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Next-to-Next-to-Leading Electroweak Logarithms in W-pair Production at ILC

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Abstract

We derive the high energy asymptotic behavior of gauge boson production cross section in a spontaneously broken $SU(2)$ gauge theory in the next-to-next-to-leading logarithmic approximation. On the basis of this result we obtain the logarithmically enhanced two-loop electroweak corrections to the differential cross section of W-pair production at ILC/CLIC up to the second power of the large logarithm.

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1 Introduction

The W-pair production at e^+e^- colliders plays a crucial role for testing the Standard Model of electroweak interactions. At LEP2 this process has been used for the determination of the W-boson mass M_W, a fundamental parameter of the standard model, through W-boson reconstruction with an uncertainty of 40 MeV [1]. Furthermore, the triple gauge boson coupling as predicted by the non-Abelian gauge theory has been verified within a few percent. The experimental study of the W-pair production at the International Linear Collider (ILC) is expected to improve the accuracy of the mass determination to 7 MeV due to much higher luminosity [2]. Moreover, the advent of ILC will give access to the new high energy domain where the cross section is increasingly sensitive to the triple gauge boson coupling and W-pair production could be used as a probe of the non-Abelian structure of the electroweak interactions and of possible gauge boson anomalous couplings. To match the experimental accuracy, the theoretical analysis has to take into account the electroweak radiative corrections. The one-loop corrections to the cross section of the on-shell W-pair production have been evaluated by different groups [3–6] already decades ago. The calculation of the one-loop corrections to the W-boson mediated $e^+e^- \to 4f$ processes has been performed in the double pole approximation in Ref. [7] and recently the full analysis has been completed in Ref. [8]. These results ensure an accuracy significantly better than one percent when the characteristic energy $\sqrt{s}$ of the process is about the gauge boson mass. However, once $\sqrt{s}$ is far larger than M_W, the cross section receives virtual corrections enhanced by powers of electroweak "Sudakov" logarithm $\ln(s/M_{W,Z}^2)$, which at the energies of about one TeV have to be controlled to two loops in order to keep the theoretical error below one percent. This is even more valid for energies of 3 TeV anticipated for the CLIC project [9]. In the case of light fermion pair production these corrections are already available through the next-to-next-to-next-to-leading logarithmic (N^{3}LL) approximation, *i.e.* including all the two-loop logarithmic terms [10–14]. For $e^+e^- \to W^+W^-$ production only the leading-logarithmic (LL) and the next-to-leading logarithms (NLL) are known so far [10,15–17].

In the present paper we extend the analysis of W-pair production to the next-to-next-to-leading logarithmic (NNLL) approximation following the approach developed in Refs. [11–14] for the four-fermion processes. The limit of the small-angle production, which could be interesting in the case of the transverse gauge bosons because the corresponding cross section is peaked in the forward direction, remains beyond the scope of the present paper. In the next section we outline the approach and derive the NNLL corrections to the differential cross section of the gauge boson pair production in a spontaneously broken $SU(2)$ model which emulates the massive gauge boson sector of the Standard Model of electroweak interactions. We generalize the result to the $SU(2) \times U(1)$ gauge theory with a heavy top quark in Section 3. A brief summary and conclusions are given in Section 4.

2 High energy asymptotic of the massive gauge boson production

Let us consider as a toy model the spontaneously broken $SU(2)$ gauge theory with the Higgs mechanism for gauge boson mass generation and with massless left-handed fermion doublets. The model retains the main features of the massive gauge boson sector of the Standard Model. In this case the result can be presented in a simple analytical form and constitutes the basis for the further extension to the full electroweak theory. We study the process of gauge boson pair production in fermion-antifermion annihilation at high energy and fixed angle when all the kinematical invariants are of the same order and far larger than the gauge boson mass,

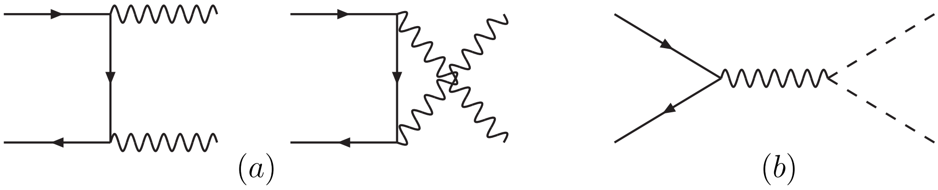


Figure 1: The diagrams represent (a) transverse and (b) longitudinal gauge boson pair production in fermion-antifermion annihilation at high energy in the Born approximation.

$|s| \sim |t| \sim |u| \gg M^2$. In this limit the asymptotic energy dependence of the field amplitudes is dominated by *Sudakov* logarithms [18,19] and governed by the evolution equations. The method of the evolution equations in the context of the electroweak corrections is described in detail for the fermion pair production in Ref. [14]. It can be directly extended to the gauge boson production as briefly described below.

Due to helicity conservation a pair of either transverse or longitudinal gauge bosons can be produced in the high energy limit. The transverse gauge bosons behave like vector particles in the adjoint representation while the longitudinal gauge bosons, as a consequence of the equivalence theorem, behave like scalar particles in the fundamental representation. The structure of the Sudakov logarithms in these cases is significantly different and we consider them separately.

2.1 Transverse polarization

The transverse gauge bosons are the true vector particles and the Born amplitude in this case is given by the t-channel and u-channel fermion exchange diagrams, Fig. 1a. It is convenient to introduce the functions $\mathcal{Z}_\psi$ and $\mathcal{Z}_A$ which describe the asymptotic dependence on the large momentum transfer Q of the fermion scattering amplitude in an external singlet vector field and of the vector boson in an external scalar singlet field, *i.e.* of the respective form factors in the Euclidean region. In leading order in M^2/Q^2 these functions are known to satisfy the following linear evolution equation [20–22]

$$\frac{\partial}{\partial \ln Q^2} \mathcal{Z}_i = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \gamma_i(\alpha(x)) + \zeta_i(\alpha(Q^2)) + \xi_i(\alpha(M^2)) \right] \mathcal{Z}_i \,, \tag{1}$$

with the solution

$$\mathcal{Z}_i = \exp \left\{ \int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \left[\int_{M^2}^{x} \frac{\mathrm{d}x'}{x'} \gamma_i(\alpha(x')) + \zeta_i(\alpha(x)) + \xi_i(\alpha(M^2)) \right] \right\} \,, \tag{2}$$

which satisfies the initial condition $\mathcal{Z}_i|_{Q^2=M^2} = 1$. Here the perturbative functions $\gamma_i(\alpha)$ *etc.* are given by the series in the coupling constant $\alpha(\mu^2)$, *e.g.* $\gamma_i(\alpha) = \sum_{n=1}^{\infty} (\alpha/4\pi)^n \gamma_i^{(n)}$. For the amplitude of transverse boson production $\mathcal{A}_T$ let us introduce the reduced amplitude $\tilde{\mathcal{A}}_T$ so that

$$\mathcal{A}_T = \mathcal{Z}_\psi \mathcal{Z}_A \tilde{\mathcal{A}}_T \,. \tag{3}$$

