# Radiative return physics program within EURIDICE network *

HENRYK CZYŻ[a] AND AGNIESZKA GRZELIŃSKA[b]

a: Institute of Physics, University of Silesia, PL-40007 Katowice, Poland
b: Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

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A short review of both theoretical and experimental aspects of the radiative return method is presented with the emphasize on the results obtained within the EURIDICE network. It is shown that the method gives not only possibility of an independent from the scan method measurement of the hadronic cross section, but also can provide information concerning details of the hadronic interactions.

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## 1. Introduction

Four years of a very active physics program of the EURIDICE network was also very fruitful for the group developing the radiative return method [1] and tools necessary for the physical analysis. Major part of all theoretical investigations in that topic was done within this network, thus the review of the theoretical investigations is in fact the review of the results obtained within the EURIDICE network. Software developed and/or upgraded within the network: PHOKHARA[2, 3, 4, 5, 6, 7] and EKHARA [8] generators together with the event generators developed by the same group (EVA [9], EVA4pi [10]) prior to the starting date of the EURIDICE is successfully used by experimental groups working at meson factories BaBar and KLOE and soon will be used also at BELLE.

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The method originally developed for the hadronic cross section measurement was successfully used by KLOE [11] to obtain $\sigma(e^+e^- \to \pi^+\pi^-)$ with the precision competitive to the one obtained by means of the scan method and by BaBar to obtain cross sections of many hadronic channels which were measured with poor accuracy or were not measured at all. Let us mention only few of them: narrow resonances studies [12], big improvement in the accuracy of the three pion [13] and four charged mesons (pions and kaons) [14] cross sections measurements and proton form factors extraction [15].

The paper starts with a description of the radiative return method in Section 2 and the ingredients necessary for the precision physics in Section 3. The basic obstacle in using of the radiative return method, final state photon emission (FSR), is discussed in Section 4, together with methods how to handle it and be able to perform precision physics despite the encountered problems. Selected topics beyond the hadronic cross section measurements are described in Section 5.

## 2. The basics of the radiative return method

Simple and innovative observation made some years ago [1] lead, with series of papers which started with [9] and [10], to a development of a radiative return method giving today many valuable physical results. The method gives an access to information contained in the hadronic cross section $d\sigma(e^+e^- \to \text{hadrons})$ through a measurement of the hadronic invariant mass distribution in the reaction $e^+e^- \to \text{hadrons} + \text{photons}$. Historically the process of $e^+e^-$ annihilation to a pair (or arbitrary number) of particles plus one photon was investigated earlier [16, 17, 18], but the scope of that papers was not to provide with a method to measure the hadronic cross section.

To illustrate in details how the method works lets consider the lowest order contribution to the radiative return cross section. The contributing diagrams are shown in Fig. 1, where only initial state radiation (ISR) is taken into account. The complications caused by final state radiation (FSR), as well as the methods to overcome them, are discussed in Section 4.

The corresponding ISR matrix element has the following form

$$\mathcal{M} \sim \bar{v}\,(p_+) \left[\gamma^\nu \frac{1}{\not{p}_- - \not{k} - m} \not{\epsilon}^*(k) + \not{\epsilon}^*(k) \frac{1}{\not{k} - \not{p}_+ - m} \gamma^\nu \right] u(p_-)\ \frac{1}{Q^2}\ J_\nu^{em}\ ,(1)$$

where $J_\nu^{em}$ is the electromagnetic hadronic current present also in the matrix element describing the process $e^+e^- \to \text{hadrons}$. It is clear from it that a factorization, allowing separation of the hadronic part will take place even

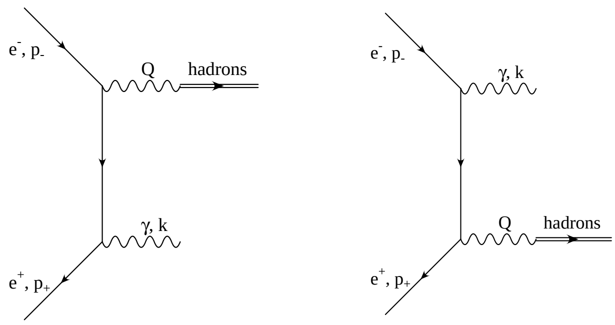


Fig. 1. The leading order diagrams contributing to the radiative return cross section.

