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Resolving the sign ambiguity in $\Delta\Gamma_s$ with $B_s \to D_s K$

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Abstract

The analysis of tagged $B_s \to J/\psi\phi$ decays determines the CP phase ϕ_s in $B_s - \overline{B}_s$ mixing with a two–fold ambiguity. The solutions differ in the sign of $\cos\phi_s$ which equals the sign of the width difference $\Delta\Gamma_s$ among the two B_s mass eigenstates. We point out that this ambiguity can be removed with the help of $B_s \to D_s K$ decays. We compare untagged and tagged strategies and find the tagged analysis more promising. The removal of the sign ambiguity in $\Delta\Gamma_s$ can be done with relatively low statistics and could therefore be a target for the early stage of $B_s \to D_s K$ studies.

The theoretical description of $B_s - \overline{B}_s$ mixing involves the elements M_{12}^s and Γ_{12}^s of the mass and decay matrices, respectively [1]. The precise measurement of the mass difference $\Delta M_s = M_H - M_L$ between the heavy and light mass eigenstates of the B_s system determines $|M_{12}^s|$ [2]. Yet theoretical uncertainties still permit new–physics contributions to $|M_{12}^s|$ of order 30%. The new contributions to M_{12}^s can even exceed the Standard Model value in magnitude, because Standard–Model and new–physics contributions generally come with an arbitrary relative complex phase. To probe new physics in M_{12}^s further, it is therefore mandatory to study the phase of M_{12}^s experimentally. The width difference between the heavy and light mass eigenstates of the B_s system is given by

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H = 2|\Gamma_{12}^s|\cos\phi_s \qquad \text{with} \qquad \phi_s = \arg\left(-\frac{M_{12}^s}{\Gamma_{12}^s}\right). \tag{1}$$

Since Γ_{12}^s is unaffected by new physics, measurements of ϕ_s probe new physics in M_{12}^s. It is useful to decompose ϕ_s as $\phi_s = \phi_s^{\rm SM} + \phi_s^\Delta$ with the two terms denoting the Standard–Model and new–physics contributions to ϕ_s, respectively. Currently most experimental information on ϕ_s^Δ stems from the decay $B_s \to J/\psi\phi$. The CP phase in this decay is the difference $\phi_s^\Delta - 2\beta_s$ between $\arg M_{12}^s$ and twice the phase of the $b \to c\overline{c}s$ decay amplitude with $\beta_s = \arg[-V_{ts}V_{tb}^*/(V_{cs}V_{cb}^*)] = 0.020 \pm 0.005 = 1.1^\circ \pm 0.3^\circ$. While it is safe to neglect $\phi_s^{\rm SM} = (4.2 \pm 1.4) \cdot 10^{-3} = 0.24^\circ \pm 0.08^\circ$ [3, 4] and to identify ϕ_s with ϕ_s^Δ, we keep $2\beta_s$ non–zero in our formulae. The untagged decay $\overset{(-)}{B_s} \to J/\psi\phi$ provides information on $\Delta\Gamma_s\cos(\phi_s^\Delta - 2\beta_s)$ and $|\sin(\phi_s^\Delta - 2\beta_s)|$ [5]. These measurements have been combined with experimental constraints on the semileptonic CP asymmetry $a_{\rm sl}$ [6] to determine the allowed ranges for ϕ_s^Δ [4, 7]. Recently, the CDF collaboration has presented a tagged analysis of $B_s \to J/\psi\phi$ [8] with the two solutions

$$\phi_s^\Delta - 2\beta_s \in [-1.36, -0.24] \qquad \text{or} \qquad \phi_s^\Delta - 2\beta_s \in [-2.90, -1.78] \qquad @68\%\,\text{CL}. \tag{2}$$

