The FORM project

J.A.M. Vermaseren[a*]

[a]Nikhef,
Kruislaan 409, 1098 SJ Amsterdam, The Netherlands

The necessity of the FORM project is discussed. Then the evolutionary needs in particle physics are considered, looking at the trends over the years. A guess is made at what will be needed in the (near) future. The whole is concluded with some critical remarks concerning the publication of results and programs.

1. Why FORM?

In particle theory we have categories of calculations that are particularly demanding on hardware and software facilities. So much so that particle theory has stood at the cradle of symbolic computation and also afterwards has made large contributions to it. Yet, as soon as a system becomes bigger and bigger and commercially interesting, it often leaves its origins and it becomes more and more difficult to influence its development.

Hence we need one or more systems of which we can influence the development. This way it can be optimized or close to optimized for our needs. The best case is if the author(s) is/are involved in our type of calculations. In the next best case we should be in the position to adapt a system by ourselves in order to avoid a very lengthy cycle of interaction with the authors. This asks for an open source system that is properly documented to make it as easy as possible for people to make additions. Moreover, a system should be readily available to all researchers.

FORM [1–3] is supposed to fit these requirements. To some extent it has already been like this, because I have been involved in many types of projects that other people in particle phenomenology are also engaged in. Like GiNaC [4] it is an 'in house' particle theory project with applications to other fields of science. The fact that FORM isn't open source is being worked at.

One should also realize that FORM is heavily optimized for speed and the handling of very large expressions. Commercial systems have usually a different optimization target. An overwhelming fraction of commercial users doesn't have problems that explore the limits of what is possible.

2. Trends in Loops and Legs

If one looks at the history of calculations in particle theory one sees a development over the years.

At first the symbolic manipulation was to combine tensors and four vector dotproducts and manipulate gamma matrices. This was what Schoonschip [5] was designed for and also one of the first things that FORM could do. Because there are still people who think in these terms (nowadays mainly people who are not in particle phenomenology) FORM has been stigmatized as a program that is only suitable for particle physics.

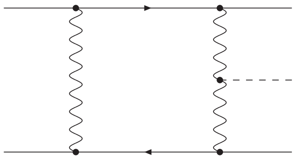
An example of a reaction that was topline research in 1976 [6]: $\gamma p \rightarrow \tau^-\tau^+X \rightarrow e^-\mu^+\nu\overline{\nu}\nu\overline{\nu}X$.

```
*
*   gamma+proton -> tau- tau+ X -> e- (nu nubar)
*                                  mu+ (nu nubar) X
*   Narrow width approximation and full
*   spin-spin correllations.
*
S   mtau,mmu,me,mnut,mnum,mnue;
I   j1,j2,j3,j4,e1;
V   pa,pb,q1,q2,e2,p1,p2,p3,p4,p5,p6,pe,pm;

L   F =
```

*Currently on leave at Institut für Theoretische Teilchenphysik, Universität Karlsruhe D-76131 Karlsruhe, Germany

```
  (g_(1,p1)+mnut*gi_(1))*
  (g_(2,pe)+me*gi_(2))*
  (g_(3,p4)+mnum*gi_(3))*(
    +g_(1,j1)*g7_(1)*(g_(1,q1)+mtau*gi_(1))
      *g_(1,e1) *(g_(1,q1)-g_(1,pa)
         +mtau*gi_(1))*g_(1,e2)
      *(-g_(1,q2)+mtau*gi_(1))
      *g_(1,j2)*g7_(1)*(-1/2)/q1.pa
      *g_(2,j1)*g7_(2)*g_(3,j2)*g7_(3)
    +g_(1,j1)*g7_(1)*(g_(1,q1)+mtau*gi_(1))
      *g_(1,e2)*(g_(1,pa)-g_(1,q2)
         +mtau*gi_(1))*g_(1,e1)
      *(-g_(1,q2)+mtau*gi_(1))
      *g_(1,j2)*g7_(1)*(-1/2)/q2.pa
      *g_(2,j1)*g7_(2)*g_(3,j2)*g7_(3)
  )*
  (g_(1,p6)-mnut*gi_(1))*
  (g_(2,p3)-mnue*gi_(2))*
  (g_(3,pm)-mmu*gi_(3))*(
    +g_(1,j4)*g7_(1)*(-g_(1,q2)+mtau*gi_(1))
      *g_(1,e2)*(g_(1,q1)-g_(1,pa)
         +mtau*gi_(1))*g_(1,e1)
      *(g_(1,q1)+mtau*gi_(1))
      *g_(1,j3)*g7_(1)*(-1/2)/q1.pa
      *g_(2,j3)*g7_(2)*g_(3,j4)*g7_(3)
    +g_(1,j4)*g7_(1)*(-g_(1,q2)+mtau*gi_(1))
      *g_(1,e1)*(g_(1,pa)-g_(1,q2)
         +mtau*gi_(1))*g_(1,e2)
      *(g_(1,q1)+mtau*gi_(1))
      *g_(1,j3)*g7_(1)*(-1/2)/q2.pa
      *g_(2,j3)*g7_(2)*g_(3,j4)*g7_(3)
  )/2^16;
 Trace4,3;
 Trace4,2;
 Trace4,1;
 id  q1.q1 = mtau^2;
 id  q2.q2 = mtau^2;
 id  pa.pa = 0;
 Print +f +s;
 .end

Time =     0.21 sec    Generated terms =    1992
             F         Terms in output =     176
                       Bytes used      =    8552
```

