Tan β-enhanced supersymmetric corrections to the anomalous magnetic moment of the muon

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We report on a two-loop supersymmetric contribution to the magnetic moment $(g-2)_\mu$ of the muon which is enhanced by two powers of $\tan\beta$. This contribution arises from a shift in the relation between the muon mass and Yukawa coupling and can increase the supersymmetric contribution to $(g-2)_\mu$ sizably. As a result, if the currently observed 3σ deviation between the experimental and SM theory value of $(g-2)_\mu$ is analyzed within the Minimal Supersymmetric Standard Model (MSSM), the derived constraints on the parameter space are modified significantly: If $(g-2)_\mu$ is used to determine $\tan\beta$ as a function of the other MSSM parameters, our corrections decrease $\tan\beta$ by roughly 10% for $\tan\beta = 50$.

The anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ of the muon is one of the most precisely measured and calculated quantities in particle physics — and recently it has developed into one of the observables with the most significant deviations between the experimental value and the corresponding Standard Model (SM) theory prediction. The review [1] obtains

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 29.5(8.8)\times 10^{-10}, \tag{1}$$

a 3.4σ deviation between $a_\mu^{\rm exp}$ and $a_\mu^{\rm SM}$, the experimental [2] and SM theory value, respectively.

Eq. (1) represents dramatic progress. It has been made possible by better determinations of the hadronic e^+e^- cross section by SND, CMD-II, KLOE and BaBar [3]. These are crucial ingredients for all recent evaluations of the hadronic vacuum polarization contribution to $a_\mu^{\rm SM}$ [4–6]. In Ref. [7], further progress on the "τ-puzzle" has been achieved, confirming the e^+e^--based result (1). Now all evaluations of the SM theory prediction have a smaller error than ever before and agree very well. With this progress the case for physics beyond the SM in a_μ has become stronger. Generically, contributions from new physics with characteristic mass scale $M_{\rm BSM}$ are suppressed as $(M_W/M_{\rm BSM})^2$ compared to the SM electroweak contribution of $a_\mu^{\rm weak} = 15.4(0.2)\times 10^{-10}$, which is only half as large as the observed deviation. Thus some parametric enhancement of the new contribution is required.

Supersymmetry (SUSY), implemented in the Minimal Supersymmetric Standard Model (MSSM), can naturally explain the observed deviation for two reasons: First, the masses of smuons and charginos, the most relevant SUSY particles, can be as small as $M_{\rm SUSY} \sim \mathcal{O}(100\text{ GeV})$ without contradicting current experimental data, allowing a rather mild suppression factor $(M_W/M_{\rm SUSY})^2$. Second, the SUSY contributions to a_μ are enhanced by the parameter

$$\tan\beta = \frac{v_2}{v_1}, \tag{2}$$

the ratio of the vacuum expectation values (vevs) of the two Higgs doublets $H_{1,2}$ in the MSSM, which governs the size of the down-type Yukawa couplings. We normalize the vevs as $v \equiv \sqrt{v_1^2+v_2^2} = 174$ GeV. Since a_μ involves a chirality-flip, it is proportional to the muon Yukawa coupling y_μ. Large values $\tan\beta \sim 50$ lead to similar top and bottom Yukawa couplings and are therefore preferred in scenarios with Yukawa unification. Remarkably, naive multi-Higgs doublet models fail to explain the deviation in Eq. (1), because the corresponding loop diagrams involve at least three powers of the small coupling y_μ.

The SUSY contributions to a_μ are approximately given by

$$a_\mu^{\rm SUSY} \approx 13\times 10^{-10}\left(\frac{100\,\text{GeV}}{M_{\rm SUSY}}\right)^2 \tan\beta\,\text{sign}(\mu), \tag{3}$$

if the SUSY parameters for the smuon, gaugino and Higgsino masses have a common scale $M_{\rm SUSY}$, see [8] and references therein. The sign of the contributions is given by the sign of the Higgsino mass parameter μ (choosing the gaugino mass parameters M_1, M_2 positive). We restrict our analysis to the case of real μ, M_1 and M_2, because sizable CP-violating phases of these parameters are in conflict with the bounds on electric dipole moments, if $M_{\rm SUSY}$ is in the range needed to accommodate $a_\mu^{\rm exp}$. The SUSY contributions explain the entire deviation of 29.5×10^{-10} if $\tan\beta$ is given by approximately $2.3(M_{\rm SUSY}/100\,\text{GeV})^2$.

