DESY 08-021
TTP08-10
LPSC 08-020
February 2008

ISSN 0418-9833

Ghost contributions to charmonium production in polarized high-energy collisions

M. Klasen[a,*], B. A. Kniehl[b,†], L. N. Mihailă[c,‡], M. Steinhauser[c,§]

[a] *Laboratoire de Physique Subatomique et de Cosmologie, Université Joseph Fourier/CNRS-IN2P3/INPG, 53 Avenue des Martyrs, 38026 Grenoble, France*

[b] *II. Institut für Theoretische Physik, Universitä t Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany*

[c] *Institut für Theoretische Teilchenphysik, Universität Karlsruhe, Engesserstraße 7, 76131 Karlsruhe, Germany*

Abstract

In a previous paper [Phys. Rev. D **68**, 034017 (2003)], we investigated the inclusive production of prompt J/ψ mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions in the factorization formalism of nonrelativistic quantum chromodynamics providing compact analytic results for the double longitudinal-spin asymmetry $\mathcal{A}_{LL}$. For convenience, we adopted a simplified expression for the tensor product of the gluon polarization four-vector with its charge conjugate, at the expense of allowing for ghost and anti-ghosts to appear as external particles. While such ghost contributions cancel in the cross section asymmetry $\mathcal{A}_{LL}$ and thus were not listed in our previous paper, they do contribute to the absolute cross sections. For completeness and the reader's convenience, they are provided in this addendum.

PACS: 12.38.Bx, 13.60.Le, 13.85.Ni, 14.40.Gx

*klasen@lpsc.in2p3.fr
†kniehl@desy.de
‡luminita@particle.uni-karlsruhe.de
§matthias.steinhauser@uka.de

The factorization formalism of nonrelativistic QCD (NRQCD) [1] provides a rigorous theoretical framework for the description of heavy-quarkonium production and decay. This formalism implies a separation of process-dependent short-distance coefficients, to be calculated perturbatively as expansions in the strong-coupling constant α_s, from supposedly process-independent long-distance matrix elements (MEs), to be extracted from experiment, and takes into account the complete structure of the $Q\overline{Q}$ Fock space, which is spanned by the states $n = {}^{2S+1}L_J^{(C)}$ with definite spin S, orbital angular momentum L, total angular momentum J, and color multiplicity $C = 1, 8$. By velocity scaling rules, the MEs are predicted to scale with a definite power of the heavy-quark (Q) velocity $v \ll 1$, so that a small number of these non-perturbative parameters should allow for meaningful predictions in practice.

In Ref. [2], we applied the NRQCD factorization formalism to the inclusive production of prompt J/ψ mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions and provided compact analytic results for the double longitudinal-spin asymmetry $\mathcal{A}_{LL}$, defined in Eq. (2.1) of Ref. [2]. Specifically, we considered inclusive J/ψ production in polarized pp, γd, and $\gamma\gamma$ collisions, appropriate for the RHIC-Spin experiments at the BNL Relativistic Heavy Ion Collider (RHIC), the SLAC fixed-target experiment E161, and the TeV-Energy Superconducting Linear Accelerator (TESLA) operated in the e^+e^- and $\gamma\gamma$ modes, respectively. We took the J/ψ mesons to be unpolarized.

There is a technical subtlety related to the definition of the polarization four-vector $\varepsilon(p,\xi)$ of an external gluon, with four-momentum p and helicity quantum number $\xi = \pm 1$, which is potentially prone to create confusion. As for the tensor product of $\varepsilon(p,\xi)$ with its charge conjugate, a natural choice, which avoids the introduction of unphysical degrees of gluon polarization, is

$$\varepsilon_\mu(p,\xi)\varepsilon_\nu^*(p,\xi) = \frac{1}{2}\left(-g_{\mu\nu} + \frac{p_\mu\eta_\nu + p_\nu\eta_\mu}{k\cdot\eta} + i\xi\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho\eta^\sigma}{p\cdot\eta}\right), \tag{1}$$

