PRECISE CHARM AND BOTTOM QUARK MASSES

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New data for the total cross section $\sigma(e^+e^- \to \text{hadrons})$ in the charm and bottom threshold region are combined with an improved theoretical analysis, which includes recent four-loop calculations, to determine the short distance $\overline{\text{MS}}$ charm and bottom quark masses. The final result for the $\overline{\text{MS}}$-masses, $m_c(3\text{ GeV}) = 0.986(13)$ GeV and $m_b(10\text{ GeV}) = 3.609(25)$ GeV is consistent with but significantly more precise than a similar previous study.

The strong coupling constant and the quark masses are the fundamental input parameters of the theory of strong interaction. Quark masses are an essential input for the evaluation of weak decay rates of heavy mesons and for quarkonium spectroscopy. Decay rates and branching ratios of a light Higgs boson, suggested by electroweak precision measurements, depend critically on the masses of the charm and bottom quarks, m_c and m_b. Last not least, confronting the predictions for these masses with experiment is an important task for all variants of Grand Unified Theories. To deduce the values in a consistent way from different experimental investigations and with utmost precision is thus a must for current phenomenology.

A detailed analysis of m_c and m_b based on the ITEP sum rules [1] has been performed several years ago[2] and lead to $m_c(m_c) = 1.304(27)$ GeV and $m_b(m_b) = 4.191(51)$ GeV. During the past years new and more precise data for $\sigma(e^+e^- \to \text{hadrons})$ have become available in the low energy region, in particular for the parameters of the charmonium and bottomonium resonances. Furthermore, the error in the strong coupling constant $\alpha_s(M_Z) = 0.1189 \pm 0.0020$ has been reduced. Last not least, the vacuum polarization induced by massive quarks has recently been computed in four-loop approximation[3,4]; more precisely: its first derivative at $q^2 = 0$ has been evaluated, which corresponds to the lowest moment of the familiar R-ratio. With the help of the traditional integration-by-parts method in combination with Laporta's algorithm[5,6] all four-loop integrals were reduced to a small set of master integrals which were taken from Refs. [7,8]. Based on these developments a new determination of the quark masses has been performed in Ref. [9].

The extraction of m_Q from low moments of the cross section $\sigma(e^+e^- \to Q\bar{Q})$ exploits its sharp rise close to the threshold for open charm and bottom production and the importance of the contributions from the narrow quarkonium resonances. By evaluating the moments

$$\mathcal{M}_n \equiv \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s)\,, \tag{1}$$

with low values of n, the long distance contributions are averaged out and $\mathcal{M}_n$ involves short distance physics only, with a characteristic scale of order $E_{\text{threshold}} = 2m_Q$. Through dispersion

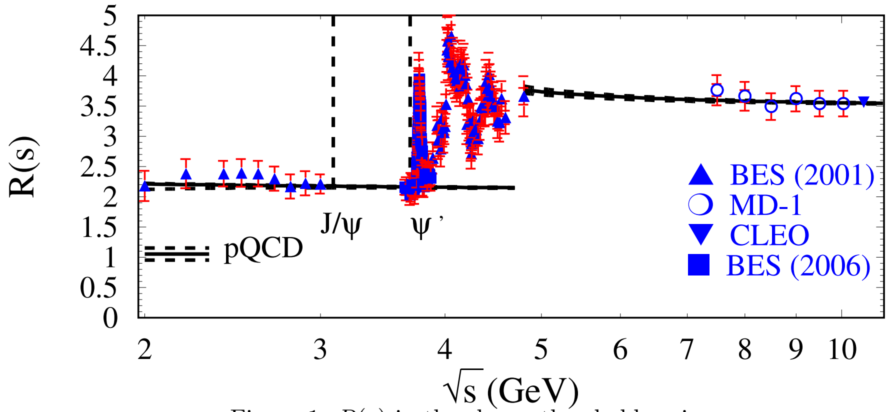

Figure 1: $R(s)$ in the charm threshold region.

relations the moments are directly related to derivatives of the vacuum polarization function at $q^2 = 0$,

$$\mathcal{M}_n = \frac{12\pi^2}{n!} \left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\Big|_{q^2=0}, \tag{2}$$

which can be evaluated in perturbative QCD.

