Precise quark masses from sum rules

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In this contribution an improved analysis is described to extract precise charm and bottom quark masses from experimental and theoretical moments of the photon polarization function. The obtained $\overline{\mathrm{MS}}$ mass values read $m_c(3\text{ GeV}) = 0.986(13)$ GeV and $m_b(10\text{ GeV}) = 3.609(25)$ GeV.

1 Introduction

The theory of strong interaction has the strong coupling constant and the quark masses as fundamental input parameters. The latter constitute an essential input for the evaluation of weak decay rates of heavy mesons and for quarkonium spectroscopy. Furthermore, decay rates and branching ratios of a light Higgs boson — as suggested by electroweak precision measurements — depend critically on the masses of the charm and bottom quarks. Last not least, confronting the predictions for these masses with experiment is an important task for all variants of Grand Unified Theories. To deduce the values in a consistent way from different experimental investigations and with utmost precision is thus a must for current phenomenology.

The method described in this contribution goes back to 1977 [1] and was applied to next-to-next-to-leading order (NNLO) in Ref. [2]. The NNNLO analysis, including updated experimental input, was presented in Ref. [3].

2 Moments

The basic object which enters our analysis is the photon polarization function defined through

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right) \Pi(q^2) \quad = \quad i \int \mathrm{d}x \, e^{iqx} \langle 0|T j_\mu(x) j_\nu^\dagger(0)|0\rangle \,, \tag{1}$$

with j_μ being the electromagnetic current. The normalized total cross section for hadron production in e^+e^- annihilation is then given by

$$R(s) \quad = \quad \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_{\text{pt}}} \quad = \quad 12\pi \, \mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right] \,, \tag{2}$$

where $\sigma_{\text{pt}} = 4\pi\alpha^2/(3s)$. In the following we add a subscript Q to indicate the contribution from the heavy quark Q.

The idea for extracting a quark mass value m_Q is based on moments constructed from Π_Q. On one hand one can compute the Taylor expansion of $\Pi_Q(q^2)$ around $q^2 = 0$ and obtain the so-called "theory-moments" from

$$\mathcal{M}_n \quad = \quad \frac{12\pi^2}{n!} \left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\bigg|_{q^2=0} \,. \tag{3}$$

The three-loop contribution to $\Pi_Q(q^2)$ up to $n = 8$ within QCD has been computed in Refs. [4, 5] and the four-loop calculation for $n = 0$ and $n = 1$ has been performed in Refs. [6, 7]. In the analysis of Ref. [3] also two-loop QED corrections and non-perturbative contributions have been considered. The latter shows a visible effect only in the case of the charm quark.

From dimensional considerations we have $m_Q \sim (\mathcal{M}_n)^{\frac{1}{2n}}$ which implies a stronger dependence of m_Q on variations of $\mathcal{M}_n$ for smaller values of n. Furthermore, higher values of n require a careful theoretical treatment of the threshold region and the construction of an effective theory. The analysis performed in Ref. [3] is restricted to $n = 1, 2, 3$ and 4. Note that precise mass values can only be obtained for the three lowest moments since the non-perturbative contributions become too big already for $n = 4$.

One of the major advantages of the method discussed in this paper is that we can adopt the $\overline{\mathrm{MS}}$ scheme for the quark mass entering Eq. (3) and thus directly extract the corresponding value for the mass.

In order to extract experimental moments one exploits the analyticity of Π_Q and arrives at

$$\mathcal{M}_n = \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s)\,, \tag{4}$$

where R_Q naturally divides into three parts: At lower energies one has the narrow resonances which are the J/Ψ and Ψ' for charm the $\Upsilon(nS)$ $(n = 1, \ldots, 4)$ in the case of the bottom quark. The corresponding contributions to $\mathcal{M}_n$ are obtained with the help of the narrow width approximation for $R(s)$

$$R^{\mathrm{res}}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)\,, \tag{5}$$

where the electronic widths Γ_{ee} are known at the 1-2% level.

The second part is called threshold region and extends in the case of the charm quark from 3.73 GeV to about 5 GeV. In this region the cross section shows a rapid variation and can not be described by perturbation theory. Measurements from the BES collaboration from 2001 [8] and 2006 [9] provide excellent data for $R(s)$ with an uncertainty of about 4%. In order to obtain R_c one has to subtract the contribution from the light quarks which is explained in detail in Ref. [3].

The treatment of the bottom threshold region is quite similar. Measurements of R from threshold up to 11.24 GeV have been performed by the CLEO Collaboration more than 20 years ago [10], with a systematic error of 6%. No radiative corrections were applied. The average value derived from the four data points below threshold amounts to $\bar{R} = 4.559 \pm 0.034(\mathrm{stat.})$ which is 28% larger than the prediction from perturbative QCD (pQCD). However, a later result of CLEO [11] at practically the same energy, $R(10.52\ \mathrm{GeV}) = 3.56 \pm 0.01 \pm 0.07$, is significantly more precise and in perfect agreement with theory. Applying a rescaling factor of 1/1.28 to the old CLEO data not only enforces agreement between old and new CLEO data and pQCD in the region below the $\Upsilon(4S)$, it leads, in addition, also to excellent agreement between theory and experiment above threshold around 11.2 GeV where pQCD should be applicable also to bottom production. Further support to our approach is provided by the CLEO measurement of the cross section for bottom quark production at $\sqrt{s} = 10.865$ GeV which is given by $\sigma_b(\sqrt{s} = 10.865\ \mathrm{GeV}) = 0.301 \pm 0.002 \pm 0.039$ nb [12].

