Spin asymmetries and correlations in Λ–pair production through the radiative return method*

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Exclusive baryon pair production through the radiative return is a unique tool to determine the electric and the magnetic form factors in the timelike region over a wide range of energies. The decay of Λ-baryons into $p + \pi^-$ can be used to measure their spin and to analyze spin-spin correlations in the $\bar{\Lambda}\Lambda$ system. We evaluate the spin dependent hadronic tensor, which describes this reaction, calculate the corresponding cross section for $e^+e^- \to \Lambda\bar{\Lambda}$ and $e^+e^- \to \Lambda\bar{\Lambda}\gamma$ with subsequent decays of Λ and $\bar{\Lambda}$ and present a compact formula for the correlated distribution. We demonstrate that the relative phase between the electric and the magnetic form factor can be determined in this analysis by considering single spin asymmetries and correlated decay distributions. The reaction is implemented in the event generator PHOKHARA. It is demonstrated that the relative phase can be measured with the event sample presently accumulated at B- factories.

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I. INTRODUCTION

Measurements of the proton–antiproton production through the radiative return, as suggested in [1], have lead to a determination of the proton electric and magnetic form factors, G_E and G_M, in the time-like region over an energy which ranges from threshold to nearly 3 GeV [2]. The study of the angular distributions allows to separate the contributions from $|G_M|^2$ and $|G_E|^2$ to the cross section. In the absence of longitudinally polarized beams and without measuring of the polarization of the produced baryons, the relative phase between G_M and G_E does not enter the predictions [1]. In this paper we demonstrate that $\Lambda\bar{\Lambda}$– production through the radiative return can be used to fully determine modulus and relative phase $\Delta\phi$ of Λ electric and magnetic form factors. We present a compact form for the hadronic tensor

$$H_{\mu\nu} = \langle\bar{\Lambda}\Lambda|\mathcal{J}_\mu|0\rangle \, \langle 0|\mathcal{J}_\nu^\dagger|\bar{\Lambda}\Lambda\rangle \; = \; J_\mu J_\nu^\dagger \; , \qquad (1)$$

including its spin dependent part and analyze single–spin asymmetries and spin–spin correlations for $\Lambda\bar{\Lambda}$ production through e^+e^- annihilation and through the radiative return. The self analyzing decays $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$ can be used to directly translate the spin asymmetries and correlations into predictions for the decay products. The spin component normal to the production plane is a measure of $\sin(\Delta\phi)$, while spin–spin correlations can be used to resolve the remaining twofold ambiguity and determine the relative phase completely.

In addition to this theoretical study we construct an event generator (an extension of the event generator PHOKHARA [1, 3–9]), which includes the spin correlated decays of Λ and $\bar{\Lambda}$. We adopt a simple model for the form factor and demonstrate, that the determination of the phase is indeed feasible. Once required by an experiment, the approach can be trivially extended to the full baryon octet.

II. THE METHOD

The reactions $e^+(p_1)e^-(p_2) \to \bar{\Lambda}(q_1,S_1)\Lambda(q_2,S_2)$ and $e^+(p_1)e^-(p_2) \to \bar{\Lambda}(q_1,S_1)\Lambda(q_2,S_2)\gamma(k)$ (with photon emitted from one of the initial states) are both in Born approximation fully described by electromagnetic current with two independent form factors

$$J_\mu = -ie\cdot\bar{u}(q_2,S_2) \times\left(F_1^\Lambda(Q^2)\gamma_\mu - \frac{F_2^\Lambda(Q^2)}{4m_\Lambda}[\gamma_\mu,\not{Q}\,]\right)\; v(q_1,S_1), \quad (2)$$

related to the electric G_E and magnetic G_M form factors by

$$G_M = F_1^\Lambda + F_2^\Lambda,\; G_E = F_1^\Lambda + \tau F_2^\Lambda, \qquad (3)$$
$$\text{with } \tau = \frac{Q^2}{4m_\Lambda^2} \text{ and } Q = q_1 + q_2.$$

The $q_i, S_i,\; i = 1,2$ are the four momenta and the spin four vectors of $\bar{\Lambda}$ and Λ with the properties $S_i.q_i = 0,\; S_i^2 = -1$. We will not consider the initial lepton polarizations and denote their four momenta by p_1 (positron) and p_2 (electron). The resulting squared amplitudes, which describe the two processes under consideration, are then given by

$$|M^{0,1}|^2 = L_{\mu\nu}^{0,1} H^{\mu\nu} \; , \qquad (4)$$

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where 0(1) stands for the process without (with) photon. The hadronic tensor, which is identical for both cases, and defined as

$$H_{\mu\nu} = J_\mu J_\nu^\dagger = H^S_{\mu\nu} + H^A_{\mu\nu} \ , \tag{5}$$

is naturally splitted into symmetric ($H^S_{\mu\nu}$) and anti-symmetric ($H^A_{\mu\nu}$) parts defined as

$$H^S_{\mu\nu} = \frac{1}{2}\left(J_\mu J_\nu^\dagger + J_\mu^\dagger J_\nu\right) , \ H^A_{\mu\nu} = \frac{1}{2}\left(J_\mu J_\nu^\dagger - J_\mu^\dagger J_\nu\right) \ . \tag{6}$$

