Loops for ILC

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This contribution summarizes the on-going activities connected to the evaluation of higher order radiative corrections in the context of a future international linear collider (ILC).

1 Introduction

The purpose of this contribution is two-fold. The primary task is to present a summary of the activities discussed in the parallel session "loops" of the Linear Collider Workshop (LCWS) 2007 at DESY in Hamburg. As a second aim we try to provide an overview of higher order corrections performed in the context of the ILC. It is clear that a brief review like the present one can not be complete and has to be restricted to the most important issues. For further related activities we want to refer to the summaries of the Top/QCD, Higgs, SUSY and extra dimensions parallel sessions which can also be found in these proceedings [1].

2 Bhabha scattering

Let us in a first step discuss the activities in the context of the next-to-next-to-leading order (NNLO) corrections to the Bhabha scattering which serves as an important luminosity monitor for basically all electron-positron colliders. The uncertainty in the luminosity enters into many observables and thus needs to be determined with the highest possible precision. This is in particular true for the Giga-Z option of the ILC.

In the recent years various groups have started the NNLO calculation to the Bhabha scattering which constitutes a highly non-trivial task since next to the kinematic variables s and t also the mass of the electron, m_e, has to be kept non-zero. As far as the dependence of the scattering cross section on m_e is concerned, it is only necessary to keep the logarithmic dependence and neglect the terms suppressed by m_e^2/s.

The calculation of the cross section $\sigma(e^+e^- \to e^+e^-)$ for $m_e = 0$ has been performed in Ref. [2]. In Ref. [3] this result has been used in order to perform a matching to the case where the infra-red singularities are regularized by a photon mass and the collinear ones by the electron mass. In this way the NNLO corrections for the purely photonic correction to the Bhabha scattering could be obtained. A similar approach has been elaborated in Ref. [4] where, however, the infrared divergences are still regularized dimensionally leading to more flexibility, in particular in view of applications within QCD (see also Ref. [5]).

The fermionic corrections which are defined by the presence of a closed lepton loop have been considered in Ref. [6] for the case of an electron loop. Recently, the results for a muon and tau have been obtained in Ref. [7]. In the approach used in this paper a reduction of the full multi-scale problem to master integrals is performed. Afterwards the latter are expanded in the desired kinematical limit. The results of Ref. [7] have been confirmed in Ref. [4] (see also Ref. [8]).

There are various further contributions which are still missing to complete the NNLO corrections. Among them is the computation of the one-loop corrections where an additional photon is radiated. Progress on the evaluation of the underlying five-point integrals have been presented at this workshop [9].

3 NLO corrections to multi-particle production

In the recent years there have been important developments concerning the techniques for one-loop calculations involving many external legs (see, e.g., Ref. [10] and references therein). However, many of the proposed methods still have to prove their applicability to real processes.

Up to date there are only two groups who performed a full one-loop calculation to a realistic $2 \to 4$ process. In Ref. [11] the process $e^+e^- \to 4f$ has been considered and in Ref. [12] electroweak corrections to $e^+e^- \to \nu\bar{\nu}HH$ have been obtained using the GRACE system (see, e.g., Ref. [13]).

In a contribution [14] to the present workshop an effective-theory approach has been introduced, based on a double-expansion in the fine structure constant and the ratio of width and mass of the W boson. In the threshold region, which is the validity range of the effective theory, good agreement with the results of Ref. [11] has been found for the cross section of the process $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}$.

In contribution [15] new developments for the GRACE system has been discussed. Among them there is an interface to FORM, the implementation of one-loop calculations in the MSSM and the proper treatment of infrared divergences in QCD processes. Furthermore, there is a new attempt to obtain octuple (or even a higher) precision in the numerical routines.

4 Sudakov logarithms

With the ILC it will be possible to consider the corrections of virtual W and Z bosons to exclusive reactions like the production of two quarks or two W bosons. Since the center-of-mass energy is significantly higher than the masses of the gauge bosons a conceptually new phenomenon occurs: in each loop-order quadratic logarithms of the form $\ln^2(s/M_{W/Z}^2)$ arise which can easily lead to corrections of order 30% at one and 5% at two loops. For recent papers dealing with this topic we refer to Refs. [16].

At LCWS07 a recent calculation has been presented [17] which deals with the complete two-loop NLL corrections to processes like $f_1 f_2 \to f_3 \ldots f_n$ involving n fermions. Furthermore, a new approach has been discussed which allows for the introduction of finite quark masses for the final state particles.

