–suppressed weak loops of the SM. Anticipating the smallness of the off–diagonal elements in $M_{\tilde{d},\tilde{u}}^2$ one can alternatively work in the super–CKM basis and treat the $\Delta_{ij}^{\tilde{q}XY}$'s (with $X, Y = L$ or $R$) as perturbations [1–4]. It is customary to define the dimensionless quantities

$$\delta_{ij}^{q\,XY} = \frac{\Delta_{ij}^{\tilde{q}XY}}{\frac{1}{6}\sum\limits_{s}\left[M_{\tilde{q}}^2\right]_{ss}}. \tag{3}$$

In the current era of precision flavour physics stringent bounds on these parameters have been derived from FCNC processes, by requiring that the gluino–squark loops do not exceed the measured values of the considered observables [1, 5–10]. In the recent analysis of Ref. [9] the strongest constraint has been obtained on $\delta_{23}^{\tilde{d}LR}$ with $\left|\delta_{23}^{\tilde{d}LR}\right| < 10^{-3}$ for $\sqrt{\left[M_{\tilde{q}}^2\right]_{ss}} = m_{\tilde{g}} = 350\,\text{GeV}$.

In this paper we show that charged–current processes give competitive bounds on the $\delta_{ij}^{\tilde{q}LR}$'s. This is surprising, because here a supersymmetric loop competes with a SM tree–level coupling. However, the flavour structure of the SM is governed by very small Yukawa couplings: In a weak basis with a diagonal up–type Yukawa matrix $Y^u$ the off–diagonal elements of the down–type Yukawa matrix $Y^d$ range from $|Y_{31}^d| \sim 10^{-7}$ to $|Y_{23}^d| \sim 6\cdot 10^{-4}$ at the relevant scale of $M_{\text{SUSY}} = \mathcal{O}(M_s^{\tilde{q}}, m_{\tilde{g}})$. The impact of supersymmetric loop corrections to these couplings is most easily understood in the decoupling limit $M_{\text{SUSY}} \gg v$: The tree–level and loop–induced Higgs couplings to down–type quarks are shown in Fig. 1. After electroweak symmetry breaking all diagrams contribute to the quark mass matrix. The loop–induced contributions are comparable in size to the tree–level term $Y_{fi}^q v_q$ if roughly $|A_{fi}^q|/M_{\text{SUSY}} \approx 16\pi^2|Y_{fi}^q|$ or (for down-type quarks) $|Y_{fj}^d \Delta_{ji}^{\tilde{d}LL}\mu|\tan\beta/M_{\text{SUSY}}^3 \approx 16\pi^2|Y_{fi}^d|$, respectively. (Here $\mu$ is the Higgsino mass parameter.)

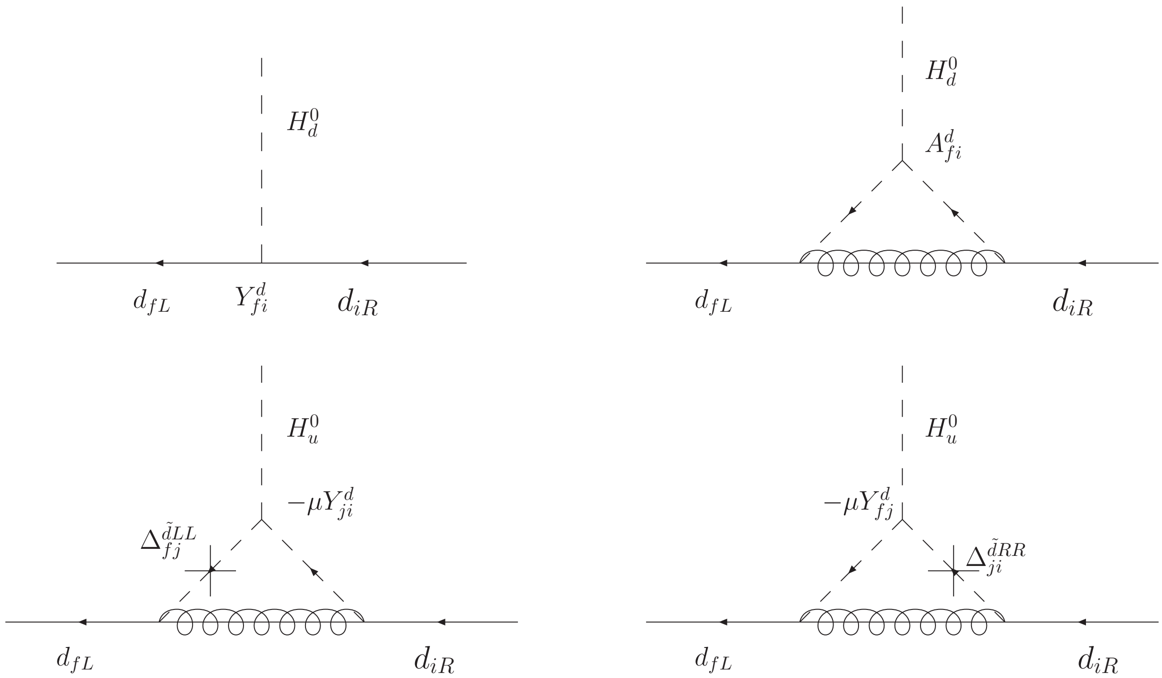


Figure 1: Tree–level coupling with $Y^d_{ij}$ and FCNC loop corrections with $A^d_{fi}$ (upper row) and $\Delta^{\tilde{d}\,LL,RR}_{fi}$ (lower row) in the mass insertion approximation for $M_{\text{SUSY}} \gg v$. Replacing the Higgs fields by their vevs gives the contributions to the down–type quark mass matrix. The lower diagrams contribute to the mass matrix with an enhancement factor of $\tan\beta = v_u/v_d$ compared to the other two contributions.

In Ref. [11] it was pointed out that such corrections constitute an important modification of the relation between the mass and the Yukawa coupling of the bottom quark. Subsequent papers studied the analogous corrections to the whole down-quark mass matrix for the case of minimal flavour violation (MFV), i.e. diagonal matrices $M^2_{\tilde{u}}$ and $M^2_{\tilde{d}}$ in the super-CKM basis [12–15]. Here the key effect of the supersymmetric loop correction is the generation of effective FCNC couplings of neutral Higgs bosons. One can proceed along these lines to calculate the shift in the CKM elements induced by squark-gluino loops: The quark mass matrix calculated from the diagrams in Fig. 1 is diagonalised in the usual way yielding the loop-corrected CKM matrix $V$. This has been done for the MFV case in Ref. [16] and for the generic case in Ref. [17]. As a disadvantage, this method is only valid in the decoupling limit $M_{\text{SUSY}} \gg v$. In particular, this is a questionable approximation for the top quark, whose mass must be set to zero in the diagrams of Fig. 1. Another difficulty is the appearance of the Yukawa matrices in the result of these diagrams, while we only have experimental information on the CKM matrix and the quark masses. To calculate the Yukawa matrices from the latter, one has to invert the relations between the loop-corrected mass matrices and the Yukawa couplings. But in a phenomenological application it is desirable to have the loop-corrected $V$ directly expressed in terms of CKM elements and (s)particle masses. All these drawbacks can be avoided if one renormalises the CKM matrix directly, as it has been done within the Standard Model in Ref. [18]. This method further involves only physical quantities and thereby bypasses another pitfall of the aforementioned calculation from the diagrams in Fig. 1: For instance, one might be tempted to derive strong bounds on $\delta^{\tilde{d}\,RR}_{ij}$ from the lower right diagram of Fig. 1. But the result of this diagram can be absorbed into an unphysical rotation in flavour space of the right-handed quarks and no such bounds can be found.

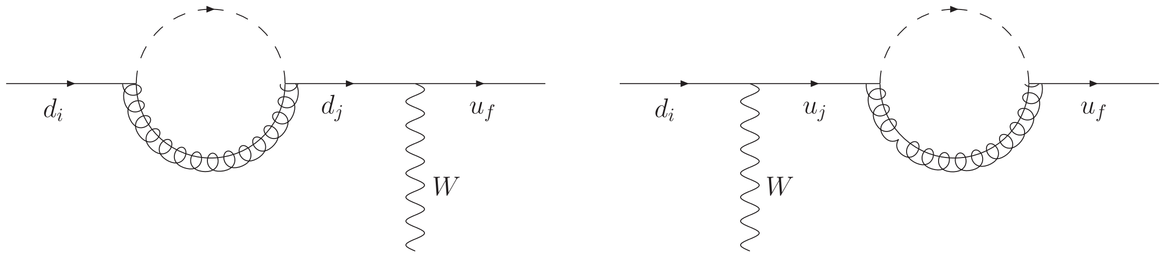


Figure 2: One-loop corrections to the CKM matrix from the down and up sectors. We denote the results of the left and right diagrams by $D_{L\,fi}$ and $D_{R\,fi}$, respectively.

Our paper is organised as follows: In Sect. 2 we calculate the one-loop renormalisation of the CKM matrix by supersymmetric QCD effects. In Sect. 3 we use our results to derive constraints on the elements $\Delta^{\tilde{q}\,LR}_{ij}$ and $\Delta^{\tilde{q}\,LL}_{ij}$ of the squark mass matrices in Eq. (1). Here we also reappraise the idea that flavour violation solely originates from supersymmetry breaking. In Sect. 4 we apply our results to the renormalisation of charged-Higgs and chargino couplings to quarks and squarks. Finally we conclude. Conventions and Feynman rules are collected in an appendix.

## 2 Renormalisation of the CKM matrix

To calculate the desired renormalisation of the CKM matrix we must consider squark-gluino loop corrections to the coupling of the $W$ boson to quarks. There are two possible contributions, the self-energy diagrams of Fig. 2 and the proper vertex correction. In the limit $M_{\text{SUSY}} \gg v$ the self-energy contributions reproduce the results of the diagrams in Fig. 1. (For a discussion of this feature in the MFV case see Refs. [11, 19].) From the considerations in the Introduction we know that we need some parametric enhancement (by e.g. a factor of $|A^q_{fi}|/(M_{\text{SUSY}}|Y^q_{fi}|) \gg 1$) to compensate the loop suppression and the diagrams of Fig. 2 involve such enhancement factors. The vertex diagrams involving a $W$ coupling to squarks are not enhanced and moreover suffer from gauge cancellations with non-enhanced pieces from the self-energies. Therefore we only need to consider self-energies, just as in the case of the electroweak renormalisation of $V$ in the SM [18].

From now on we work in the super-CKM basis unless stated otherwise. Since we work beyond tree-level, we have to clarify how we define the super-CKM basis in the presence of radiative corrections: Starting from some weak basis with Yukawa matrices $\mathbf{Y^d}$ and $\mathbf{Y^u}$ we perform the usual rotations in flavour space

$$d_{L,R} \;\rightarrow\; U^{(0)\,d}_{L,R} d_{L,R}, \qquad\qquad u_{L,R} \;\rightarrow\; U^{(0)\,u}_{L,R} u_{L,R} \tag{4}$$

to diagonalise $\mathbf{Y^q}$ and the tree-level mass matrices $\mathbf{m_q^{(0)}} = \mathbf{Y^q} v_q$ and apply the same rotations to $\widetilde{d}_{L,R}$ and $\widetilde{u}_{L,R}$. This defines the super-CKM basis in which the elements of $M^2_{\tilde{q}}$ in Eq. (1) are defined. The tree-level CKM matrix is then given by

$$V^{(0)} = U^{(0)\,u\dagger}_L U^{(0)\,d}_L \tag{5}$$

To fix the relation of $V^{(0)}$ to the physical CKM matrix $V$ we must define a renormalisation scheme.