Due to the factorization property of the Sudakov logarithms associated with the *collinear* divergences of the massless theory [23] the reduced amplitude satisfies the simple renormalization group like equation [24–26]

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}}_T = \boldsymbol{\chi}_T(\alpha(Q^2))\tilde{\mathcal{A}}_T\,, \tag{4}$$

where $Q^2 = -s$ and $\boldsymbol{\chi}_T$ is the soft anomalous dimension matrix acting in the space of the isospin amplitudes. The solution of the above equation is given by the path-ordered exponent

$$\tilde{\mathcal{A}}_T = \mathrm{Pexp}\left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\boldsymbol{\chi}_T(\alpha(x))\right]\mathcal{A}_{0T}(\alpha(M^2))\,, \tag{5}$$

where $\mathcal{A}_{T0}$ determines the initial conditions for the evolution equation at $Q = M$. By calculating the functions entering the evolution equations order by order in α one gets the logarithmic approximations for the amplitude. For example, the LL approximation includes all the terms of the form $\alpha^n\mathcal{L}^{2n}$ and is determined by the one-loop coefficients $\gamma_i^{(1)}$. The NLL approximation includes all the terms of the form $\alpha^n\mathcal{L}^{2n-m}$ with $m = 0,\ 1$ and requires the one-loop coefficients $\zeta_i^{(1)}$, $\xi_i^{(1)}$, and $\boldsymbol{\chi}^{(1)}$ as well as the one-loop running of α in $\gamma_i(\alpha)$, and so on. To get the NNLL terms $\alpha^n\mathcal{L}^{2n-2}$ one needs in addition the two-loop coefficient $\gamma_i^{(2)}$, the two-loop running of α in $\gamma_i(\alpha)$ and the one-loop contribution to $\mathcal{A}_{T0}$.

The anomalous dimensions $\gamma(\alpha)$, $\zeta(\alpha)$ and $\boldsymbol{\chi}(\alpha)$ are mass-independent and can be associated with the infrared divergences of the massless (unbroken) theory. From the QCD result (see *e.g.* [27] and references therein) adopted for the specific case of $SU(2)$ gauge group, n_f chiral quarks, and one scalar in the fundamental representation we get

$$\gamma_\psi^{(1)} = -3/2\,, \qquad \gamma_\psi^{(2)} = -\frac{65}{3} + \pi^2 + \frac{5}{6}n_f\,, \qquad \zeta_\psi^{(1)} = \frac{9}{4}\,, \qquad \zeta_A^{(1)} = 0\,, \tag{6}$$

and $\gamma_A^{(n)} = 8\gamma_\psi^{(n)}/3$. The matrix $\boldsymbol{\chi}_T^{(1)}$ can be extracted from the results of Refs. [28,29]. In the isospin basis $(\boldsymbol{\sigma}^a\boldsymbol{\sigma}^b,\ \boldsymbol{\sigma}^b\boldsymbol{\sigma}^a,\ \delta^{ab}\cdot\mathbf{1})$ it takes the form

$$\boldsymbol{\chi}_T^{(1)} = \begin{pmatrix} -2(\ln(x_-)+i\pi) & 0 & \ln(\frac{x_+}{x_-})) \\ 0 & -2(\ln(x_+)+i\pi) & \ln(\frac{x_-}{x_+}) \\ (\ln(x_+)+i\pi) & (\ln(x_-)+i\pi) & 0 \end{pmatrix}\,, \tag{7}$$

where $x_\pm = (1\pm\cos\theta)/2$ and θ is the production angle. Note that in this basis the Born amplitude up to a common factor is given by the vector $(1/x_-, 1/x_+, 0)$. At the same time the functions $\xi_i(\alpha)$ and $\mathcal{A}_T(\alpha)$ do depend on the infrared structure of the model and require the calculation in the spontaneously broken phase. For example $\xi_\psi^{(1)} = 0$ [12] and from the result of Ref. [6] we obtain $\xi_A^{(1)} = 0$. To emulate the $e^+e^- \to W_T^+W_T^-$ process one has to project the amplitude on the relevant initial and final isospin states, which is straightforward. The Born cross section in the high energy limit reads

$$\frac{\mathrm{d}\sigma_T^B}{\mathrm{d}\Omega} = \frac{\alpha^2(M^2)}{4s}\frac{x_+(x_+^2+x_-^2)}{x_-}\,, \tag{8}$$

and is peaked in the forward direction. We define the perturbative expansion for the differential cross section in the $\overline{\rm MS}$ renormalized coupling constant $\alpha \equiv \alpha(M^2)$ as follows

$$\frac{d\sigma}{d\Omega} = \left[1 + \left(\frac{\alpha}{4\pi}\right)\delta^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \delta^{(2)} + \ldots\right] \frac{d\sigma^B}{d\Omega}\,. \tag{9}$$