The quoted ranges correspond to the analysis in Ref. [8] which constrains $\Delta\Gamma_s$ in Eq. (1) with the theoretical value $|\Gamma_{12}| = 0.048 \pm 0.018\,\text{ps}^{-1}$ [3, 4, 9]. Eq. (1) implies $\text{sign}\,\Delta\Gamma_s = \text{sign}\,\cos\phi_s$, so that the two solutions in Eq. (2) correspond to $\Delta\Gamma_s > 0$ and $\Delta\Gamma_s < 0$, respectively. Lifetime measurements in the components of the angular distributions of an untagged $B_s \to J/\psi\phi$ sample determine $|\Delta\Gamma_s|$, which implies a four–fold ambiguity in ϕ_s^Δ [4, 10, 11]. Neglecting the small β_s for a moment, all quantities which can be extracted from $B_s \to J/\psi\phi$ and also $a_{\rm sl}$ suffer from the same two–fold ambiguity $\phi_s^\Delta \leftrightarrow \pi - \phi_s^\Delta$ visible in Eq. (2). Thus at present we do not have any information on the sign of $\Delta\Gamma_s$. The two solutions in Eq. (2) correspond to different values of $\cos\delta_{1,2}$, where δ_1 and δ_2 are strong phases. In order to resolve the ambiguity in sign $\cos\phi_s$ = sign $\Delta\Gamma_s$ one must determine sign $\cos\delta_{1,2}$. If this is done with naive factorisation [12], the second solution in Eq. (2) is obtained. However, if the strong phases measured in $B_d \to J/\psi K^*$ [13] are used, one finds the first solution in Eq. (2) (see the discussion in [8]). It should be noted that the $SU(3)_{\rm F}$ symmetry links $B_d \to J/\psi K^*$ only partially to $B_s \to J/\psi\phi$. Only the component of the ϕ meson with U–spin equal to 1 belongs to the symmetry multiplet of the K^*. The decay amplitude into the equally large U–spin–zero component cannot be related to $B_d \to J/\psi K^*$. Since there is also no reason to trust naive factorisation in $B_s \to J/\psi\phi$, we conclude that the sign ambiguity in $\Delta\Gamma_s$ is unresolved.

The tagged decays $\overset{(-)}{B}_s \to D_s^\mp K^\pm$ were proposed to determine the angle γ of the unitarity triangle (UT). They do not involve any penguin pollution and are therefore hadronically very clean [14]. However, these decays are also sensitive to a possible new phase in $B_s - \overline{B}_s$ mixing and really determine $\phi_s^\Delta - 2\beta_s + \gamma$ (up to a tiny correction of order $0.1°$). An exhaustive study of $\overset{(-)}{B}_s \to D_s^\mp K^\pm$ including the effects of a non–zero ϕ_s^Δ can be found in Ref. [15], which also focuses on the determination of γ assuming that ϕ_s^Δ has been determined unambigously with other methods. In this paper we propose to view $\overset{(-)}{B}_s \to D_s^\mp K^\pm$ from a different angle: We exploit that γ is well–measured at B factories and show that $\overset{(-)}{B}_s \to D_s^\mp K^\pm$ can be used to discriminate between the two solutions with $\cos\phi_s > 0$ and $\cos\phi_s < 0$ in Eq. (2). This information can be found with relatively low statistics, much before studies of this decay mode at LHCb become competitive for the determination of γ. The resolution of the sign ambiguity in $\Delta\Gamma_s$ can be achieved either with a lifetime measurement in untagged $\overset{(-)}{B}_s \to D_s^\mp K^\pm$ decays or by inspecting the sign of the oscillating term in a tagged $B_s \to D_s^\mp K^\pm$ data sample. We will discuss both strategies below. Since the CDF experiment has already gathered more than 100 $B_s \to D_s^\mp K^\pm$ events [16], the determination of sign $\Delta\Gamma_s$ maybe even within reach of the Tevatron.