First note that all radiative corrections discussed in this paper are finite, so that the notion of minimal renormalisation means that all counter-terms are simply equal to zero. Two possibilities come to mind:

i) Minimal renormalisation of $V$: The Lagrangian contains diagonal Yukawa matrices and $V^{(0)}$ without counter-terms, while the measured CKM matrix $V$ differs from $V^{(0)}$ by the radiative corrections in Fig. 2. Recall that for $m_j \neq m_i$ one can treat the diagrams of Fig. 2 in the same way as genuine vertex corrections, i.e. there is no need to truncate such diagrams or to introduce matrix-valued wave function renormalisations [20].

ii) On-shell renormalisation of $V$: The Lagrangian contains finite counter-terms to cancel the flavour-changing self-energies of Fig. 2. These counter-terms arise from a perturbative unitary rotation of the quark fields in flavour space, $q_{L,R} \to [1 + \delta U^q_{L,R}] q_{L,R}$ [18]. This in turn induces a counter-term

$$\delta V = \delta U_L^{u\dagger} V^{(0)} + V^{(0)} \delta U_L^d \tag{6}$$

to the CKM matrix. In the on-shell scheme we can identify $V = V^{(0)}$, but after the extra rotation of the quark fields we are no more in the super-CKM basis and the bare Yukawa matrices $Y^d$ and $Y^u$ are no more diagonal.[1]

We choose method i), because it involves the super-CKM basis, so that we can immediately use the $\Delta^{\tilde{q}\,XY}_{ij}$'s defined in Eq. (1), permitting a direct comparison with FCNC analyses. This issue of the definition of $\Delta^{\tilde{q}\,XY}_{ij}$ formally goes beyond the one-loop order, but is numerically highly relevant, because the tree-level elements $V^{(0)}_{ij}$ and the finite counter-terms $[\delta U^q_L]_{ij}$ are similar in size: If one works in an alternative basis in which the (s)quark superfields are rotated by $[1 + \delta U^q_{L,R}]$, the off-diagonal elements $\Delta^{\tilde{q}\,XY}_{ij}$ of the squark mass matrices can substantially differ from those of our definition of the super-CKM basis.

We also need to address the renormalisation scheme used for the quark masses: The supersymmetric loops are subtracted on-shell, so that we can use the masses which are extracted from measurements using SM formulae. While we do not consider gluonic QCD corrections in this paper, we assume that an $\overline{\mathrm{MS}}$ prescription is used for the latter. That is, we take $\overline{\mathrm{MS}}$ values for the quark masses in our numerical analyses. This procedure is guided by the decoupling limit discussed in the Introduction: The Yukawa couplings of Fig. 1 enter these diagrams as short-distance quantities defined in a mass-independent scheme such as $\overline{\mathrm{MS}}$ and are evaluated at a scale of order $M_{\mathrm{SUSY}}$. The effective couplings are then evolved down to a low scale at which the quark masses and $V$ are calculated, yielding quark masses in the $\overline{\mathrm{MS}}$ scheme, yet with decoupled (i.e. on-shell subtracted) SUSY loops.

The self-energies can be divided into a helicity-flipping and a helicity-conserving part ($q = u, d$ and $i, f = 1, 2, 3$ labels the incoming and outgoing quark flavours, respectively):

$$\Sigma^q_{fi}(p) = \Sigma^{q\,RL}_{fi}\left(p^2\right) P_L + \Sigma^{q\,LR}_{fi}\left(p^2\right) P_R + \not{p}\left[\Sigma^{q\,LL}_{fi}\left(p^2\right) P_L + \Sigma^{q\,RR}_{fi}\left(p^2\right) P_R\right] \tag{7}$$

Since the SUSY particles are much heavier than the five lightest quarks, it is possible to expand in the external momentum, unless one external quark is the top. In the following we consider the self-energies with only light external quarks and return to the case with a top quark at the end of this section.

[1] That is, in our Feynman diagrammatic approach the FCNC Higgs couplings of Refs. [12–15] enter the Lagrangian through a finite FCNC counter-term to Yukawa couplings.

We now write the result of the left diagram of Fig. 2 (omitting external spinors) as

$$i\frac{g_2}{\sqrt{2}}\gamma^\mu P_L\, D_{L\,fi}$$

with

$$D_{L\,fi} = \sum_{j\neq i} V_{fj} \frac{m_{d_j}\left(\Sigma_{ji}^{d\,RL} + m_{d_i}\Sigma_{ji}^{d\,RR}\right) + m_{d_i}\left(\Sigma_{ji}^{d\,LR} + m_{d_j}\Sigma_{ji}^{d\,LL}\right)}{m_{d_i}^2 - m_{d_j}^2}. \tag{8}$$

The diagram with $j = i$ is treated as in the case without flavour mixing, i.e. the self-energy is truncated and contributes to the LSZ factor in the usual way. The right diagram $D_{R\,fi}$ involving $\Sigma_{fj}^u$ instead is obtained in a similar way. Since the quarks are light compared to the heavy SUSY particles, we can evaluate the self-energies in Eq. (8) at $p^2 = 0$. $\Sigma_{fi}^{q\,RL}$ and $\Sigma_{fi}^{q\,LR}$ have mass dimension 1, while $\Sigma_{fi}^{q\,LL}$ and $\Sigma_{fi}^{q\,RR}$ are dimensionless. The helicity-flipping self-energies involve at least one power of $v$ and at least one factor of a trilinear term or a Yukawa coupling multiplied by $\mu$. To first order in flavour-changing SUSY parameters these factors contribute to $D_L$ with a parametric enhancement of $|A_{ji}^q| v_d/(M_{\mathrm{SUSY}} \max(m_{q_i}, m_{q_j}))$ or (using $m_{q_i}^{(0)} = Y_{ii}^q v_q$) of $|\Delta_{ji}^{\tilde{d}\,LL}\mu| \tan\beta/M_{\mathrm{SUSY}}^3$. Thus we find the enhancement factors which we inferred earlier from Fig. 1. Clearly, the terms with $\Sigma_{fi}^{d\,LL}$ and $\Sigma_{fi}^{d\,RR}$ in Eq. (8) are suppressed by $m_{d_{i,j}}/M_{\mathrm{SUSY}}$ compared to the helicity-flipping contributions and are therefore negligible.

The $LR$ and $RL$ self-energies are

$$\Sigma_{fi}^{q\,RL,LR}(p^2 = 0) = \frac{2m_{\tilde{g}}}{3\pi}\alpha_s(M_{\mathrm{SUSY}}) \sum_{s=1}^{6} V_{s\,fi}^{(0)\,q\,RL,LR} B_0\left(m_{\tilde{g}}, m_{\tilde{q}_s}\right), \tag{9}$$

satisfying $\Sigma_{fi}^{q\,RL}(0) = \Sigma_{if}^{q\,LR*}(0)$. In Eq. (9) we have diagonalised the squark mass matrix, with the eigenvalues denoted by $m_{q_s}$. The quantities $V_{s\,fi}^{q\,RL}$ and $V_{s\,fi}^{q\,LR}$ are combinations of the rotation matrices of quarks and squarks fields and are defined in Eq. (28) of the appendix, where also our conventions for the loop functions $B_0$, $C_0$ and $D_0$ are listed. In the second order of the mass insertion approximation (neglecting terms with more than one flavour change) Eq. (9) becomes (for $f \neq i$)

$$\begin{aligned}
\Sigma_{fi}^{q\,LR}(p^2 = 0) \;=\; & \alpha_s(M_{\mathrm{SUSY}})\frac{2m_{\tilde{g}}}{3\pi}\Big[\Delta_{fi}^{\tilde{q}\,LR} C_0\left(m_{\tilde{g}}, \left[M_{\tilde{q}}^2\right]_{ff}, \left[M_{\tilde{q}}^2\right]_{i+3,i+3}\right) \\
& + \Delta_{fi}^{\tilde{q}\,LL}\,\Delta_{ii}^{\tilde{q}\,LR}\, D_0\left(m_{\tilde{g}}, \left[M_{\tilde{q}}^2\right]_{ff}, \left[M_{\tilde{q}}^2\right]_{ii}, \left[M_{\tilde{q}}^2\right]_{i+3,i+3}\right) \\
& + \Delta_{ff}^{\tilde{q}\,LR}\,\Delta_{fi}^{\tilde{q}\,RR}\, D_0\left(m_{\tilde{g}}, \left[M_{\tilde{q}}^2\right]_{ff}, \left[M_{\tilde{q}}^2\right]_{f+3,f+3}, \left[M_{\tilde{q}}^2\right]_{i+3,i+3}\right)\Big].
\end{aligned} \tag{10}$$

Even if the FCNC elements $\Delta_{fi}^{\tilde{q}\,LL}$ are small, the approximation of Eq. (10) breaks down, if $\Delta_{jj}^{\tilde{q}\,LR}$ is of the order of $(M_{jL,R}^{\tilde{q}})^2$ for either $j = i$ or $j = f$, i.e. for large flavour-diagonal left-right mixing. This is the default situation for the top squarks and also happens with the bottom squarks if $\tan\beta$ is large. We therefore work with the exact formula of Eq. (9).

We now collect the results of the diagrams in Fig. 2 with the simplification that we neglect all small ratios of quark masses such as $m_s/m_b$. One finds

$$D_{L\,fi} = \sum_{j=1}^{3} V_{fj}^{(0)} \left[\Delta U_L^d\right]_{ji}, \qquad D_{R\,fi} = \sum_{j=1}^{3} \left[\Delta U_L^{u\dagger}\right]_{fj} V_{ji}^{(0)} \tag{11}$$

with the matrices

$$\Delta U_L^q = \begin{pmatrix} 0 & \frac{1}{m_{q_2}} \Sigma_{12}^{q\,LR} & \frac{1}{m_{q_3}} \Sigma_{13}^{q\,LR} \\ \frac{-1}{m_{q_2}} \Sigma_{21}^{q\,RL} & 0 & \frac{1}{m_{q_3}} \Sigma_{23}^{q\,LR} \\ \frac{-1}{m_{q_3}} \Sigma_{31}^{q\,RL} & \frac{-1}{m_{q_3}} \Sigma_{32}^{q\,RL} & 0 \end{pmatrix} \tag{12}$$

In our super-CKM scheme i) the inclusion of the radiative correction is equivalent to the use of the tree-level coupling in the $\bar{u}W^+d$ vertex with the replacement

$$V^{(0)} \to V = \left(1 + \Delta U_L^{u\dagger}\right) V^{(0)} \left(1 + \Delta U_L^d\right) \tag{13}$$

and we identify $V$ with the physical CKM matrix. In the on-shell scheme ii) the counter-terms $\delta U_L^d = -\Delta U_L^d$ and $\delta U_L^u = -\Delta U_L^u$ cancel the loops and $V^{(0)} = V$ is maintained. It is crucial that $1 + \delta U_L^q$ is unitary, otherwise the unitarity of $V$ (and electroweak gauge invariance) would be spoiled [18]. To our one-loop order this means that $\delta U_L^q$ is anti-hermitian. We can easily verify from Eq. (12) that $\Delta U_L^d$ fulfills this criterion owing to $\Sigma_{fi}^{q\,RL}(0) = [\Sigma_{if}^{q\,LR}(0)]^*$. The self-energies do not decouple for $M_{\mathrm{SUSY}} \to \infty$ and, in accordance with the decoupling theorem [21], we find that their mere effect is the renormalisation of the CKM matrix, as implemented in scheme ii).