They are equal to

$$\begin{aligned}
\tfrac{1}{e^2} H^S_{\mu\nu} = \ & \tfrac{|G_M|^2}{2}\Bigg(Q.S_2 \ \{Q_\mu, S_{1\nu}\}_+ + Q.S_1 \ \{Q_\mu, S_{2\nu}\}_+ - S_1.S_2 \ Q_\mu \ Q_\nu - Q^2 \ \{S_{1\mu}, S_{2\nu}\}_+ \Bigg) \\
& + |G_M|^2 \big(\tfrac{Q^2}{2} \ S_1.S_2 - Q.S_1 \ Q.S_2\big) \ g_{\mu\nu} + \Bigg(\tfrac{|G_M - G_E|^2}{m^2_\Lambda (\tau-1)^2} \ Q.S_1 \ Q.S_2 \ + \tfrac{2\tau}{\tau-1}\Bigg(|G_M|^2 - \tfrac{1}{\tau}|G_E|^2 \Bigg) \ S_1.S_2 \Bigg) \ q_\mu \ q_\nu \\
& + \tfrac{\tau |G_M|^2 - Re(G_M G_E^*)}{\tau-1} \big(Q.S_2 \ \ \{q_\mu, S_{1\nu}\}_+ - Q.S_1 \ \ \{q_\mu, S_{2\nu}\}_+ \big) + \ \tfrac{Im(G_M G_E^*)}{m_\Lambda(\tau-1)} \ \{\varepsilon_{\beta\gamma\delta\mu} \ Q^\beta q^\gamma \ (S_1^\delta + S_2^\delta), q_\nu\}_+ + \tfrac{1}{4} H^U_{\mu\nu} \ ,
\end{aligned} \tag{7}$$

$$\begin{aligned}
\tfrac{1}{e^2} H^A_{\mu\nu} = \ & i \tfrac{Im(G_M G_E^*)}{\tau-1} \big(Q.S_2 \ \ \{q_\mu, \ S_{1\nu}\}_- - Q.S_1 \ \ \{q_\mu, \ S_{2\nu}\}_- \big) \\
& - \ i \ \tfrac{Re(G_M G_E^*)}{m_\Lambda(\tau-1)} \ \{\varepsilon_{\beta\gamma\delta\mu} \ Q^\beta q^\gamma \ (S_1^\delta + S_2^\delta), q_\nu\}_- + \ i \ \tfrac{|G_M|^2}{2 m_\Lambda (\tau-1)} \varepsilon_{\beta\gamma\mu\nu} \ Q^\beta q^\gamma \ (Q.S_2 - Q.S_1) \ ,
\end{aligned} \tag{8}$$

where $q = (q_2 - q_1)/2$, $\{a_\mu, b_\nu\}_\pm \equiv a_\mu b_\nu \pm b_\mu a_\nu$ and

$$\begin{aligned}
H^U_{\mu\nu} = \ & 2|G_M|^2 (Q_\mu Q_\nu - g_{\mu\nu} Q^2) \\
& - \tfrac{8\tau}{\tau-1} \Bigg(|G_M|^2 - \tfrac{1}{\tau}|G_E|^2 \Bigg) q_\mu q_\nu
\end{aligned} \tag{9}$$

is the unpolarized hadronic tensor.

In the first step, for illustration and further reference, we shall discuss baryon production in electron–positron annihilation without photon radiation. The application to the radiative return will be presented subsequently.

The lowest order leptonic tensor, when one neglects the electron mass, is equal to

$$\begin{aligned}
L^0_{\mu\nu} \ = \ & \frac{\pi\alpha}{s^2} \left(-2 s g^{\mu\nu} + 4\{p_1^\mu, p_2^\nu\}_+\right) \\
= \ & \frac{2\pi\alpha}{s} \ \ \mathrm{diag}(0,1,1,0) \ ,
\end{aligned} \tag{10}$$

while the explicit form of $L^1_{\mu\nu}$ (describing the radiative return), together with definitions and notation, can be found for instance in [5, 10]. In both cases the leptonic tensor is symmetric (apart from small imaginary part coming from the radiative corrections), hence the asymmetric part of the hadronic tensor does not contribute to the cross section