γ is well–known from the decay $B_d \to \rho^+\rho^-$, which measures the UT angle α plus a potential phase of new physics in B_d–$\overline{B}_d$ mixing: If the measured CP asymmetries in $B_d \to \rho^+\rho^-$ and $B_d \to J/\psi K_s$ are combined to solve for $\gamma = \pi - \alpha - \beta$, any new physics in $B_d - \overline{B}_d$ mixing drops out. Exploiting the smallness of the penguin pollution in $B_d \to \rho^+\rho^-$ and using QCD factorisation [17] Ref. [18] finds $\gamma = 71° \pm 5°$. For simplicity we define

$$\gamma_s \;\equiv\; \gamma \,-\, 2\beta_s \;=\; 69° \pm 5°. \tag{3}$$

Alternatively one can include other $B \to \rho\rho$ modes and use isospin symmetry to control the penguin pollution [19]. In principle this analysis comes with discrete ambiguities for γ as well. However, the global analysis of the UT only permits the one solution for γ quoted above.

The time–dependent $\overset{(-)}{B}_s \to D_s^\mp K^\pm$ decay rates involve [1, 14, 15]

$$\begin{aligned}
\lambda_{D_s^- K^+} &= \frac{q}{p}\frac{\langle D_s^- K^+ | \overline{B}_s\rangle}{\langle D_s^- K^+ | B_s\rangle} = |\lambda_{D_s^- K^+}|\, e^{-i\,(\gamma_s + \phi_s^\Delta - \delta)} \\
\lambda_{D_s^+ K^-} &= \frac{q}{p}\frac{\langle D_s^+ K^- | \overline{B}_s\rangle}{\langle D_s^+ K^- | B_s\rangle} = \frac{1}{|\lambda_{D_s^- K^+}|}\, e^{-i\,(\gamma_s + \phi_s^\Delta + \delta)}.
\end{aligned}$$

Here q/p encodes $B_s - \overline{B}_s$ mixing in the usual way and δ is a strong phase which equals zero if the matrix elements are computed in the factorisation approximation [15]. The $\overset{(-)}{B_s} \to D_s^{\mp} K^{\pm}$ decays are colour–allowed tree level processes and the factorisation approximation is exact in the limit of a large number N_c of colours. We point out that the only $1/N_c$ corrections to the matrix elements stem from annihilation topologies which are empirically known to be small. The remaining corrections to the large–N_c limit are quadratic in $1/N_c$, see e.g. [20]. Of course the full-flesh tagged analysis can determine δ [14, 15], but for our purposes it is sufficient to know that δ is small. We conservatively assume $|\delta| < 0.2$.

We introduce the shorthand notation

$$b = \frac{2\,|\lambda_{D_s^- K^+}|}{1 + |\lambda_{D_s^- K^+}|^2} \tag{4}$$

and note that $0 < b \leq 1$. For realistic values $|\lambda_{D_s^- K^+}| \approx 0.4$ one finds $b \approx 0.7$.

The time–dependent decay rates for the four relevant processes are [1]

$$\begin{aligned}
\Gamma\left(B_s(t) \to D_s^{\mp} K^{\pm}\right) &= N e^{-\Gamma_s t}\Big[\cosh(\frac{\Delta\Gamma_s\, t}{2}) \pm \Big(1 - b\,|\lambda_{D_s^- K^+}|\Big)\cos(\Delta M_s\, t) \\
&\quad - b\,\cos(\gamma_s + \phi_s^{\Delta} \mp \delta)\sinh(\frac{\Delta\Gamma_s\, t}{2}) \;+\; b\,\sin(\gamma_s + \phi_s^{\Delta} \mp \delta)\sin(\Delta M_s\, t)\Big], \\
\Gamma\left(\overline{B}_s(t) \to D_s^{\mp} K^{\pm}\right) &= N e^{-\Gamma_s t}\Big[\cosh(\frac{\Delta\Gamma_s\, t}{2}) \mp \Big(1 - b\,|\lambda_{D_s^- K^+}|\Big)\cos(\Delta M_s\, t) \\
&\quad - b\,\cos(\gamma_s + \phi_s^{\Delta} \mp \delta)\sinh(\frac{\Delta\Gamma_s\, t}{2}) \;-\; b\,\sin(\gamma_s + \phi_s^{\Delta} \mp \delta)\sin(\Delta M_s\, t)\Big].
\end{aligned} \tag{5}$$