for the squared matrix element. Indeed, integrating the cross section over the hadrons phase space $d\bar{\Phi}_n$ ('bar' indicates that all statistical factors are included in its definition) one gets

$$\int J_\mu^{em}(J_\nu^{em})^* \;\; d\bar{\Phi}_n(Q;q_1,\ldots,q_n) = \frac{1}{6\pi}\Big(Q_\mu Q_\nu - g_{\mu\nu}Q^2\Big)\; R(Q^2)\ , \qquad (2)$$

where

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma_{\rm point}}\ ,\ \sigma_{\rm point} = \frac{4\pi\alpha^2}{3s}\ . \qquad (3)$$

That leads to the relation between cross sections with and without photon emission

$$d\sigma(e^+e^- \to \mathrm{hadrons} + \gamma(\mathrm{ISR})) \ = \\ H(Q^2,\theta_\gamma)\ d\sigma(e^+e^- \to \mathrm{hadrons}, s=Q^2)\ , \quad (4)$$

where $Q^2$ is the invariant mass of the hadronic system. As it is clear from Eq. (1) and Eq. (2) the factorization of the Eq. (4) remains valid at any order provided that only ISR is considered. The function $H(Q^2,\theta_\gamma)$ at relatively low energies of meson factories is given with high accuracy by QED only and thus it is well known. That means that by measurement of the differential in $Q^2$ cross section of the reaction $e^+e^- \to$ hadrons+photons one can extract the cross section $e^+e^- \to$ hadrons for the energies from the production threshold to almost the nominal energy of a given experiment, provided that one is able to overcome complications described in Section 4. The cross section of the reaction with photon emission is lower then the one without photon emission, thus requires higher luminosity then the ones necessary for scan experiments for similar statistical accuracy. However it does not require to build a dedicated experiments and it can use data collected at any of the meson factories, where luminosity is not a problem.

### 3. Ingredients necessary for a precise analysis

An extraction of the hadronic cross section via the radiative return requires a few basic ingredients:

- an accurate calculation of the ISR including appropriate radiative corrections
- an adequate, tested experimentally, model of the FSR
- a Monte Carlo event generator to be able to use the theoretical information in a realistic experimental set up
- $e^+e^-$ scattering experiment with a high luminosity and a good detector

The virtual radiative corrections to ISR were calculated in [19] while the real emission was included in the developed Monte Carlo generator PHOKHARA in [2] and [3]. The estimated physical accuracy of the ISR contributions, 0.5%, was adequate at the time of the release of the generator even for the precise KLOE pion form factor extraction [11]. The new data collected at DAPHNE by KLOE collaboration [20, 21] together with an improvement in the theoretical description of the Monte Carlo event generator BABAYAGA [22] requires however further work and an inclusion of the NNLO contributions, at least in the leading logarithmic approximation, is a necessary condition to be able to fully profit from the new very accurate data.

The FSR modeling requires close collaboration between experimental and theory groups to reach required precision in a short time and will be discussed in the next Section.

A development of a reliable Monte Carlo generator requires not only the precise knowledge of both ISR and FSR, but a continuous work on efficiency of the generator and its tests for each added hadronic channel. Only this way one can assure reliability of the developed product used afterwords by demanding experimental groups.

### 4. Final state photon emission: the problems and how to overcome them

The final state emission (FSR) forms a potential problem for the application of the radiative return method and it has to be studied carefully to assure adequate accuracy of the description. At B- factories the region of hadronic masses of physical interests, below 4 GeV, lays far from the nominal energy of the experiments and an emission of a hard photon is required to reach it. It means that the typical kinematic configuration of an event consists of a photon emitted in one direction and hadrons going opposite to it. That suppresses the FSR contributions, which are large for photons emitted parallel to the direction of a charged hadron in the final state, and makes the measurement of the hadronic cross section easier. For the $\phi$-factory DAPHNE, where the region of interest is not far from the nominal energy of the experiment, that natural separation between the emitted photon and the hadrons does not take place and one has to suppress FSR by an appropriate event selection. As a result one has to control the uncertainty due to the model dependence of the final state emission. That forms a challenge, as the existing models were not tested with the adequate precision prior to the DAPHNE results. Let's discuss that problem on the basis of the $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$ reaction, where the accuracy is the most demanding. The solution was first proposed in [9] and further elaborated in [4]. A similar investigations is possible for other hadronic final states, however till now it was not performed.

