Dimensional Reduction Applied to Non-SUSY Theories

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Abstract. We consider regularisation of a Yang-Mills theory by Dimensional Reduction (DRED). In particular, the anomalous dimensions of fermion masses and gauge coupling are computed to four-loop order. We put special emphasis on the treatment of evanescent couplings which appear when DRED is applied to non-supersymmetric theories. We highlight the importance of distinguishing between the evanescent and the real couplings. Considering the special case of a Super-Yang-Mills theory, we find that Dimensional Reduction is sufficient to preserve Supersymmetry in our calculations.

PACS. 11.10.Gh Renormalisation – 11.30.Pb Supersymmetry

1 Introduction

When calculating higher orders in perturbation theory, Dimensional Regularisation (DREG) [1,2] is the regularisation procedure of choice. Its convenience stems from the fact that it automatically preserves gauge invariance: the finite part of the effective action satisfies the Ward identities of the gauge symmetry, without the need to introduce additional finite local counterterms.

When applied to supersymmetric theories, however, the Ward identities of supersymmetry (SUSY) are violated by the use of DREG. That is because invariance of a given action under supersymmetry transformations only hold for specific values of the space-time dimension, and DREG alters this value.

Dimensional Reduction (DRED)[3] was proposed as a way of reconciling DREG and SUSY. The essence of this method is to restrict the momenta to a D-dimensional subspace of the 4-dimensional space-time, while keeping all vector fields 4-dimensional. Thus, the momentum integrals can be regularised without meddling with the number of degrees of freedom of the gauge fields, which is what breaks SUSY in DREG.

In the present talk, we will discuss the application of DRED to a Yang-Mills Theory with arbitrary gauge group. In particular, we will outline the calculation of renormalisation group coefficients in a gauge theory with fermions, using DRED with minimal subtraction, which is known as the $\overline{DR}$ scheme. We will emphasise on the non-supersymmetric case and the subleties that arise therein, namely the appearance of evanescent couplings. The calculations have been done up to the four-loop order.

The correct application of DRED to non-supersymmetric theories is an important issue in phenomenological studies when one wants to connect parameters in a supersymmetric theorie valid at high energies with their counterparts in a non-supersymmetric low-energy effective theory. A recent example is the treatment of the running of the strong coupling constant and the bottom mass in the MSSM in Ref. [4].

2 Evanescent Couplings

Consider a non-abelian gauge theory with gauge fields W^a_μ and a multiplet of two-component fermions $\psi^A_\alpha(x)$ transforming according to a representation R of the gauge group $\mathcal{G}$.

The bare Lagrangian density, with covariant gauge fixing and ghost (C, C^*) terms is

$$\begin{aligned}L_B = -\frac{1}{4}G^2_{\mu\nu} - \frac{1}{2\alpha}(\partial^\mu W_\mu)^2 + C^{a*}\partial^\mu D^{ab}_\mu C^b \\ +i\overline{\psi}_{\dot{\alpha}A}\overline{\sigma}^{\mu\dot{\alpha}\alpha}(D_\mu)^A{}_B\psi^B_\alpha\end{aligned} \tag{1}$$

where

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{abc}W^b_\mu W^c_\nu \tag{2}$$

and

$$(D_\mu)^A{}_B = \delta^A{}_B\partial_\mu - ig(R^a)^A{}_B W^a_\mu. \tag{3}$$

For the case when the theory admits a gauge invariant fermion mass term we will have $L_B \to L_B + L^m_B$, where

$$L^m_B = \frac{1}{2}m_{AB}\psi^{\alpha A}\psi^B_\alpha + \text{c.c.} \tag{4}$$

Applying DRED amounts to imposing that all field variables depend only on D out of 4 space-time dimensions, where $D = 4 - 2\epsilon$. We can then make the decomposition

$$W^a_\mu(x^j) = W^a_i(x^j) \oplus W^a_\sigma(x^j) \tag{5}$$

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where μ is an index of a 4-dimensional, i and j are indices of a D-dimensional, and σ is an index of a 2ϵ-dimensional vector space. An explicit construction of these vector spaces can be found in Ref. [5]. The Lagrange density then takes the form

$$L_B = L_B^d + L_B^\epsilon \tag{6}$$

where

$$L_B^d = -\frac{1}{4}G_{ij}^2 - \frac{1}{2\alpha}(\partial^i W_i)^2 + C^*\partial^i D_i C + i\overline{\psi}\overline{\sigma}^i D_i \psi \tag{7}$$

and

$$\begin{aligned} L_B^\epsilon = &\frac{1}{2}(D_i W_\sigma)^2 - g\overline{\psi}\overline{\sigma}_\sigma R^a \psi W_\sigma^a \\ &-\frac{1}{4}g^2 f^{abc} f^{ade} W_\sigma^b W_{\sigma'}^c W_\sigma^d W_{\sigma'}^e . \end{aligned} \tag{8}$$

Under gauge transformations, the W_σ-fields transform as scalars, and they are commonly called ε-scalars. Also, each term in L_B^ϵ is separately invariant under gauge transformations, so there is no reason to expect the form of Eq. (8) to be preserved under renormalisation – except in the case of a supersymmetric theory, where invariance under supersymmetry transformations requires L_B^ϵ to take the form of Eq. (8). In general, the coupling of the ε-scalars to the fermions will not be governed by the gauge coupling g, but by a different coupling g_e, called an evanescent coupling.

