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Running of α_s and m_b in the MSSM

R.V. Harlander[(a)], L. Mihaila[(b)], M. Steinhauser[(b)]

(a) Fachbereich C, Theoretische Physik, Universität Wuppertal, 42097 Wuppertal, Germany

(b) Institut für Theoretische Teilchenphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany

Abstract

A consistent evolution of the strong coupling constant α_s from M_Z to the GUT scale is presented, involving three-loop running and two-loop decoupling. The two-loop transition from the $\overline{\rm MS}$- to the $\overline{\rm DR}$-scheme is properly taken into account. In the second part of the paper, the bottom quark mass in the $\overline{\rm DR}$ scheme at the electroweak/SUSY scale is evaluated with four-loop accuracy. We find that the three-loop effects are comparable to the experimental uncertainty both for α_s and m_b.

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1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) is one of the most studied extensions of the Standard Model (SM). The observation that the gauge couplings of the strong, electromagnetic and weak interaction tend to unify in the MSSM at a high energy scale $\mu_{\rm GUT} \simeq 10^{16}$ GeV [1–5] consistent with Grand Unification Theories (GUT) is probably the most compelling hint in favour of supersymmetry (SUSY). Constraints from the Yukawa sectors of SUSY-GUT models yield appealing predictions for SM parameters, including the top quark mass and the ratio of the bottom quark to the tau lepton masses [6–9].

Nevertheless, SUSY can only be an approximate symmetry in nature. Several scenarios for the mechanism of SUSY breaking, based on very different physical models, have been proposed. A possibility to constrain the type and scale of SUSY breaking is to study, with very high precision, the relations between the MSSM parameters evaluated at the electroweak and the GUT scales. The extrapolation over many orders of magnitude requires high-precision experimental data at the low energy scale. A first set of precision measurements is expected from the CERN Large Hadron Collider (LHC) with an accuracy at the percent level. A comprehensive high-precision analysis can be performed at the International Linear Collider (ILC), for which the estimated experimental accuracy is at the per mill level. It is obvious that the same precision must be reached also on the theory side in order to match with the data [10]. Running analyses based on full two-loop renormalization group equations (RGEs) [11–14] for all parameters and one-loop threshold corrections [15] are currently implemented in the public programs ISAJET [16], SOFTSUSY [17], SPHENO [18], SuSpect [19]. The agreement between the different codes is in general within one percent [20]. A first three-loop running analysis, based, however, only on one-loop threshold effects, was carried out in Ref. [21, 22]. While the three-loop running has little effect on the weakly interacting particle sector of the MSSM, this effect can be comparable to that of two loop running for strongly interacting particles, as for example the squark mass spectrum [22].

In this paper, we elaborate an earlier investigation [23] of the evolution of the strong coupling α_s in MSSM, based on three-loop RGEs [24] and two-loop threshold corrections. On the one hand, the three-loop corrections reduce significantly the dependence on the scale at which heavy particles are integrated out and on the other hand, they are essential for phenomenological studies, because they are as large as, or greater than, the effects induced by the current experimental accuracy of $\alpha_s(M_Z)$. Our aim is to compute α_s at a high-energy scale $\mu \simeq \mathcal{O}(\mu_{\rm GUT})$ with three-loop accuracy, starting from the SM parameter $\alpha_s(M_Z)$ as input. Additionally, we compare the predictions obtained within the above mentioned approach with those based on the leading-logarithmic (LL) approximation suggested in Ref. [10].

In the context of $SO(10)$ GUT [9, 25] the third-family Yukawa couplings may unify for a large ratio of the Higgs vacuum expectation values $\tan\beta \simeq 50$. Moreover, in all SUSY models with large $\tan\beta$, the supersymmetric particle spectrum and the Higgs boson masses are sensitive to the bottom Yukawa coupling [26, 27]. In turn, the relation between the running bottom mass and the bottom Yukawa coupling is affected in such theories by large supersymmetric radiative corrections [28]. For the calculation of higher order corrections in the framework of SUSY theories, the $\overline{\rm DR}$ scheme, based on regularization by dimensional reduction and modified minimal subtraction, is most convenient. On the other hand, the values of the SM parameters are presently extracted from the experimental data for the $\overline{\rm MS}$ renormalization scheme. It is obvious that translation formulae from one scheme to the other to the appropriate order in the perturbative expansion have to be employed for phenomenological studies.

Owing to these considerations, a precise determination of the running bottom mass in the $\overline{\rm DR}$ scheme is of great importance for SUSY theories sensitive to the bottom quark Yukawa coupling. Ref. [29] proposed an approach to the determination of $m_b^{\overline{\rm DR}}(M_Z)$, which avoids the infrared sensitivity problem related with the use of the bottom-quark pole mass. The procedure consists of employing the RGEs of QCD and the relation between $\overline{\rm MS}$ and $\overline{\rm DR}$ masses in order to calculate $m_b^{\overline{\rm DR}}$, using as input parameter the accurately known $\mu_b = m_b^{\overline{\rm MS}}(\mu_b)$. A second purpose of this paper is to extend the study of Ref. [29] to three- and even to four-loop order [30, 31], in order to take full advantage of the recent determination of $m_b^{\overline{\rm MS}}(\mu_b)$ with four-loop accuracy [32]. Furthermore, we study the phenomenological significance of evanescent couplings related with the application of the $\overline{\rm DR}$ scheme to QCD.

The remainder of the paper is organized as follows: in Section 2 we discuss the evaluation of the strong coupling at the GUT scale from the knowledge of $\alpha_s(M_Z)$ and propose a consistent prescription based on three-loop running and two-loop decoupling. Section 3 deals with the running of the bottom quark mass from $\mu = \mu_b$ to a decoupling scale of supersymmetric particles.

2 Running and decoupling of α_s

The value of the strong coupling constant, measured at the mass of the Z boson M_Z, is a central quantity in high energy physics. Usually, what is being quoted in the literature as $\alpha_s(M_Z)$ is its value in a theory with five quark flavours, renormalized in the $\overline{\mathrm{MS}}$ scheme. In order to be precise, we will refer to this quantity as $\alpha_s^{\overline{\mathrm{MS}},(5)}(M_Z)$.

On the contrary, what is meant by the value of $\alpha_s(\mu_{\mathrm{GUT}})$ is the strong coupling constant in a supersymmetric theory, renormalized in the $\overline{\mathrm{DR}}$ scheme. We will denote this quantity by $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_{\mathrm{GUT}})$ in what follows. It is the purpose of this section to study the relation between $\alpha_s^{\overline{\mathrm{MS}},(5)}(M_Z)$ and $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_{\mathrm{GUT}})$ at three-loop level. This requires the consistent combination of (a) the renormalization group evolution of α_s; (b) the transition from the $\overline{\mathrm{MS}}$ to the $\overline{\mathrm{DR}}$ scheme; (c) the transition from five-flavour QCD to the full SUSY theory. All these issues will be discussed in detail in what follows.