The main tool in the tests of the model(s) of the photon emission from the final pions is the charge asymmetry. For ISR emission of any number of photons the two-pion state is produced in C=-1 and with an odd orbital angular momentum. For one real photon emitted from the final pions the two-pion state is produced in C=1 and with an even orbital angular momentum. As a result, the initial-final state interference for one photon emission is odd under $\pi^+ \leftrightarrow \pi^-$ interchange and the integrals for charge blind event selections are equal to zero. In the same time the interference is the only source of the charge asymmetry and allows for tests of the models of the final state emission. The charge asymmetry depends on the invariant mass of the two-pion system and that allows for detailed tests of the model(s) of the FSR emission. In short, the tests should be done in the following way: First one compares the experimental data for the asymmetry with the Monte Carlo where the tested model was implemented. That has to be performed for an event selection which enhance the FSR as compared to the ISR. Once the implemented model agrees with the data one chooses an event selection, which suppresses the FSR and makes the radiative cross section measurement. That guaranties that the ISR and the FSR contributions are separately well under control. For the case of untagged photons a specific background, $e^+e^- \to \pi^+\pi^-e^+e^-$, has to be also taken into account [23, 8] as the final leptons are not vetoed and even if they are, the major part of them escapes detection being emitted at small angles.

The reaction $e^+e^- \to \pi^+\pi^-\gamma$, with the photon emitted from the pions, does contribute also to dispersion integrals for evaluation of $a_\mu$ and $\alpha_{QED}$ and in the former case its theoretically estimated value [4] is about 1.5 times higher then the size of the present theoretical uncertainty [24, 25] and thus numerically important. The sketched program was successfully undertaken by KLOE and resulted in a sound extraction of the $\sigma(e^+e^- \to \pi^+\pi^-)$ [11] together with the mentioned photon corrections. The most general form of the photon emission from scalar particles produced in $e^+e^-$ annihilation, with three form factors was investigated already in [16], where it was shown also that only one form factor is relevant in the limit of soft photon emission. Physical program initiated there was undertaken in [26, 27] with a special emphasize on the FSR tests at KLOE.

Another source of complications for using of the radiative return method at DAPHNE are the radiative $\phi$ decays. That problem was considered for the first time in [28] and it is discussed in more details in the next section.

## 5. Not only the hadronic cross section

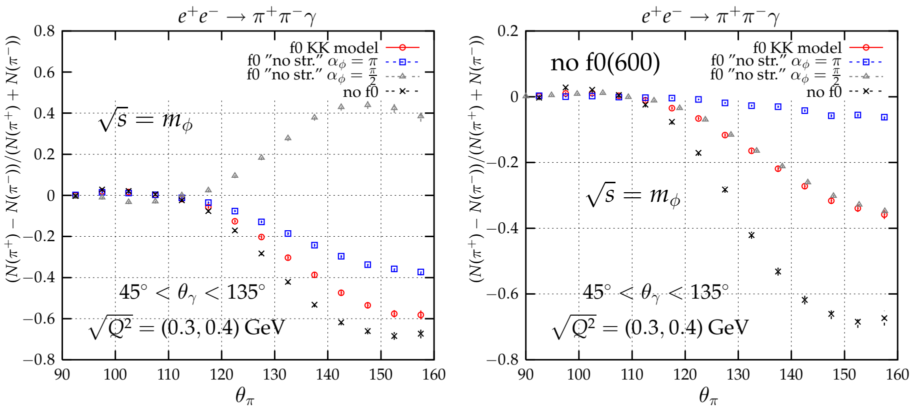


Fig. 2. Charge asymmetries for three different models of the radiative $\phi$ decays compared to the result based on sQED only (no $f_0$). Left plot with $f_0(980)$ and $f_0(600)$, right plot with $f_0(980)$ only.

The study of the $\phi \to \pi\pi\gamma$ decays at DAPHNE is a subject, where the problem of FSR emission in the pion form factor extraction is interlinked with the possibility of using of the radiative return method to study hadronic models of the $\phi \to \pi\pi\gamma$ decays. In [6] charge asymmetries were proposed to test both topics. The sensitivity to the model parameters, which can be reached this way is definitely better then the one obtained by the fit to the $Q^2$ spectra. An example is shown in Fig.2, where charge asymmetries predicted within a number of models of the radiative $\phi$ decays is shown for models with and without $f_0(600)$ contribution. Very distinct asymmetries promise deep insight in the details of the constructed models. The asymmetry was partially used by KLOE [29] to cross check the fit of the model parameters to the $Q^2$ spectrum of the pion pair invariant masses. More detailed tests are expected at KLOE, when both data taken at the $\phi$ resonance and below it will be analyzed.