It is important to stress that the replacement rule in Eq. (13) only absorbs the effects of the self-energy diagrams of Fig. 2 correctly, if both quark lines are external lines. If some $\overline{u}_f W^+ d_i$ vertex appears in a loop diagram, one or both self-energies are probed off-shell and one must work with $V^{(0)}$ and must calculate the loop diagram with the nested self-energy explictly.

We can now understand how to treat self-energies with a top quark in Eq. (12): If the top quark appears on the internal line of the right diagram in Fig. 2, that is $j = 3$, the self-energy involved must be evaluated at $p^2 = 0$, because the external quark is up or charm. The unitarity of $V$ now forces us to evaluate $\Sigma_{31}^{u\,RL}$ and $\Sigma_{32}^{u\,RL}$ at $p^2 = 0$ as well. Interestingly, from today's precision data in K and B physics one can determine $V$ from tree-level data only [22]. Of course, none of these measurements involves top decays, so that the values of $V_{ts}$ and $V_{td}$ inferred from these measurements (through unitarity of $V$) indeed correspond to the definition in Eq. (13), with self-energies $\Sigma_{3i}^{u\,RL}$ evaluated at $p^2 = 0$. While FCNC processes of K and B mesons involve $V_{ts}$ or $V_{td}$, we cannot determine these CKM elements from FCNC processes in a model-independent way, because new particles (in our case squarks and gluinos) will affect the FCNC loops directly. Clearly nothing can be learned from measuring the $\bar{t}W^+d_i$ couplings (in, for instance, single top production or top decays) if $M_{\mathrm{SUSY}} \gg m_{u_3} = m_t$. However, if $m_t \sim M_{\mathrm{SUSY}}$ any on-shell $t \to s$ or $t \to d$ transition involves

$$\Delta\sigma_i^t \equiv \frac{\Sigma_{3i}^{u\,RL}(m_t^2) - \Sigma_{3i}^{u\,RL}(0)}{m_t} \qquad \text{with } i = 1 \text{ or } 2. \tag{14}$$

Here the first self-energy enters the calculated $\bar{t}W^+d_i$ process explicitly, while $\Sigma_{3i}^{u\,RL}(0)$ stems from the relationship between $V$ and $V^{(0)}$. $\Delta\sigma_i^t$ decouples as $m_t^2/M_{\mathrm{SUSY}}^2$, but can be sizable for

$\mathcal{O}(200\,\mathrm{GeV})$ superpartners, since it involves poorly-constrained FCNC squark mass terms. We conclude that the flavour structure of tree-level top couplings can help to study new physics entering helicity-flipping self-energies, while this effect is unobservable in charged-current processes of light quarks: Here the helicity-flipping self-energies merely renormalise the CKM matrix; the physical effect in a charged-current processes with external quark $q$ is suppressed by a factor of $m_q^2/M_{\mathrm{SUSY}}^2$. The experimental signature would be an apparent violation of CKM unitarity, since the measured value of $V_{ts}$ or $V_{td}$ would be in disagreement with the value inferred from CKM unitarity. Unitarity is restored, once the correction $\Delta\sigma_i^t$ is taken into account.

Since inverse quark masses enter Eq. (12), we must address the proper definition of these masses in the presence of ordinary QCD corrections. If we worked in the decoupling limit and calculated the diagrams of Fig. 1, we would encounter the $\overline{\mathrm{MS}}$-renormalised Yukawa couplings evaluated at the renormalisation scale $Q = M_{\mathrm{SUSY}}$, at which the heavy SUSY particles are integrated out. Translating that result into the language of Sect. 2 amounts to the evaluation of the inverse quark masses in the $\overline{\mathrm{MS}}$ scheme at $Q = M_{\mathrm{SUSY}}$. One can derive this (somewhat surprising) result entirely in the diagrammatic language of Sect. 2, by studying QCD corrections to the diagrams of Fig. 2 [23]. The first element in this proof is the observation that e.g. $\Sigma_{fi}^{q\,LR}$, viewed as the Wilson coefficient of the two-quark operator $\overline{q}_f P_R q_i$, renormalises in the same way as the quark mass, so that the ratios $\Sigma_{fi}^{q\,LR}/m_{q_i}$ in Eq. (12) are independent of $Q$. Since the SUSY parameters entering $\Sigma_{fi}$ are defined at the high scale $Q = M_{\mathrm{SUSY}}$, our constraints derived in the next section will involve $m_{q_i}(M_{\mathrm{SUSY}})$. The second element in the proof is the explicit analysis of gluonic corrections to the diagrams of Fig. 2. While at intermediate steps a quark pole mass enters through the Dirac equation $\not{p}\, q_i = m_{q_i}^{\mathrm{pole}} q_i$, gluonic self-energies add to $m_{q_i}^{\mathrm{pole}}$ in such a way that the final result only involves the properly defined $\overline{\mathrm{MS}}$ mass $m_{q_i}$, see [23].

We close this section by recalling the relationship between the Yukawa matrices $\mathbf{Y}^q = \mathrm{diag}\,(Y^{q_1}, Y^{q_2}, Y^{q_3})$ and the quark masses [11, 19]:

$$Y^{q_i} \;=\; \frac{m_{q_i}}{v_q\,(1+\Delta_{q_i})}. \tag{15}$$

For down-type quarks $\Delta_{q_i}$ is enhanced by a factor of $\tan\beta$ and can be of order 1 for $\tan\beta \sim 50$. In the case of up-type quarks the correction $\Delta_{u_i}$ is enhanced if $A_{ii}^u/m_{q_i}$ is large. Eq. (15) holds in the super-CKM scheme i), which has the advantage that no FCNC Yukawa couplings occur. In the on-shell scheme ii) the rotations of the quark fields in flavour space lead to the loop-induced finite FCNC Yukawa couplings of Refs. [12–15]. In the super-CKM scheme these effects are reproduced from diagrams with flavour-diagonal Yukawa couplings and FCNC self-energies. Finally note that $\Delta_{q_i}$ can be complex, so that the entries of $\mathbf{Y}^q$ (and $m_q^{(D)} = \mathbf{Y}^q v_q$ entering the squark mass matrices in Eq. (30)) are not necessarily real.

## 3 Numerical Analysis

Large accidental cancellations between the SM and supersymmetric contributions are, as already mentioned in the introduction, unlikely and from the theoretical point of view undesirable. Requiring the absence of such cancellations is a commonly used fine-tuning argument, which is also employed in standard FCNC analyses of the $\delta_{ij}^{q\,XY}$'s [1, 5–10]. Analogously, we assume that the corrections due to flavour-changing SQCD self-energies do not exceed the experimentally measured values for the CKM matrix elements quoted in the Particle Data Table (PDT) [24]. To this end we set the tree-level CKM matrix $V^{(0)}$ (working in our super-CKM scheme i)) equal to the unit matrix and generate the measured values radiatively. For simplicity, we consider the up and down sector separately. We parameterise the squark mass matrix according to Eq. (1). If not mentioned otherwise, we choose all diagonal elements of the squark mass matrix to be equal and denote them by $m_{\tilde{q}}^2$. For small off-diagonal elements $m_{\tilde{q}}$ is in good approximation equal to the physical squark mass.

The quantities $\delta_{ij}^{q\,LR}$ and $\delta_{ij}^{q\,RL}$ as defined in Eq. (3) violate SU(2) and are proportional to the vacuum expectation value (vev) of a Higgs field. This means that if one scales the SUSY spectrum by a common factor $a$, the helicity-flipping elements $\delta_{ij}^{q\,LR,RL}$ will scale like $1/a$ rather than staying constant as one might naively expect for a dimensionless quantity. This leads to somewhat counter-intuitive bounds on $\delta_{ij}^{q\,LR}$ and $\delta_{ij}^{q\,RL}$, which become stronger for a larger scale parameter $a$. However, the inferred bounds on the trilinear terms $A_{ij}^q$ stay constant for $a \gg 1$ as expected for a bound derived from a non-decoupling quantity. Since it is customary in nearly all treatments of non-minimal flavour violation to constrain $\delta_{ij}^{q\,XY}$ [1,5–10], we will also follow this convention rather than quoting constraints on $A_{ij}^q$.

We use the standard parameterisation for the CKM matrix and quark masses defined in the $\overline{\mathrm{MS}}$ scheme with the central PDT values [24]. As discussed at the end of Sect. 2, the masses enter the loop contributions to $V$ in Eq. (12) at the renormalisation scale $Q = M_{\mathrm{SUSY}}$, at which the self-energies are calculated.

Because of the V–A structure of the W vertex only self-energies $\Sigma_{fi}^{q\,LR}$ with $f < i$ enter the renormalisation of the CKM matrix in the approximation $m_{q_i} = 0$ (see Eq. (12)). This implies that for $f < i$ only $\delta_{fi}^{q\,LR}$ is constrained in the leading order of the mass insertion approximation in Eq. (10). In order to constrain $\delta_{fi}^{q\,LL}$ or $\delta_{fi}^{q\,RR}$ an additional insertion of some $\delta_{jk}^{q\,LR}$ is needed. Even three insertions are needed to involve $\delta_{fi}^{q\,RL}$. These diagrams with multiple insertions of $\delta_{jk}^{q\,XY}$ can give useful bounds on one of these quantities only if the $\delta_{jk}^{q\,LR}$ providing the needed helicity-flip is large. The only candidate is the flavour-diagonal mass insertion $\delta_{jj}^{q\,LR}$ and the corresponding contributions to $\Sigma_{fi}^{q\,LR}$ can be read off from the last two terms in Eq. (10). Indeed, we find useful bounds on $|\delta_{13}^{d\,LL}|$ in the large-$\tan\beta$ scenario, where $\delta_{33}^{d\,LR}/m_{d_3}$ is large. The analogous contributions involving $\delta_{ij}^{d\,RR}$ are suppressed with respect to those with $\delta_{ij}^{d\,LL}\delta_{jj}^{d\,LR}$ by a small ratio of quark masses. The upper limits on $\delta_{jk}^{q\,RR}$ and $\delta_{jk}^{q\,RL}$ from vacuum stability [26], electric dipole moments and FCNC processes are stronger than ours.