Expanding the Sudakov exponents to NNLL order for the one- and two-loop corrections we get

$$\begin{aligned}
\delta^{(1)}_T &= \left(\gamma^{(1)}_\psi + \gamma^{(1)}_A\right)\mathcal{L}^2(s) + 2\left[\zeta^{(1)}_\psi + \zeta^{(1)}_A + \xi^{(1)}_\psi + \xi^{(1)}_A + t^{(1)}_{11} + t^{(1)}_{31} + \frac{x_-}{x_+}\left(t^{(1)}_{12} + t^{(1)}_{32}\right)\right] \\
&\quad \times\mathcal{L}(s) + \delta^{(1)}_{0T} \qquad (10) \\
\delta^{(2)}_T &= \frac{1}{2}\left(\gamma^{(1)}_\psi + \gamma^{(1)}_A\right)^2\mathcal{L}^4(s) + 2\left[\zeta^{(1)}_\psi + \zeta^{(1)}_A + \xi^{(1)}_\psi + \xi^{(1)}_A + t^{(1)}_{11} + t^{(1)}_{31} + \frac{x_-}{x_+}\left(t^{(1)}_{12} + t^{(1)}_{32}\right)\right. \\
&\quad \left. -\frac{1}{6}\beta_0\right]\left(\gamma^{(1)}_\psi + \gamma^{(1)}_A\right)\mathcal{L}^3(s) + \left[\gamma^{(2)}_\psi + \gamma^{(2)}_A + 2\left(\zeta^{(1)}_\psi + \zeta^{(1)}_A + \xi^{(1)}_\psi + \xi^{(1)}_A\right)\left(\zeta^{(1)}_\psi + \zeta^{(1)}_A\right.\right. \\
&\quad \left. +\xi^{(1)}_\psi + \xi^{(1)}_A + 2\left(t^{(1)}_{11} + t^{(1)}_{31} + \frac{x_-}{x_+}\left(t^{(1)}_{12} + t^{(1)}_{32}\right)\right)\right) - \beta_0\left(t^{(1)}_{11} + t^{(1)}_{31} + \frac{x_-}{x_+}\left(t^{(1)}_{12} + t^{(1)}_{32}\right)\right. \\
&\quad \left.\zeta^{(1)}_\psi + \zeta^{(1)}_A\right) + t^{(1)\,2}_{11} + t^{(1)}_{12}t^{(1)}_{21} + t^{(1)}_{11}t^{(1)}_{31} + t^{(1)}_{13}t^{(1)}_{31} + t^{(1)}_{21}t^{(1)}_{32} + t^{(1)}_{31}t^{(1)}_{33} + \frac{x_-}{x_+} \\
&\quad \times\left(t^{(1)}_{12}\left(t^{(1)}_{11} + t^{(1)}_{22} + t^{(1)}_{31}\right) + t^{(1)}_{32}\left(t^{(1)}_{13} + t^{(1)}_{22} + t^{(1)}_{33}\right)\right) + \left(t^{(1)}_{11} + t^{(1)}_{31} + \frac{x_-}{x_+}\left(t^{(1)}_{12} + t^{(1)}_{32}\right)\right)^2 \\
&\quad \left. +\delta^{(1)}_{0T}\left(\gamma^{(1)}_\psi + \gamma^{(1)}_A\right)\right]\mathcal{L}^2(s) \qquad (11)
\end{aligned}$$

where $\beta_0 = 43/6 - n_f/3$ is the one-loop beta-function, $t^{(1)}_{ij} \equiv {\rm Re}[\boldsymbol{\chi}^{(1)}_T]_{ij}$, $\mathcal{L}(s) \equiv \ln(s/M^2)$, and $\delta^{(1)}_{0T}$ is the nonlogarithmic part of the one-loop corrections which can be extracted from the result of Ref. [6]. For the Higgs boson mass $M_H = M$ it takes a simple form

$$\begin{aligned}
\delta^{(1)}_{0T} &= \left(\frac{5 + 3x_-}{2(x_-^2 + x_+^2)} - \frac{5}{x_+}\right)\ln^2(x_-) + \frac{3x_-}{x_-^2 + x_+^2}\ln(x_+^2) + \frac{4}{x_+}\ln(x_-)\ln(x_+) \\
&\quad + \frac{9 - 19x_-}{2(x_-^2 + x_+^2)}\ln(x_-) - \frac{5x_+}{2(x_-^2 + x_+^2)} - \frac{7\pi^2}{18} - \frac{13\pi}{3\sqrt{3}} + \frac{455}{36} - \frac{10}{9}n_f.
\end{aligned} \tag{12}$$

Substituting the values of the coefficients for $n_f = 12$ we obtain

$$\begin{aligned}
\delta^{(1)}_T &= -\frac{11}{2}\mathcal{L}^2(s) + \left[\left(-8 + \frac{4x_-}{x_+}\right)\ln(x_-) + 4\ln(x_+) + \frac{9}{2}\right]\mathcal{L}(s) \\
&\quad + \left(\frac{5 + 3x_-}{2(x_-^2 + x_+^2)} - \frac{5}{x_+}\right)\ln^2(x_-) + \frac{3x_-}{x_-^2 + x_+^2}\ln(x_+^2) + \frac{4}{x_+}\ln(x_-)\ln(x_+)
\end{aligned}$$

$$+\frac{9-19x_-}{2(x_-^2+x_+^2)}\ln(x_-)-\frac{5x_+}{2(x_-^2+x_+^2)}-\frac{7\pi^2}{18}-\frac{13\pi}{3\sqrt{3}}-\frac{25}{36}\,, \tag{13}$$

$$\begin{aligned}
\delta_T^{(2)} &= \frac{121}{8}\mathcal{L}^4(s)+\left[\left(44-\frac{22x_-}{x_+}\right)\ln(x_-)-22\ln(x_+)-\frac{341}{18}\right]\mathcal{L}^3(s)\\
&+\left[\left(32+\frac{4x_-^2}{x_+^2}-\frac{55+33x_-}{4(x_-^2+x_+^2)}+\frac{55-40x_-}{2x_+}\right)\ln^2(x_-)+\left(8-\frac{33x_-}{2(x_-^2+x_+^2)}\right)\ln^2(x_+)\right.\\
&-\left(28+\frac{22-4x_-}{x_+}\right)\ln(x_-)\ln(x_+)+\left(-\frac{70}{3}+\frac{35x_-}{3x_+}-\frac{99-209x_-}{4(x_-^2+x_+^2)}\right)\ln(x_-)\\
&\left.+\frac{35}{3}\ln(x_+)+\frac{55x_+}{4(x_-^2+x_+^2)}+\frac{209\pi^2}{36}+\frac{143\pi}{6\sqrt{3}}-\frac{863}{24}\right]\mathcal{L}^2(s)\,.
\end{aligned} \tag{14}$$