where η is an arbitrary light-like four-vector orthogonal to p, with $\eta^2 = 0 \neq p\cdot\eta$. An obvious disadvantage of Eq. (1) is that it introduces a host of terms involving η in intermediate results. In practical calculations such as the one performed in Ref. [2], it is therefore advantageous to omit the second term on the right-hand side of Eq. (1) and to identify η with the four-momentum p' of another external parton [3], so that

$$\varepsilon_\mu(p,\xi)\varepsilon_\nu^*(p,\xi) = \frac{1}{2}\left(-g_{\mu\nu} + i\xi\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho p'^\sigma}{p\cdot p'}\right), \tag{2}$$

at the expense of endowing the gluon with unphysical degrees of polarization, which must be eliminated by subtracting contributions arising from the presence of its ghost h and anti-ghost $\overline{h}$ as external particles. Since such ghost contributions cancel in the cross section differences appearing in the numerator of $\mathcal{A}_{LL}$, as illustrated below, we did not list them in Ref. [2]. However, they are necessary to recover the well-known expressions for the unpolarized cross sections entering the denominator of $\mathcal{A}_{LL}$, as we did. In this sense, they were included in our numerical analysis.

Recently, there has been renewed interest in charmonium production by polarized hadron-hadron and photon-hadron collisions [4, 5]. In Ref. [4], the J/ψ and ψ' polarizations were predicted for polarized pp collisions at RHIC-Spin. In Ref. [5], the squares of the helicity amplitudes $\mathcal{M}(a,b,c)$ of the partonic subprocesses $\gamma(a)+g(b)\to Q\overline{Q}[n]+g(c)$ and $g(a)+g(b)\to Q\overline{Q}[n]+g(c)$ were listed for $n={}^1S_0^{(C)},{}^3S_1^{(C)},{}^1P_1^{(C)},{}^3P_J^{(C)}$ with $J=0,1,2$ and $C=1,8$. The longitudinally-polarized differential cross sections evaluated from these helicity amplitudes were found to agree with our results [2] after properly subtracting the ghost contributions mentioned above, which we had provided to the authors of Ref. [5] via private communication. Since these contributions may be useful for applications by other authors as well, we decided to publish them in this addendum to Ref. [2].

In the following, we present the differential cross sections $\mathrm{d}\sigma/\mathrm{d}t$ of the partonic subprocesses

$$\{\gamma,g\}h\to c\overline{c}[n]h. \tag{3}$$

Here and in the following, s, t, and u denote the usual Mandelstam variables. The results for $\{\gamma,g\}\overline{h}\to c\overline{c}[n]\overline{h}$ are identical by charge-conjugation invariance, while those for $h\{\gamma,g\}\to c\overline{c}[n]h$ and $h\overline{h}\to c\overline{c}[n]\{\gamma,g\}$ are related by crossing symmetry, as indicated below. As usual, $\mathrm{d}\sigma/\mathrm{d}t$ is evaluated from the absolute square of the transition matrix element $\mathcal{M}$ through multiplication with factors for flux, phase space, spin, and color, as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}=\frac{1}{2s}\,\frac{1}{8\pi s}\,\frac{1}{4}\left(\frac{1}{8}\right)^i|\mathcal{M}|^2, \tag{4}$$

where $i=1,2$ is the number of color-octet partons (gluons or ghosts) in the initial state.