$$d\sigma(e^+e^- \to \bar\Lambda\Lambda) = \frac{1}{2s} L^0_{\mu\nu} H^{\mu\nu} d\Phi_2(p_1 + p_2; q_1, q_2) \ , \tag{11}$$

which can be written in the following compact form

$$\begin{aligned}
L^0_{\mu\nu} H^{\mu\nu} = \tfrac{4\pi^2\alpha^2}{s^2} \Bigg\{ \ & 16 \ q.p \ \tfrac{\tau |G_M|^2 - Re(G_M G_E^*)}{\tau-1} \ (Q.S_1 \ p.S_2 - Q.S_2 \ p.S_1) + 2 \ s \ |G_M|^2 (s + 4 \ p.S_1 \ p.S_2) \\
& - \Bigg[\tfrac{2\tau s}{\tau-1} \Bigg(|G_M|^2 - \tfrac{1}{\tau}|G_E|^2 \Bigg) + \ \tfrac{8|G_M - G_E|^2}{m^2_\Lambda(\tau-1)^2} \ (q.p)^2 \Bigg] \ Q.S_1 \ Q.S_2 \\
& - \tfrac{2\tau}{\tau-1} \Bigg(|G_M|^2 - \tfrac{1}{\tau}|G_E|^2 \Bigg) \ (8 \ (q.p)^2 + 2 \ s \ m^2_\Lambda - \tfrac{s^2}{2})(S_1.S_2 - 1) - \ q.p \ \tfrac{16 \ Im(G_M G_E^*)}{m_\Lambda(\tau-1)} \ \varepsilon_{\beta\gamma\mu\nu} \ Q^\beta q^\gamma (S_1^\mu + S_2^\mu) p^\nu \Bigg\},
\end{aligned} \tag{12}$$

where $p = (p_2 - p_1)/2$ and $s = (p_1 + p_2)^2 = Q^2$. This result becomes more transparent, if we use the spin vectors of the Λ and $\bar{\Lambda}$ in their respective rest frame

$$L^0_{\mu\nu}H^{\mu\nu} = 4\pi^2\alpha^2\Bigg\{|G_M|^2\left(1+\cos^2\theta_{\bar{\Lambda}}\right) + \tfrac{1}{\tau}|G_E|^2\sin^2\theta_{\bar{\Lambda}} + \tfrac{Im(G_M G_E^*)}{\sqrt{\tau}}\sin(2\theta_{\bar{\Lambda}})\left(S^y_\Lambda + S^y_{\bar{\Lambda}}\right)$$
$$-\tfrac{Re(G_M G_E^*)}{\sqrt{\tau}}\sin(2\theta_{\bar{\Lambda}})\left(S^z_{\bar{\Lambda}}S^x_{\Lambda} + S^z_{\Lambda}S^x_{\bar{\Lambda}}\right) + \left(\tfrac{1}{\tau}|G_E|^2 + |G_M|^2\right)\sin^2\theta_{\bar{\Lambda}}\quad S^x_{\bar{\Lambda}}S^x_{\Lambda}$$
$$+\left(\tfrac{1}{\tau}|G_E|^2 - |G_M|^2\right)\sin^2\theta_{\bar{\Lambda}}\quad S^y_{\bar{\Lambda}}S^y_{\Lambda} - \left(\tfrac{1}{\tau}|G_E|^2\sin^2\theta_{\bar{\Lambda}} - |G_M|^2\left(1+\cos^2\theta_{\bar{\Lambda}}\right)\right)\quad S^z_{\bar{\Lambda}}S^z_{\Lambda}\Bigg\}\ , \tag{13}$$

where S^i_Λ ($S^i_{\bar{\Lambda}}$) is the ith component of the unit vector pointing into the direction of the Λ ($\bar{\Lambda}$) spin, calculated in the Λ ($\bar{\Lambda}$) rest frame, and $\theta_{\bar{\Lambda}}$ is the $\bar{\Lambda}$ polar angle in the e^+e^- center of mass frame. The y-direction is perpendicular to the production plane with the positive direction defined through $\bar{e}_y = \bar{e}_{e^+} \times \bar{e}_{\bar{\Lambda}}$, where $\bar{e}_{e^+}$ and $\bar{e}_{\bar{\Lambda}}$ are the unit vectors pointing into the positron and $\bar{\Lambda}$ momenta directions respectively (see Fig. 1).

