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NLO QCD corrections to $\mathbf{pp \to t\bar{t} + jet + X}$

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We discuss the production of top–anti-top quark pairs in association with a hard jet at the Tevatron and at the LHC and we report on the calculation of the next-to-leading order QCD corrections to this process. Numerical results for the $t\bar{t}$+jet cross section and the forward–backward charge asymmetry are presented. The corrections stabilize the leading-order prediction for the cross section. In contrast, the charge asymmetry receives large corrections. The dependence of the cross section as well as the asymmetry on the minimum transverse momenta used to define the additional jet is studied in detail for the Tevatron.

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1. Introduction

The top quark is the heaviest of the known elementary particles. More than ten years after its discovery, the dynamics and many properties of the top quark, such as its electroweak quantum numbers, are not yet precisely measured. It is widely believed that the top quark plays a key role in extensions of the Standard Model. This renders experimental investigations of the top quark particularly important. Up to now the main (direct) source of information on top quarks are top-quark pairs produced at the Tevatron. Only recently first evidence for single-top production has been found [1]. It is important to note that in the inclusive $\mathrm{t\bar{t}}$ sample a significant fraction comprises $\mathrm{t\bar{t}}$+jet events. An investigation of the process of $\mathrm{t\bar{t}}$ production in association with a hard jet can thus improve our knowledge about the top quark.

In this context, the forward–backward charge asymmetry of the top (or anti-top) quark [2, 3, 4, 5] is of particular interest. In inclusive $\mathrm{t\bar{t}}$ production it appears first at one loop, because it results from interferences of C-odd with C-even parts of double-gluon exchange between initial and final states. This means that the available prediction for $\mathrm{t\bar{t}}$ production—although of one-loop order—describes this asymmetry only at leading-order (LO) accuracy. In $\mathrm{t\bar{t}}$+jet production the asymmetry appears already in LO. Thus, the next-to-leading order (NLO) calculation described in the following provides a true NLO prediction for the asymmetry. Our calculation will, therefore, be an important tool in the experimental analysis of this observable at the Tevatron where the asymmetry is measureable as discussed in Ref. [5].

Measuring the cross section of the related process of $\mathrm{t\bar{t}}+\gamma$ production provides direct access to the electric charge of the top quark. Obviously NLO QCD predictions to this process are important for a reliable analysis. They can be obtained from $\mathrm{t\bar{t}}$+jet production presented here via simple substitutions. Finally, a signature of $\mathrm{t\bar{t}}$ in association with a hard jet represents an important background process for searches at the LHC, such as the search for the Higgs boson in the weak-vector-boson fusion or $\mathrm{t\bar{t}H}$ channels.

The above-mentioned issues clearly underline the case for an NLO calculation for $\mathrm{t\bar{t}}$+jet production at hadron colliders. We report here on a first calculation of this kind as presented in Ref. [6].

2. Details of the NLO calculation

At LO, hadronic $\mathrm{t\bar{t}}$+jet production receives contributions from the partonic processes $q\bar{q} \to \mathrm{t\bar{t}}g$, $qg \to \mathrm{t\bar{t}}q$, $\bar{q}g \to \mathrm{t\bar{t}}\bar{q}$, and $gg \to \mathrm{t\bar{t}}g$. The first three channels are related by crossing symmetry to the amplitude $0 \to \mathrm{t\bar{t}}q\bar{q}g$. Evaluating $2 \to 3$ particle processes at the NLO level, is non-trivial, both in the analytical and numerical parts of the calculation. In order to prove the correctness of our results we have evaluated each ingredient twice using independent calculations based—as far as possible—on different methods, yielding results in mutual agreement.

2.1 Virtual corrections

The virtual corrections modify the partonic processes that are already present at LO. At NLO these corrections are induced by self-energy, vertex, box (4-point), and pentagon (5-point) corrections. The most complicated diagrams are the pentagon diagrams.