The case of the quartic ε-scalar interaction is even more complicated. Gauge invariance does not require the f-f tensor structure, but allows the quartic coupling to take the form

$$-\frac{1}{4}\sum_{r=1}^{p} \lambda_r H_r^{abcd} W_\sigma^a W_{\sigma'}^c W_\sigma^b W_{\sigma'}^d \tag{9}$$

where H^{abcd} are tensors which are non-vanishing when symmetrised with respect to (ab) and (cd) interchange. The number p of such tensors which are linearly independent depends on the group $\mathcal{G}$ and can be up to 4. For instance, one could choose

$$\begin{aligned} 2H_1 &= f^{ace} f^{bde} + f^{ade} f^{bce} , \\ 2H_2 &= \delta^{ab}\delta^{cd} , \\ 2H_3 &= \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc} , \\ 2H_4 &= f^{aef} f^{bfg} f^{cgh} f^{dhe} + f^{aef} f^{bfg} f^{dgh} f^{che} . \end{aligned} \tag{10}$$

Corresponding to these evanescent couplings, we define the coupling constants

$$\alpha_s^{\overline{\rm DR}} = \frac{g^2}{4\pi} , \qquad \alpha_e = \frac{g_e^2}{4\pi} , \qquad \eta_r = \frac{\lambda_r}{4\pi} . \tag{11}$$

3 Relating the $\overline{DR}$ to the $\overline{MS}$ scheme

Considering two viable renormalisation schemes, it is possible to translate calculations done in one scheme to the other scheme by finite shifts of the renormalised parameters. In Ref. [6], we derived the relation between $\alpha_s^{\overline{\rm DR}}$ and $\alpha_s^{\overline{\rm MS}}$ at the two-loop level using a method mentioned in Ref. [7], which relies on the fact that the value of α_s in a physical renormalisation scheme should not depend on the regularisation procedure:

$$\begin{aligned} &\alpha_s^{\rm ph} = \left(z_s^{\rm ph,X}\right)^2 \alpha_s^{\rm X} , \qquad z_s^{\rm ph,X} = Z_s^{\rm X}/Z_s^{\rm ph,X} , \\ &\qquad \text{where } {\rm X} \in \{\overline{\rm MS}, \overline{\rm DR}\} \\ &\Rightarrow \alpha_s^{\overline{\rm DR}} = \left(\frac{Z_s^{\rm ph,\overline{DR}}\, Z_s^{\overline{\rm MS}}}{Z_s^{\rm ph,\overline{MS}}\, Z_s^{\overline{\rm DR}}}\right)^2 \alpha_s^{\overline{\rm MS}} , \end{aligned} \tag{12}$$

where $Z_s^{\overline{\rm MS}/\overline{\rm DR}}$ are the charge renormalisation constants using minimal subtraction in DREG/DRED. For $Z_s^{\rm ph,\overline{MS}/\overline{DR}}$, on the other hand, we used DREG/DRED combined with a physical renormalisation condition.

The two-loop result of Ref. [6] reads, for the case of QCD,

$$\alpha_s^{\overline{\rm MS}} = \alpha_s^{\overline{\rm DR}} \left[1 - \frac{\alpha_s^{\overline{\rm DR}}}{4\pi} - \frac{5}{4}\left(\frac{\alpha_s^{\overline{\rm DR}}}{\pi}\right)^2 + \frac{\alpha_s^{\overline{\rm DR}}\alpha_e}{12\pi^2}\, n_f\right] . \tag{13}$$

At the three-loop level, the quartic ε-scalar interaction starts to contribute. In Ref. [8], the three-loop term in the conversion relation was calculated for the case of QCD, and in Ref. [9] it was possible to calculate it for an arbitrary gauge group.

In addition to the conversion formulae for α_s, also conversion formulae for the quark mass in the $\overline{MS}$ and $\overline{DR}$ scheme have been found in Ref. [6,8,9], using the same technique.

4 Renormalisation Group Coefficients

The dependence of the coupling constants (11) and the quark mass on the renormalisation scale μ is given by their β functions

$$\begin{aligned} \beta_s^{\overline{DR}}(\alpha_s^{\overline{\rm DR}}, \alpha_e, \{\eta_r\}) &= \mu^2 \frac{\rm d}{{\rm d}\mu^2} \alpha_s^{\overline{\rm DR}} , \\ \beta_e(\alpha_s^{\overline{\rm DR}}, \alpha_e, \{\eta_r\}) &= \mu^2 \frac{\rm d}{{\rm d}\mu^2} \alpha_e , \\ \beta_{\eta_r}(\alpha_s^{\overline{\rm DR}}, \alpha_e, \{\eta_r\}) &= \mu^2 \frac{\rm d}{{\rm d}\mu^2} \eta_r , \\ \gamma_m^{\overline{DR}}(\alpha_s^{\overline{\rm DR}}, \alpha_e, \{\eta_r\}) &= \mu^2 \frac{\rm d}{{\rm d}\mu^2} m , \end{aligned} \tag{14}$$