We use the following input parameters [24]:

$$\begin{aligned}
\overline{m}_s(2\,\mathrm{GeV}) &= 0.095\,\mathrm{GeV}, & \overline{m}_c(\overline{m}_c) &= 1.25\,\mathrm{GeV}, \\
\overline{m}_b(\overline{m}_b) &= 4.2\,\mathrm{GeV}, & \overline{m}_t(\overline{m}_t) &= 166\,\mathrm{GeV}, \\
|V_{us}| &= 0.227, \quad |V_{ub}| = 0.00396, & |V_{cb}| &= 0.0422.
\end{aligned} \tag{16}$$

## 3.1 Down-sector

We present our bounds on $|\delta_{ij}^{d\,LR}|$ and $|\delta_{ij}^{d\,LL}|$ in Sects. 3.1.1 and 3.1.2, respectively.

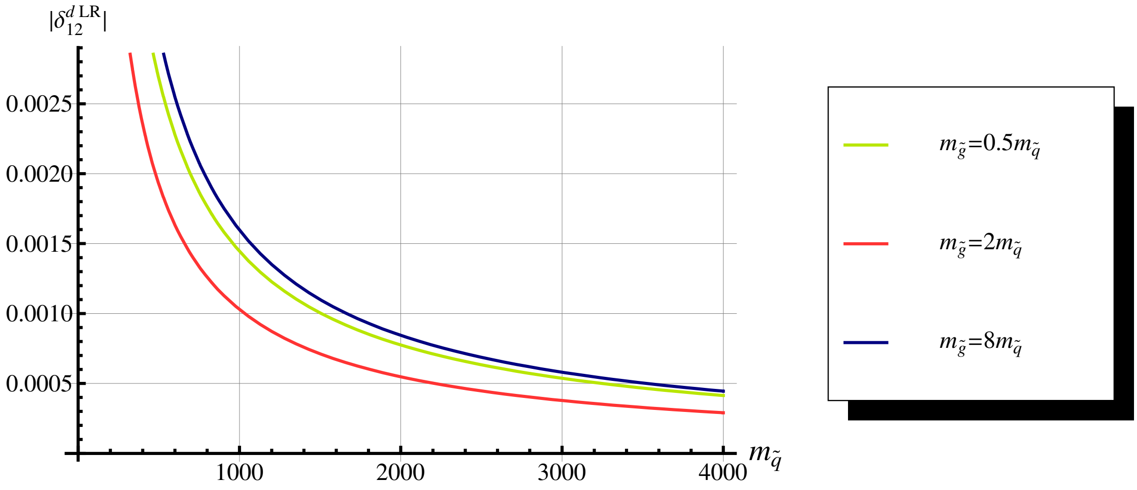


Figure 3: Constraints on $|\delta_{12}^{d\,LR}|$ from $V_{us}$ or $V_{cd}$ as a function of the squark mass. The constraints become stronger with growing $M_{\mathrm{SUSY}}$, because $\delta_{ij}^{q\,LR} \propto \dfrac{v}{M_{\mathrm{SUSY}}}$.

#### 3.1.1 Constraints on $|\delta_{\mathbf{ij}}^{\mathbf{d\,LR}}|$

**Constraints from $\mathbf{V_{us}}$,$\mathbf{V_{cd}}$:** $V_{us}$ and $V_{cd}$ are experimentally well known. Their absolute values are nearly equal and they have opposite sign in the standard parameterisation, which is respected by the corrections Eq. (12). Fig. 3 shows the dependence of the constraints on the squark mass with different ratios $m_{\tilde{g}}/m_{\tilde{q}}$. In the approximation $m_{d_1} = 0$, only $\delta_{12}^{d\,LR}$ is constrained. Looking at the dependence on the gluino mass (Fig. 4), it is interesting to note that there is a minimum for $m_{\tilde{g}} \approx 1.5 m_{\tilde{q}}$.

**Constraints from $\mathbf{V_{cb}}$,$\mathbf{V_{ts}}$:** In this case, the situation is nearly the same as in the case of $V_{us}$, except that the constraints are weaker (see Fig. 5), because $m_b$ is much larger than $m_s$. $|V_{ts}|$ is essentially fixed by the measured value of $|V_{cb}|$ through CKM unitarity.

**Constraints from $\mathbf{V_{ub}}$:** The last pair of of CKM elements to be discussed is $(V_{ub}, V_{td})$. In this case $|V_{ub}|$ does not fix $|V_{td}|$, but $|V_{td}|$ is largely affected by the CKM phase. Now $|V_{ub}|$ is experimentally better known than $|V_{td}|$, because $V_{td}$ is extracted from FCNC loop processes by comparing the corresponding experimental result with the SM prediction. In beyond-MFV scenarios, this is not a valid procedure anymore, because the new particles will alter the FCNC loop processes. Therefore we can only exploit the constraint from $|V_{ub}|$. The anti-hermiticity of $\Delta U_L^d$ in Eq. (12) implies that $\delta_{13}^{d\,LR}$ gives a negative contribution of the same size to $V_{td}$. This cannot be the whole contribution, since $|V_{ub}| \neq |V_{td}|$ and in the standard CKM parameterisation $\mathrm{Re}\, V_{td}$ and $\mathrm{Re}\, V_{ub}$ are both positive. The hierarchical structure of the CKM matrix is responsible for a second contribution: Since $V_{ub}$ and $V_{td}$ are of third order in the Wolfenstein parameter $\lambda$, the two-loop process in Fig. 7 involving the loop-contributions to $V_{us} \propto \lambda$ and $V_{cb} \propto \lambda^2$ is important as well. This diagram adds a contribution of $V_{us}V_{cb} = 0.0088$ to $V_{td}$. Together with the one-loop contribution from $\delta_{13}^{d\,LR}$, this yields the correct value for $V_{td}$. We stress that this does not imply any additional constraint on the SUSY parameters entering the self-energies, to order $\lambda^3$ we just reproduced a unitarity relation of the CKM elements:

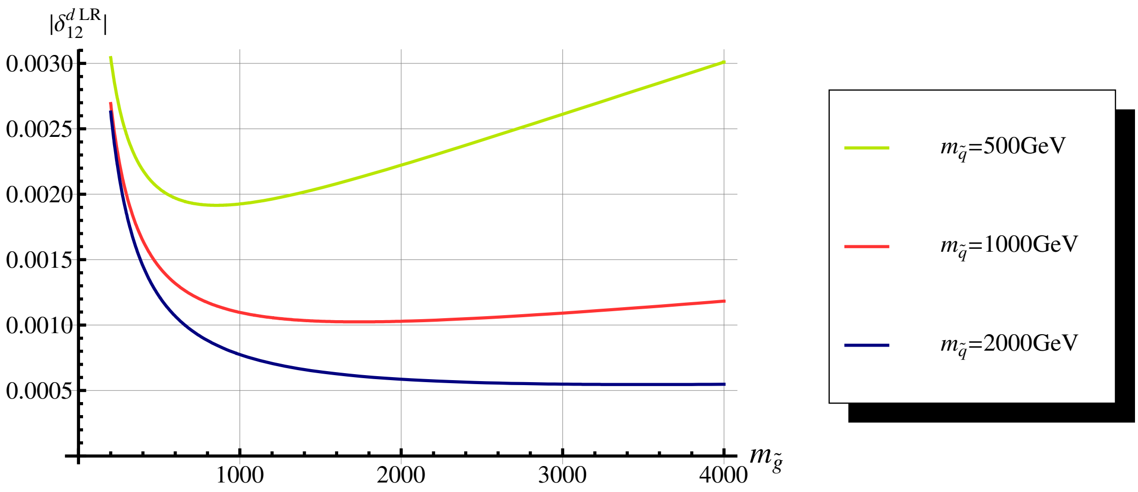


Figure 4: Constraints on $\delta_{12}^{d\,LR}$ from $V_{us}$ (or $V_{cd}$) as a function of the gluino mass.

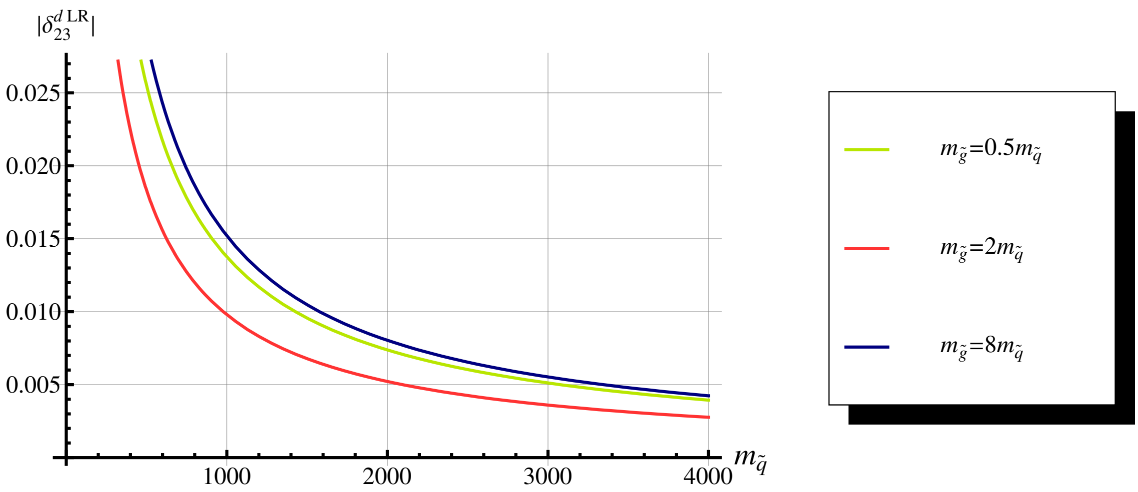


Figure 5: Constraints on $\delta_{23}^{d\,LR}$ from $|V_{cb}|$ (or $|V_{ts}|$) as a function of the squark mass.

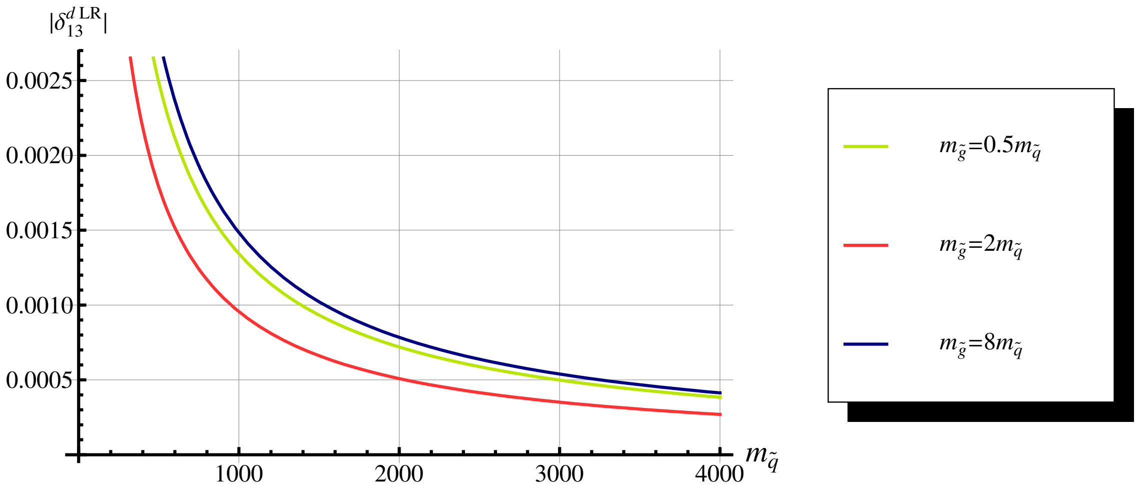


Figure 6: Constraints on $\delta_{13}^{d\,LR}$ from $|V_{ub}|$ as a function of the squark mass.