Next came the manipulation of loop integrals. At first the one loop integrals and their reduction to scalar loop integrals. Here is a very advanced example of a Feynman diagram calculated around the year 1983 [7]:



Then (1989-2000) [8–12] came also the manipulation of the three loop propagator graphs, done by many relations based on integration by parts [13]. The expansions in terms of ϵ posed additional requirements. These were the days of version 2 of FORM.

In the late nineties new trends were emerging. Not only new techniques for the rewriting of the diagrams in terms of master integrals were developed [14–16], but also new methods for the treatment of the integrals themselves saw the daylight. Most notoriously methods with nested sums [17] and harmonic polylogarithms [18]. The rewriting started to need the solving of large sets of equations. Additionally the occasional calculation of more and more complicated color factors [19] required some additional types of topological pattern matching. These last methods have not yet been explored very much as they will be needed mostly when there are very many loops. But they can also be very useful for complicated tensor algebra. Also the use of very large tables inspired new and original solutions. Version 3 of FORM has these methods in mind.

By now the traces of the gamma matrices form just a very small corner in the space of its capabilities.

Several types of calculations will always be at the limits of what is possible. A good example is the project at Karlsruhe of Baikov, Chetyrkin and Kühn [20]. Given more power, they can do deeper calculations. Hence it is important to have FORM as powerful as possible. This is adressed by the ParFORM [2] project and more recently this has led to TFORM [3].

Another example of something that is in principle open ended is the expression of multiple zeta values into a minimal set of variables [21]. This is the status at 2007[2]

[2] We make no attempt to be exhaustive here.

weight	variables	equations	left	time(sec)
1	2	1	2	
2	6	10	1	
3	18	38	1	
4	54	138	1	
5	162	462	2	
6	486	1486	2	
7	1458	4730	4	8.8
8	4374	15110	5	46.8
9	13122	48558	8	306
10	39366	158602	11	2382
11	118098	515858	18	28906
12	354294	1669610	25	1243191

Time is real time on a computer with 8 Xeon cores at 2.3 GHz and 2 Gbytes of memory per core, running TFORM.

The recent developments in massless multiparticle one loop amplitudes [22] hasn't led yet to particular symbolic projects. It is not clear whether it is needed. The methods with sector decomposition [23] are under development and again, it isn't 100% clear whether they need new developments in the symbolic sector. Possibly internal capabilities for treating combinations of sums, theta functions, delta functions and the splitting of factors with denominators could speed up the nested sums considerably. This would be very useful for the current methods based on the Mellin-Barnes approach [24]. This is however not entirely trivial, unless it will be too specific for a single problem. The automatic calculations as in GRACE [25] and CompHEP [26] can definitely use some new facilities in the field of code simplification.

I am probably forgetting a few things here.

3. The current status

What is the status of the FORM project?

First in the field of manpower. Of course I myself work at the moment almost full time on FORM. In addition Misha Tentyukov is involved in the ParFORM project. He has been making other additions in the past citeExternal. At the moment Jens Vollinga is on a three year postdoc position at Nikhef which involves also work for FORM. He has already made the code for systems independent .sav files and is working on a failsafe system that (at some cost of course) allows one to set up checkpoints from which one can restart after a computer failure. He is also setting up a framework for documentation and provide better installation using the 'make install' approach. As part of a project grant of FOM we have the money to get a programmer for 18 months (starting in the autumn) to help with the project of code simplification. This is to facilitate the FORM version of GRACE citeGraceForm.

Over the past few years TFORM [3] has been developed as a complement to ParFORM [2]. Each of the two has restrictions. ParFORM can operate on clusters and TFORM works only on multi core systems with shared memory. Because of the shared memory some things are much easier in TFORM. Much administrative work needs only a single copy. Much multiple reading of files can be done with a relatively simple locking system. ParFORM has the advantage that clusters can have many more processors. But the communication is much more complicated. Optimization of the programs is a field of research and may need some extra manpower. The ideal would be a system that can use clusters of multi core machines. The problem is to reduce the bottlenecks so that for N processors the execution time comes as close as possible to $1/N$ times the time needed on a single processor. Currently it is rather hard to get beyond $1/5$ on 8 processors and beyond $1/10$ on 32 processors. A careful study and inventive solutions will be needed.