5 NNLO calculation to $e^+e^- \to 3$ jets

An accurate determination of the strong coupling can be obtained by the measurement of the 3-jet cross section in e^+e^- annihilation. Currently the error on α_s from this method is dominated by the theoretical uncertainties which is mainly due to the unknown NNLO corrections to $e^+e^- \to 3$ jets.

There are basically three ingredients contributing to $e^+e^- \to 3$ jets: (i) the two-loop virtual corrections, (ii) the one-loop corrections to the real radiation of a parton, and (iii) the double real radiation which involves five partons in the final state. The individual contributions are known since many years (see contribution [18] to this workshop). However, up to very recently a proper combination of the individual pieces has not been achieved. The main reason for this are the infrared divergences inherent to the contributions (i), (ii) and (iii) which only cancel in the proper combination. In the recent years different approaches have been developed which are either based on the construction of appropriate subtraction terms or on direct numerical integration. The latter essentially relies on sector decomposition.

In Ref. [19] the first physical NNLO result has been presented for the thrust distribution defined through $T = \max_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p}_i \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p}_i|}$. The corrections turn out to be moderate leading to a significant reduction of the theoretical uncertainty on the thrust distribution.

6 Four-loop integrals

At the forefront of multi-loop calculations one also has to mention the contributions to four-loop vacuum integrals and four-loop massless two-point functions. The former integrals, often also denoted as "bubbles", are reduced with the help of the so-called Laporta-algorithm [20] to master integrals. The latter are evaluated with various methods based, e.g., on difference equations or on asymptotic expansion (see, e.g., Refs. [21, 22]).

Two applications have been presented at the LCWS07. In the first one the four-loop corrections to the ρ parameter have been studied [23, 24, 25, 26]. The new terms induce a shift in the W boson mass of about 2 MeV which is of the same order as the anticipated accuracy reached with the GIGA-Z option of the ILC. The latter is estimated to 6 MeV.

The second application [27] concerns the extraction of precise values for the charm and bottom quark masses which in the $\overline{\mathrm{MS}}$ are given by [28] $m_c(m_c) = 1.286(13)$ GeV and $m_b(m_b) = 4.164(25)$ GeV. The analysis performed in Ref. [28] is based on improved experimental data to $\sigma(e^+e^- \to \text{hadrons})$ and new four-loop contributions to the photon polarization function [29, 30].

Also the four-loop massless two-point functions have various applications where the most important one is the order α_s^4 correction to the cross section $\sigma(e^+e^- \to \text{hadrons})$ (see, e.g., Ref. [31] for a recent publication). Their evaluation is based on Baikov's method [32] where the reduction to master integrals is established via an integral representation for the coefficients of the individual master integrals. The parameter integrals are solved in the limit of large space-time dimension, d. Due to the fact that the coefficients are rational functions of d it is possible to reconstruct the exact d dependence, provided sufficient expansion terms are available.

7 Further loops

There have been four further contributions which shall be mentioned in this Section.

New two-loop electroweak corrections to the partial decay width of the Higgs boson to bottom quarks have been presented in contribution [33] (see also Ref. [34]). Although the new terms are enhanced by a factor $(G_F m_t^2)^2$ the change of the partial decay rate is tiny and amounts to only 0.05%.

In contribution [35] new three-loop corrections to the relation between the $\overline{\mathrm{MS}}$ and on-shell quark mass have been presented. In contrast to the previously known terms an additional mass scale from closed quark loop is allowed [36] where the main phenomenological applications are charm quark corrections to the bottom quark mass. The reduction of all occurring integrals leads to 27 master integrals which involve two mass scales. They have been computed both with the help of the Mellin-Barnes and the differential equation technique.

The production of a Higgs boson at LHC in the so-called vector-boson fusion channel is very promising for its discovery. At LO in perturbation theory the gauge bosons are radiated off the quarks and combine in order to produce the Higgs boson. There is no colour exchange between the quarks and thus it is expected that two jets are observed at high rapidity whereas the decay products of the Higgs boson can be found at low rapidity. Thus, it is possible to apply cuts which allow for a huge suppression of the background. The exchange of colour between the quark lines occurs for the first time at NNLO. In contribution [37] the NNLO corrections originating from squared one-loop amplitudes with gluons in the initial state have been considered. Preliminary results have been presented which show that the numerical effect is small if the so-called "vector-boson fusion" cuts are applied.