2.2 Longitudinal polarization

The equivalence theorem relates the cross section of the longitudinal gauge boson production $e^+e^- \to W_L^+W_L^-$ to the production of the Goldstone bosons $e^+e^- \to \phi^+\phi^-$. The Born amplitude is now given by the s-channel annihilation diagram, Fig. 1b. The analysis of the high energy asymptotic for the longitudinal gauge boson production goes along the line described in the previous section and is very similar to the one for fermion pair production [12]. Instead of $\mathcal{Z}_A$ one should use the function $\mathcal{Z}_\phi$ which correspond to the scalar particle scattering in an external singlet vector field. The necessary parameters of the evolution equation read $\gamma_\phi^{(n)}=\gamma_\psi^{(n)}$, $\zeta_\phi^{(1)}=3$, and $\xi_\phi^{(1)}=0$. The structure of the reduced amplitude is also different. In the isospin basis $(\boldsymbol{\sigma}^a\otimes\boldsymbol{\sigma}^a, \mathbf{1}\otimes\mathbf{1})$ the one-loop matrix of the soft anomalous dimensions takes the form

$$\boldsymbol{\chi}_L^{(1)}=\begin{pmatrix} -4\left(\ln(x_+)+i\pi\right)+2\ln\left(\frac{x_+}{x_-}\right) & 4\ln\left(\frac{x_+}{x_-}\right)\\ \frac{3}{4}\ln\left(\frac{x_+}{x_-}\right) & 0\end{pmatrix}. \tag{15}$$

Note that in this basis the Born amplitude is given by the vector $(1,0)$. The corresponding Born cross section reads

$$\frac{\mathrm{d}\sigma_L^B}{\mathrm{d}\Omega}=\frac{\alpha^2(s)}{4s}\frac{x_+x_-}{4}\,. \tag{16}$$

It has a maximum at $\theta=90^o$. We proceed as in the case of transverse polarization and obtain the one- and two-loop NNLL corrections to the differential cross section

$$\delta_L^{(1)} = \left(\gamma_\psi^{(1)}+\gamma_\phi^{(1)}\right)\mathcal{L}^2(s)+2\left[\zeta_\psi^{(1)}+\zeta_\phi^{(1)}+\xi_\psi^{(1)}+\xi_\phi^{(1)}+l_{11}^{(1)}+4l_{21}^{(1)}\right]\mathcal{L}(s)+\delta_{0L}^{(1)} \tag{17}$$

$$\begin{aligned}
\delta_L^{(2)} &= \frac{1}{2}\left(\gamma_\psi^{(1)}+\gamma_\phi^{(1)}\right)^2\mathcal{L}^4(s)+2\left[\zeta_\psi^{(1)}+\zeta_A^{(1)}+\xi_\psi^{(1)}+\xi_\phi^{(1)}+l_{11}^{(1)}+4l_{21}^{(1)}-\frac{1}{6}\beta_0\right]\left(\gamma_\psi^{(1)}+\gamma_\phi^{(1)}\right)\\
&\times\mathcal{L}^3(s)+\left[\gamma_\psi^{(2)}+\gamma_\phi^{(2)}+2\left(\zeta_\psi^{(1)}+\zeta_\phi^{(1)}+\xi_\psi^{(1)}+\xi_\phi^{(1)}\right)\left(\zeta_\psi^{(1)}+\zeta_\phi^{(1)}+\xi_\psi^{(1)}+\xi_\phi^{(1)}\right.\right.\\
&\left.+2l_{11}^{(1)}+8l_{21}^{(1)}\right)-\beta_0\left(l_{11}^{(1)}+4l_{21}^{(1)}+\zeta_\psi^{(1)}+\zeta_\phi^{(1)}\right)+{l_{11}^{(1)}}^2+l_{21}^{(1)}\left(4l_{11}^{(1)}+l_{12}^{(1)}+4l_{22}^{(1)}\right)
\end{aligned}$$

$$+\left(l_{11}^{(1)}+4l_{21}^{(1)}\right)^2+\delta_{0L}^{(1)}\left(\gamma_\psi^{(1)}+\gamma_\phi^{(1)}\right)\Big]\mathcal{L}^2(s) \tag{18}$$

where $l_{ij}^{(1)}\equiv\mathrm{Re}[\boldsymbol{\chi}_L^{(1)}]_{ij}$. From the result of Ref. [6] for $M_H=M$ we obtain

$$\delta_{0L}^{(1)} = -\frac{5}{2x_+}\ln^2(x_-)+\frac{1}{2x_-}\ln^2(x_+)-\frac{7\pi^2}{3}+\frac{32\pi}{3\sqrt{3}}-\frac{25}{36}-\frac{10}{9}n_f\,. \tag{19}$$

For $n_f=12$ this gives

$$\delta_L^{(1)} = -3\mathcal{L}^2(s)+\left[-10\ln(x_-)+2\ln(x_+)+\frac{21}{2}\right]\mathcal{L}(s)^2 -\frac{5}{2x_+}\ln^2(x_-)+\frac{1}{2x_-}\ln^2(x_+)-\frac{7\pi^2}{3}+\frac{32\pi}{3\sqrt{3}}-\frac{505}{36}\,, \tag{20}$$

$$\delta_L^{(2)} = \frac{9}{2}\mathcal{L}^2(s)+\left[30\ln(x_-)-6\ln(x_+)-\frac{85}{3}\right]\mathcal{L}^3(s) +\left[\left(38+\frac{15}{2x_+}\right)\ln^2(x_-)+\left(2-\frac{3}{2x_-}\right)\ln^2(x_+)-8\ln(x_-)\ln(x_+) -\frac{535}{6}\ln(x_-)+\frac{107}{6}\ln(x_+)+9\pi^2-\frac{32\pi}{\sqrt{3}}+\frac{229}{4}\right]\mathcal{L}^2(s)\,. \tag{21}$$

3 W-pair production in e^+e^- annihilation

In the standard electroweak model the perturbative expansion involves the $SU_L(2)$ coupling constant α_{ew} and the $U(1)$ hypercharge coupling constant α_Y. We eliminate the latter by means of the relation $\alpha_Y=\tan^2\theta_W\,\alpha_{ew}$, where θ_W is the weak mixing angle, and consider the one-parameter series for the cross section in α_{ew} of the form of Eq. (9). In the high energy limit the transverse gauge bosons are produced only in annihilation of the left-handed electron-positron pair. The corresponding Born cross section is given by Eq. (8) with α replaced by α_{ew}. The longitudinal gauge bosons can be produced in the annihilation of the electron-positron pair of both chiralities. In the case of the left-handed initial state fermions the Born cross section gets the contribution from the $SU_L(2)$ and the hypercharge virtual gauge bosons and reads

$$\frac{\mathrm{d}\sigma_{-L}^B}{\mathrm{d}\Omega}=\frac{1}{\cos^4\theta_W}\frac{\alpha_{ew}^2(s)}{4s}\frac{x_+x_-}{4}\,. \tag{22}$$

For the right-handed initial state fermions the Born cross section is saturated by the hypercharge gauge boson

$$\frac{\mathrm{d}\sigma_{+L}^B}{\mathrm{d}\Omega}=4\sin^4\theta_W\,\frac{\mathrm{d}\sigma_{-L}^B}{\mathrm{d}\Omega}\,. \tag{23}$$

The analysis of the radiative corrections in the Standard Model is complicated by the presence of the mass gap and mixing in the gauge sector and by the large top quark Yukawa coupling.