Here $\Gamma_s = (\Gamma_L + \Gamma_H)/2$ and N is a normalisation constant. Now the untagged decay rate for the decay mode $\overset{(-)}{B_s} \to D_s^{\pm} K^{\mp}$ reads

$$\begin{aligned}
\Gamma\left[D_s^{\mp} K^{\pm}, t\right] &\equiv \Gamma\left(B_s(t) \to D_s^{\mp} K^{\pm}\right) \;+\; \Gamma\left(\overline{B}_s(t) \to D_s^{\mp} K^{\pm}\right) \\
&= 2N e^{-\Gamma_s t}\left[\cosh(\frac{\Delta\Gamma_s t}{2}) \;-\; b\,\cos(\gamma_s + \phi_s^{\Delta} \mp \delta)\sinh(\frac{\Delta\Gamma_s t}{2})\right],
\end{aligned} \tag{6}$$

which is just the familiar two–exponential formula with the time–dependent factors $\exp[-\Gamma_L t]$ and $\exp[-\Gamma_H t]$. In practice one can determine two quantities from the untagged decay rate in Eq. (6), the branching fraction and the lifetime measured in the considered mode $\overset{(-)}{B_s} \to D_s^{\pm} K^{\mp}$. The normalisation constant N can be related to the CP–averaged branching fraction [1, 11]:

$$\begin{aligned}
\mathcal{B}(\overset{(-)}{B_s} \to D_s^{\mp} K^{\pm}) &\equiv \frac{\mathcal{B}(B_s \to D_s^{\mp} K^{\pm}) + \mathcal{B}(\overline{B}_s \to D_s^{\mp} K^{\pm})}{2} \\
&= \frac{N\,\Gamma_s}{\Gamma_s^2 - (\Delta\Gamma_s)^2/4}\left[1 - b\cos(\gamma_s + \phi_s^{\Delta} \mp \delta)\frac{\Delta\Gamma_s}{2\Gamma_s}\right].
\end{aligned} \tag{7}$$

From Eq. (7) one finds

$$\frac{\mathcal{B}(\overset{(-)}{B_s} \to D_s^+ K^-) - \mathcal{B}(\overset{(-)}{B_s} \to D_s^- K^+)}{\mathcal{B}(\overset{(-)}{B_s} \to D_s^+ K^-) + \mathcal{B}(\overset{(-)}{B_s} \to D_s^- K^+)} = b\,\frac{\sin(\gamma_s + \phi_s^{\Delta})\,\sin\delta}{1 - \cos(\gamma_s + \phi_s^{\Delta})\cos\delta}\,\frac{\Delta\Gamma_s}{2\Gamma_s}. \tag{8}$$

Once this ratio of branching fractions is measured at the level of a few percent, it will be useful to place tighter bounds on $|\sin(\gamma_s + \phi_s^{\Delta})\sin\delta|$ and may help the tagged analysis.