In contribution [38] (see also Ref. [39]) a new method has been proposed to extract a precise top quark mass value from jet observables. It is based on a sequence of effective field theories which allows to derive a factorization theorem for the top quark invariant mass spectrum. The factorization theorem allows for a separation of perturbative and non-perturbative effects which in turn is the basis of the extraction of the so-called "jet mass".

For the evaluation of higher order quantum corrections it is crucial to have appropriate tools which facilitate the calculations [40]. As far as one-loop corrections are concerned one should mention `FeynArts` [41] and `FormCalc` [42] which have been applied to a variety of processes in the electroweak sector of the Standard Model but also in its extensions. Beyond one-loop the programs in general aim for specific tasks of the whole calculation. E.g., the program `AIR` [43] implements the Laporta algorithm, the `Mathematica` codes `AMBRE` [44] and `MB` [45] can be used to evaluate Feynman integrals with the Mellin-Barnes method, and the program `exp` [46] allows for the application of an Euclidian asymptotic expansion for a given hierarchy in the mass scales involved in the problem. A tool which nowadays is indispensable in higher order calculations is the algebra program `FORM` [47] enabling large computations in a quite effective way. Also its parallel versions, `ParFORM` [48] and `TFORM` [49], have proven to substantially extend the capability of `FORM`.

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References

[1] J. Hewett, these proceedings; S. Kanemura, these proceedings.

[2] Z. Bern, L. J. Dixon and A. Ghinculov, Phys. Rev. D **63** (2001) 053007 [arXiv:hep-ph/0010075].

[3] A. A. Penin, Phys. Rev. Lett. **95** (2005) 010408 [arXiv:hep-ph/0501120]; Nucl. Phys. B **734** (2006) 185 [arXiv:hep-ph/0508127].

[4] T. Becher and K. Melnikov, JHEP **0706** (2007) 084 [arXiv:0704.3582 [hep-ph]].

[5] A. Mitov and S. Moch, JHEP **0705** (2007) 001 [arXiv:hep-ph/0612149].

[6] R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi and J. J. van der Bij, Nucl. Phys. B **701** (2004) 121 [arXiv:hep-ph/0405275]; Nucl. Phys. B **716** (2005) 280 [arXiv:hep-ph/0411321].

[7] S. Actis, M. Czakon, J. Gluza and T. Riemann, arXiv:0704.2400 [hep-ph].

[8] R. Bonciani, A. Ferroglia and A. A. Penin, arXiv:0710.4775 [hep-ph].

[9] T. Riemann, these proceedings.

[10] R. K. Ellis, W. T. Giele and G. Zanderighi, JHEP **0605** (2006) 027 [arXiv:hep-ph/0602185]; T. Binoth, G. Heinrich, T. Gehrmann and P. Mastrolia, Phys. Lett. B **649** (2007) 422 [arXiv:hep-ph/0703311]; Z. Bern, L. J. Dixon and D. A. Kosower, Annals Phys. **322**, 1587 (2007) [arXiv:0704.2798 [hep-ph]].

[11] A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Phys. Lett. B **612** (2005) 223 [arXiv:hep-ph/0502063]; Nucl. Phys. B **724** (2005) 247 [arXiv:hep-ph/0505042].

[12] F. Boudjema *et al.*, [arXiv:hep-ph/0510184].

[13] G. Belanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato and Y. Shimizu, Phys. Rept. **430** (2006) 117 [arXiv:hep-ph/0308080].

[14] C. Schwinn, these proceedings; arXiv:0708.0730 [hep-ph].

[15] Y. Yasui *et al.*, these proceedings; arXiv:0710.2957 [hep-ph].

[16] B. Jantzen, J. H. Kühn, A. A. Penin and V. A. Smirnov, Nucl. Phys. B **731** (2005) 188 [Erratum-ibid. B **752** (2006) 327] [arXiv:hep-ph/0509157]; A. Denner, B. Jantzen and S. Pozzorini, Nucl. Phys. B **761** (2007) 1 [arXiv:hep-ph/0608326]; J. H. Kühn, F. Metzler and A. A. Penin, arXiv:0709.4055 [hep-ph].