In next section we use the method of Ref. [10,14] to separate the electroweak and QED Sudakov logarithms. In Sect. 3.2 we extend the evolution equation approach to the Yukawa enhanced contributions. In Sect. 3.3 we present the final numerical results for the two-loop NNLL corrections to the differential cross sections.

3.1 Separating QED Sudakov logarithms

The electroweak Standard Model with the spontaneously broken $SU_L(2)\times U(1)$ gauge group involves both the massive W and Z bosons and the massless photon. The corrections to the fully exclusive cross sections due to the virtual photon exchange are infrared divergent and should be combined with soft real photon emission to obtain infrared finite physical observables. We regularize the infrared divergences by giving the photon a small mass λ. Thus besides the electroweak Sudakov logarithms discussed above the radiative corrections contain the QED Sudakov logarithms of the form $\ln(Q^2/\lambda^2)$. To disentangle the electroweak and QED logarithms we use the approach of Ref. [10,12,14]. While the dependence of the amplitudes on the large momentum transfer is governed by the *hard* evolution equations (*c.f.* Eqs. (1, 4)), their dependence on the photon mass is governed by the *infrared* evolution equations [10]. In the limit $\lambda^2 \ll M_W^2,\ m_t^2 \ll Q^2$ the infrared evolution equations in the full theory are the same as in QED and the solution to the NNLL accuracy in the massless fermion approximation $m_f = 0$ $(f \neq t)$ is given by the factor

$$\begin{aligned}\mathcal{U} &= U_0(\alpha_e)\exp\left\{-\frac{\alpha_e(\lambda^2)}{4\pi}\left[\left(2-\left(\frac{290}{27}+\frac{40}{9}\ln\left(\frac{x_+}{x_-}\right)\right)\frac{\alpha_e}{\pi}\right)\ln^2\left(\frac{Q^2}{\lambda^2}\right)\right.\right.\\ &\left.\quad -\left(3+4\ln\left(\frac{x_+}{x_-}\right)\right)\ln\left(\frac{Q^2}{\lambda^2}\right)+\frac{40}{27}\frac{\alpha_e}{\pi}\ln^3\left(\frac{Q^2}{\lambda^2}\right)-\left(\ln\left(\frac{M_W^2}{\lambda^2}\right)-1\right)^2\right]\\ &\left.\quad +\mathcal{O}(\alpha_e^3)\right\}, \end{aligned} \tag{24}$$

where α_e is the $\overline{\rm MS}$ QED coupling constant. The NNLL approximation for $\mathcal{U}$ can be obtained from the result for the fermion-antifermion production [12] by proper modification of the QED anomalous dimensions. It is convenient to normalize the QED factor so that $\mathcal{U}(\alpha_e)|_{s=\lambda^2=M_W^2} = 1$. In order to cancel the singular dependence on the photon mass, the QED Sudakov exponent (24) should be combined with the the real photon emission, which is also of pure QED nature if the energy of emitted photons is much smaller than M_W.

Two sets of equations completely determine the dependence of the amplitudes on two dimensionless variables Q/M_W and Q/λ up to the initial conditions which are fixed through the matching procedure. To get the purely weak logarithms one subtracts the QED exponent (24) from the exponent given by the solution of the hard evolution equation. This can naturally be formulated in terms of the functions parameterizing the solution. The functions γ, ζ, and $\boldsymbol{\chi}$ are mass-independent. Therefore the anomalous dimensions parameterizing the purely weak logarithms can be obtained by subtracting the QED contribution from the result of the unbroken symmetry phase calculation to all orders in the coupling constants. In the order of interest they can be found in or easily derived from the result of Ref. [12]. For example, we have

$$\gamma_A^{(2)} = \gamma_A^{(2)}|_{SU(2)} - \frac{800}{27}\sin^2\theta_W\,, \tag{25}$$

$$\gamma_\phi^{(2)} = \gamma_\phi^{(2)}|_{SU(2)} + \frac{52}{9}\tan^2\theta_W - \frac{800}{27}\sin^2\theta_W\,, \tag{26}$$

and so on. Here the $SU(2)$ contributions are given by the results of Sect. 2 with $n_f = 12$. The only new ingredient in comparison with the light fermion pair production [12] is the effect of the large Yukawa coupling of the third generation quarks on the longitudinal gauge boson production which is considered in the next section.

On the other hand the functions ξ and $\mathcal{A}_0$ are infrared sensitive and require the use of the true mass eigenstates of the Standard Model in the perturbative calculation. In the NNLL approximation one needs the one-loop contribution to these quantities which can be extracted from the result of Ref. [6]. For example, for the left-handed initial state fermions we find

$$\begin{aligned}
\xi_\psi^{(1)} + \xi_A^{(1)} &= \frac{1-4\cos^2\theta_W + 8\cos^4\theta_W}{2\cos^2\theta_W}\ln\left(\frac{M_Z^2}{M_W^2}\right)\,, \\
\xi_\psi^{(1)} + \xi_\phi^{(1)} &= \frac{(1-2\cos^2\theta_W)^2}{\cos^2\theta_W}\ln\left(\frac{M_Z^2}{M_W^2}\right)\,.
\end{aligned} \tag{27}$$

At the same time the $\mathcal{A}_0^{(1)}$ term results in the one-loop nonlogarithmic contribution to the cross section. The corresponding expression directly follows from Ref. [6] and is rather lengthy so we do not give it explicitly. Note that in Ref. [6] the result is presented in the on-shell renormalization scheme and in the limit $\lambda \ll m_e$. We convert it to $\overline{\rm MS}$ scheme and to the massless electron approximation using the formulae of Refs. [30,31].