The narrow charmonium resonances J/Ψ, $\Psi(2S)$ and the higher excitations will obviously contribute to the moments. Open charm production exhibits a sharp rise, nearly like a step function. Beyond the $\Psi(3770)$-resonance a few oscillations are observed which quickly level out into a fairly flat energy dependence. Around and above approximately 5 GeV the cross section is well approximated by perturbative QCD (pQCD) and, furthermore, mass terms can be considered as small corrections[10,11]. The sensitivity to m_Q is, therefore, concentrated on the small region from J/Ψ up to approximately 5 GeV.

We therefore distinguish three energy regions: First, the region of the narrow resonances J/Ψ and $\Psi(2S)$, second, the "charm threshold region" starting from the D-meson threshold at 3.73 GeV up to approximately 5 GeV, where the cross section exhibits rapid variations and, third, the continuum region where pQCD and local duality are expected to give reliable predictions. For the threshold region we use the data from the BES collaboration [12,13], shown in Fig. 1 together with data from MD-1 [14] and CLEO [15]. Evidently pQCD provides an excellent description of all the data in the continuum region. The description of the perturbative continuum includes the complete mass dependence up to $\mathcal{O}(\alpha_s^2)$ plus the dominant mass dependent $\mathcal{O}(\alpha_s^3)$ terms [9] which were used to extrapolate R_{uds} from the region below charm threshold up to 4.8 GeV.

In its domain of analyticity $\Pi_c(q^2)$ can be cast into the form

$$\Pi_c(q^2) = Q_c^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \left(\frac{q^2}{4m_c^2}\right)^n, \tag{3}$$

where $m_c = m_c(\mu)$ is the $\overline{\mathrm{MS}}$ charm quark mass at the scale μ. The perturbative series for the coefficients $\bar{C}_n$ in order α_s^2 was evaluated in Ref. [16],the four-loop contributions to $\bar{C}_0$ and $\bar{C}_1$ in Refs. [3,4]. The coefficients depend on α_s and on the charm quark mass through logarithms of the form $l_{m_c} \equiv \ln(m_c^2(\mu)/\mu^2)$. Combining Eqs. (1), (2) and (3), the charm quark mass can be obtained:

$$m_c(\mu) = \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\mathrm{exp}}}\right)^{1/(2n)}. \tag{4}$$