2.1 Renormalization group evolution

The energy dependence of the strong coupling constant is governed by the RGE

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\alpha_s(\mu^2) = \beta(\alpha_s)\,, \qquad \beta(\alpha_s) = -\alpha_s^2 \sum_{n\geq 0} \beta_n \alpha_s^n\,. \tag{1}$$

The β function depends on the underlying theory and on the renormalization scheme. In QCD with n_f quark flavours, it is known in the $\overline{\mathrm{MS}}$ scheme through four loops [33]; the first three coefficients read

$$\begin{aligned} \beta_0 &= \frac{1}{4}\left(11 - \frac{2}{3}n_f\right)\,, \qquad \beta_1 = \frac{1}{16}\left(102 - \frac{38}{3}\,n_f\right)\,, \\ \beta_2 &= \frac{1}{64}\left(\frac{2857}{2} - \frac{5033}{18}\,n_f + \frac{325}{54}n_f^2\right)\,. \end{aligned} \tag{2}$$

The QCD β function in the $\overline{\mathrm{DR}}$ scheme involves evanescent couplings (see Section 2.3 below) at three loops and higher. Explicit results through four loops are given in Refs. [30, 31].

In SUSY-QCD, the β function has been evaluated in the $\overline{\mathrm{DR}}$ scheme through three loops from the NSVZ formula [34,35] by a proper shift in the strong coupling constant [24]. For six (s)quark flavours one finds

$$\beta_0 = \frac{3}{4}\,, \qquad \beta_1 = -\frac{7}{8}\,, \qquad \beta_2 = -\frac{347}{192}\,. \tag{3}$$

2.2 Decoupling of the SUSY particles

For mass independent renormalization schemes like $\overline{\text{MS}}$ or $\overline{\text{DR}}$, the decoupling of heavy particles has to be performed explicitely. In practice, this means that intermediate effective theories are introduced by integrating out the heavy degrees of freedom. The parameters of the various effective theories are related by so-called decoupling (or matching) constants.

When going from a high to a low energy scale, one may separately integrate out every particle at its individual threshold. This multi-scale approach (sometimes denoted as "step approximation") is suited for SUSY models with a severely split mass spectrum [36]. As pointed out long time ago, the intermediate effective theories with "smaller" symmetry raise the problem of introducing new couplings, each governed by its own RGE. This is the case in the Yukawa sector if the $SU(2)$ symmetry is broken, for example. It also happens for the "evanescent" couplings, related with the application of DRED to non-supersymmetric theories. A comprehensive multi-scale decoupling procedure in the context of the MSSM is currently not available, although it could be of phenomenological relevance [37].

On the other hand, in SUSY models with roughly degenerate mass spectrum at a scale $\tilde{M}$, one can consider the MSSM as the full theory that is valid from the GUT scale μ_{GUT} down to $\tilde{M}$, which we assume to be around 1 TeV. Integrating out all SUSY particles at this common scale, one directly obtains the SM as the effective theory, valid at low energies. As already mentioned above, the strong coupling constants in SUSY and in QCD are then related by a decoupling constant ζ_s. The transition between the two theories can be done at an arbitrary decoupling scale μ:[1]

$$\alpha_s^{\overline{\text{DR}},(n_f)}(\mu) = \zeta_s^{(n_f)}\,\alpha_s^{\overline{\text{DR}},(\text{full})}(\mu)\,. \tag{4}$$

ζ_s depends logarithmically on the scale μ, which is why one generally chooses $\mu \sim \tilde{M}$. In Eq. (4), $n_f = 6$ means that only the SUSY particles are integrated out, while for $n_f = 5$ at the same time the top quark is integrated out. This procedure, also known as "common scale approach" [37], has the advantage that it avoids the occurrence of intermediate non-supersymmetric effective theories. Most of the present codes computing the SUSY spectrum [17–19] follow this approach by applying the one-loop approximation of Eq. (4) and setting $n_f = 5$ and $\mu = M_Z$.

In this paper, we apply the corresponding two-loop approximation, as it is required by a consistent three-loop evolution of the coupling constant [38]. Furthermore, in contrast to most analyses performed up to now, we allow for a general decoupling scale $\mu = \mu_{\text{dec}}$ where the heavy particles are decoupled.

The decoupling coefficients of Eq. (4) have been evaluated through two loops in Eqs. (26) and (29) of Ref. [23]:

$$\zeta_s^{(n_f)} = 1 + \frac{\alpha_s}{\pi}\,\zeta_{s1}^{(n_f)} + \left(\frac{\alpha_s}{\pi}\right)^2 \zeta_{s2}^{(n_f)} + \mathcal{O}\big(\alpha_s^3\big)\ , \tag{5}$$

[1] In principle the decoupling constants should also carry a label $\overline{\text{MS}}$ or $\overline{\text{DR}}$. However, in this paper decoupling is always performed in the $\overline{\text{DR}}$ scheme, and therefore we omit the label.

where $\alpha_s \equiv \alpha_s^{\overline{\rm DR},({\rm full})}(\mu)$, and

$$\begin{aligned}
\zeta_{s1}^{(6)} &= -L_{\tilde{M}}\,, \qquad \zeta_{s1}^{(5)} = -\frac{1}{6}L_t - L_{\tilde{M}}\,,\\
\zeta_{s2}^{(6)} &= -\frac{65}{32} - \frac{5}{2}L_{\tilde{M}} + L_{\tilde{M}}^2\,,\\
\zeta_{s2}^{(5)} &= -\frac{215}{96} - \frac{19}{24}L_t - \frac{5}{2}L_{\tilde{M}} + \left[\frac{1}{6}L_t + L_{\tilde{M}}\right]^2\\
&\quad + \left(\frac{m_t}{\tilde{M}}\right)^2\left(\frac{5}{48} + \frac{3}{8}L_{t\tilde{M}}\right) - \frac{7\pi}{36}\left(\frac{m_t}{\tilde{M}}\right)^3\\
&\quad + \left(\frac{m_t}{\tilde{M}}\right)^4\left(\frac{881}{7200} - \frac{1}{80}L_{t\tilde{M}}\right) + \frac{7\pi}{288}\left(\frac{m_t}{\tilde{M}}\right)^5 + \ldots\,.
\end{aligned} \tag{6}$$

$\tilde{M}$ is the common mass of the SUSY particles, m_t is the top quark mass, and

$$L_{\tilde{M}} = \ln\frac{\mu^2}{\tilde{M}^2}\,, \qquad L_t = \ln\frac{\mu^2}{m_t^2}\,, \qquad L_{t\tilde{M}} = \ln\frac{m_t^2}{\tilde{M}^2}\,. \tag{7}$$

Although the formula for $\zeta_{s2}^{(5)}$ in Eq. (6) has been obtained in the limit $m_t \ll \tilde{M}$, it has been shown that it holds at the per cent level for all values of $m_t \leq \tilde{M}$ [23]. The exact limit $m_t = \tilde{M}$ is also given in Ref. [23].

2.3 $\overline{\rm MS}$–$\overline{\rm DR}$ conversion and evanescent couplings

In order to use Eq. (4), one needs to transform the input value for α_s from the $\overline{\rm MS}$ to the $\overline{\rm DR}$ scheme. The corresponding relation has been evaluated through three loops in Ref. [31]. Here, we only need the two-loop expression [30]:

$$\alpha_s^{\overline{\rm MS}} = \alpha_s^{\overline{\rm DR}}\left[1 - \frac{\alpha_s^{\overline{\rm DR}}}{4\pi} - \frac{5}{4}\left(\frac{\alpha_s^{\overline{\rm DR}}}{\pi}\right)^2 + \frac{\alpha_s^{\overline{\rm DR}}\alpha_e}{12\pi^2}\,n_f + \ldots\right], \tag{8}$$

where $\alpha_s^{\overline{\rm DR}} \equiv \alpha_s^{\overline{\rm DR},(n_f)}(\mu)$ and $\alpha_s^{\overline{\rm MS}} \equiv \alpha_s^{\overline{\rm MS},(n_f)}(\mu)$.