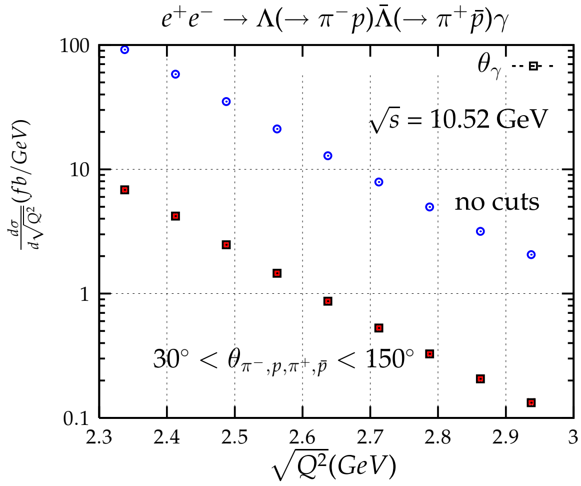


Fig. 3. The differential, in the invariant mass of the $\Lambda\bar{\Lambda}$ pairs ($Q^2$), cross section of the process $e^+e^- \to \Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)\gamma$ at B-factories energy: circles - with no cuts applied, crosses - angular cuts on the pions and protons only, squares (overlapped with crosses) - angular cuts on pions, protons and the photon.

Another example of using of the radiative return method to study hadron properties is the baryon form factors extraction. It was shown in [5] that at B-factories it is possible to extract the nucleon form factors up to their phases for a wide range of the invariant masses of the nucleon-antinucleon pairs. That is possible through studies of the nucleon angular distributions. In a properly chosen reference frame in which they are studied, a rest frame of the nucleon-antinucleon system with z-axis along the emitted photon, that studies are particularly simple as the angular distributions resemble there the nucleon angular distributions of the process without a photon emission. That measurement was successfully performed by BaBar collaboration [15].

For the production of unstable baryon pairs, the decay products carry information about their spins and serve as spin analyzers providing with complete information about the production process. In [30] it was studied in details for the process $e^+e^- \to \Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)\gamma$. The feasibility of such measurement at B-factories is obvious from Fig.3 as one expects about one hundred events per 100 fb$^{-1}$ in the range of BaBar detector.

The direction of pions coming from lambda decays is strictly correlated with the spin of the decaying lambdas and by observing them one can measure both spin asymmetries and spin correlations in the process of production of the lambda-antilambda pairs. In Fig.4 (left) the spin asymmetry, which is proportional to the sine of the phase difference of the magnetic and electric lambda form factors is shown for $\Delta\phi = \frac{\pi}{2}$. In Fig.4 (right) the xz-spin correlation (please look [30] for the reference frames definition), which is proportional to cosine of the phase difference of the magnetic and electric lambda form factors is shown for $\Delta\phi = \pi$. It is enough to measure one of them to determine the phase difference of the lambda form factors up to a twofold sign ambiguity and the other serve to determine that sign. The analysis can be applied also to other members of the baryon octet. As almost nothing is known about their form factors, B-factories can provide valuable physical information allowing to test symmetries of the underlying models.

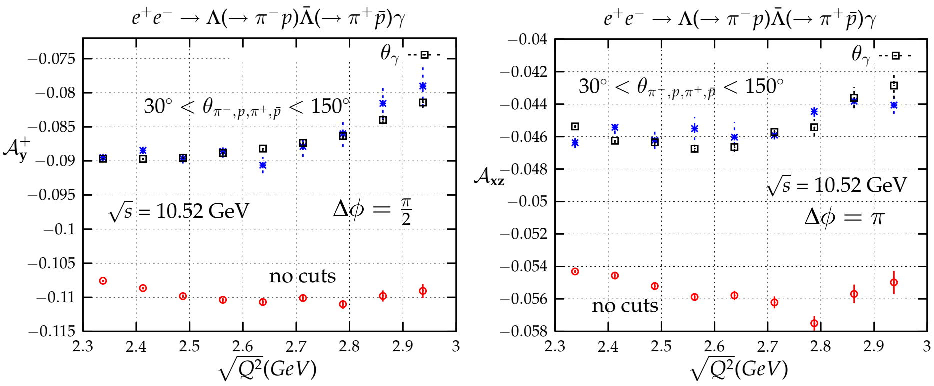


Fig. 4. Spin asymmetry (left) for the relative phase between magnetic and electric lambda form factors $\Delta\phi = \frac{\pi}{2}$ and spin correlations (right) for $\Delta\phi = \pi$ in the process $e^+e^- \to \Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)\gamma$: circles - with no cuts applied, crosses - angular cuts on the pions and protons only, squares - angular cuts on pions, protons and the photon.

## 6. Summary

The radiative return research program carried within the EURIDICE network was outlined. It was shown that just by theoretical and experimental analysis of the data of the existing meson factories one can get rich information concerning hadronic physics, which is not limited to the hadronic cross section measurement.

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