Recently we have started to make use of the GMP (GNU Multiple Precision) [29] library for some of the computations with large integers. This is because the size of integers and fractions has become larger and larger and is often way beyond what was envisioned originally. We use only the low level routines for multiplication, division and GCD calculations. The gain in speed isn't impressive though, because the algorithms inside FORM are rather decent (especially after the improvements found in the end of 2006). But the GMP can do some things more efficiently because it has some assembler routines and in assembler one can do a number of things far more efficiently than in C. The need to convert from FORM notation to GMP notation introduces an overhead. We still need to experiment with what is the optimal size below which we should use the original routines and above which we should use the GMP library. If one does calculations that involve fractions with very large integers (like hundreds of digits) one will find that the more recent versions (2007 and on) of FORM are noticeably faster.

4. What to expect and hope for

The systems of equations that need to be solved are asking often for capabilities with rational polynomials. This is particularly the case with the Laporta algorithm [16]. It is something that FORM doesn't have currently. Hence it has rather high priority to build this in. And to build this in in a rather efficient way. There exist publicly available libraries for the manipulation of polynomials in a single variable, some of them claiming great efficiency, but there are no equivalent libraries for polynomials in many variables. In addition there is the problem of notation. Too much time spent on conversion will not be beneficial. Currently the problem is under study. Most univariate algorithms (in particular the GCD) have been implemented in various methods. This is by now reasonably fast. Factorization is less urgent, but can come in handy when constructing a system for simplification.

It is also important to deal with multivariate rational polynomials efficiently when one likes to create a system for computing Gröbner bases. There are however several ways to deal with polynomials and each way needs its own solution:

- Small polynomials: when they take a small amount of space they can be kept inside the argument of a function. There may be billions of such polynomials. They should be treated inside the regular workspace. Univariate polynomials will usually be in this category. An improvement in efficiency will be to tabulate a number of them. This is especially the case for factorization which is relatively expensive.

- Intermediate polynomials: these could be handled by means of memory allocations as is done with the dollar variables. One could have hundreds or even thousands of them. Typically not billions.

- Large polynomials: These are complete expressions that could have billions of terms. Calculating their GCD would have to use the same mechanisms as by which expressions are treated. There should be only very few of these.

An example of something that works already: PolyRatFun is an experimental statement that is similar to the PolyFun statement, but now the function needs two arguments: a numerator and a denominator.

```
Symbols x,y;
CFunction pacc;
PolyRatFun pacc;
L   F = pacc(x^2+x-3,(x+1)*(x+2))*y
       +pacc(x^2+3*x+1,(x+3)*(x+2))*y^2;
Print +s;
.sort

F =
  + y*pacc(x^2 + x - 3,x^2 + 3*x + 2)
  + y^2*pacc(x^2 + 3*x + 1,x^2 + 5*x + 6)
   ;

id  y = 1;
Print;
.end

F =
   pacc(2*x^2 + 4*x - 4,x^2 + 4*x + 3);
```

Sometimes one would like to have quick private additions for things that are extremely hard to program at the FORM level. Such things are often either of combinatoric nature or special patterns. It is of course impossible to forsee what some people will need. Hence FORM should be structured in such a way that it is possible to make such additions, even though this won't be for beginners. The first requirement for this is a good documentation of the inner workings, including a number of examples. The second requirement is code that can be understood and is structured properly. Due to these two requirements FORM hasn't been released yet as open source. We hope to be this far in about two years time.

As mentioned before, we like to have a way to introduce code simplification. This would be relevant for all outputs that would need further numerical evaluation in the languages Fortran and C. If it is possible we would like to extend this to the regular output for as far as factorization is concerned. Already some things can be done at the FORM level, but this is usually rather slow. One can for instance make a procedure 'tryfactor' which would work like

```
#do i = -100,100
#call tryfactor(acc,x+`i')
#enddo
B acc;
Print;
```

and the answer might be like

```
+acc(x-27)*acc(x+6)*acc(x+67)*(.......)
```

Because this is very slow and requires guessing the factors, this is far from ideal.

Something that the community should think about: At a given moment Nikhef, and/or I may not be able to take the responsibility for FORM any longer. Which institute/individual(s) can take over this responsibility? Would FORM disappear? It is not a good idea to depend on the free time of some individuals. The open source project may help, but this is probably not sufficient. There should be a professional commitment. Of course, if someone can come with a better product, evolution will take its course. But that would require a large investment as well. Good ideas are needed here, because it doesn't look like CERN (which would be the most natural choice) is volunteering.