$\alpha_e \equiv \alpha_e^{(n_f)}(\mu)$ is one of the so-called evanescent coupling constants that occur when DRED is applied to non-supersymmetric theories (QCD in this case). In particular, it describes the coupling of the 2ε-dimensional components (so-called ε-scalars) of the gluon to a quark. Other evanescent couplings that occur in QCD are η_r ($r = 1, 2, 3$), related to the vertex of four ε-scalars (see, e.g., Ref. [31] for details). In the following, we will assume that QCD is obtained by integrating out the heavy degrees of freedom (squarks and gluinos) from SUSY-QCD. In this case, the evanescent couplings can be related to the gauge coupling α_s as follows:[2]

$$\begin{aligned}
\alpha_e^{({\rm full})}(\mu) &= \alpha_s^{\overline{\rm DR},({\rm full})}(\mu) = \eta_1^{({\rm full})}(\mu)\,,\\
\eta_2^{({\rm full})}(\mu) &= \eta_3^{({\rm full})}(\mu) = 0\,.
\end{aligned} \tag{9}$$

[2]Assuming that DRED preserves supersymmetry.

The evanescent couplings in n_f-flavour QCD, i.e. $\alpha_e^{(n_f)}$ and $\eta_r^{(n_f)}$ are then obtained by decoupling relations analogous to Eq. (4).

2.4 Decoupling for α_e

For the one-loop decoupling relation that connects the values of α_e in five-flavour QCD and in SUSY-QCD, we find by a calculation similar to Ref. [23]:

$$\begin{aligned}
\alpha_e^{(5)}(\mu) &= \zeta_e \alpha_e^{(\mathrm{full})}(\mu)\,, \\
\zeta_e &= 1 + \frac{\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu)}{\pi}\Bigg[T_F\left(-\frac{1}{2}L_t\right) + C_A\left(-\frac{1}{4}L_{\tilde{g}} + \frac{\left(m_{\tilde{g}}^2(1+L_{\tilde{g}}) - m_{\tilde{q}}^2(1+L_{\tilde{q}})\right)}{2(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)}\right) \\
&\quad + C_F\left(\frac{m_{\tilde{q}}^2 - 3m_{\tilde{g}}^2}{4(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)} + \frac{m_{\tilde{q}}^2(2m_{\tilde{g}}^2 - m_{\tilde{q}}^2)L_{\tilde{q}} - m_{\tilde{g}}^4 L_{\tilde{g}}}{2(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^2}\right)\Bigg] + \mathcal{O}\left(\alpha_s^2\right)\,, \qquad (10)
\end{aligned}$$

where we have used the short-hand notation

$$L_x = \ln\frac{\mu^2}{m_x^2}\,, \qquad x \in \{t, \tilde{g}, \tilde{q}\}\,. \qquad (11)$$

Furthermore, the mixing angle between the squark mass eigenstates $m_{\tilde{q}1}$ and $m_{\tilde{q}2}$ has been set to zero. Note that Eq. (10) depends on a specific (s)quark flavour q. This means that actually ζ_e and therefore also α_e should carry an additional label q:

$$\alpha_e^{q,(5)}(\mu) = \zeta_e^q\, \alpha_e^{(\mathrm{full})}(\mu)\,.$$

For example, $\alpha_e^{u,(5)}$ is the coupling constant for the $\bar{u}u\varepsilon$ vertex, where the decoupling constant ζ_e^u depends on $m_{\tilde{u}}$. Therefore, if all squark masses are different, one needs a separate evanescent coupling for each quark flavour. We remark that gauge invariance prevents the occurrence of such a flavour dependence for $\alpha_s^{\overline{\mathrm{DR}},(5)}$. If all squarks have the same mass $m_{\tilde{q}}$, then all evanescent couplings are the same, $\alpha_e^{u,(5)} = \alpha_e^{d,(5)} = \cdots = \alpha_e^{b,(5)}$, and one can drop the flavour label. In the limit $m_{\tilde{g}} = m_{\tilde{q}} = \tilde{M}$, Eq. (10) reads

$$\zeta_e = 1 + \frac{\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu)}{\pi}\left[T_F\left(-\frac{1}{2}L_t\right) + C_A\left(\frac{1}{4}L_{\tilde{M}}\right) + C_F\left(-\frac{1}{2}L_{\tilde{M}}\right)\right]. \qquad (12)$$

α_e occurs typically at higher orders, and in fact, as we will now argue, one can easily eliminate it perturbatively from the three-loop running of α_s.

Because of Eq. (9), one can write

$$\alpha_e^{(n_f)}(\mu) = \xi^{(n_f)}\, \alpha_s^{\overline{\mathrm{DR}},(n_f)}(\mu)\,, \qquad (13)$$

with

$$\xi^{(n_f)} = \frac{\zeta_e^{(n_f)}}{\zeta_s^{(n_f)}} = 1 + \frac{\alpha_s^{\overline{\mathrm{DR}},(n_f)}(\mu)}{\pi}\,\xi_1^{(n_f)} + \mathcal{O}(\alpha_s^2)\,, \qquad (14)$$

where the dependence on $\alpha_s^{\overline{\rm DR},({\rm full})}$ has been eliminated by using Eq. (4). In case all heavy particles have a common mass $\tilde{M}$, the quantity $\xi_1^{(n_f)}$ only depends on $\ln(\mu^2/\tilde{M}^2)$ and thus for $\mu \equiv \mu_{\rm dec} \approx \tilde{M}$ no numerically enhanced perturbative coefficients appear. This is even true for a mass spectrum where all mass ratios are within the range of one or two orders of magnitude. This absence of artificially large logarithms allows us to write

$$\alpha_e^{(n_f)}(\mu_{\rm dec}) = \alpha_s^{\overline{\rm DR},(n_f)}(\mu_{\rm dec}) + \mathcal{O}\big(\alpha_s^2\big)\ . \tag{15}$$

In this way, we can replace α_e by $\alpha_s^{\overline{\rm DR}}$ in the two-loop term of Eq. (8).

2.5 Evaluation of $\alpha_s(\mu_{\rm GUT})$ from $\alpha_s(M_Z)$

There are various ways to go from $\alpha_s^{\overline{\rm MS},(n_f)}(M_Z)$ to $\alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm GUT})$. They most importantly differ in the degree to which evanescent couplings are needed. We propose the following method:

$$\begin{aligned}\alpha_s^{\overline{\rm MS},(n_f)}(M_Z) \quad &\stackrel{(i)}{\to} \quad \alpha_s^{\overline{\rm MS},(n_f)}(\mu_{\rm dec}) \quad \stackrel{(ii)}{\to} \quad \alpha_s^{\overline{\rm DR},(n_f)}(\mu_{\rm dec}) \\ &\stackrel{(iii)}{\to} \quad \alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm dec}) \quad \stackrel{(iv)}{\to} \quad \alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm GUT})\,.\end{aligned} \tag{16}$$

The individual steps require

- (i) $\beta(\alpha_s)$ in QCD through three loops [Eqs. (1), (2)]
- (ii) the $\overline{\rm MS}$–$\overline{\rm DR}$ relation through order α_s^2 [Eq. (8)]; due to Eq. (15) one can set $\alpha_e = \alpha_s^{\overline{\rm MS}}$ at this order
- (iii) decoupling through order α_s^2 [Eqs. (4) with $n_f = 5$, Eq. (6)]; no evanescent couplings appear at this order (for a "common scale approach"). Note that if the decoupling of the supersymmetric particles were performed in several steps, α_e would appear in the two-loop decoupling coefficients
- (iv) $\beta(\alpha_s)$ through three loops in SUSY [Eqs. (1), (3)]

An advantage of this procedure as compared to a multi-scale approach is that the RGEs are only one-dimensional (there is only one coupling constant), and that for α_e one can apply Eq. (9) and Eq. (15).