[17] B. Jantzen, these proceedings; arXiv:0709.2311 [hep-ph].

[18] T. Gehrmann, these proceedings; arXiv:0709.0351 [hep-ph].

[19] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, arXiv:0707.1285 [hep-ph].

[20] S. Laporta and E. Remiddi, Phys. Lett. B **379** (1996) 283 [arXiv:hep-ph/9602417]; S. Laporta, Int. J. Mod. Phys. A **15** (2000) 5087 [arXiv:hep-ph/0102033].

[21] Y. Schroder and A. Vuorinen, JHEP **0506** (2005) 051 [arXiv:hep-ph/0503209].

[22] K. G. Chetyrkin, M. Faisst, C. Sturm and M. Tentyukov, Nucl. Phys. B **742** (2006) 208 [arXiv:hep-ph/0601165].

[23] C. Sturm, these proceedings.

[24] Y. Schroder and M. Steinhauser, Phys. Lett. B **622** (2005) 124 [arXiv:hep-ph/0504055].

[25] K. G. Chetyrkin, M. Faisst, J. H. Kuhn, P. Maierhofer and C. Sturm, Phys. Rev. Lett. **97** (2006) 102003 [arXiv:hep-ph/0605201].

[26] R. Boughezal and M. Czakon, Nucl. Phys. B **755** (2006) 221 [arXiv:hep-ph/0606232].

[27] M. Steinhauser, these proceedings; arXiv:0709.2020 [hep-ph].

[28] J. H. Kühn, M. Steinhauser and C. Sturm, Nucl. Phys. B **778** (2007) 192 [arXiv:hep-ph/0702103].

[29] K. G. Chetyrkin, J. H. Kühn and C. Sturm, Eur. Phys. J. C **48** (2006) 107 [arXiv:hep-ph/0604234].

[30] R. Boughezal, M. Czakon and T. Schutzmeier, Phys. Rev. D **74** (2006) 074006 [arXiv:hep-ph/0605023].

[31] P. A. Baikov, K. G. Chetyrkin and J. H. Kühn, Nucl. Phys. Proc. Suppl. **157** (2006) 27 [arXiv:hep-ph/0602126].

[32] P. A. Baikov, Nucl. Phys. Proc. Suppl. **116** (2003) 378.

[33] M. Butenschön, these proceedings.

[34] M. Butenschön, F. Fugel and B. A. Kniehl, Nucl. Phys. B **772** (2007) 25 [arXiv:hep-ph/0702215].

[35] S. Bekavac and D. Seidel, these proceedings; arXiv:0710.0245 [hep-ph].

[36] S. Bekavac, A. Grozin, D. Seidel and M. Steinhauser, arXiv:0708.1729 [hep-ph].

[37] M. Weber, these proceedings; arXiv:0709.2668 [hep-ph].

[38] S. Fleming, A. H. Hoang, S. Mantry and I. W. Stewart, these proceedings; arXiv:0710.4205 [hep-ph].

[39] S. Fleming, A. H. Hoang, S. Mantry and I. W. Stewart, arXiv:hep-ph/0703207.

[40] R. Harlander and M. Steinhauser, Prog. Part. Nucl. Phys. **43** (1999) 167 [arXiv:hep-ph/9812357].

[41] T. Hahn, Comput. Phys. Commun. **140** (2001) 418 [arXiv:hep-ph/0012260].

[42] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118** (1999) 153 [arXiv:hep-ph/9807565].

[43] C. Anastasiou and A. Lazopoulos, JHEP **0407** (2004) 046 [arXiv:hep-ph/0404258].

[44] J. Gluza, K. Kajda and T. Riemann, Mellin-Barnes arXiv:0704.2423 [hep-ph].

[45] M. Czakon, Comput. Phys. Commun. **175** (2006) 559 [arXiv:hep-ph/0511200].

[46] R. Harlander, T. Seidensticker and M. Steinhauser, Phys. Lett. B **426** (1998) 125 [hep-ph/9712228]; T. Seidensticker, hep-ph/9905298.

[47] J. A. M. Vermaseren, arXiv:math-ph/0010025.

[48] M. Tentyukov, H. M. Staudenmaier and J. A. M. Vermaseren, Nucl. Instrum. Meth. A **559** (2006) 224.

[49] M. Tentyukov and J. A. M. Vermaseren, arXiv:hep-ph/0702279.