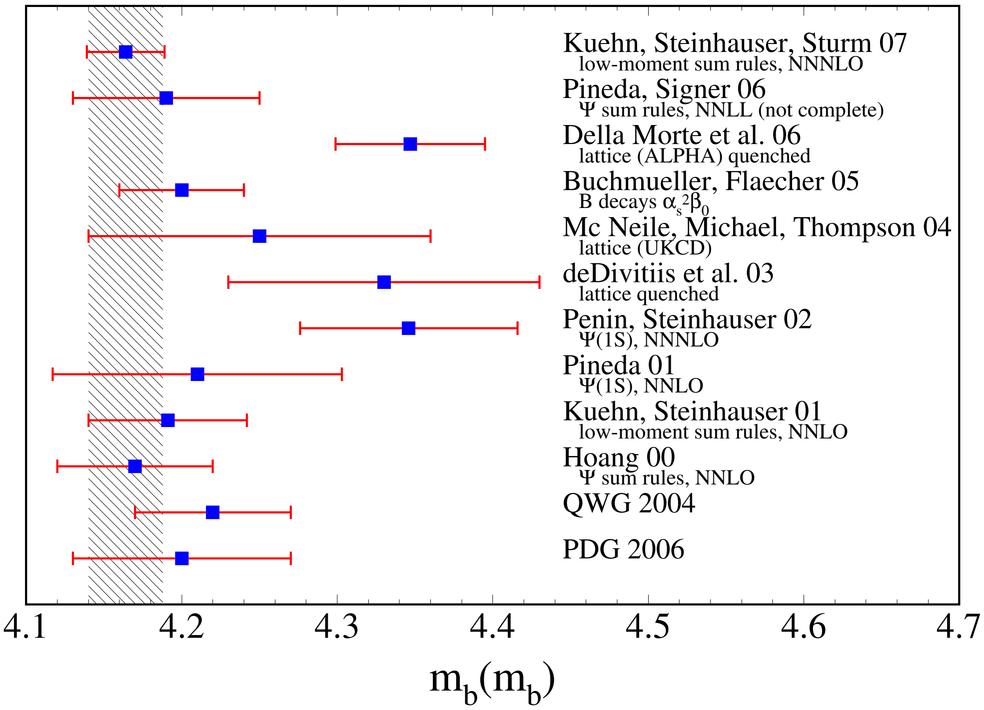

Figure 2: Comparison of recent determinations of $m_b(m_b)$

In the charm threshold region (which includes $\Psi(3770)$) we have to identify the contribution from the charm quark, i.e. we have to subtract the parts arising from the light u, d and s quark. In the continuum region above $\sqrt{s} = 4.8$ GeV data are sparse and imprecise. On the other hand, pQCD provides reliable predictions for $R(s)$. Thus in this region we replace data by the theoretical prediction.

We use the results for the moments to obtain in a first step $m_c(3\text{ GeV})$. The moment with $n = 1$ is least sensitive to non-perturbative contributions from condensates, to the Coulombic higher order effects, the variation of μ and the parametric α_s dependence. We therefore adopt

$$m_c(3\text{ GeV}) \quad = \quad 0.986(13)\text{ GeV}\,, \tag{5}$$

as our final result. Transforming this to the scale-invariant mass $m_c(m_c)$ one finds $m_c(m_c) = 1.286(13)$ GeV. Using the three-loop relation [17,18] between pole- and $\overline{\text{MS}}$-mass this leads to $M_c^{(3-\text{loop})} = 1.666$ GeV.

The same approach is also applicable to the determination of m_b. Just as in the charm case, a remarkable consistency and stability is observed. For $n = 1$ the error is dominated by the experimental input. For $n = 3$ we obtain ± 0.010 from the experimental input, ± 0.014 from α_s and ± 0.006 from the variation of μ. The three results based on $n = 1, 2$ and 3 are of comparable precision. The relative size of the contributions from the threshold and the continuum region decreases for the moments $n = 2$ and 3. On the other hand, the theory uncertainty estimated from the variation of μ and the unknown four-loop contribution is still acceptable. Therefore the result from $n = 2$ is taken as the final answer,

$$m_b(10\text{ GeV}) = 3.609(25)\text{ GeV}\,, \tag{6}$$

corresponding to $m_b(m_b) = 4.164(25)$ GeV and a pole mass of $M_b^{(3-\text{loop})} = 4.800$ GeV. A comparison with a few selected determinations is shown in Fig. 2.

For various applications, either related to Z-boson decays or in connections to Grand Unified Theories (GUTs), the values of $m_b(\mu)$ at M_Z and $m_t(m_t) = 161.8 \pm 2.0$ GeV (as derived from $M_t = 171.4 \pm 2.1$ GeV [19]) are of interest:

$$m_b(M_Z) = 2.834 \pm 0.019 \pm 0.017 \text{ GeV}\,, \qquad m_b(161.8) = 2.703 \pm 0.018 \pm 0.019 \text{ GeV}\,. \tag{7}$$

The first error reflects the combined error from Eq. (6) and the second one the uncertainty due to α_s. The ratio $m_t(m_t)/m_b(m_t) = 59.8 \pm 1.3$ should be a useful input for Grand Unified Theories.

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