Let us remark that in principle it is possible to decouple the top quark separately:

$$\alpha_s^{\overline{\rm MS},(5)}(M_Z) \quad \stackrel{(i')}{\to} \quad \alpha_s^{\overline{\rm MS},(5)}(\mu_t) \quad \stackrel{(ii')}{\to} \quad \alpha_s^{\overline{\rm MS},(6)}(\mu_t) \quad \stackrel{(iii')}{\to} \quad \alpha_s^{\overline{\rm MS},(6)}(\mu_{\rm dec}) \tag{17}$$

and continue with steps (ii)–(iv) of Eq. (16), but now using $n_f = 6$ in all the formulas. μ_t is an additional decoupling scale to be chosen of the order of m_t. The only new ingredient needed is the decoupling constant for going from five to six quark flavours in the $\overline{\rm MS}$ scheme in step (ii'). It has been evaluated through three and four loops in

Refs. [39] and [40, 41], respectively. In any case, for a mass spectrum as given by the benchmark point SPS1a′ [10], for example, the separate decoupling of the top quark implies a numerically small effect. This can also be established by comparing "Scenario D" and "Scenario C" in Ref. [23].

For a direct application of Eqs. (4), (6), and (8) to steps (*ii*) and (*iii*) of Eq. (16), these equations need to be inverted. In fact, it may be convenient for practical phenomenological analyses to combine steps (*ii*) and (*iii*) of Eq. (16) into a single formula:

$$\begin{aligned}\alpha_s^{\overline{\rm DR},({\rm full})} = \alpha_s^{\overline{\rm MS},(n_f)}\Bigg\{1 &+ \frac{\alpha_s^{\overline{\rm MS},(n_f)}}{\pi}\left(\frac{1}{4}-\zeta_{s1}^{(n_f)}\right) \\ &+\left(\frac{\alpha_s^{\overline{\rm MS},(n_f)}}{\pi}\right)^2\left[\frac{11}{8}-\frac{n_f}{12}-\frac{1}{2}\,\zeta_{s1}^{(n_f)}+2\,(\zeta_{s1}^{(n_f)})^2-\zeta_{s2}^{(n_f)}\right]\Bigg\}\,,\end{aligned} \tag{18}$$

where the scale $\mu_{\rm dec}$ has been suppressed in the notation. The coefficients $\zeta_{s1}^{(n_f)}$ and $\zeta_{s2}^{(n_f)}$ are given in Eq. (6). The numerical deviation of Eq. (18) from the two-step procedure is well below 0.2%.

2.6 Numerical results

The result for $\alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm GUT}=10^{16}\text{ GeV})$, obtained using $M_Z = 91.1876$ GeV and

$$m_t = 170.9\pm 1.9\text{ GeV}\,,\quad \alpha_s^{\overline{\rm MS},(5)}(M_Z)=0.1189\,,\quad \tilde{M}=m_{\tilde q}=m_{\tilde g}=1000\,\text{GeV} \tag{19}$$

as input parameters is shown if Fig. 1. The dotted, dashed and solid line are based on Eq. (16), where n-loop running is combined with $(n-1)$-loop decoupling, as it is required for consistency ($n = 1, 2, 3$, respectively). We find a nice convergence when going from one to three loops, with a very weakly $\mu_{\rm dec}$–dependent result at three-loop order.

For comparison, we show the result obtained from the formula given in Eq. (21) of Ref [10]. It corresponds to the dash-dotted line in Fig. 1. In this case, the evolution is done in two steps:

$$\alpha_s^{\overline{\rm MS},(5)}(M_Z) \quad \xrightarrow{(i)} \quad \alpha_s^{\overline{\rm DR},({\rm full})}(M_Z) \quad \xrightarrow{(ii)} \quad \alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm GUT})\,. \tag{20}$$

In (*i*), $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$ is converted into $\alpha_s^{\overline{\rm DR},({\rm full})}(M_Z)$ by means of resummed one-loop contributions originating from both the change of scheme (from $\overline{\rm MS}$ to $\overline{\rm DR}$) and the decoupling of heavy particles. In step (*ii*), the evolution from M_Z to $\mu_{\rm GUT}$ is performed using the one-loop MSSM RGE in the $\overline{\rm DR}$ scheme (β_0 from Eq. (3)). A remarkable feature of this combination is that the $\mu_{\rm dec}$–dependence drops out explicitely. However, the difference between our three-loop result with two-loop decoupling (upper solid line) and the one-loop formula given in Ref. [10] exceeds the experimental uncertainty by almost a factor of four for sensible values of $\mu_{\rm dec}$. This uncertainty is indicated by the hatched band, derived from $\delta\alpha_s(M_Z) = \pm 0.001$ [42]. The formulae of Ref. [10] should therefore be taken only