3.2 Top quark Yukawa coupling effects

The large Yukawa coupling of the third generation quarks to the scalar (Higgs and Goldstone) bosons results in specific logarithmic corrections proportional to m_t^2/M_W^2. These corrections are determined by the ultraviolet quark and scalar boson field renormalization and can be taken into account through the modification of the evolution equations for the corresponding $\mathcal{Z}$-functions. The analysis is straightforward but complicated because the Yukawa interaction mixes quark and scalar boson evolution and in general does not commute with the $SU(2)$ and hypercharge couplings. However, due to the factorization property of the collinear logarithms the Yukawa enhanced contribution to NLL approximation is given simply by the product of the one-loop Yukawa corrections and the double logarithmic exponent as observed in Ref. [32]. The structure of the NNLL contribution is much more complicated and we restrict the analysis to a simplified model with $\sin\theta_W = 0$, *i.e.* with no hypercharge interaction. Let us introduce the following five-component vector in the space of $\mathcal{Z}$-functions

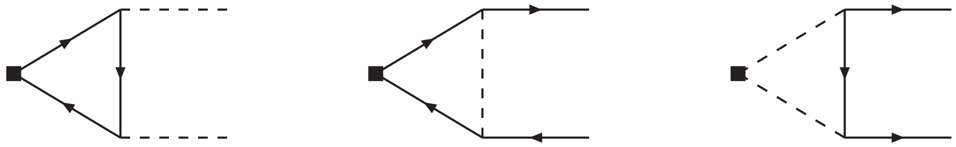


Figure 2: The one-loop diagrams contributing to the anomalous dimension matrix $\boldsymbol{\zeta}$. The arrow lines correspond to the third generation quarks. The dashed lines correspond to the Higgs, neutral or charged Goldstone bosons. The black square represent an external singlet vector field

$\boldsymbol{\mathcal{Z}} = (\mathcal{Z}_\phi,\ \mathcal{Z}_\chi,\ \mathcal{Z}_{b-},\ \mathcal{Z}_{t-},\ \mathcal{Z}_{t+})$, where the subscript + (−) stand for the right (left) quark fields and $\mathcal{Z}_\chi$ corresponds to the transition of the Higgs boson into the neutral Goldstone boson in the external singlet vector field. The evolution equation for this vector takes the form

$$\frac{\partial}{\partial \ln Q^2}\boldsymbol{\mathcal{Z}} = \left[\int_{M_W^2}^{Q^2}\frac{\mathrm{d}x}{x}\boldsymbol{\gamma}(\alpha_{ew}(x)) + \boldsymbol{\zeta}(\alpha_{ew}(Q^2),\alpha_{Yuk}(Q^2)) + \boldsymbol{\xi}(\alpha_{ew}(M_W^2))\right]\boldsymbol{\mathcal{Z}}\,, \tag{28}$$

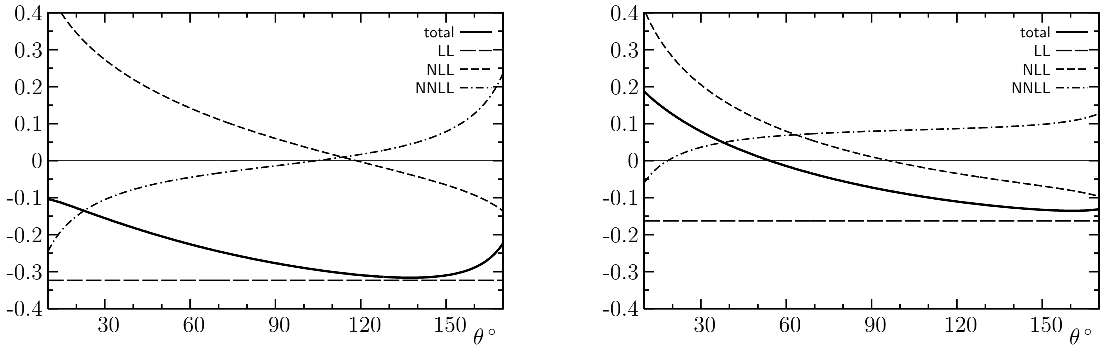
with the solution

$$\boldsymbol{\mathcal{Z}} = \mathrm{Pexp}\left\{\int_{M_W^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M_W^2}^{x}\frac{\mathrm{d}x'}{x'}\boldsymbol{\gamma}(\alpha_{ew}(x')) + \boldsymbol{\zeta}(\alpha_{ew}(x),\alpha_{Yuk}(x)) + \boldsymbol{\xi}(\alpha_{ew}(M_W^2))\right]\right\}\boldsymbol{\mathcal{Z}}_0\,, \tag{29}$$

where $\boldsymbol{\gamma}^{(1)} = (-3/2)\cdot\mathbf{1}$ and $\boldsymbol{\xi} = 0$. The anomalous dimension matrix $\boldsymbol{\zeta}$ includes all the dependence on the Yukawa coupling α_{Yuk}. We eliminate the latter by means of the relation $\alpha_{Yuk} = \frac{m_t^2}{2M_W^2}\alpha_{ew}$, and consider the one-parameter series for the anomalous dimension in α_{ew}. The one-loop coefficient reads

$$\zeta^{(1)} = \frac{1}{4}\begin{pmatrix} 12 & 0 & 0 & 0 & 0\\ 0 & 12 & 0 & 0 & 0\\ 0 & 0 & 9 & 0 & 0\\ 0 & 0 & 0 & 9 & 0\\ 0 & 0 & 0 & 0 & 9\end{pmatrix} + \frac{m_t^2}{4M_W^2}\begin{pmatrix} 0 & 0 & 6 & 0 & -6\\ 0 & 0 & 0 & 6 & -6\\ 1 & 0 & 0 & 0 & -1\\ 0 & 1 & 0 & 0 & -1\\ -1 & -1 & -1 & -1 & 0\end{pmatrix}, \tag{30}$$

where the first term representing the pure $SU_L(2)$ contribution follows from the result of Sect. 2.1 and the second term represents the Yukawa contribution, Fig. 2. It can be extracted from the known one-loop result (see *e.g.* Ref. [32,33]). The proper initial condition which corresponds to the Born amplitudes of the quark and scalar boson production in e^+e^- annihilation is given by the vector $\boldsymbol{\mathcal{Z}}_0 = (1,-1,-1,1,0)$. In NNLL approximation one needs also the one-loop running of the Yukawa coupling with the corresponding beta-function $\beta_0^{Yuk} = \frac{9}{4} - \frac{3m_t^2}{4M_W^2}$. By expanding the solution for the component $\mathcal{Z}_\phi$ we obtain the two-loop corrections enhanced by the second or fourth power of the top quark mass. Note that in the production amplitude one has to take into account also the interference between the one-loop Yukawa contribution to $\mathcal{Z}_\phi$ and the one-loop logarithmic term in the reduced amplitude and