Clearly, a_μ plays an eminent role in studies of the MSSM parameter space, see e.g. [9]. In Ref. [10] a preference of a constrained version of the MSSM over the SM is found from a global fit to collider and electroweak precision data. This conclusion is primarily driven by a_μ. In Refs. [11, 12] a future $\tan\beta$-determination using LHC-data combined with a_μ is outlined. Owing to its importance, a_μ therefore deserves a theoretical precision analysis including radiative corrections.

So far, all SUSY one-loop contributions are known [13]. At the two-loop level two kinds of relevant SUSY contributions have been identified: QED-logarithms $\log(M_{\rm SUSY}/m_\mu)$ arising from SUSY one-loop diagrams with additional photon exchange have been evaluated in [14] and amount to -7% to -9% of the one-loop contributions. Two-loop diagrams involving closed loops of either sfermions (stops, sbottoms, etc) or charginos/neutralinos have been evaluated in [15]. They amount to about 2% of the one-loop contributions if all SUSY masses are degenerate but can be much larger, if e.g. smuon masses are very heavy but stops and/or charginos and Higgs bosons are light.

All these known SUSY contributions to a_μ share the feature of Eq. (3): For large $\tan\beta$ they are linear in $\tan\beta$,

$$a_\mu^{\text{SUSY, known}} \propto \alpha^l \left(\frac{m_\mu}{M_{\text{SUSY}}}\right)^2 \tan\beta, \tag{4}$$

where $l = 1,2$ denotes the loop order. In this paper we identify and discuss a SUSY contribution $a_\mu^{\text{SUSY, }\Delta_\mu}$ which is quadratic in $\tan\beta$, i.e. of the order

$$a_\mu^{\text{SUSY, }\Delta_\mu} \propto \alpha^2 \left(\frac{m_\mu}{M_{\text{SUSY}}}\right)^2 \tan^2\beta, \tag{5}$$

and can therefore be a significant correction in the large-$\tan\beta$ region.

The physical origin of these $\tan^2\beta$-corrections is a shift in the muon Yukawa coupling y_μ due to $\tan\beta$-enhanced one-loop effects. In the computation of a_μ^{SUSY} beyond the one-loop level this shift appears in the muon mass renormalization constant δm_μ, defined in the on-shell scheme:

$$\begin{aligned} m_\mu + \delta m_\mu &= \frac{m_\mu}{1+\Delta_\mu} + \text{non-}\tan\beta\text{-enhanced terms}, \\ y_\mu &= \frac{m_\mu}{v\cos\beta(1+\Delta_\mu)}\left(1+\mathcal{O}(\cot\beta)\right), \end{aligned} \tag{6}$$

where m_μ is the physical, pole-mass of the muon and where the shift $\Delta_\mu \propto \alpha\tan\beta$ will be given below. This type of $\tan\beta$-enhanced corrections has been studied intensely in the down-quark sector [16, 17]. In the standard approach one employs the limit $M_{\text{SUSY}} \gg v_2$ and derives an effective loop-induced coupling of H_2 to down-type fermions, which results in relations between masses and Yukawa couplings of the type in Eq. (6) [16]. For a_μ, however, this procedure fails, because a_μ^{SUSY} vanishes in the limit $M_{\text{SUSY}} \gg v_2$, so that the important corrections associated with Δ_μ were overlooked so far. In the case of a_μ one must resort to the method of Ref. [17], which explicitly identifies $\tan\beta$-enhanced loop diagrams and resums them to all orders in perturbation theory for the case of interest $M_{\text{SUSY}} \sim v_2$. Eq. (6) contains the desired effect to all orders $\alpha^l\tan^l\beta$, $l = 1,2,\ldots$. For the phenomenology of a_μ only the term with $l=1$, contributing to a_μ^{SUSY} at the two-loop level, is relevant.