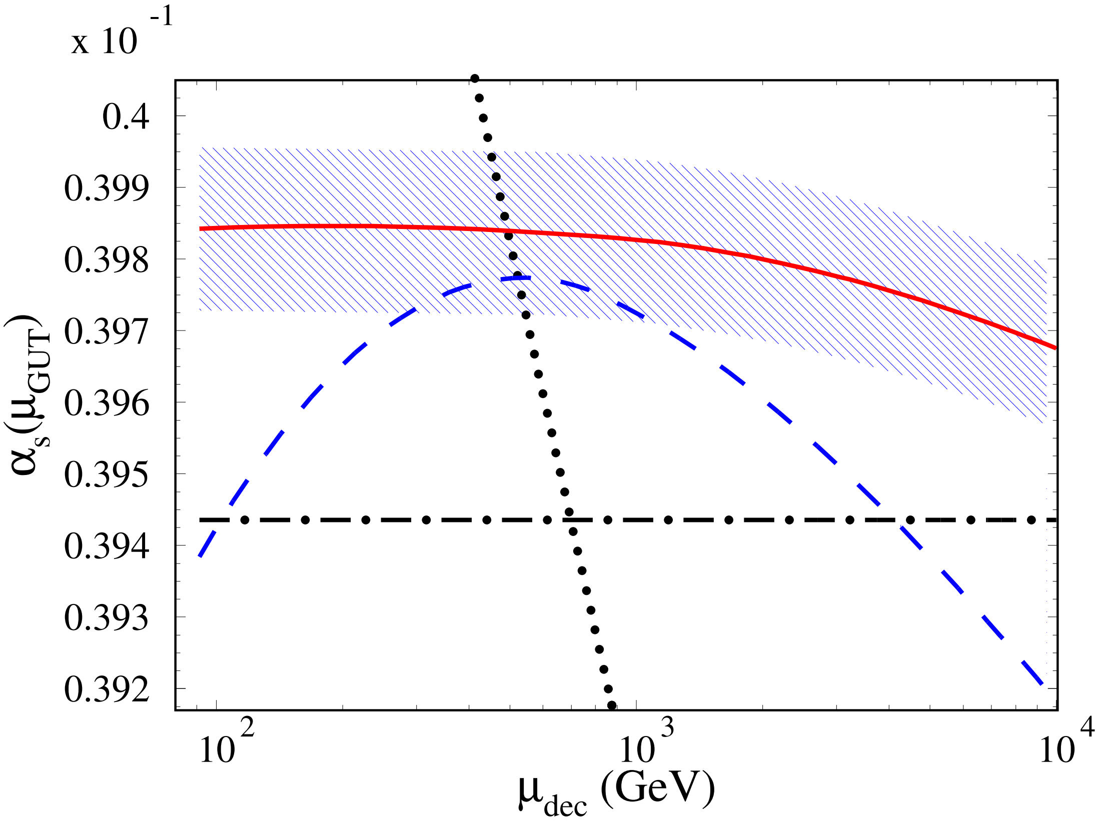

Figure 1: $\alpha_s(\mu_{\rm GUT})$ as a function of $\mu_{\rm dec}$. Dotted, dashed and solid line: prescription discussed in Eq. (16) for one, two, and three loops, respectively. The hatched band denotes the uncertainty from the input value $\alpha_s(M_Z)$. Dash-dotted line: one-loop running and one-loop decoupling as described in Ref. [10].

as rough estimates; once precision studies are required, one should rely on the consistent treatment of running, decoupling, and $\overline{\rm MS}$–$\overline{\rm DR}$ conversion, as it is outlined here.

In Fig. 2 we show $\alpha_s(\mu_{\rm GUT})$ as a function of $\tilde{M}$ where $\mu_{\rm dec} = \tilde{M}$ has been adopted. Dotted, dashed and full curve correspond again to the one-, two- and three-loop analysis and the uncertainty form $\alpha_s(M_Z)$ is indicated by the hatched band. One observes a variation of 10% as $\tilde{M}$ is varied between 100 GeV and 10 000 GeV. This shows that the actual SUSY scale can significantly influence the unification, respectively, the non-unification behaviour of the three couplings at the GUT scale. For definite conclusions also the influence of $\tilde{M}$ on the electro-magnetic and the weak coupling has to be carefully studied, of course.

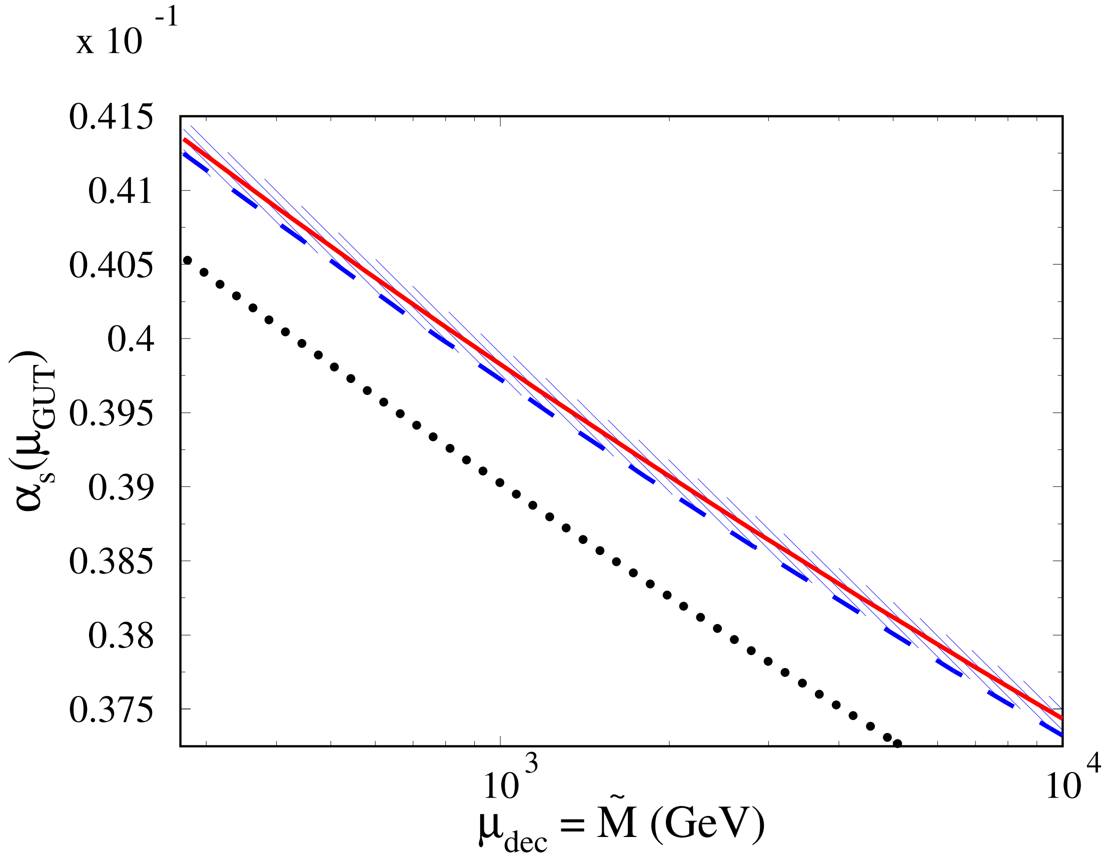

Figure 2: $\alpha_s(\mu_{\rm GUT})$ as a function of $\tilde{M}$ where $\mu_{\rm dec} = \tilde{M}$ has been chosen. The notation is adopted from Fig. 1.

3 Evaluating $m_b(\mu)$ in the $\overline{\rm DR}$ scheme

Large values of $\tan\beta$ could enhance the importance of the bottom Yukawa coupling in the MSSM enormously as compared to the SM. If predictions within this model are calculated in the $\overline{\rm DR}$ scheme, then it is certainly necessary to know $m_b^{\overline{\rm DR}}$ with the highest possible precision. Experimental data concerning the measurement of the bottom mass are typically converted into μ_b defined by the recursive equation

$$\mu_b = m_b^{\overline{\rm MS},(5)}(\mu_b)\,. \tag{21}$$

In the following, we will provide a precise relation between this value and $m_b^{\overline{\rm DR},(5)}(\mu_S)$, where μ_S may be an energy scale between the electroweak gauge boson masses and a few TeV.

The quantity $m_b^{\overline{\rm DR},(5)}(M_Z)$ has also been considered in Ref. [29] at two-loop level. We will comment on the differences between this and our result at the end of this section.

3.1 Evolution and scheme conversion of $m_b^{(5)}$

We follow two different methods. The first one is given by the chain

$$m_b^{\overline{\rm MS},(5)}(\mu_b) \quad \overset{(i)}{\to} \quad m_b^{\overline{\rm MS},(5)}(\mu_S) \quad \overset{(ii)}{\to} \quad m_b^{\overline{\rm DR},(5)}(\mu_S)\,. \tag{22}$$

The RGEs of QCD required for step (i) are known to four-loop accuracy [43, 44], and three-loop corrections are available [30] for the $\overline{\rm MS}$–$\overline{\rm DR}$ conversion relation in step (ii). A peculiar feature of this mass conversion is that the evanescent coupling α_e occurs already at one-loop level:

$$m^{\overline{\rm DR}} \;=\; m^{\overline{\rm MS}}\left(1 - \frac{\alpha_e}{3\pi} + \ldots\right)\,, \tag{23}$$

where the dots denote higher orders in α_e, η_r and α_s.