(a) (b)

Figure 3: The one-loop logarithmic corrections to the differential cross section relative to the Born approximation at $\sqrt{s} = 1$ TeV as functions of the production angle for (a) transverse and (b) longitudinal polarization of the gauge bosons.

the electron $\mathcal{Z}_\psi$ function. The two-loop NNLL Yukawa enhanced contribution to the cross section reads

$$\delta^{(2)}_{NNLL}\Big|_{Yuk} = \left[-\frac{3}{4}\frac{m_t^4}{M_W^4} + \frac{m_t^2}{M_W^2}\left(30\ln(x_-) - 6\ln(x_+) - \frac{45}{4}\right)\right]\mathcal{L}^2(s)\,. \tag{31}$$

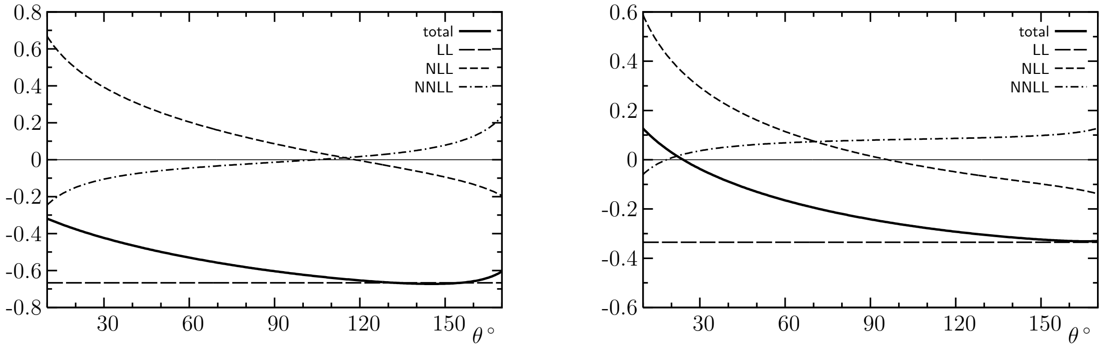
This expression approximates the full result up to the terms suppressed by $\sin^2\theta_W \sim 0.2$.

3.3 Numerical results

We adopt the following input values [34] $M = M_W = 80.41$ GeV, $M_H = 117$ GeV, $m_t = 174$ GeV for the masses and $\sin^2\theta_W = 0.231$, $\alpha_{ew} = 3.38\cdot 10^{-2}$ for the $\overline{\rm MS}$ coupling constants renormalized at the scale of the gauge boson mass. Note that the coupling constants in the Born cross section of the longitudinal gauge boson production are renormalized at the scale $\sqrt{s}$.

Transverse polarization. We obtain the following one and two-loop NNLL corrections to the cross section

$$\begin{aligned}
\delta^{(1)}_T &= -4.73\,\mathcal{L}^2(s) + \left[\left(-6.15 + 4.00\,\frac{x_-}{x_+}\right)\ln(x_-) + 2.15\,\ln(x_+) + 4.43\right]\mathcal{L}(s) \\
&\quad + \left(-\frac{4.70}{x_+} + \frac{2.35 + 1.95\,x_-}{x_-^2 + x_+^2}\right)\ln^2(x_-) + \frac{4.00}{x_+}\ln(x_-)\ln(x_+) + \frac{3.00\,x_-}{x_-^2 + x_+^2}\ln^2(x_+) \\
&\quad + \left(0.54 + \frac{4.95 - 9.65\,x_-}{x_-^2 + x_+^2}\right)\ln(x_-) - 0.54\ln(x_+) - \frac{2.35\,x_+}{x_-^2 + x_+^2} - 6.59\,, \qquad (32) \\
\delta^{(2)}_T &= 11.17\,\mathcal{L}^4(s) + \left[\left(29.08 - \frac{18.91\,x_-}{x_+}\right)\ln(x_-) - 10.17\,\ln(x_+) - 14.49\right]\mathcal{L}^3(s)
\end{aligned}$$



(a) (b)

Figure 4: The same as Fig. 3 but for $\sqrt{s} = 3$ TeV.

$$
\begin{aligned}
&+\left[\left(18.92+\frac{22.21-12.61\,x_-}{x_+}+\frac{4.00\,x_-^2}{x_+^2}-\frac{11.11+9.22\,x_-}{x_-^2+x_+^2}\right)\ln^2(x_-)\right.\\
&-\left(9.24+\frac{18.91}{x_+}+\frac{3.39\,x_-}{x_+}\right)\ln(x_-)\ln(x_+)+\left(2.32-\frac{14.18\,x_-}{x_-^2+x_+^2}\right)\ln^2(x_+)\\
&-\left(15.26-\frac{11.40\,x_-}{x_+}+\frac{23.40-45.61\,x_-}{x_-^2+x_+^2}\right)\ln(x_-)+3.87\,\ln(x_+)+\frac{11.11\,x_+}{x_-^2+x_+^2}\\
&\left.+24.92\right]\mathcal{L}^2(s)\,. \qquad (33)
\end{aligned}
$$

The one-loop result is well known [6] and is given here to show the structure of the logarithmic expansion, see Figs. 3*a* and 4*a*. In Figs. 5*a* and 6*a* the values of different logarithmic two-loop contributions as well as their sum are plotted as functions of the production angle at the center of mass energy of 1 TeV and 3 TeV, respectively. The two-loop subleading contributions exceed the LL one in absolute value in the small angle region. However, due to the partial cancellation between the NLL and NNLL terms the total NNLL approximation is close to the LL one. It has a fairly flat angular dependence and amount to about 5% for $\sqrt{s} = 1$ TeV and 20% for $\sqrt{s} = 3$ TeV.