Version 1 of the virtual corrections is essentially obtained following the method described in Ref. [7], where $t\bar{t}H$ production at hadron colliders was considered. Feynman diagrams and amplitudes have been generated with the *FeynArts* package [8, 9] and further processed with in-house *Mathematica* routines, which automatically create an output in *Fortran*. The IR (soft and collinear) singularities are analytically separated from the finite remainder as described in Refs. [7, 10]. The tensor integrals appearing in the pentagon diagrams are directly reduced to box integrals following Ref. [11]. This method does not introduce inverse Gram determinants in this step, thereby avoiding notorious numerical instabilities in regions where these determinants become small. Box and lower-point integrals are reduced à la Passarino–Veltman [12] to scalar integrals, which are either calculated analytically or using the results of Refs. [13, 14, 15]. Sufficient numerical stability is already achieved in this way. Nevertheless the integral evaluation is currently further refined by employing the more sophisticated methods described in Ref. [16] in order to numerically stabilize the tensor integrals in exceptional phase-space regions.

Version 2 of the evaluation of loop diagrams starts with the generation of diagrams and amplitudes via *QGRAF* [17], which are then further manipulated with *Form* [18] and eventually automatically translated into *C++* code. The reduction of the the 5-point tensor integrals to scalar integrals is performed with an extension of the method described in Ref. [19]. In this procedure also inverse Gram determinents of four four-momenta are avoided. The lower-point tensor integrals are reduced using an independent implementation of the Passarino–Veltman procedure. The IR-finite scalar integrals are evaluated using the *FF* package [20, 21].

2.2 Real corrections

The matrix elements for the real corrections are given by $0 \to t\bar{t}gggg$, $0 \to t\bar{t}q\bar{q}gg$, $0 \to t\bar{t}q\bar{q}q'\bar{q}'$ and $0 \to t\bar{t}q\bar{q}q\bar{q}$. The various partonic processes are obtained from these matrix elements by all possible crossings of light particles into the initial state.

The evaluation of the real-emission amplitudes is performed in two independent ways. Both evaluations employ the dipole subtraction formalism [22, 23, 24] for the extraction of IR singularities and for their combination with the virtual corrections.

Version 1 results from a fully automated calculation based on helicity amplitudes, as described in Ref. [25]. Individual helicity amplitudes are computed with the help of Berends–Giele recurrence relations [26]. The evaluation of color factors and the generation of subtraction terms is automated. For the channel $gg \to t\bar{t}gg$ a dedicated soft-insertion routine [27] is used for the generation of the phase space.

Version 2 uses for the LO $2 \to 3$ processes and the $gg \to t\bar{t}gg$ process optimized code obtained from a Feynman diagramatic approach. As in version 1 standard techniques like color decomposition and the use of helicity amplitudes are employed. For the $2 \to 4$ processes including light quarks, *Madgraph* [28] has been used. The subtraction terms according to Ref. [24] are obtained in a semi-automatized manner based on an in-house library written in *C++*.

3. Numerical results

In the following we consistently use the CTEQ6 [29, 30] set of parton distribution functions (PDFs). In detail, we take CTEQ6L1 PDFs with a 1-loop running α_s in LO and CTEQ6M PDFs

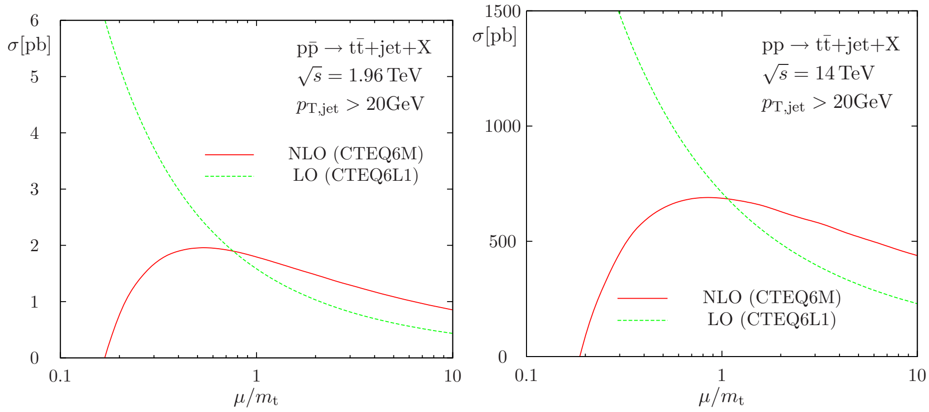

Figure 1: Scale dependence of the LO and NLO cross sections for $\mathrm{t\bar{t}}$+jet production at the Tevatron (left) and at the LHC (right) as taken from Ref. [6], where the renormalization scale (μ_r) and the factorization scale (μ_f) are set equal to μ.