The second method consists of the following sequence:

$$m_b^{\overline{\rm MS},(5)}(\mu_b) \quad \overset{(i')}{\to} \quad m_b^{\overline{\rm DR},(5)}(\mu_b) \quad \overset{(ii')}{\to} \quad m_b^{\overline{\rm DR},(5)}(\mu_S)\,. \tag{24}$$

Again, three-loop terms are known for the relation used in step (i'). The running in the $\overline{\rm DR}$ scheme in step (ii') involves the non-diagonal RGEs of $\alpha_s^{\overline{\rm DR},(5)}$ and the evanescent couplings $\alpha_e^{(5)}$ and $\eta_r^{(5)}$. The corresponding β functions are known to four-, three-, and one-loop order, respectively [30].

Because of these multi-dimensional RGEs required for Eq. (24), the sequence of Eq. (22) is preferable to the latter in practical calculations. However, it may be interesting to see how the perturbative series behaves for the two methods, and to which extent they are numerically equivalent. In particular, this may provide an estimate of the uncertainty in the relation between $m_b^{\overline{\rm MS},(5)}(\mu_b)$ and $m_b^{\overline{\rm DR},(5)}(\mu_S)$.

Just as for the strong coupling α_s, the consistent evolution of m_b requires that n-loop running is combined with $(n-1)$-loop $\overline{\rm MS}$–$\overline{\rm DR}$ conversion. Assuming that the couplings $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$, $\alpha_s^{\overline{\rm DR},(5)}(M_Z)$, $\alpha_e^{(5)}(M_Z)$ and $\eta_r^{(5)}(M_Z)$ are known, all operations are within five-flavour QCD and do not require any decoupling. This allows it to apply Eq. (22) at four-loop order. This is not strictly the case for Eq. (24), because the β functions for the evanescent couplings are not known to four loops. We ignore this fact when evaluating the three- and four-loop evolution along Eq. (24), expecting these effects to be small.

3.2 Input values for α_e and η_r

In addition to $m_b^{\overline{\rm MS},(5)}(\mu_b)$, the only input value for the determination of $m_b^{\overline{\rm DR},(5)}(\mu_S)$ is $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$. However, either of the two methods of Eqs. (22) and (24) requires to switch to the $\overline{\rm DR}$ parameters $\alpha_s^{\overline{\rm DR},(5)}$, $\alpha_e^{(5)}$, and $\eta_r^{(5)}$ at some point.

In principle, any choice of $\alpha_s^{\overline{\rm DR},(5)}(M_Z)$, $\alpha_e^{(5)}(M_Z)$ and $\eta_r^{(5)}(M_Z)$ is allowed which obeys Eq. (8) for the given value of $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$. Any such choice would simply correspond to a particular renormalization scheme. But let us assume that QCD is the low energy effective theory of SUSY-QCD, and thus Eq. (9) holds. Then all $\overline{\rm DR}$ couplings of five-flavour standard QCD are uniquely determined by the corresponding decoupling relations. In contrast to Section 2, we now want to go to the four-loop level, and thus we take the one-loop decoupling relation for α_e into account[3], see Eq. (10). The η_r, on the other hand, occur only at four-loop level and can be decoupled trivially by setting $\eta_r^{(\rm full)}(\mu_{\rm dec}) = \eta_r^{(5)}(\mu_{\rm dec})$.

Since $\alpha_s^{\overline{\rm DR},(\rm full)}$ is not known *a priori*, one cannot use Eq. (10) directly in order to derive $\alpha_e^{(5)}$. Rather, we start with a trial value for $\alpha_s^{\overline{\rm DR},(\rm full)}(\mu_{\rm dec})$ and obtain the corresponding $\alpha_e^{(5)}(\mu_{\rm dec})$ through Eq. (10), as well as $\alpha_s^{\overline{\rm DR},(5)}(\mu_{\rm dec})$ through Eq. (6). Then we evaluate $\alpha_s^{\overline{\rm MS},(5)}(\mu_{\rm dec})$ through Eq. (8), and from that $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$. The trial value for $\alpha_s^{\overline{\rm DR},(\rm full)}(\mu_{\rm dec})$ is systematically varied until the resulting $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$ agrees with the experimental input.

As indicated above, current knowledge of the renormalization group functions allows us to follow the sequences Eq. (22) and Eq. (24) at four-loop level. Strictly speaking, this requires also that $\alpha_s^{\overline{\rm DR},(5)}(M_Z)$, $\alpha_e^{(5)}(M_Z)$, and $\eta_r^{(5)}(M_Z)$ are derived from $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$ at four-loop level. The decoupling of the SUSY particles mentioned above should thus be performed at three-loop level, which is currently not known for any of the three couplings. However, we may consider $\alpha_s^{\overline{\rm DR},(5)}(M_Z)$, $\alpha_e^{(5)}(M_Z)$, and $\eta_r^{(5)}(M_Z)$ as input, and their derivation from $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$ only as a guideline, allowing us to neglect higher order decoupling effects. We do not expect that a fully consistent evaluation of these parameters will change our numerical results for $m_b^{\overline{\rm DR}}$ significantly.

3.3 Numerical results

Taking Eq. (19) as well as $m_b^{\overline{\rm MS},(5)}(\mu_b) = 4.164$ GeV [32] as input values and following steps (*i*) and (*ii*) of Eq. (22), we find (the numbers refer to $\mu_{\rm dec} = \mu_S = M_Z$):

(*i*) The difference between two- and three-loop running on $m_b^{\overline{\rm DR},(5)}(M_Z)$ is 23 MeV, while going to four loops further modifies the result by less than 2 MeV.

(*ii*) The one-loop term in the $\overline{\rm MS}$–$\overline{\rm DR}$ transition amounts for $\mu = M_Z$ to about 29 MeV, while the two-loop term is only about 2 MeV.

Following steps (i') and (ii') of Eq. (24), on the other hand, one finds:

(i') The one-loop term in the $\overline{\rm MS}$–$\overline{\rm DR}$ transition amounts for $\mu = \mu_b$ to about 66 MeV, while the two-loop term is only about 6 MeV.

(ii') The difference between two- and three-loop running on $m_b^{\overline{\rm DR},(5)}(M_Z)$ is 18 MeV, while going to four loops further modifies the result by less than 0.5 MeV.

[3] Since we take the three-loop beta function for α_e into account we would need the two-loop corrections at this point. However, we assume that the numerical effect is small.