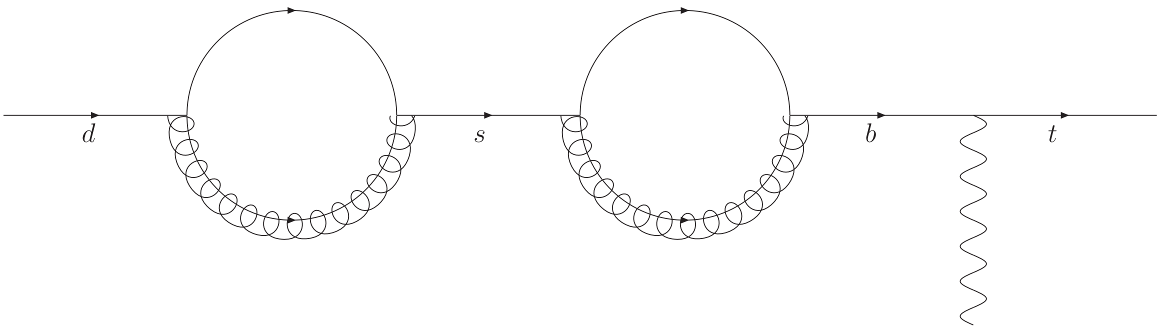


Figure 7: Two-loop correction to $V_{td}$.

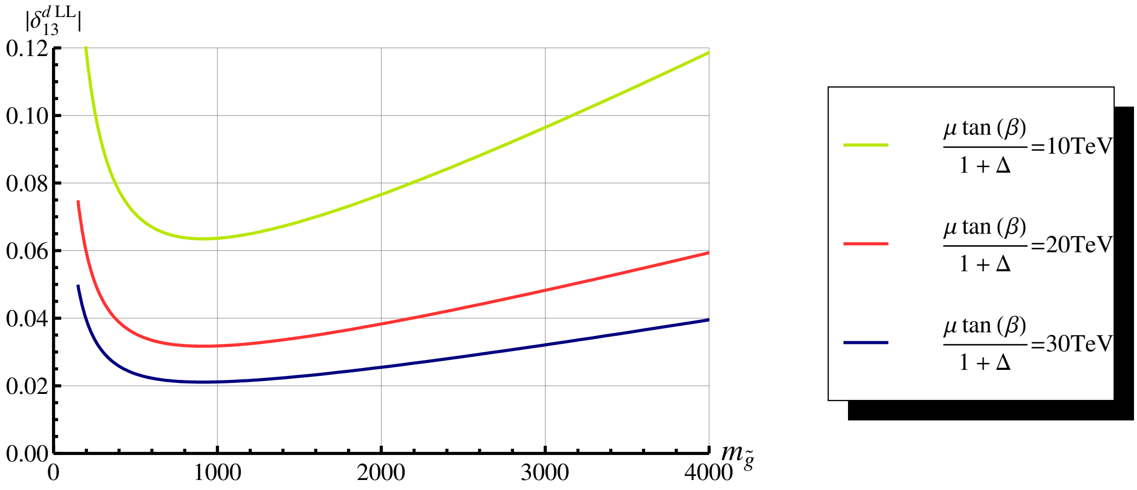


Figure 8: Constraints on $\delta_{13}^{d\,LL}$ from $|V_{ub}|$ as a function of the gluino mass for different values of $\mu \tan\beta$.

$V_{td} = -V_{ub}^* + V_{cd}V_{ts} \simeq -V_{ub}^* - V_{us}^* V_{ts}$, which (with insertion of $V_{tb} \simeq V_{ud} \simeq 1$) equals the product of the first and third rows of the CKM matrix.

The last possible constraint is the phase of the CKM matrix, which one could infer from $\gamma = \arg(-V_{ub}^* V_{ud}/(V_{cb}^* V_{cd}))$. However, since $\gamma$ is large, no fine-tuning argument can be applied to derive bounds. Only in a given scenario of radiatively generated CKM elements, the measured value of $\gamma$ can be used to derive a constraint on the complex phases in the mass matrix of Eq. (1).

### 3.1.2 Constraints on $\delta_{\mathrm{ij}}^{\mathrm{d\,LL}}$

In the presence of large helicity-flipping flavour-diagonal elements in the squark mass matrix, also $\delta_{ij}^{q\,LL}$ can be constrained. This is the case for large $A_{jj}^q$ terms or (if $q = d$) for a large value of $\mu \tan\beta$. Here we only consider the second possibility, which is widely studied in the literature. The strongest constraints are obtained for $\delta_{13}^{d\,LL}$, because $V_{ub}$ is the smallest entry of the CKM matrix. We have included the correction term $\Delta_b$ of Eq. (15) in our analysis. Our result is shown in Fig. 8. Our constraint is compatible with the experimental bound on $Br(B_d \to \mu^+\mu^-)$ for values of $\tan\beta$ around 30 or below [25].

We next discuss the constraint on $\delta_{23}^{d\,LL}$: It is clear that our bound will be looser by a factor of $|V_{cb}/V_{ub}|$. Furthermore, for large $\tan\beta$ and typical values of the massive SUSY parameters we find $Br(B_s \to \mu^+\mu^-)$ more constraining. To find bounds on $\delta_{23}^{d\,LL}$ from $|V_{cb}|$ which comply with $Br(B_s \to \mu^+\mu^-)$ we need a smaller value of $\tan\beta$ around 20 and therefore a quite large value for $\mu$, if the masses of the non-standard Higgs bosons are around 500 GeV. We do not include a bound on $\delta_{23}^{d\,LL}$ in our table of results in Sect. 3.3.

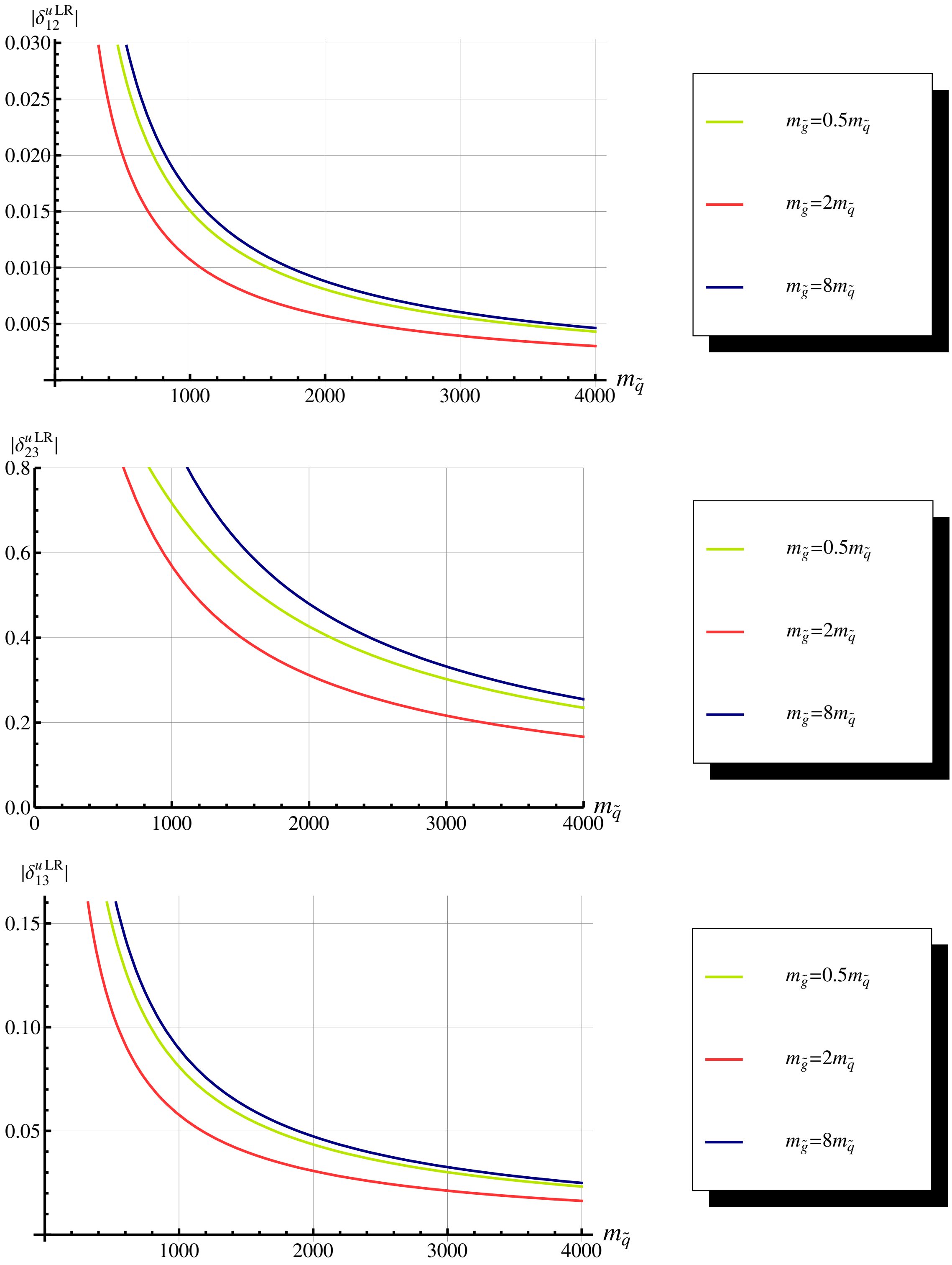


Figure 9: Constraints on $\delta_{12}^{u\,LR}$, $\delta_{23}^{u\,LR}$ and $\delta_{13}^{u\,LR}$ as a function of the squark mass.

### 3.2 Up-sector

In the up-sector, everything is in straight analogy to the down-sector. The only difference is that the constraints are weaker because of the larger charm and top masses. But the upper bounds are still restrictive, except the ones obtained from $V_{ts}$ or $V_{cb}$ (see Fig. 9). Remarkably, we now have a powerful constraint on $\delta_{13}^{u\,LR}$ from the second diagram in Fig. 2 with $u_f = u$ and $d_i = b$.