The only non-vanishing ghost contributions read

$$\begin{aligned}
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^1S_0^{(8)}]h) &= \frac{24e^2g_s^4\langle\mathcal{O}[{}^1S_0^{(8)}]\rangle Q_c^2 su}{Mt(s+u)^2},\\
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_0^{(8)}]h) &= \frac{32e^2g_s^4\langle\mathcal{O}[{}^3P_0^{(8)}]\rangle Q_c^2 su}{M^3t(s+u)^4}[(2t+3u)^2+6s(2t+3u)+9s^2],\\
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_1^{(8)}]h) &= \frac{32e^2g_s^4\langle\mathcal{O}[{}^3P_1^{(8)}]\rangle Q_c^2}{M^3(s+u)^4}[u^2(t+u)-su^2+s^2(t-u)+s^3],\\
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_2^{(8)}]h) &= \frac{32e^2g_s^4\langle\mathcal{O}[{}^3P_2^{(8)}]\rangle Q_c^2}{5M^3t(s+u)^4}[3tu^2(t+u)+su(8t^2+21tu+12u^2)\\
&\quad+3s^2(t^2+7tu+8u^2)+3s^3(t+4u)],\\
|\mathcal{M}|^2(gh\to c\overline{c}[{}^1S_0^{(1)}]h) &= \frac{4g_s^2\langle\mathcal{O}[{}^1S_0^{(1)}]\rangle}{3e^2Q_c^2\langle\mathcal{O}[{}^1S_0^{(8)}]\rangle}|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^1S_0^{(8)}]h),\\
|\mathcal{M}|^2(gh\to c\overline{c}[{}^3P_J^{(1)}]h) &= \frac{4g_s^2\langle\mathcal{O}[{}^3P_J^{(1)}]\rangle}{3e^2Q_c^2\langle\mathcal{O}[{}^3P_J^{(8)}]\rangle}|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_J^{(8)}]h),\\
|\mathcal{M}|^2(gh\to c\overline{c}[{}^1S_0^{(8)}]h) &= \frac{5g_s^2}{12e^2Q_c^2}|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^1S_0^{(8)}]h),
\end{aligned}$$

$$
\begin{aligned}
|\mathcal{M}|^2(gh \to c\overline{c}[^3S_1^{(8)}]h) &= \frac{3g_s^6\langle\mathcal{O}[^3S_1^{(8)}]\rangle}{4M^5stu(s+u)^2}[tu^2(t+u)^2(3t-u)+su(-2t^4+2t^3u \\
&\quad +7t^2u^2+4tu^3+u^4)+s^2(3t^4+t^3u-4t^2u^2-3tu^3+u^4) \\
&\quad +s^3(4t^3+7t^2u-2tu^2-u^3)+s^4(t^2+6tu-u^2)], \\
|\mathcal{M}|^2(gh \to c\overline{c}[^1P_1^{(8)}]h) &= \frac{g_s^2\langle\mathcal{O}[^1P_1^{(8)}]\rangle}{e^2Q_c^2\langle\mathcal{O}[^1S_0^{(8)}]\rangle M^2}|\mathcal{M}|^2(\gamma h \to c\overline{c}[^1S_0^{(8)}]h), \\
|\mathcal{M}|^2(gh \to c\overline{c}[^3P_J^{(8)}]h) &= \frac{5g_s^2}{12e^2Q_c^2}|\mathcal{M}|^2(\gamma h \to c\overline{c}[^3P_J^{(8)}]h), \qquad (5)
\end{aligned}
$$

where $e = \sqrt{4\pi\alpha}$, with α being Sommerfeld's fine-structure constant, and $g_s = \sqrt{4\pi\alpha_s}$ are the electromagnetic and strong gauge couplings, Q_c and m_c are the fractional electric charge and mass of the c quark, and $M = 2m_c$. By four-momentum conservation, we have $s+t+u = M^2$.

We now explain how the unpolarized and polarized results of Refs. [5, 6] may be recovered from the results of Ref. [2] in combination with Eqs. (4) and (5), considering $\gamma g \to c\overline{c}[^1S_0^{(8)}]g$ as an example. The unpolarized and polarized results of Eqs. (A4) and (A5) in Ref. [6] are obtained from Eq. (4) by inserting