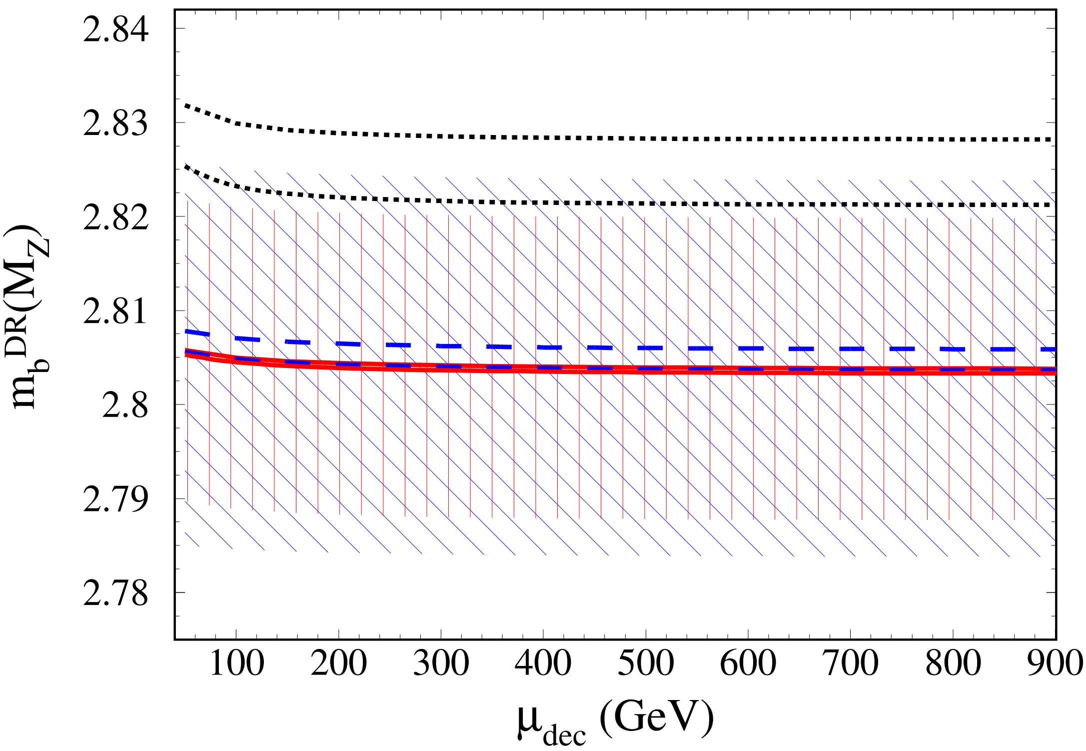

Figure 3: $m_b^{\overline{\rm DR}}(M_Z)$ as a function of $\mu_{\rm dec}$ for two-, three- and four-loop running (dotted, dashed and solid lines). The upper (lower) curves correspond to the prescriptions (i) and (ii) ((i') and (ii')). The band describes the uncertainty from α_s or m_b (see text).

In Fig. 3, $m_b^{\overline{\rm DR}}(M_Z)$ is shown as a function of $\mu_{\rm dec}$ where the dotted, dashed and solid lines correspond to two-, three- and four-loop running.[4] At each loop level, the upper curve corresponds to the path given in Eq. (22), the lower one to Eq. (24). The smaller hatched band reflects the uncertainty induced by the current experimental accuracy of $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$, equal to $\delta\alpha_s = 0.001$ [42], and the larger one corresponds to $\delta m_b = 25$ MeV [32]. The total uncertainty from both m_b and α_s is thus obtained by adding in quadrature the individual uncertainties.

As expected, the difference between paths Eq. (22) and Eq. (24) decreases as higher order corrections are included. At four-loop level, the lines are practically indistinguishable. One notices a slightly better convergence of the perturbative terms when following the path of Eq. (24). Note also that both two-loop curves lie on the upper edge or even

[4]For the one-loop result the difference between $(i),(ii)$ and $(i'),(ii')$ is small because no non-trivial matching terms are included at this order. The resulting value $m_b^{\overline{\rm DR}}(M_Z) \approx 3.05$ GeV lies well outside the plot.

$\mu_S = \mu_{\rm dec} = M_Z$				
(i), (ii)	1	2	3	4 (running)
$\alpha_e^{(5)}(\mu_S)$	0.1189	0.0968	0.0995	0.0995
$\alpha_s^{\overline{\rm DR},(5)}(\mu_S)$	0.1189	0.1200	0.1202	0.1202
$m_b^{\overline{\rm MS},(5)}(\mu_S)$	3.055	2.859	2.838	2.836
$m_b^{\overline{\rm DR},(5)}(\mu_S)$	3.055	2.830	2.807	2.805
(i'), (ii')				
$\alpha_e^{(5)}(\mu_b)$	0.1649	0.1501	0.1537	0.1538
$\alpha_s^{\overline{\rm DR},(5)}(\mu_b)$	0.2153	0.2316	0.2336	0.2343
$m_b^{\overline{\rm DR},(5)}(\mu_b)$	4.164	4.098	4.092	4.094
$m_b^{\overline{\rm DR},(5)}(\mu_S)$	3.055	2.823	2.805	2.805

$2\mu_S = \mu_{\rm dec} = 2M_Z$				
(i), (ii)	1	2	3	4 (running)
$\alpha_e^{(5)}(\mu_S)$	0.1133	0.1006	0.1014	0.1014
$\alpha_s^{\overline{\rm DR},(5)}(\mu_S)$	0.1189	0.1200	0.1202	0.1202
$m_b^{\overline{\rm MS},(5)}(\mu_S)$	3.055	2.859	2.838	2.836
$m_b^{\overline{\rm DR},(5)}(\mu_S)$	3.055	2.829	2.807	2.804
(i'), (ii')				
$\alpha_e^{(5)}(\mu_b)$	0.1590	0.1545	0.1559	0.1560
$\alpha_s^{\overline{\rm DR},(5)}(\mu_b)$	0.2153	0.2316	0.2336	0.2343
$m_b^{\overline{\rm DR},(5)}(\mu_b)$	4.164	4.096	4.091	4.092
$m_b^{\overline{\rm DR},(5)}(\mu_S)$	3.055	2.822	2.804	2.804

Table 1: Numerical results for the bottom quark $\overline{\rm DR}$ mass for $\mu_S = M_Z$ using $\mu_{\rm dec} = M_Z$ and $\mu_{\rm dec} = 2M_Z$, respectively. For convenience intermediate results for α_s and α_e are given resulting from the one-, two-, three- and four-loop analysis.

outside the uncertainty band of α_s and m_b, thus proving the importance of the three-loop terms. Detailed numerical results for the couplings and bottom quark masses are given in Tab. 1. We consider the choices $\mu_S = \mu_{\rm dec} = M_Z$, as well as $\mu_S = \mu_{\rm dec}/2 = M_Z$.

It is worth mentioning that the numerical effect of the decoupling of $\alpha_s^{\overline{\rm DR},(n_f)}$ (cf. Eq. (4)) and α_e (cf. Eq. (10)) on $m_b^{\overline{\rm DR},(5)}(M_Z)$ is not negligible. For $\zeta_s^{(n_f)} = 1$ and $\zeta_e = 1$, we observe a decrease of more than 6 MeV for both Eqs. (22) and (24), employed at two-, three-, and four-loop order.