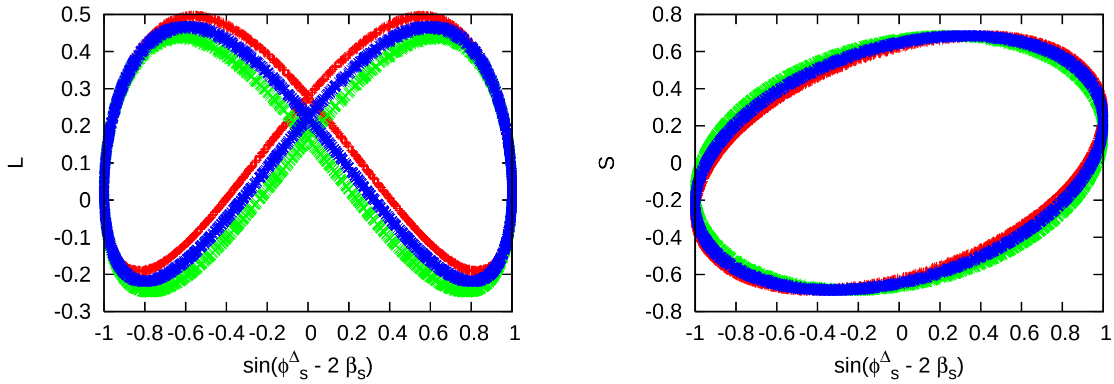

Figure 1: The tagged $\overset{(-)}{B}_s \to J/\psi\phi$ analysis gives one solution for $\sin(\phi_s^\Delta - 2\beta_s)$ corresponding to the two–fold ambiguity $\phi_s^\Delta - 2\beta_s \leftrightarrow \pi - \phi_s^\Delta + 2\beta_s$. The left plot shows how the measurement of L in Eq. (10) can resolve this ambiguity. For $\sin(\phi_s^\Delta - 2\beta_s) < 0$ the upper branch corresponds to $\cos\phi_s^\Delta > 0$ and $\Delta\Gamma_s > 0$, while the lower branch corresponds to $\cos\phi_s^\Delta < 0$ and $\Delta\Gamma_s < 0$. For $\sin(\phi_s^\Delta - 2\beta_s) > 0$ the situation is reversed. The right plot shows the coefficient S of the tagged analysis. The upper curve is for $\cos\phi_s > 0$ meaning $\Delta\Gamma_s > 0$, the lower curve corresponds to $\cos\phi_s < 0$ meaning $\Delta\Gamma_s < 0$. Both plots are for $b = 0.7$ and $|\delta| < 0.2$. The blue (dark) curves correspond to $\gamma = 71°$, the red (medium grey) and green (light) curves correspond to $\gamma = 66°$ and $\gamma = 76°$, respectively.

For our purposes we need the lifetime information: A maximum likelihood fit of a time evolution given by Eq. (6) to a single exponential $\propto \exp[-\Gamma_{D_s^\mp K^\pm} t]$ determines [11,21]:

$$\Gamma_{D_s^\mp K^\pm} = \Gamma_s \; + \; b\,\cos(\gamma_s + \phi_s^\Delta \mp \delta)\,\frac{\Delta\Gamma_s}{2} \;=\; \Gamma_s \; + \; b\,\cos(\gamma_s + \phi_s^\Delta \mp \delta)\cos\phi_s^\Delta\;|\Gamma_{12}^s|\,, \tag{9}$$

where we neglected corrections of order $(\Delta\Gamma_s)^2/\Gamma_s^2$. Comparing the rates $\Gamma_{D_s^+K^-}$ and $\Gamma_{D_s^-K^+}$ gives the same information as Eq. (8). More important for us is the average of the two widths: Defining the quantity L through

$$\frac{\Gamma_{D_s^+K^-} + \Gamma_{D_s^-K^+}}{2} - \Gamma_s \;=\; b\cos\delta\cos(\gamma_s + \phi_s^\Delta)\cos\phi_s^\Delta\,|\Gamma_{12}^s| \;\equiv\; L\,|\Gamma_{12}^s|\,, \tag{10}$$

one first realises that the dependence of L on δ is only quadratic, so that the uncertainty from δ is inessential in view of the error on γ_s in Eq. (3): $|\delta| < 0.2$ implies $0.98 < \cos\delta < 1$. Second we verify from Eq. (10) that we can resolve the ambiguity in ϕ_s^Δ by comparing the lifetime measured in $\overset{(-)}{B}_s \to D_s K$ with $1/\Gamma_s$, provided that ϕ_s^Δ differs from 0 or π. This feature is illustrated in the left plot of Fig. 1. For example, the central values in the two intervals in Eq. (2) both correspond to $\sin(\phi_s^\Delta - 2\beta_s) = -0.72$. But the solution with $\Delta\Gamma_s > 0$ comes with $L > 0$, while $\Delta\Gamma_s < 0$ implies $L < 0$. We do not recommend to fit the data to a single exponential, because Eq. (9) is only correct, if e.g. the experimental acceptance does not vary with the decay length. Instead we propose to determine ϕ_s^Δ with the exact formula in Eq. (6) with $\Delta\Gamma_s$ expressed as $\Delta\Gamma_s = 2|\Gamma_{12}^s|\cos\phi_s^\Delta$ and $|\Gamma_{12}^s| = 0.048 \pm 0.018\,\mathrm{ps}^{-1}$ [4]. Further Γ_s could be fixed to the theoretical value $\Gamma_s \simeq 1/\tau_{B_d}$.