In the following we will show that the shift in Eq. (6) is the only source of the $\tan\beta$-enhanced radiative corrections of type $\alpha^l\tan^l\beta$ and that there are no enhancement factors with even more powers of $\tan\beta$. The proof relies on an analysis of mass singularities similar to the analysis presented in [17]: The one-loop diagram proportional to y_μ gives one power of $m_\mu\tan\beta$. (The second factor of m_μ in Eqs. (4) and (5) stems from the definition of a_μ.) A genuine l-loop diagram (i.e. without counterterms) may involve n powers of $\tan\beta$ stemming from the muon Yukawa coupling y_μ or any other Yukawa coupling $y_f \propto (m_f/M_W)\tan\beta$. It will result in a desired $\tan\beta$-enhanced correction if $n \geq l$ and the loop diagram diverges as $1/(m_f^{n-1})$ for $m_f \to 0$ to compensate for the factor of m_f^n in y_f^n.

Such mass singularities can be analyzed by passing from the MSSM to an effective field theory in which all heavy particles are integrated out and only particles with mass m_f or less are retained. Heavy loops are represented by point-like interactions in the effective theory and the infrared structure of any MSSM loop diagram and its counterpart in the effective theory are the same. A novel feature compared to the analysis of Yukawa interactions in Ref. [17] is the appearance of one dimension-5 coupling, the magnetic interaction term $\overline{\mu}_L\sigma_{\nu\rho}\mu_R F^{\nu\rho}$. On dimensional grounds any loop corrections involving this term can only depend logarithmically on light fermion masses m_f. Potentially dangerous loops involve effective couplings of dimension 4 or less, since they might come with one or more inverse power of m_f. However, the only such couplings induced by heavy loops are those which are already present in low-energy QED and QCD and the effect of the heavy particles in the underlying theory can be completely absorbed into the renormalization of masses and couplings in the effective theory [18].

In conclusion the only effective diagrams with inverse powers of m_f are the known QED diagrams proportional to $1/m_\mu$ and they are unaffected by our MSSM short-distance structure. These findings only hold, if a decoupling scheme is adopted for the renormalization [18]; for our case it is important that m_μ is renormalized in the on-shell scheme. Next we inspect other diagrams involving counterterms: The counterterm for the Yukawa coupling y_μ is $\delta y_\mu = y_\mu \delta m_\mu/m_\mu \propto m_\mu\tan^2\beta$ and gives rise to the enhanced corrections in Eq. (6) [17]. The counterterms for gauge couplings and the muon and photon fields cannot be $\tan\beta$-enhanced, because unlike $\delta y_\mu/y_\mu$ they do not involve any factor of $1/m_f$. Finally the renormalization of the Higgsino mass parameter μ, the soft SUSY breaking terms and the parameter $\tan\beta$ can be chosen at will, and our statement about the absence of $\tan\beta$-enhanced corrections beyond those in Eq. (6) is valid for all renormalization schemes in which these renormalization constants are *not* $\tan\beta$-enhanced. This includes common schemes such as the $\overline{\text{DR}}$-scheme for all SUSY parameters, or a mixed scheme where $\tan\beta$ and the A-parameter are defined in the $\overline{\text{DR}}$-scheme but the smuon, chargino and neutralino masses are renormalized on-shell.

The shift Δ_μ is given by the $\tan\beta$-enhanced terms of the muon self energy. In terms of the loop function

$$I(a,b,c) = \frac{a^2b^2\log\frac{a^2}{b^2} + b^2c^2\log\frac{b^2}{c^2} + c^2a^2\log\frac{c^2}{a^2}}{(a^2-b^2)(b^2-c^2)(a^2-c^2)}, \tag{7}$$

which satisfies $I(a,a,a)=1/(2a^2)$, it can be written as

$$\begin{aligned}\Delta_\mu = &-\mu\ \tan\beta\,\frac{g_2^2\,M_2}{16\pi^2}\,I(m_1,m_2,m_{\tilde\nu_\mu})\\&-\mu\ \tan\beta\,\frac{g_2^2\,M_2}{16\pi^2}\,\frac{1}{2}I(m_1,m_2,m_{\tilde\mu_L})\\&-\mu\ \tan\beta\,\frac{g_1^2\,M_1}{16\pi^2}\,\Big[I(\mu,M_1,m_{\tilde\mu_R})\\&-\frac{1}{2}I(\mu,M_1,m_{\tilde\mu_L})-I(M_1,m_{\tilde\mu_L},m_{\tilde\mu_R})\Big].\end{aligned}\tag{8}$$