In Fig. 4 we show $m_b^{\overline{\rm DR}}(\mu)$ varying μ between 50 GeV and 5 TeV where the two-, three- and four-loop results corresponding to the steps (i) and (ii) of Eq. (22) are plotted. The decoupling scale is set to $\mu_{\rm dec} = \tilde{M} = 1$ TeV. In Tab. 2 we show for some selected

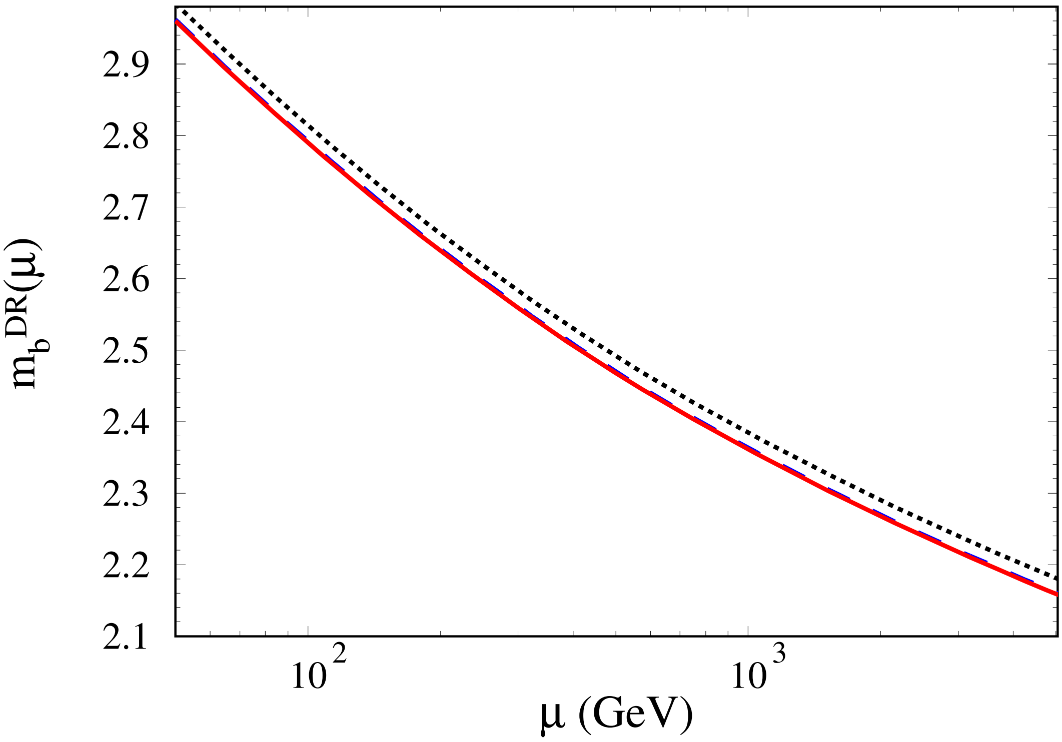

Figure 4: $m_b^{\mathrm{DR}}(\mu)$ as a function of μ. The notation is adopted from Fig. 3.

μ (GeV)	$m_b^{\overline{\mathrm{DR}}}(\mu)$
91.1876	2.804(16)(20)
350	2.528(17)(18)
500	2.467(17)(18)
800	2.394(17)(17)
1000	2.361(17)(17)
2000	2.268(17)(16)

Table 2: $m_b^{\overline{\mathrm{DR}}}$ for various values of the renormalization scale. The two numbers given in the round brackets correspond to the uncertainties induced by $\delta\alpha_s^{\overline{\mathrm{MS}}} = 0.001$ (first number) and $\delta m_b^{\overline{\mathrm{MS}}}(\mu_b) = 25$ MeV (second number), respectively.

values of μ the corresponding result for $m_b^{\overline{\mathrm{DR}}}$ together with the uncertainties arising from $\delta\alpha_s^{\overline{\mathrm{MS}}} = 0.001$ and $\delta m_b^{\overline{\mathrm{MS}}}(\mu_b) = 25$ MeV.[5]

[5]In the analysis of Ref. [32] $\delta\alpha_s^{\overline{\mathrm{MS}}} = 0.002$ has been adopted, leading to the uncertainty of 25 MeV for m_b. Still, for illustration purpose we choose $\delta\alpha_s^{\overline{\mathrm{MS}}} = 0.001$ in order to obtain the numbers in Tab. 2.

Let us now comment on the earlier two-loop calculation of $m_b^{\overline{\rm DR}}$ by Baer et al. [29]. Our results represent an improvement with respect to several issues. First of all, we increased the accuracy of the result by including the three- and the four-loop terms in the derivation of $m_b^{\overline{\rm DR}}$. In particular the three-loop terms turn out to be numerically very important, while the four-loop result indicates a nice stabilization of the perturbative expansion. In addition, we have made a significant conceptual generalization, related to the evanescent couplings. Ref. [29] sets $\alpha_s(M_Z) = \alpha_e(M_Z)$ in Eq. (23). As we have argued above, this corresponds to a particular renormalization scheme within $\overline{\rm DR}$, and the value for $m_b^{\overline{\rm DR}}$ is associated with this scheme. In contrast to that, we have systematically derived α_e by assuming that squarks and gluinos decouple at a scale $\mu_{\rm dec}$ and have studied the dependence of $m_b^{\overline{\rm DR}}$ on this scale. Finally, we have given explicit numerical results for the method described by Eq. (24). Ref. [29] correctly claims that the difference to the method of Eq. (22) is small, but no numbers or any details of the calculation are given. In particular, it is unclear which value was used for $\alpha_e(\mu_b)$ in Eq. (23).

It should be pointed out that the formulae and prescriptions of Ref. [29] have been partially taken over in the outline of the SPA project [10]. However, as in the case of the strong coupling, various orders of perturbation theory have been combined inconsistently. Thus, let us stress again that the formulae of Ref. [10] should not be taken over literally in phenomenological analyses if precision is of concern.

4 Conclusions

Application of DRED to non-SUSY theories is rather cumbersome compared to DREG because of the occurrence of evanescent couplings. However, when low energy precision data such as $\alpha_s^{\overline{\rm MS}}(M_Z)$ or $m_b^{\overline{\rm MS}}(\mu_b)$ are related to their counterparts in a SUSY theory at high energies, one may need to switch between the $\overline{\rm MS}$ and the $\overline{\rm DR}$ scheme at some energy scale. The conversion formulae will involve evanescent couplings, sometimes already at one-loop level, as in the case of the quark mass.

We have used recent three- and four-loop results for the β functions, the quark anomalous dimension, and the decoupling coefficients in order to derive $\alpha_s^{\overline{\rm DR}}(\mu_{\rm GUT})$ and $m_b^{\overline{\rm DR},(5)}(\mu)$ from $\alpha_s^{\overline{\rm MS}}(M_Z)$ and $m_b^{\overline{\rm MS}}(\mu_b)$ at three- and four-loop level, respectively.

It turns out that the three-loop terms are numerically significant both for α_s and for m_b. The dependence on where the SUSY spectrum is decoupled becomes particularly flat in this case. The theoretical uncertainty is expected to be negligible w.r.t. the uncertainty induced by the experimental input values.

Comparing our results and methods to the literature, we find that the issue of evanescent couplings has either been ignored (by assuming $\alpha_e = \alpha_s$) or circumvented by decoupling the SUSY spectrum at $\mu_{\rm dec} = M_Z$. We find that at one- and two-loop level, this choice does not allow for a good approximation of the higher order effects, if one assumes the SUSY partner masses to be of the order of 1 TeV.

To conclude, we strongly suggest that phenomenological studies concerning the implications of low energy data on Grand Unification should be done at three-loop level, and that decoupling effects and evanescent couplings are properly taken into account.

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