Next we discuss the tagged analysis: With $\Gamma(B_s(t) \to D_s K) = [\Gamma(B_s(t) \to D_s^- K^+) + \Gamma(B_s(t) \to$

$D_s^+ K^-)]/2$ we encounter the CP asymmetry

$$\frac{\Gamma(B_s(t) \to D_s K) - \Gamma(\overline{B}_s(t) \to D_s K)}{\Gamma(B_s(t) \to D_s K) + \Gamma(\overline{B}_s(t) \to D_s K)} = \frac{b \cos\delta \sin(\gamma_s + \phi_s^\Delta) \sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s/2) - b\cos\delta\cos(\gamma_s + \phi_s^\Delta)\sinh(\Delta\Gamma_s/2)} \tag{11}$$

The coefficient of the oscillating term,

$$S \equiv b \cos\delta \sin(\gamma_s + \phi_s^\Delta) \tag{12}$$

also permits the removal of the discrete ambiguity in ϕ_s^Δ, because the replacement of $\phi_s^\Delta - 2\beta_s$ by $\pi - \phi_s^\Delta + 2\beta_s$ changes S dramatically, as shown in the right plot of Fig. 1. S even discriminates between the two cases $\phi_s^\Delta = 0$ and $\phi_s^\Delta = \pi$, which are the two possible cases in the class of new physics models without new sources of CP violation.

Comparing the untagged and tagged method we find that discriminating the two branches for L in the left plot of Fig. 1 means a lifetime measurement with an accuracy of roughly 2% requiring at least 2500 events, because the difference of the two solutions for $L|\Gamma_{12}^s|/\Gamma_s$ hardly exceeds 0.04 and even vanishes if ϕ_s^Δ is close to 0 or π. The untagged measurement looks better, even though tagging costs roughly a factor of 12–20 in statistics. The two solutions with $\sin(\phi_s^\Delta - 2\beta_s) = -0.72$ correspond to $S \simeq 0.3$ and $S \simeq -0.6$ and a fairly small data sample should permit to discriminate between the two solutions. Finally we remark that one can eliminate δ altogether, if both L and S are measured precisely: Eqs. (10) and (12) combine to

$$\tan(\gamma_s + \phi_s^\Delta) = \frac{S}{L} \cos\phi_s^\Delta. \tag{13}$$

Now Eq. (13) has four solutions for ϕ_s^Δ and two of them can be eliminated with the information on the sign of S. The remaining two solutions are not related by $\phi_s^\Delta - 2\beta_s \leftrightarrow \pi - \phi_s^\Delta + 2\beta_s$, so that in combination with $B_s \to J/\Psi\phi$ the discrete ambiguity is lifted with the help of Eq. (13). We emphasize that Eq. (13) has been discussed before in Ref. [15], where $\tan(\gamma_s + \phi_s^\Delta)$ is expressed in terms of the coefficient of $\sinh(\Delta\Gamma_s t/2)$ in Eq. (5). The extraction of this coefficient has a sign ambiguity if the sign of $\Delta\Gamma_s$ is unknown. Through Eqs. (10) and (13) we have merely expressed $\tan(\gamma_s + \phi_s^\Delta)$ in terms of $|\Gamma_{12}^s|$ and $\cos\phi_s^\Delta$ to eliminate the implicit dependence on sign $\Delta\Gamma_s$.

In conclusion we have discussed the removal of the two–fold ambiguity in the extraction of ϕ_s^Δ from tagged $B_s \to J/\psi\phi$ decays. We have shown that $B_s \to D_s^\pm K^\mp$ data can be used to resolve this ambiguity. This analysis can be done with relatively low statistics, well before $B_s \to D_s^\pm K^\mp$ decays become competitive for the determination of γ. Comparing untagged with tagged analyses we find that the tagged analysis is more promising, despite of the penalty from small tagging efficiencies at hadron colliders.

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