### 3.3 Comparison with previous bounds

In this section we compare our bounds with those in the literature, derived from FCNC processes [6, 8–10] and vacuum stability (VS) bounds [26]. We take $M_{\mathrm{SUSY}} = \sqrt{\left[M_{\tilde{q}}^2\right]_{ss}} = m_{\tilde{g}} = 1000\,\mathrm{GeV}$:

| quantity | our bound | bound from FCNC's | | bound from VS [26] |
|---|---|---|---|---|
| $\lvert\delta_{12}^{d\,LR}\rvert$ | $\leq 0.0011$ | $\leq 0.006$ | $K$ mixing [6] | $\leq 1.5\,\times\,10^{-4}$ |
| $\lvert\delta_{13}^{d\,LR}\rvert$ | $\leq 0.0010$ | $\leq 0.15$ | $B_d$ mixing [8] | $\leq 0.05$ |
| $\lvert\delta_{23}^{d\,LR}\rvert$ | $\leq 0.010$ | $\leq 0.06$ | $B \to X_s\gamma; X_s l^+ l^-$ [9] | $\leq 0.05$ |
| $\lvert\delta_{13}^{d\,LL}\rvert$ | $\leq 0.032$ | $\leq 0.5$ | $B_d$ mixing [8] | – |
| $\lvert\delta_{12}^{u\,LR}\rvert$ | $\leq 0.011$ | $\leq 0.016$ | $D$ mixing [10] | $\leq 1.2\,\times\,10^{-3}$ |
| $\lvert\delta_{13}^{u\,LR}\rvert$ | $\leq 0.062$ | — | | $\leq 0.22$ |
| $\lvert\delta_{23}^{u\,LR}\rvert$ | $\leq 0.59$ | — | | $\leq 0.22$ |

Our value for $\delta_{13}^{d\,LL}$ is calculated with $\dfrac{\mu\,\tan\beta}{1+\Delta_b} = 20\,\mathrm{TeV}$. The quoted bound on $\delta_{23}^{d\,LR}$ from $b \to s\gamma$ and $B \to X_s\,l^+l^-$ has been rescaled by an approximate factor of 3 from the value quoted for $M_{\mathrm{SUSY}} = 350\,\mathrm{GeV}$ in Ref. [9]. The VS bounds on $\delta_{ij}^{u\,LR}$ have also been obtained by scaling the quoted values for $M_{\mathrm{SUSY}} = 500\,\mathrm{GeV}$ of Ref. [26] by a factor of $1/2$. The VS bounds on $\delta_{13}^{u\,LR}$ and $\delta_{23}^{u\,LR}$ are obtained by multiplying the bound on $\delta_{12}^{u\,LR}$ with $m_t/m_c$. FCNC effects are decoupling and scale as $1/M_{\mathrm{SUSY}}^2$, but the constraints on $\delta_{ij}^{q\,LR}$ are proportional to $M_{\mathrm{SUSY}}$ rather than $M_{\mathrm{SUSY}}^2$, because the definition of $\delta_{ij}^{q\,LR}$ involves a factor of $v/M_{\mathrm{SUSY}}$. Both our constraints and the VS bounds on the trilinear SUSY-breaking terms are independent of $M_{\mathrm{SUSY}}$ (i.e. non-decoupling), so that the bounds on $\delta_{ij}^{q\,LR}$ scale like $1/M_{\mathrm{SUSY}}$. We conclude that all our bounds on $\delta_{ij}^{d\,LR}$ are more restrictive than those from FCNC processes for $M_{\mathrm{SUSY}} \geq 500\,\mathrm{GeV}$, and our bound on $\delta_{12}^{u\,LR}$ is stronger than the quoted FCNC bound for $M_{\mathrm{SUSY}} \geq 900\,\mathrm{GeV}$.

Substantially stronger bounds than ours are only listed for the VS bounds on $\lvert\delta_{23}^{u\,LR}\rvert$, $\lvert\delta_{12}^{u\,LR}\rvert$ and $\lvert\delta_{12}^{d\,LR}\rvert$. However, the VS bounds related to the latter two quantities are of the form

$$A_{12}^{q} \quad < \quad Y^{q_2}\,f, \tag{17}$$

where $f = \mathcal{O}(M_{\mathrm{SUSY}})$ depends on other massive parameters of the scalar potential. The bounds are obtained by studying the scalar potential at tree level and $Y^{q_2}$ enters the analysis through the quartic coupling of strange squarks to Higgs bosons. The smallness of $Y^{q_2}$ makes this coupling sensitive to large loop corrections and the quoted bounds have to be considered as rough estimates at best. Our results for $\delta^{q\,LR}_{12}$ rest on a firmer footing.

### 3.4 Supersymmetry breaking as the origin of flavour?

The smallness of the Yukawa couplings of the first two generations (and possibly also of the bottom and tau couplings) suggest the idea that Yukawa couplings are generated through radiative corrections [27]. In the context of supersymmetric theories these loop-induced couplings arise from the diagrams of Fig. 1 or, in our approach, of Fig. 2 [28]. The B factories have confirmed the CKM mechanism of flavour violation, leaving little room for new sources of FCNC's. This seriously challenges the idea that flavour violation stems from the same source as supersymmetry breaking. The surprising discovery of Sect. 3 is the finding that this idea is still viable, with SUSY masses well below 1 TeV, if the sources of flavour violation are the trilinear terms $A^q_{ij}$. (Note that the VS bound on $|\delta^{u\,LR}_{23}|$ poses no problem, because one can generate $V_{cb}$ entirely from $\delta^{d\,LR}_{23}$.) Of course, the heaviness of the top quark requires a special treatment of $Y^t$ and the successful bottom-tau Yukawa unification suggest to keep tree-level Yukawa couplings for the third generation. This scenario has been studied in Ref. [17], where possible patterns of the dynamical breaking of flavour symmetries are discussed.

In the modern language of Refs. [29, 30] the global $[U(3)]^3$ flavour symmetry[2] of the gauge sector is broken to $[U(2)]^3 \times U(1)$ by the Yukawa couplings of the third generation. Here the three $U(2)$ factors correspond to rotations of the left-handed doublets and the right-handed down-type and up-type singlets of the first two generations in flavour space, respectively. That is, our starting point is $\mathbf{Y^u} = \text{diag}\,(0,0,Y^t)$, $\mathbf{Y^d} = \text{diag}\,(0,0,Y^b)$. We next assume that the soft breaking terms $\Delta^{\tilde{q}\,LL}_{ij}$ and $\Delta^{\tilde{q}\,RR}_{ij}$ possess the same flavour symmetry as the Yukawa sector, which implies that $\Delta^{\tilde{q}\,LL}$ and $\Delta^{\tilde{q}\,RR}$ are diagonal matrices with the first two entries being equal. That is, flavour universality holds for the first two generations. From a model-building point of view, it maybe easier to motivate that at some scale $\Delta^{\tilde{q}\,LL}$ and $\Delta^{\tilde{q}\,RR}$ are proportional to the unit matrix, meaning that they possess the full $[U(3)]^3$ flavour symmetry of the gauge sector. However, below this fundamental scale this symmetry will be reduced to $[U(2)]^3 \times U(1)$ by renormalisation group (RG) effects from the Yukawa sector and we restrict our discussion to the $[U(2)]^3 \times U(1)$ case here. Now we assume that the trilinear terms $A^q_{ij}$ are the spurion fields which break $[U(2)]^3 \times U(1)$ down to the $U(1)_B$ baryon number symmetry. (After including the lepton sector the remaining anomaly-free symmetry group is $U(1)_{B-L}$.) Note that RG effects do not destroy the symmetry of the Yukawa sector, because the soft terms do not mix into the Yukawa couplings. By contrast, in the standard MFV scenario [30] the role of flavour-symmetric and symmetry-breaking terms is interchanged, and the latter (here the Yukawa couplings) mix into the former. Still our scenario is not completely RG-invariant, because the trilinear terms mix into $\Delta^{\tilde{q}\,RR}_{ij}$ and $\Delta^{\tilde{q}\,LL}_{ij}$. We next apply the rotations of Eq. (4) to the quark supermultiplets of the first two generations to render the upper left $2 \times 2$ submatrix of $\mathbf{A^q}$ diagonal and real. That is, the Cabibbo angle arises from the misalignment of $\mathbf{A^u}$ with $\mathbf{A^d}$ in flavour space. The $u$, $d$, $s$ and $c$ quark masses all arise from supersymmetric self-energies involving $A^q_{ii}$. Note that the RG evolution destroys the $[U(2)]^3 \times U(1)$ symmetry of $\Delta^{\tilde{q}\,RR}_{ij}$ and $\Delta^{\tilde{q}\,LL}_{ij}$ due to $A^q_{11} \neq A^q_{22}$, but only generates off-diagonal $\Delta^{\tilde{q}\,LL}_{12}$ terms through tiny electroweak loops as in MFV scenarios. The vacuum stability bound in the first row of the table in Sect. 3.3 is absent and the corresponding bounds on the diagonal elements $A^q_{ii}$ can only be obtained after loop corrections to the scalar potential are included. The CKM elements of

[2] We only discuss the quark sector here.

the third row and column are obtained by calculating $\Delta U_L^q$ of Eq. (12).

The idea that flavour violation is a collateral damage of supersymmetry breaking is not only economical, it also solves one of the most urgent problems of the MSSM: It was pointed out in [31] that the phase alignment between $A_{ii}^q$ and the radiatively generated quark masses suppresses the supersymmetric contribution to the neutron electric dipole moment. Since $Y^{d_1} = 0$, the phase of $\mu$ does not enter the neutron EDM at the one-loop level.

# 4 Charged-Higgs and chargino couplings

CKM elements do not only enter the Feynman rules for $W$ couplings but also appear in the couplings of charged Higgs bosons and charginos. The Feynman rules in the super-CKM scheme i) involve $V^{(0)} = U_L^{u\,(0)\dagger} U_L^{d\,(0)}$ throughout as described in section 2. Whenever a charged Higgs boson or a chargino couples to an external quark there are chirally enhanced one-loop corrections similar to those in Figs. 2 and 7. We can include these diagrams by working with the tree-level diagrams and replacing $U_L^{q\,(0)}$ by

$$U_L^q \;=\; U_L^{q\,(0)} \left(1 + \Delta U_L^q\right), \tag{18}$$

if the external quark is left-handed. For instance, we have shown in Sect. 2 that the loop corrections to the $\bar{u_f} W^+ d_i$ coupling were correctly included by this replacement (see Eq. (13)). That is, in the case of $\bar{u_f} W^+ d_i$ coupling one simply uses the physical CKM matrix $V_{fi}$ instead of the tree-level CKM matrix $V_{fi}^{(0)}$. One immediately notices that (in the super-CKM scheme) the $\widetilde{u}_f^* W^+ \widetilde{d}_i$ coupling still involves $V_{fi}^{(0)}$, because the supersymmetric analogues of the diagrams of Fig. 2 are not chirally enhanced and will only lead to small corrections of the typical size of ordinary loop corrections. Enhanced corrections to charged-Higgs and chargino interactions have been discussed for MFV scenarios with large $\tan\beta$ in Refs. [15, 32]; in this section we derive the corresponding results for the non-MFV case using the formalism of Sect. 2.