The appearing gaugino, Higgsino, and smuon mass parameters and the Standard Model parameters $g_{1,2}$, s_W are defined as usual, see e.g. [8], and we have defined

$$\begin{aligned}m_{1,2}^2 &= \frac{1}{2}\Big[(M_2^2+\mu^2+2M_W^2)\\&\qquad\mp\sqrt{(M_2^2+\mu^2+2M_W^2)^2-4M_2^2\mu^2}\Big],\\m_{\tilde\nu_\mu}^2 &= m_{L,\tilde\mu}^2-\frac{M_Z^2}{2},\quad m_{\tilde\mu_L}^2=m_{L,\tilde\mu}^2-M_Z^2(s_W^2-\frac{1}{2}),\\m_{\tilde\mu_R}^2 &= m_{R,\tilde\mu}^2+M_Z^2 s_W^2.\end{aligned}\tag{9}$$

While the chargino contributions are exact in the large-$\tan\beta$ limit, the neutralino contributions in (8) have been simplified using the approximation $M_Z \ll \mu, M_1, M_2$. The deviation of Δ_μ as given in (8) from the exact result satisfies $|\Delta_\mu - \Delta_\mu^{\rm exact}| < 0.01$ over the entire parameter range (all supersymmetry masses are varied independently between 100 GeV and 2 TeV, $\tan\beta \le 100$) for which $|a_\mu^{\rm SUSY}| < 10^{-8}$.

Similar shifts exist for all down-type fermions, and in particular the shift of the bottom-quark Yukawa coupling Δ_b has been analyzed in detail in the literature [16, 17], and the results can be readily applied to the muon case.

The contribution $a_\mu^{\rm SUSY,\Delta_\mu}$ of the new $\tan\beta$-enhanced contributions to $a_\mu = -2m_\mu F_M(0)$ can be easily obtained by noting that the magnetic form factor $F_M(0)$ is proportional to y_μ, apart from numerically irrelevant terms with three or more powers of y_μ. Now y_μ enters F_M in two ways: First it appears explicitly in the higgsino-muon couplings or in the Higgs-smuon coupling triggering the left-right mixing in the smuon mass matrix. Second it appears implicitly through $m_\mu \propto y_\mu \cos\beta$, which arises from the application of the Dirac equation $\not p\mu_L = m_\mu \mu_R$. The second contribution is suppressed by a factor of $\cot\beta$ compared to the first. The $\tan\beta$-enhanced corrections to the first contribution are obtained by using the expression in Eq. (6) for y_μ. Therefore

$$a_\mu^{\rm SUSY,1L}+a_\mu^{\rm SUSY,\Delta_\mu}=a_\mu^{\rm SUSY,1L}\left(\frac{1}{1+\Delta_\mu}\right).\tag{10}$$

Note that this formula is only correct for the enhanced terms of order $\alpha^l\tan^l\beta$, but this is sufficient for our purposes.

Equation (10) is the main result of this paper. We are now in the position to write down the most accurate prediction for $a_\mu^{\rm SUSY}$, replacing the result given in [8] by [21] [22]

$$\begin{aligned}a_\mu^{\rm SUSY} &= a_\mu^{\rm SUSY,1L}\left(1-\frac{4\alpha}{\pi}\log\frac{M_{\rm SUSY}}{m_\mu}\right)\left(\frac{1}{1+\Delta_\mu}\right)\\&+a_\mu^{(\chi\gamma H)}+a_\mu^{(\tilde f\gamma H)}+a_\mu^{(\chi\{W,Z\}H)}+a_\mu^{(\tilde f\{W,Z\}H)}\\&+a_\mu^{\rm SUSY,ferm,2L}+a_\mu^{\rm SUSY,bos,2L}+\ldots.\end{aligned}\tag{11}$$

The first line contains the one-loop result, corrected by large QED-logarithms [14] and by the new $\tan\beta$-enhanced terms discussed here. The second and third lines contain further known two-loop contributions [15]. The terms $a_\mu^{(\tilde pVS)}$ denote contributions from diagrams where a vector boson V and scalar S couple to the muon line and which involve a closed $\tilde p$-loop; $a_\mu^{\rm SUSY,ferm,2L}$ and $a_\mu^{\rm SUSY,bos,2L}$ denote the difference of diagrams without SUSY particles between the MSSM and the SM, arising from the different Higgs sectors. The dots denote known but negligible terms computed in [15], the contributions computed partially in [19], and the remaining, unknown contributions. For analytical results see the original references and [8].