which can be calculated if one knows the corresponding renormalisation constants. For instance,

$$\begin{aligned} &\beta_s^{\overline{DR}}(\alpha_s^{\overline{\rm DR}}, \alpha_e, \{\eta_r\}) = \\ &-\left(\epsilon\frac{\alpha_s^{\overline{\rm DR}}}{\pi} + 2\frac{\alpha_s^{\overline{\rm DR}}}{Z_s^{\overline{\rm DR}}}\frac{\partial Z_s^{\overline{\rm DR}}}{\partial \alpha_e}\beta_e + 2\frac{\alpha_s^{\overline{\rm DR}}}{Z_s^{\overline{\rm DR}}}\sum_r \frac{\partial Z_s^{\overline{\rm DR}}}{\partial \eta_r}\beta_{\eta_r}\right) \\ &\quad \left(1 + 2\frac{\alpha_s^{\overline{\rm DR}}}{Z_s^{\overline{\rm DR}}}\frac{\partial Z_s^{\overline{\rm DR}}}{\partial \alpha_s^{\overline{\rm DR}}}\right)^{-1} . \end{aligned} \tag{15}$$

However, Eq. (13) and its extension to three loop-level allows to transfer the gauge and fermion mass β functions from the $\overline{MS}$ scheme, where they are known to the four-loop level [10–13] to the $\overline{DR}$ scheme via

$$\begin{aligned}
\beta_s^{\overline{DR}} &= \beta_s^{\overline{MS}} \frac{\partial \alpha_s^{\overline{\mathrm{DR}}}}{\partial \alpha_s^{\overline{\mathrm{MS}}}} + \beta_e \frac{\partial \alpha_s^{\overline{\mathrm{DR}}}}{\partial \alpha_e} + \sum_r \beta_{\eta_r} \frac{\partial \alpha_s^{\overline{\mathrm{DR}}}}{\partial \eta_r}\,, \\
\gamma_m^{\overline{DR}} &= \gamma_m^{\overline{MS}} \frac{\partial \ln m^{\overline{\mathrm{DR}}}}{\partial \ln m^{\overline{\mathrm{MS}}}} + \frac{\pi \beta_s^{\overline{MS}}}{m^{\overline{\mathrm{DR}}}} \frac{\partial m^{\overline{\mathrm{DR}}}}{\partial \alpha_s^{\overline{\mathrm{MS}}}} \\
&\quad + \frac{\pi \beta_e}{m^{\overline{\mathrm{DR}}}} \frac{\partial m^{\overline{\mathrm{DR}}}}{\partial \alpha_e} + \sum_r \frac{\pi \beta_{\eta_r}}{m^{\overline{\mathrm{DR}}}} \frac{\partial m^{\overline{\mathrm{DR}}}}{\partial \eta_r}\,. \qquad (16)
\end{aligned}$$

In Ref. [9], a four-loop result for $\beta_s^{\overline{DR}}$ and $\gamma_m^{\overline{DR}}$ was derived using Eq. (16). In addition to the $\overline{MS}$ result, the following building blocks were needed: since the dependence of $\alpha_s^{\overline{\mathrm{DR}}}$ and $m^{\overline{\mathrm{DR}}}$ on α_e starts at two- and one-loop order [6], respectively, β_e is needed up to the three-loop level. On the other hand, both $\alpha_s^{\overline{\mathrm{DR}}}$ and $m^{\overline{\mathrm{DR}}}$ depend on η_r starting from three loops and consequently only the one-loop term of β_{η_r} enters in Eq. (16).

It should be noted that the β functions of the evanescent couplings differ from the gauge β function starting already at the one-loop level. Hence, it is not possible to identify the evanescent couplings with the gauge coupling.

5 Super-Yang-Mills Checks

Starting with a Yang-Mills theory, it is possible to construct a (supersymmetric) Super-Yang-Mills theory by putting the fermions in the adjoint representation. This is useful for applying checks to the results of Ref. [6,8,9]: in a supersymmetric theory, the evanescent coupling g_e must equal the gauge coupling, so their β functions should also be the same. We checked this equality to the three-loop level, which is a strong check on our calculation and also invalidates an earlier claim [14] that β_s and β_e would differ at the three-loop level in a Super-Yang-Mills theory. In Ref. [14], the inequality of the β functions was interpreted as an example of SUSY breaking by DRED.

In Ref. [15], the four-loop gauge β function of a Super-Yang-Mills theory was presented. This provided another check to our calculations, and we did find agreement.

6 Discussion

We demonstrated that Dimensional Reduction is a viable regularisation procedure even in the non-supersymmetric case and derived explicit conversion formulae for the gauge coupling and quark mass between the $\overline{MS}$ and the $\overline{DR}$ scheme. We explained the appearance of evanescent couplings and emphasised that they cannot be identified with the gauge coupling, since the corresponding β functions differ.

We calculated gauge and fermion mass β functions for arbitrary gauge theories and applied various checks in the special case of supersymmetric theories.

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