Flavour-changing self-energies lead to antihermitian corrections to the matrices $U_L^{(0)q}$. Charged-Higgs and chargino couplings also involve right-handed fields; the corresponding corrections to $U_R^{(0)q}$ are obtained by simply exchanging the chiralities in the expressions for $\Delta U_L^q$ (cf. Eqs. (12) and (18)). The CKM matrix which enters charged-Higgs or chargino vertices is not the physical one, because in these cases $V^{(0)}$ does not add up to $V$ together with enhanced loop corrections. The charged Higgs interaction $\bar{u}_f H^+ d_i$ has the Feynman rule (up to a factor of $i$):

$$Y^{u_f *} V_{fi}^{(0)} \cos\beta P_L \;+\; V_{fi}^{(0)} Y^{d_i} \sin\beta P_R \tag{19}$$

The effect of self-energies in the external legs is included by substituting this Feynman rule with

$$\begin{aligned} \sum_{j,k=1}^{3} \quad & \Big[ \left(1 + \Delta U_R^{u\dagger}\right)_{fj} Y^{u_j *} \left(1 - \Delta U_L^{u\dagger}\right)_{jk} V_{ki} \cos\beta P_L \\ & + V_{fj} \left(1 - \Delta U_L^d\right)_{jk} Y^{d_k} \left(1 + \Delta U_R^d\right)_{ki} \sin\beta P_R . \Big] \end{aligned} \tag{20}$$

Using the explicit expression for $\Delta U^q_{L,R}$ given in Eq. (12) and expressing $Y^{q_j}$ in terms of quark masses through Eq. (15) the substitution rule of Eq. (20) becomes

$$Y^{u_f*}V^{(0)}_{fi}P_L + V^{(0)}_{fi}Y^{d_i}P_R \rightarrow$$
$$\sum_{j=1}^{3}\left[\begin{pmatrix} \frac{m_{u_1}}{1+\Delta_{u_1}} & \frac{-\Sigma^{uRL}_{12}}{1+\Delta_{u_2}} & \frac{-\Sigma^{uRL}_{13}}{1+\Delta_{u_3}} \\ \frac{-\Sigma^{uRL}_{21}}{1+\Delta_{u_2}} & \frac{m_{u_2}}{1+\Delta_{u_2}} & \frac{-\Sigma^{uRL}_{23}}{1+\Delta_{u_3}} \\ \frac{-\Sigma^{uRL}_{31}}{1+\Delta_{u_3}} & \frac{-\Sigma^{uRL}_{32}}{1+\Delta_{u_3}} & \frac{m_{u_3}}{1+\Delta_{u_3}} \end{pmatrix}_{fj} \frac{V_{ji}\cos\beta}{v_u} P_L + \frac{V_{fj}\sin\beta}{v_d} \begin{pmatrix} \frac{m_{d_1}}{1+\Delta_{d_1}} & \frac{-\Sigma^{dLR}_{12}}{1+\Delta_{d_2}} & \frac{-\Sigma^{dLR}_{13}}{1+\Delta_{d_3}} \\ \frac{-\Sigma^{dLR}_{21}}{1+\Delta_{d_2}} & \frac{m_{d_2}}{1+\Delta_{d_2}} & \frac{-\Sigma^{dLR}_{23}}{1+\Delta_{d_3}} \\ \frac{-\Sigma^{dLR}_{31}}{1+\Delta_{d_3}} & \frac{-\Sigma^{dLR}_{32}}{1+\Delta_{d_3}} & \frac{m_{d_3}}{1+\Delta_{d_3}} \end{pmatrix}_{ji} P_R\right] \tag{21}$$

We observe a cancellation between the inverse quark masses in $\Delta U^q_L$ (see Eq. (12)) and the factors of $m_{q_i}$ from the $Y^{q_i}$'s in the effective off-diagonal couplings.

In the case of chargino interactions we must take into account that a squark never comes with an enhanced self-energy, even if the squark line is an external line of the considered Feynman diagram. Here four different couplings (and their hermitian conjugates) occur. Neglecting numerical factors and chargino mixing matrices, the Feynman rules for the chargino couplings contain the following flavour structures:

$$\begin{aligned}
&\bar{u}_{fL}\,\tilde{\chi}^+\,\tilde{d}_s: && \sum_{j=1}^{3} V^{(0)}_{fj}Y^{d_j}W^{\tilde{d}}_{j+3,s}P_R \quad \text{and} \quad g_w\sum_{j=1}^{3} V^{(0)}_{fj}W^{\tilde{d}}_{js}P_R,\\
&\bar{u}_{fR}\,\tilde{\chi}^+\,\tilde{d}_s: && \sum_{j=1}^{3} Y^{u_f*}V^{(0)}_{fj}W^{\tilde{d}}_{js}P_L,\\
&\tilde{u}^*_t\,\tilde{\chi}^+\,d_{i\,L}: && \sum_{j=1}^{3} W^{\tilde{u}*}_{t,j+3}Y^{u_j*}V^{(0)}_{ji}P_L \quad \text{and} \quad g_w\sum_{j=1}^{3} W^{\tilde{u}*}_{tj}V^{(0)}_{ji}P_L,\\
&\tilde{u}^*_t\,\tilde{\chi}^+\,d_{i\,R}: && \sum_{j=1}^{3} W^{\tilde{u}*}_{tj}V^{(0)}_{ji}Y^{d_i}P_R.
\end{aligned} \tag{22}$$

Here $g_w$ is the SU(2) gauge coupling and $W^{\tilde{q}}_{ij}$ is defined in Eq. (24). Again we include the self-energy corrections, and express $V^{(0)}$ in terms of the physical CKM matrix. Then the expressions in Eq. (22) become:

$$\sum_{l,m=1}^{3} V_{fl}\left(1-\Delta U^d_L\right)_{lm} Y^{d_m}W^{\tilde{d}}_{m+3,s}P_R$$

and $$g_w\sum_{l,m=1}^{3} V_{fl}\left(1-\Delta U^d_L\right)_{lm} W^{\tilde{d}}_{ms}P_R,$$

$$\sum_{j,k,l,m=1}^{3}\left(1+\Delta U^{u\dagger}_L\right)_{fj} Y^{u_j*}\left(1-\Delta U^{u\dagger}_L\right)_{jk} V_{kl}\left(1-\Delta U^d_L\right)_{lm} W^{\tilde{d}}_{ms}P_L,$$

$$\sum_{j,k=1}^{3} W^{\tilde{u}*}_{t,j+3}Y^{u_j*}\left(1-\Delta U^{u\dagger}_L\right)_{jk} V_{ki}P_L$$

and $$g_w\sum_{j,k=1}^{3} W^{\tilde{u}*}_{tj}\left(1-\Delta U^{u\dagger}_L\right)_{jk} V_{ki}P_L,$$

$$\sum_{j,k,l,m=1}^{3} W_{tj}^{\tilde{u}*} \left(1 - \Delta U_L^{u\dagger}\right)_{jk} V_{kl} \left(1 - \Delta U_L^{d}\right)_{lm} Y^{d_m} \left(1 + \Delta U_R^{d}\right)_{mi} P_R. \tag{23}$$

We have seen in this section that in the case of non-minimal flavour violation the CKM matrix (including loop corrections) entering charged Higgs and quark-squark-chargino vertices is not simply the physical one. Instead it has to be corrected according to Eq. (20) or Eq. (21) and Eq. (23), leading to potentially large effects.

# 5 Conclusions

We have computed the renormalisation of the CKM matrix by chirally-enhanced flavour-changing SQCD effects in the MSSM with generic flavour structure. Our paper extends the work of [16], which considered the MFV case. We have worked beyond the decoupling limit $M_{\mathrm{SUSY}} \gg v$ and our results are valid for arbitrary left-right mixing and arbitrary flavour mixing among squarks. Subsequently we have derived upper bounds on the flavour-changing off-diagonal elements $\Delta_{ij}^{\tilde{q}\,XY}$ of the squark mass matrix by requiring that the supersymmetric corrections do not exceed the measure values of the CKM elements. For $M_{\mathrm{SUSY}} \geq 500\,\mathrm{GeV}$ our constraints on *all* elements $\Delta_{ij}^{\tilde{d}\,LR}$, $i < j$, are stronger than the constraints from FCNC processes. We were further able to derive a strong bound on $\Delta_{13}^{u\,LR}$, a quantity which is not constrained by FCNC's. For a large value of $\tan\beta$ one can constrain $\Delta_{13}^{d\,LL}$ as well.

As an important consequence, we conclude that it is possible to generate the observed CKM elements completely through finite supersymmetric loop diagrams [17, 28] without violating present-day data on FCNC processes. In this scenario the Yukawa sector possesses a higher flavour symmetry than the trilinear SUSY breaking terms. Most naturally, first an exact $[U(2)]^3$ symmetry is imposed on the quark supermultiplets of the first two generations: Then the corresponding Yukawa couplings $Y_{ij}^q$ vanish and the squark mass terms $\Delta_{ij}^{\tilde{q}\,LL}$, $\Delta_{ij}^{\tilde{q}\,RR}$ are universal for the first two generations. In the second step the trilinear terms $A_{ij}^q$ are chosen to break the flavour symmetry softly and generate light quark masses and off-diagonal CKM elements radiatively.

As another application of our results, we have derived supersymmetric loop corrections to the couplings of charged Higgs bosons and charginos to quarks and squarks. In these couplings the squark-gluino loops which renormalise the CKM elements are physical and can have a significant numerical impact because of their chiral enhancement. We have further pointed out that the calculated flavour-changing self-energies can have observable effects in the $W$-mediated production or decay of the top quark, with the SUSY effects decoupling as $m_t^2/M_{\mathrm{SUSY}}^2$ for $M_{\mathrm{SUSY}} \to \infty$.


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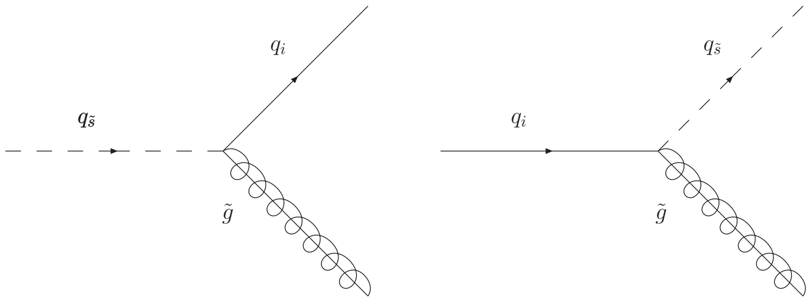


Figure 10: Quark-squark-gluino vertex. The Feynman rules for the left and right diagrams are given in Eqs. (26) and (25), respectively.