In order to discuss the phenomenological impact of our new contributions, we start by noting that

$$\Delta_\mu = -0.0018\tan\beta\ {\rm sign}\,\mu\tag{12}$$

in the case where all SUSY masses are equal and much larger than M_W. Hence in the interesting region with $\tan\beta\sim 50$ the value of $\tan\beta$ extracted from $a_\mu^{\rm exp}$ will be off by roughly 10%, if $a_\mu^{\rm SUSY,\Delta_\mu}$ is omitted in Eq. (10). Fig. 1 shows the impact of the new contribution on the dependence of a_μ on $\tan\beta$.

Importantly, as a dimensionless quantity Δ_μ does not decouple for arbitrarily large SUSY masses. For slight mass splittings, Δ_μ can be even larger than in Eq. (12). For example, for $\tan\beta = 50$ and $m_{L,\tilde\mu} = 300$, $m_{R,\tilde\mu} = 500$, $M_2 = 650$, $\mu = 800$ GeV and $M_1 = M_2/2$ one obtains a correction of $+14\%$ for $a_\mu^{\rm SUSY}$.

Among the SPS SUSY benchmark parameter points [20] large effects are obtained at SPS 4 with $\tan\beta = 50$ ($+8\%$), and at SPS 1b with $\tan\beta = 30$ ($+6\%$). In particular, for SPS 4, which is already experimentally disfavoured by a_μ, the contribution rises from $a_\mu^{\rm SUSY}(\text{SPS 4}) = 49\times10^{-10}$ to 53×10^{-10} by including the new $\tan\beta$-enhanced correction. This corresponds to a rise of the deviation from the experimental value from 2.2σ to 2.6σ.

In conclusion we have identified a new $\tan\beta$-enhanced contribution to a_μ which first enters at the two-loop level. In scenarios with large values of $\tan\beta$ the new term $a_\mu^{\rm SUSY,\Delta_\mu}$ alters the MSSM phenomenology by typically 10%, but can have an even larger impact in certain regions of the parameter space. Our contribution is typically larger than the previously known supersymmetric two-loop corrections and should be included in global fits of electroweak precision data to the MSSM.

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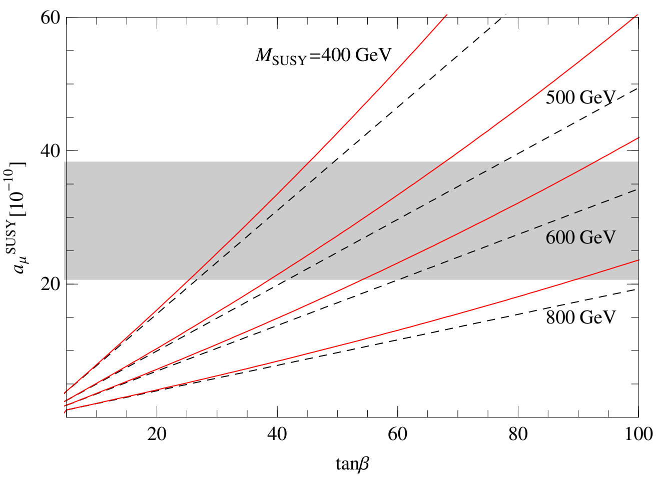

FIG. 1: a_μ as a function of $\tan\beta$ for four different values of degenerate SUSY masses. Solid (red) lines: correct a_μ as in Eq. (11). Dashed (black) lines: a_μ without $a_\mu^{\rm SUSY,\Delta_\mu}$. Gray band: 1σ range of Eq. (1).

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[21] Note that there is no double-counting between the non-$\tan^2\beta$-enhanced terms implicitly contained in (10) and the terms in the second and third line of (11).
[22] In our numerical analysis we parametrize the one-loop result in terms of the muon decay constant G_μ, i.e. we replace $\pi\alpha/s_W^2 \to \sqrt{2}G_\mu M_W^2$, in order to absorb further universal two-loop corrections. This gives rise to slight numerical differences compared to [8].