$$
\begin{aligned}
|\mathcal{M}|^2_{\mathrm{unpol}}(\gamma g \to c\overline{c}[^1S_0^{(8)}]g) &= \sum_{\xi_a,\xi_b=\pm1}|\mathcal{M}|^2_{\xi_a,\xi_b}(\gamma g \to c\overline{c}[^1S_0^{(8)}]g) - |\mathcal{M}|^2(\gamma h \to c\overline{c}[^1S_0^{(8)}]h) \\
&\quad - |\mathcal{M}|^2(\gamma\overline{h} \to c\overline{c}[^1S_0^{(8)}]\overline{h}), \\
|\mathcal{M}|^2_{LL}(\gamma g \to c\overline{c}[^1S_0^{(8)}]g) &= \sum_{\xi_a,\xi_b=\pm1}(-1)^{\xi_a\xi_b}|\mathcal{M}|^2_{\xi_a,\xi_b}(\gamma g \to c\overline{c}[^1S_0^{(8)}]g), \qquad (6)
\end{aligned}
$$

respectively, where $|\mathcal{M}|^2_{\xi_a,\xi_b}(\gamma g \to c\overline{c}[^1S_0^{(8)}]g)$ may be gleaned from Eq. (A5) of Ref. [2] and $|\mathcal{M}|^2(\gamma h \to c\overline{c}[^1S_0^{(8)}]h) = |\mathcal{M}|^2(\gamma\overline{h} \to c\overline{c}[^1S_0^{(8)}]\overline{h})$ is given in Eq. (5) above. As mentioned above, all ingredients entering $|\mathcal{M}|^2_{LL}(\gamma g \to c\overline{c}[^1S_0^{(8)}]g)$ are contained in Ref. [2]. By crossing symmetry, we have

$$
\begin{aligned}
|\mathcal{M}|^2(h\gamma \to c\overline{c}[^1S_0^{(8)}]h) &= |\mathcal{M}|^2(\gamma h \to c\overline{c}[^1S_0^{(8)}]h)\Big|_{t\leftrightarrow u}, \\
|\mathcal{M}|^2(h\overline{h} \to c\overline{c}[^1S_0^{(8)}]\gamma) &= |\mathcal{M}|^2(\gamma h \to c\overline{c}[^1S_0^{(8)}]h)\Big|_{s\leftrightarrow t}. \qquad (7)
\end{aligned}
$$

Similar relationships hold for the other partonic subprocesses involving two external gluons considered in Ref. [2].

In conclusion, we complemented the partonic cross sections for the inclusive production of prompt J/ψ mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions listed in the Appendix of Ref. [2] by providing the ghost contributions, which cancel in the cross section differences entering $\mathcal{A}_{LL}$, but contribute to absolute cross sections, including the unpolarized ones.

We thank Jack Smith for carefully comparing the results of Ref. [5] with ours [2]. The work of B.A.K. was supported in part by the German Federal Ministry for Education and

Research BMBF through Grant No. 05 HT6GUA, by the German Research Foundation DFG through Grant No. KN 365/6–1, and by the Helmholtz Association through Grant No. HA 101.

References

[1] W. E. Caswell and G. P. Lepage, Phys. Lett. B **167**, 437 (1986); G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997) [arXiv:hep-ph/9407339].

[2] M. Klasen, B. A. Kniehl, L. N. Mihaila, and M. Steinhauser, Phys. Rev. D **68**, 034017 (2003) [arXiv:hep-ph/0306080].

[3] I. Bojak and M. Stratmann, Phys. Lett. B **433**, 411 (1998) [arXiv:hep-ph/9804353]; Nucl. Phys. B **540**, 345 (1999) [arXiv:hep-ph/9807405].

[4] G. C. Nayak and J. Smith, Phys. Rev. D **73**, 014007 (2006) [arXiv:hep-ph/0509335].

[5] M. M. Meijer, J. Smith, and W. L. van Neerven, arXiv:0710.3090 [hep-ph].

[6] F. Yuan, H.-S. Dong, L.-K. Hao, and K.-T. Chao, Phys. Rev. D **61**, 114013 (2000) [arXiv:hep-ph/9909221].