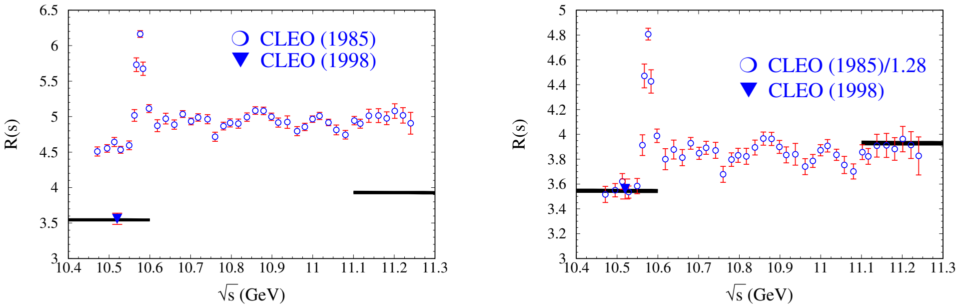

Figure 1: In the left plot the data for $R(s)$ are shown as published in Refs. [10] (circles) and [11] (triangle). The black curves are the predictions from pQCD outside the resonance region. In the right plot the older data from [10] are rescaled by a factor 1/1.28.

The central value can be converted to $R_b(10.865\text{ GeV}) = 0.409$. On the other hand, if one extracts $R_b(10.865\text{ GeV})$ from the rescaled CLEO data from 1984 [10] one obtains $R_b(10.865\text{ GeV}) = 0.425$ which deviates by less than 4% from the recent result [12].

In Fig. 1 the original and the rescaled data from [10] is shown and compared to pQCD and data point from [11]. We thus extract the threshold contribution to the moments from the interval $10.62\text{ GeV} \leq \sqrt{s} \leq 11.24\text{ GeV}$ by applying the rescaling factor to the data, subtract the "background" from u, d, s and c quarks and attribute a systematic error of 10% to the result.

The third contribution to the experimental moment is provided by the so-called continuum region which for the charm and bottom quark starts above 4.8 GeV and 11.24 GeV, respectively. In both cases there is no precise experimental data available. On the other hand, pQCD is supposed to work very well in these energy regions, in particular since $R_Q(s)$ is known to order α_s^2 including the full quark mass dependence and to order α_s^3 including quartic mass effects. For recent compilations we refer to Refs. [13, 14, 15] and would like to mention the `Fortran` program `rhad` [15] which provides a convenient platform to access easily the various radiative corrections.

3 Quark masses

Equating the theoretical and experimental moments of Eqs. (3) and (4), adopting $\mu = 3$ GeV ($\mu = 10$ GeV) for the charm (bottom) quark and solving for the quark mass leads to the results which are shown in Fig. 2 in graphical form.[a] It is nicely seen that the results for m_Q further stabilize when going from three to four loops. At the same time the uncertainty is considerably reduced. Furthermore, the preference for the first three moments is clearly visible. Also the analysis for $n = 2$ and $n = 3$ leads to small errors, even if we include the uncertainty from the yet uncalculated four-loop contributions.[b] We emphasize the remarkable consistency between the three results which we consider as additional confirmation of our approach.

[a]The numerical results including a detailed error analysis can be found in Ref. [3].

[b]See Ref. [3] for details.

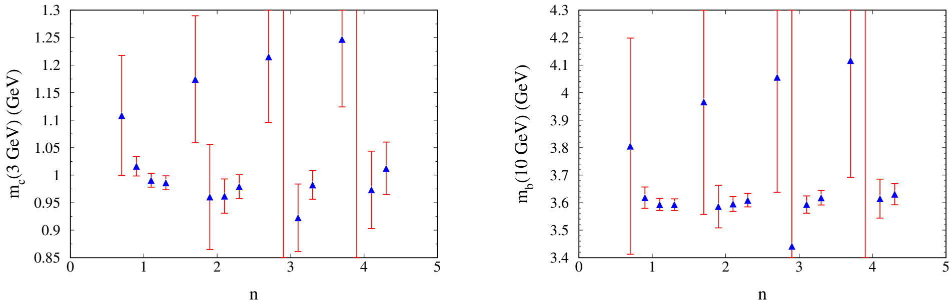

Figure 2: $m_c(3\text{ GeV})$ (left) and $m_b(10\text{ GeV})$ (right) for $n = 1, 2, 3$ and 4. For each value of n the results from left to right correspond the inclusion of terms of order α_s^0, α_s^1, α_s^2 and α_s^3 to the theory-moments. Note, that for $n = 3$ and $n = 4$ the uncertainties can not be determined in those cases where only the two-loop corrections of order α_s are included into the coefficients $\bar{C}_n$ as the equation cannot be solved for the quark mass.

The final result for the $\overline{\text{MS}}$-masses read $m_c(3\text{ GeV}) = 0.986(13)$ GeV and $m_b(10\text{ GeV}) = 3.609(25)$ GeV. They can be translated into $m_c(m_c) = 1.286(13)$ GeV and $m_b(m_b) = 4.164(25)$ GeV. This analysis is consistent with but significantly more precise than a similar previous study.

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