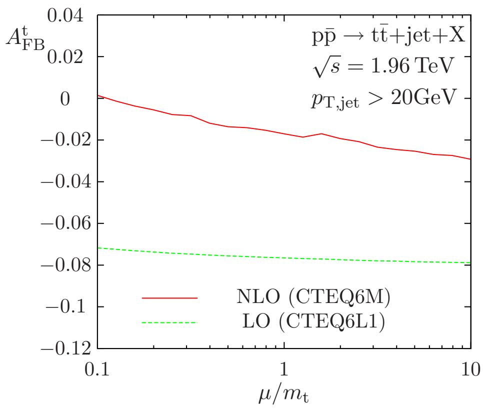

Figure 2: Scale dependence of the LO and NLO forward–backward charge asymmetry of the top quark in $\mathrm{p\bar{p}} \to \mathrm{t\bar{t}}$+jet+$X$ at the Tevatron as taken from Ref. [6] with $\mu = \mu_f = \mu_r$.

with a 2-loop running α_s in NLO. The number of active flavours is $N_F = 5$, and the respective QCD parameters are $\Lambda_5^{\mathrm{LO}} = 165\,\mathrm{MeV}$ and $\Lambda_5^{\overline{\mathrm{MS}}} = 226\,\mathrm{MeV}$. Note that the top-quark loop in the gluon self-energy is subtracted at zero momentum. In this scheme the running of α_s is generated solely by the contributions of the light quark and gluon loops. The top-quark mass is renormalized in the on-shell scheme, as numerical value we take $m_t = 174\,\mathrm{GeV}$.

We apply the jet algorithm of Ref. [31] with $R = 1$ for the definition of the tagged hard jet. Unless stated otherwise we require a transverse momentum of $p_{\mathrm{T,jet}} > p_{\mathrm{T}}^{\mathrm{cut}} = 20\,\mathrm{GeV}$ for the hardest jet. The outgoing (anti-)top quarks are neither affected by the jet algorithm nor by the phase-space cut. Note that the LO prediction and the virtual corrections are not influenced by the jet algorithm, but the real corrections are.

In Figure 1 the scale dependence of the NLO cross sections is shown. For comparison, the LO results are included as well. As expected, the NLO corrections significantly reduce the scale dependence compared to LO. We observe that arround $\mu \approx m_t$ the NLO corrections are of moderate size for the chosen setup.

We have also studied the forward–backward charge asymmetry of the top quark at the Tevatron. In LO the asymmetry is defined by

$$A^{\mathrm{t}}_{\mathrm{FB,LO}} = \frac{\sigma^{-}_{\mathrm{LO}}}{\sigma^{+}_{\mathrm{LO}}}, \quad \sigma^{\pm}_{\mathrm{LO}} = \sigma_{\mathrm{LO}}(y_{\mathrm{t}}{>}0) \pm \sigma_{\mathrm{LO}}(y_{\mathrm{t}}{<}0), \tag{3.1}$$

where y_{t} denotes the rapidity of the top quark. Cross-section contributions $\sigma(y_{\mathrm{t}} \gtrless 0)$ correspond to top quarks in the forward or backward hemispheres, respectively, where incoming protons fly into the forward direction by definition. Denoting the corresponding NLO contributions to the cross sections by $\delta\sigma^{\pm}_{\mathrm{NLO}}$, we define the asymmetry at NLO by

$$A^{\mathrm{t}}_{\mathrm{FB,NLO}} = \frac{\sigma^{-}_{\mathrm{LO}}}{\sigma^{+}_{\mathrm{LO}}} \left(1 + \frac{\delta\sigma^{-}_{\mathrm{NLO}}}{\sigma^{-}_{\mathrm{LO}}} - \frac{\delta\sigma^{+}_{\mathrm{NLO}}}{\sigma^{+}_{\mathrm{LO}}} \right), \tag{3.2}$$

i.e. via a consistent expansion in α_{s}. Note, however, that the LO cross sections in Eq. (3.2) are evaluated in the NLO setup (PDFs, α_{s}).