## Appendix: Conventions and Feynman rules

We denote the tree-level quark mass matrix by $\mathbf{m}_q^{(0)} = \mathbf{Y}^q v_q$. The unitary matrices diagonalising these matrices and the squark mass matrices are denoted by $U_{L,R}^{(0)\,u,d}$ and $W^{\tilde{u},\tilde{d}}$, respectively:

$$\begin{array}{rr} U_L^{(0)\,u\dagger}\mathbf{m}_u^{(0)}U_R^{(0)\,u} = \mathbf{m}_u^{(D)}, & U_L^{(0)\,d\dagger}\mathbf{m}_d^{(0)}U_R^{(0)\,d} = \mathbf{m}_d^{(D)} \\ W^{\tilde{u}\dagger}\mathbf{M}_{\tilde{u}}^2 W^{\tilde{u}} = \mathbf{M}_{\tilde{u}}^{2(D)}, & W^{\tilde{d}\dagger}\mathbf{M}_{\tilde{d}}^2 W^{\tilde{d}} = \mathbf{M}_{\tilde{d}}^{2(D)} \end{array} \tag{24}$$

The superscript $(D)$ in Eq. (24) indicates diagonal matrices. That is, the mass eigenstates of the quarks and squarks are obtained from the original fields by unitary rotations in flavour space involving the matrices $U_L^{(0)\,u,d}$, $U_R^{(0)\,u,d}$ and $W^{\tilde{u},\tilde{d}}$ as defined in [33]. The Feynman rules for the quark-squark-gluino vertices in the basis of mass eigenstates then read

$$-i\sqrt{2}g_s T^a \sum_{j=1}^{3}\left(U_{L\,ji}^{(0)\,q} W_{js}^{\tilde{q}*} P_R - U_{R\,ji}^{(0)\,q} W_{j+3,s}^{\tilde{q}*} P_L\right) \tag{25}$$

for an incoming quark and

$$-i\sqrt{2}g_s T^a \sum_{j=1}^{3}\left(U_{L\,ji}^{(0)\,q*} W_{js}^{\tilde{q}} P_L - U_{R\,ji}^{(0)\,q*} W_{j+3,s}^{\tilde{q}} P_R\right) \tag{26}$$

for an outgoing quark. The interaction vertices are depicted in Fig. 10. Eqs. (25) and (26) hold in any basis for the quark and squark fields, provided the quark-squark-gluino coupling is flavour-diagonal in the original basis, in which the mass matrices $m_{u,d}^{(0)}$ and $M_{\tilde{u},d}^2$ are defined. This condition is not only fulfilled if the original basis consists of weak (s)quark eigenstates but also for the super-CKM basis.

Our starting point is a basis of weak eigenstates: The squark mass term in the Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\tilde{m}} \;=\; & -(\tilde{d}_L^*, \tilde{s}_L^*, \tilde{b}_L^*, \tilde{d}_R^*, \tilde{s}_R^*, \tilde{b}_R^*)\mathrm{M}_{\tilde{d}}^{\mathrm{w}2}(\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R, \tilde{s}_R, \tilde{b}_R)^T \\ & -(\tilde{u}_L^*, \tilde{c}_L^*, \tilde{t}_L^*, \tilde{u}_R^*, \tilde{c}_R^*, \tilde{t}_R^*)\mathrm{M}_{\tilde{u}}^{\mathrm{w}2}(\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R)^T \end{aligned}$$

with

$$\mathrm{M}_{\tilde d}^{\mathrm{w}2} = \begin{pmatrix} \mathbf{M}_{\tilde q}^2 - M_Z^2\left(1+\frac{1}{3}\sin^2\theta_W\right)\cos 2\beta\mathbf{1} + \mathbf{m}_d^{(0)}\mathbf{m}_d^{(0)\dagger} & v_d\mathbf{A}_w^d - \mathbf{m}_d^{(0)}\mu\tan\beta \\ v_d\mathbf{A}_w^{d\,\dagger} - \mu^*\tan\beta\mathbf{m}_d^{(0)\dagger} & \mathbf{M}_{\tilde d}^2 - \frac{1}{3}M_Z^2\cos 2\beta\sin^2\theta_W\mathbf{1} + \mathbf{m}_d^{(0)\dagger}\mathbf{m}_d^{(0)} \end{pmatrix}$$

$$\mathrm{M}_{\tilde u}^{\mathrm{w}2} = \begin{pmatrix} \mathbf{M}_{\tilde q}^2 + M_Z^2\left(1+\frac{2}{3}\sin^2\theta_W\right)\cos 2\beta\mathbf{1} + \mathbf{m}_u^{(0)}\mathbf{m}_u^{(0)\dagger} & v_u\mathbf{A}_w^u - \mathbf{m}_u^{(0)}\mu\cot\beta \\ v_u\mathbf{A}_w^{u\,\dagger} - \mu^*\cot\beta\mathbf{m}_u^{(0)\dagger} & \mathbf{M}_{\tilde u}^2 + \frac{2}{3}M_Z^2\cos 2\beta\sin^2\theta_W\mathbf{1} + \mathbf{m}_u^{(0)^\dagger}\mathbf{m}_u^{(0)} \end{pmatrix} \tag{27}$$

The physical CKM matrix differs from $V^{(0)} = U_L^{(0)\,u\,\dagger}U_L^{(0)\,d}$ by the corrections from the finite squark-gluino self-energies, which are the subject of this paper. In physical processes with external quarks the matrices of Eq. (24) appear in pairs and it is useful to define:

$$\begin{aligned} V_{s\,fi}^{(0)\,q\,RL} &\equiv \sum_{j,k=1}^{3} U_{R\,fj}^{(0)\,q*}W_{j+3,s}^{\tilde q}U_{L\,ki}^{(0)\,q}W_{ks}^{\tilde q*} \\ V_{s\,fi}^{(0)\,q\,LR} &\equiv \sum_{j,k=1}^{3} U_{L\,fj}^{(0)\,q*}W_{js}^{\tilde q}U_{R\,ki}^{(0)\,q}W_{k+3,s}^{\tilde q*} \\ V_{s\,fi}^{(0)\,q\,LL} &\equiv \sum_{j,k=1}^{3} U_{L\,jf}^{(0)\,q*}W_{js}^{\tilde q}U_{L\,ki}^{(0)\,qL}W_{k,s}^{\tilde q*} \\ V_{s\,fi}^{(0)\,q\,RR} &\equiv \sum_{j,k=1}^{3} U_{R\,jf}^{(0)\,q*}W_{j+3,s}^{\tilde q}U_{R\,ki}^{(0)\,q}W_{k+3,s}^{\tilde q*} \end{aligned} \tag{28}$$

Flavour violation in the squark mass matrices is usually quantified in the super-CKM basis, in which $m_u^{(0)} = m_u^{(D)}$ and $m_d^{(0)} = m_d^{(D)}$. Then we can use Eq. (28) and the Feynman rules of Eqs. (25) and (26) with the substitutions

$$\sum_{j=1}^{3} U_{L\,ji}^{(0)\,q*}W_{js}^{\tilde q} \to W_{is}^{\tilde q}, \qquad \sum_{j=1}^{3} U_{R\,ji}^{(0)\,q*}W_{j+3,s}^{\tilde q} \to W_{i+3,s}^{\tilde q}. \tag{29}$$

Next we relate the quantities of Eq. (1), which are defined in the super-CKM basis, to the parameters of the MSSM Lagrangian, which are defined in a weak basis. To this end we have to specify a weak basis as our starting point and we choose a basis with $m_d^{(0)} = m_d^{(D)}$, $U_R^{(0)\,u} = U_L^{(0)\,d} = U_R^{(0)\,d} = 1$ and $U_L^{(0)\,u} = V^{(0)}$. Note that $m_q^{(D)}$ can be complex, if the threshold corrections $\Delta_{q_i}$ in Eq. (15) are complex. The transition from this weak basis to the super-CKM basis only involves a rotation of the left-handed up-type supermultiplets with $V^{(0)}$. Therefore the down squark mass matrix is unchanged, $\mathrm{M}_{\tilde d}^2 = \mathrm{M}_{\tilde d}^{\mathrm{w}2}$, while

$$\mathrm{M}_{\tilde u}^2 = \begin{pmatrix} V^{(0)\dagger}\mathbf{M}_{\tilde q}^2V^{(0)} + M_Z^2\left(1+\frac{2}{3}\sin^2\theta_W\right)\cos 2\beta\mathbf{1} + \mathbf{m}_u^{(D)}\mathbf{m}_u^{(D)\,\dagger} & V^{(0)\dagger}\mathbf{A}_w^u - \mathbf{m}_u^{(D)}\mu\cot\beta \\ \mathbf{A}_w^{u\,\dagger}V^{(0)} + \mu^*\cot\beta\mathbf{m}_u^{(D)\dagger} & \mathbf{M}_{\tilde u}^2 + \frac{2}{3}M_Z^2\cos 2\beta\sin^2\theta_W\mathbf{1} + \mathbf{m}_u^{(D)\,\dagger}\mathbf{m}_u^{(D)} \end{pmatrix} \tag{30}$$

Thus the trilinear terms of the super-CKM basis, $\mathbf{A}^q$, and those in the weak basis are related as

$$\mathbf{A}^d = \mathbf{A}_w^d, \qquad \mathbf{A}^u = V^{(0)\dagger}\mathbf{A}_w^u, \tag{31}$$

and $\Delta^{\tilde{q}\,LR}_{ij}$ is expressed in terms of $A^q_{ij}$ in Eq. (2). SU(2) symmetry enforces a relation between the upper left $3\times 3$ sub-matrices of $\mathrm{M}^2_{\tilde{u}}$ and $\mathrm{M}^2_{\tilde{d}}$, since they both involve $\mathbf{M}^2_{\tilde{q}}$. The corresponding relation in the super-CKM basis is read off from Eq. (30):

$$\Delta^{\tilde{d}\,LL}_{ij} = \left[V^{(0)\dagger}\mathrm{M}^2_{\tilde{u}}V^{(0)}\right]_{ij} \tag{32}$$

for $i,j=1,2,3$ and $i\neq j$. Eq. (32) was derived in Ref. [3] with $V^{(0)}=V$, i.e. Eq. (32) generalises the latter result to the case that radiative corrections to $V$ are included.

The bounds on $\Delta^{\tilde{q}\,XY}_{ij}$ derived in this paper assume that the squark-gluino loops at most saturate the measured elements of $V$. The extremal values for $\Delta^{\tilde{q}\,XY}_{ij}$ correspond to the case $V^{(0)}=1$, for which the super-CKM basis coincides with a weak basis. This limiting case is realised in the scenario of Sect. 3.4, in which all of $V$ is generated through supersymmetric loops.

Finally we quote our conventions for the two-point, three-point and four-point one-loop functions $B_0$, $C_0$ and $D_0$:

$$\begin{aligned}
B_0\left(m_1,m_2\right) &= 1+\frac{m_1^2\ln\left(\frac{Q^2}{m_1^2}\right)-m_2^2\ln\left(\frac{Q^2}{m_2^2}\right)}{m_1^2-m_2^2}\\
C_0\left(m_1,m_2,m_3\right) &= \frac{B_0(m_1,m_2)-B_0(m_1,m_3)}{m_2^2-m_3^2}\\
&= \frac{m_1^2m_2^2\ln\left(\frac{m_1^2}{m_2^2}\right)+m_2^2m_3^2\ln\left(\frac{m_2^2}{m_3^2}\right)+m_3^2m_1^2\ln\left(\frac{m_3^2}{m_1^2}\right)}{\left(m_1^2-m_2^2\right)\left(m_2^2-m_3^2\right)\left(m_3^2-m_1^2\right)}\\
D_0\left(m_1,m_2,m_3,m_4\right) &= \frac{C_0(m_1,m_2,m_3)-C_0(m_1,m_2,m_4)}{m_3^2-m_4^2}
\end{aligned}$$

The two-point function $B_0$ is UV-divergent, our definition above is $\overline{\mathrm{MS}}$-subtracted. UV divergence and the renormalisation scale $Q$ drop out from our results thanks to the super-GIM mechanism.

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