5. Some critical remarks

Some people prefer to use expensive commercial systems and give their results in terms of routines for these systems. I believe this to be very shortsighted.

Years ago we had a preprint system, and only the top universities would get the preprints and be up to date. Poor universities would not be able to be up to date and hence meaningful up to date research could only be done at a limited number of places.

Now with the internet, everybody can be up to date and meaningful research can be done everywhere. If however we present our results in the form of programs for very expensive software systems, we take a big step back. It is a form of elitism.

I am not pushing here for my own program. What I want to say is that it is in the interest of science that all results are freely available and freely accessible, and that the threshold for using the results is as low as possible.

If someone isn't happy with the facilities offered by the free software, spend some effort or resources on helping with providing such facilities. That is something that everybody can benefit of.

Another thing (Remember Babylon):

The situation becomes really chaotic when there are many complementary results from different authors set up for different systems. It becomes rapidly impossible to combine such results.

6. Acknowledgements

The author wishes to thank the Humboldt foundation for its generous support and the university of Karlsruhe for its kind hospitality.

REFERENCES

1. J.A.M. Vermaseren, math-ph/0010025.
2. D. Fliegner, A. Retey, and J.A.M. Vermaseren, hep-ph/9906426, hep-ph/0007221, M. Tentyukov et al., CS/0407066.
3. M. Tentyukov, J.A.M. Vermaseren, HEP-PH/0702279.
4. C. W. Bauer, A. Frink and R. Kreckel, cs/0004015.
5. A program by M. Veltman. See H. Strubbe, Comp. Phys. Commun. **8** (1974) 1.
6. J. Smith, A. Soni, and J.A.M. Vermaseren, Phys. Rev. **D15** (1977) 648.
7. W.L. van Neerven, and J.A.M. Vermaseren, Phys. Lett. **B142** (1984) 80.
8. S.G. Gorishnii, A.L. Kataev and S.A. Larin, Phys. Lett. **B259** (1991) 144.
9. S. G. Gorishnii, S.A. Larin, L.R. Surguladze

and F.V. Tkachov, Compt. Phys. Commun. **55** (1989) 381.
10. S.A. Larin, F.V. Tkachov, and J.A.M. Vermaseren, NIKHEF-H-91-18.
11. M. Steinhauser, Comput. Phys. Comm. **134** (2001) 335.
12. S.A. Larin, Paulo Nogueira, T. van Ritbergen, and J.A.M. Vermaseren, Nucl. Phys. **B492** (1997) 338.
13. F. Tkachov, Phys. Lett. **B200** (1981) 65. K.G. Chetyrkin and F. Tkachov, Nucl. Phys. **B192** (1981) 159.
14. O.V. Tarasov, Nucl. Phys. **B502** (1997) 455. O.V. Tarasov, Nucl. Phys. Proc. Suppl. **89** (2000) 237.
15. T. Gehrmann and E. Remiddi, Nucl. Phys. **B580**: 485, 2000.
16. S. Laporta, Int. J. Mod. Phys. **A15** (2000) 5087.
17. J.A.M. Vermaseren, Int. J. Mod. Phys, **A14** (1999) 2037.
18. E. Remiddi and J.A.M. Vermaseren, Int. J. Mod. Phys, **A15** (2000) 725.
19. T. van Ritbergen, A. N. Schellekens and J. A. M. Vermaseren, Int. J. Mod. Phys, **A14** (1999) 41.
20. P.A. Baikov, Phys. Lett. **B385** (1996) 404. P.A.Baikov, K.G. Chetyrkin and J.H. Kühn, Phys. Rev. Lett. **88** (2002) 012001.
21. J. Blümlein, D. Broadhurst and J.A.M. Vermaseren: in preparation.
22. See for instance the talks by D. Kosower, S. Badger and G. Rodrigo at this workshop.
23. G. Heinrich, Int. J. Mod. Phys. **A23** (2008) 1457.
24. J. B. Tausk, Phys. Lett. **B469** (1999) 225.
25. F. Yuasa *et al.*, Prog. Theor. Phys. Suppl. **138** (2000) 18.
26. E.E. Boos *et al.*, hep-ph/9503280, A. Pukhov *et al.*, hep-ph/9908288.
27. M. Tentyukov, J.A.M. Vermaseren, Comput. Phys. Commun. **176** (2007) 385-405.
28. J. Fujimoto et al., Nucl. Phys. Proc. Suppl. **160** (2006) 150-154.
29. The GNU Multi Precision Arithmetic library. See the site http://gmplib.org