Longitudinal polarization. For the left-handed initial state fermions the one and two-loop NNLL corrections to the cross section read

$$
\begin{aligned}
\delta_L^{(1)} &= -2.38\,\mathcal{L}^2(s)+[-6.91\,\ln(x_-)+0.75\,\ln(x_+)-3.48]\,\mathcal{L}(s)\\
&\quad-\frac{2.19}{x_+}\ln^2(x_-)+\frac{0.65}{x_-}\ln^2(x_+)+0.19\,(\ln(x_-)-\ln(x_+))+31.07\,, \qquad (34)\\
\delta_L^{(2)} &= 2.82\,\mathcal{L}^4(s)+[16.41\,\ln(x_-)-1.79\,\ln(x_+)+11.87]\,\mathcal{L}^3(s)\\
&\quad+\left[\left(18.38+\frac{5.20}{x_+}\right)\ln^2(x_-)+\left(0.28-\frac{1.55}{x_-}\right)\ln^2(x_+)\right.
\end{aligned}
$$

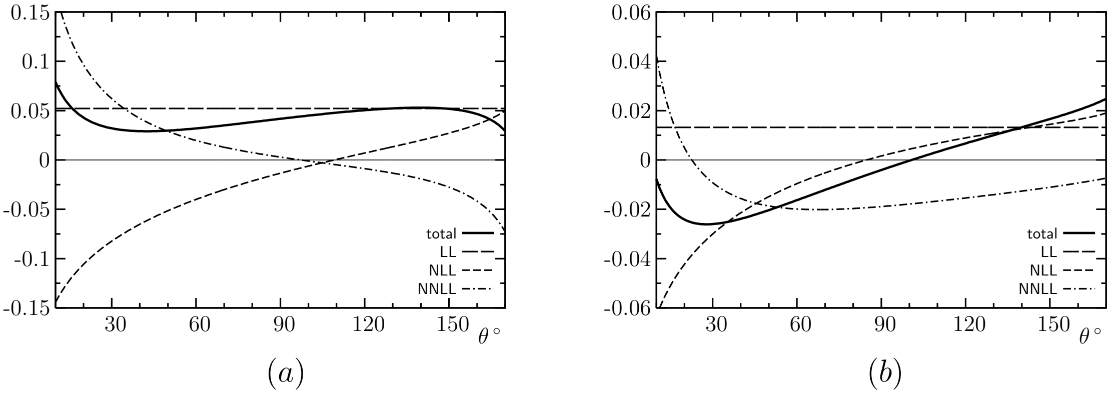


Figure 5: The two-loop logarithmic corrections to the differential cross section relative to the Born approximation at $\sqrt{s} = 1$ TeV as functions of the production angle for (a) transverse and (b) longitudinal polarization of the gauge bosons.

$$+3.11\,\ln(x_-)\ln(x_+) + 49.(10.)\,\ln(x_-) - 18.(4.)\,\ln(x_+) - 95.(20.)\Big]\mathcal{L}^2(s)\,. \quad (35)$$

In the two-loop NNLL contribution the error bars indicate the uncertainty due to our approximation of the Yukawa enhanced contribution. The structure of the logarithmic corrections differs from the case of the transverse polarization as one can see on Figs. 3b, 4b and 5b. The sum of the two-loop logarithmic terms is strongly angular dependent and varies between -2% and 2% for $\sqrt{s} = 1$ TeV and between -2% and 8% for $\sqrt{s} = 3$ TeV.

For the right-handed initial state fermions the Born cross section is suppressed by the factor $4\sin^4\theta_W \approx 0.2$ in comparison to the left-handed case. Moreover the two-loop logarithmic corrections turned out to be about $3 \cdot 10^{-3}$ for all the scattering angles. Thus they are of no phenomenological importance and are not presented here.

4 Summary

In the present paper we employed the evolution equation approach to analyze the high energy asymptotic behavior of the gauge boson production through the annihilation of fermion-antifermion pair in the spontaneously broken $SU(2)$ gauge model. The result has been used to compute the two-loop NNLL electroweak corrections to the differential cross section of W-pair production in e^+e^- annihilation. The corrections are comparable and even exceed the LL terms depending on the production angle. The structure of the corrections is different for the transverse and longitudinal boson production. In the first case we observe the cancellation between the huge NLL and NNLL contributions so that the sum is dominated by the LL term and amounts of about 5% at $\sqrt{s} \sim 1$ TeV and 20% at $\sqrt{s} \sim 3$ TeV. For the longitudinal bosons the corrections exhibit significant cancellation between the LL, NLL and NNLL terms so that the sum does not exceed 2% in absolute value for $\sqrt{s} \sim 1$ TeV. The cancellation

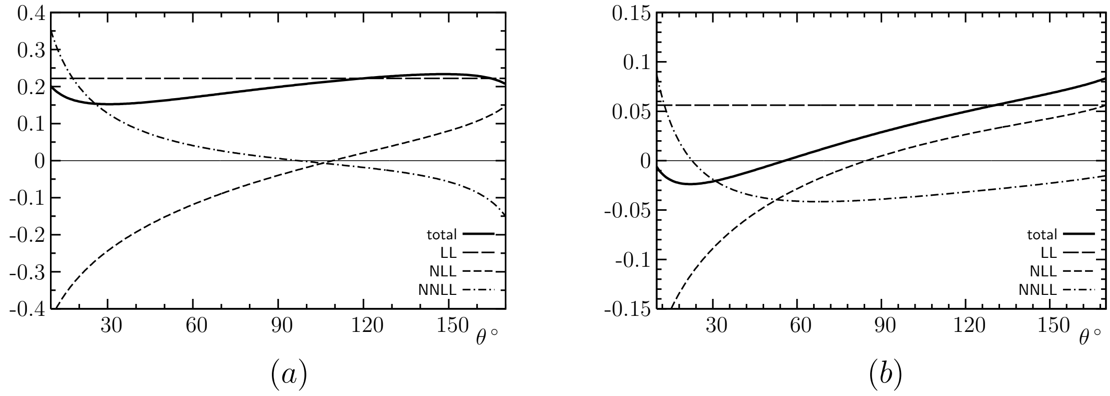


Figure 6: The same as Fig. 5 but for $\sqrt{s} = 3$ TeV.

becomes less pronounced at higher energy. The uncertainty of the theoretical prediction for the on-shell W-pair production at ILC is now determined by the unknown two-loop linear logarithmic terms. For the fermion pair production such terms are know to contribute about 1–2% of the cross section [14]. This value can be used as an estimate of the accuracy of our approximation. We should emphasize that our approximation breaks down at small production angles where the *Regge* logarithms $\ln(-t/s)$ become large.

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