Figure 2 shows the scale dependence of the asymmetry at LO and NLO. The LO result for the asymmetry is of order α_s^0 and does therefore not depend on the renormalization scale. The plot for the LO result shows a mild residual dependence on the factorization scale, but the size of this variation does not reflect the theoretical uncertainty, which is much larger. The NLO corrections to the asymmetry are of order α_s^1 and depend on the renormalization scale. It is therefore natural to expect a stronger scale dependence of the asymmetry at NLO than at LO, as seen in the plot. The size of the asymmetry, which is about -7% at LO, is drastically reduced by the NLO corrections. To investigate the origin of the large NLO corrections to the asymmetry we have studied the dependence on the cut value $p_{\mathrm{T}}^{\mathrm{cut}}$ used to define the minimal p_{T} of the additional jet. The results are shown in Table 1. We observe that both the NLO cross section as well as the NLO asymmetry dependent strongly on $p_{\mathrm{T}}^{\mathrm{cut}}$. This is related to the fact that the cross section becomes ill-defined in the limit $p_{\mathrm{T}}^{\mathrm{cut}} \to 0$ due to the appearance of IR divergencies. On the other hand, the LO prediction for the asymmetry shows only a mild dependence on $p_{\mathrm{T}}^{\mathrm{cut}}$.

	cross section [pb]		charge asymmetry [%]	
$p_{\mathrm{T}}^{\mathrm{cut}}$ [GeV]	LO	NLO	LO	NLO
20	$1.583(2)^{+0.96}_{-0.55}$	$1.791(1)^{+0.16}_{-0.31}$	$-7.69(4)^{+0.10}_{-0.085}$	$-1.77(5)^{+0.58}_{-0.30}$
30	$0.984(1)^{+0.60}_{-0.34}$	$1.1194(8)^{+0.11}_{-0.20}$	$-8.29(5)^{+0.12}_{-0.085}$	$-2.27(4)^{+0.31}_{-0.51}$
40	$0.6632(8)^{+0.41}_{-0.23}$	$0.7504(5)^{+0.072}_{-0.14}$	$-8.72(5)^{+0.13}_{-0.10}$	$-2.73(4)^{+0.35}_{-0.49}$
50	$0.4670(6)^{+0.29}_{-0.17}$	$0.5244(4)^{+0.049}_{-0.096}$	$-8.96(5)^{+0.14}_{-0.11}$	$-3.05(4)^{+0.49}_{-0.39}$

Table 1: Cross section and forward-backward charge asymmetry at the Tevatron for different values of $p_{\mathrm{T}}^{\mathrm{cut}}$ used to define the minimal transverse momentum p_{T} of the additional jet ($\mu = \mu_f = \mu_r = m_t$). The upper and lower indices are the shifts towards $\mu = m_t/2$ and $\mu = 2m_t$.

4. Conclusions

Predictions for $\mathrm{t\bar{t}}$+jet production at hadron colliders have been reviewed at NLO QCD. For the cross section the NLO corrections drastically reduce the scale dependence of the LO predictions, which is of the order of 100%. The charge asymmtry of the top quarks, which is going to be measured at the Tevatron, is significantly decreased at NLO and is almost washed out by the residual scale dependence. In addition we have also studied the $p_{\mathrm{T}}^{\mathrm{cut}}$-dependence of the NLO predictions. Further refinements of the precise definition of the forward-backward asymmetry are required to stabilize the asymmetry with respect to higher order corrections.

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