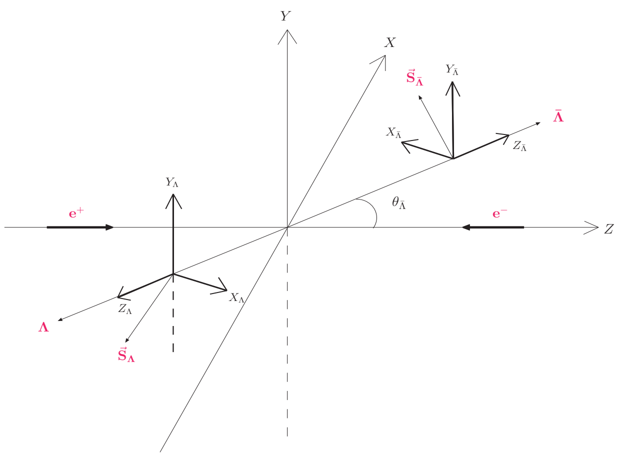

FIG. 1: (color online). Definition of the three reference frames used to obtain Eq. (13).

The linear polarization is a consequence of the relative phase between electric and magnetic form factors and points into the direction normal to the production plane [11, 12] (and references therein). The correlation coefficients $C_{x,z}, C_{x,x}, C_{y,y}, C_{z,z}$ are in general non vanishing even if the relative phase between G_E and G_M is zero or π.

The absolute values of the electric and magnetic form factors can be measured without measuring the polarization of the baryons via the Rosenbluth method, i.e. by analyzing the angular distributions of the baryons. In previous investigations it was shown [11] that the relative phase can be determined by using longitudinally polarized beams. From Eq.(13) it is obvious that a measurement of S^y, which can be performed for Λ, as discussed below, determines that phase modulo π. The remaining twofold ambiguity can be eliminated through a study of the correlation between $S^z_{\bar{\Lambda}}$ and S^x_{Λ} (or S^x_{Λ} and $S^z_{\bar{\Lambda}}$), which is proportional to $Re(G_M G_E^*)$.

The measurement of the subsequent two body decays of $\Lambda \to \pi^- p$, $\pi^0 n$ and its charge conjugate allow for a straightforward spin analysis of the decaying Λs. The neutral modes are experimentally more demanding. In the remaining part of this work we will therefor refer only to the charged modes even if the theoretical description of both is identical. The decay distribution, when the spin of the proton is not measured, is proportional to

$$R_\Lambda = 1 - \alpha_\Lambda\ \bar{S}_\Lambda \cdot \bar{n}_{\pi^-}\ , \tag{14}$$

where $\bar{n}_{\pi^-}$ is a unit vector in the direction of the π^- momentum. The factor $\alpha_\Lambda = 0.642(13)$ characterizes the analyzing power of the angular distribution. For the antiparticle $\alpha_{\bar{\Lambda}} = -\alpha_\Lambda$.

To combine production and decay, one sums over the intermediate spins of the Λ and $\bar{\Lambda}$ and uses the fact that the sums over the Λ polarizations can be performed easily,

$$\sum_{pol.}\bar{S}_\Lambda = 0\ , \qquad \sum_{pol.} S^i_\Lambda S^j_\Lambda = \delta^{ij}\ . \tag{15}$$

The spin vectors in the Eq.(13) are then effectively replaced by $\bar{S}_\Lambda \to -\alpha_\Lambda \bar{n}_{\pi^-}$ and $\bar{S}_{\bar{\Lambda}} \to -\alpha_{\bar{\Lambda}}\bar{n}_{\pi^+}$.

As a result the relation between cross sections of the reaction $e^+e^- \to \bar{\Lambda}\Lambda$ and the reaction $e^+e^- \to \bar{\Lambda}(\to \pi^+\bar{p})\Lambda(\to \pi^- p)$, when the spins of the final proton and antiprotons are not measured, is simple. Using the narrow width approximation one finds

$$\begin{aligned} d\sigma\left(e^+e^- \to \bar{\Lambda}(\to \pi^+\bar{p})\Lambda(\to \pi^- p)\right) = \\ d\sigma\left(e^+e^- \to \bar{\Lambda}\Lambda\right)\left(S_{\Lambda,\bar{\Lambda}} \to \mp\alpha_\Lambda n_{\pi^\mp}\right) \\ \times\ d\bar{\Phi}_2(q_1; p_{\pi^+}, p_{\bar{p}})d\bar{\Phi}_2(q_2; p_{\pi^-}, p_p) \\ \times\ \mathrm{Br}(\bar{\Lambda} \to \pi^+\bar{p})\mathrm{Br}(\Lambda \to \pi^- p)\ , \end{aligned} \tag{16}$$

where $d\bar{\Phi}_2$ is the two body phase space normalized to 1, α_Λ is the asymmetry parameter of the Λ and $n_{\pi^+}(n_{\pi^-})$

is a four vector, which in the $\bar{\Lambda}$ (Λ) rest frame is equal to $(0, \bar{n}_{\pi^+})$ ($(0, \bar{n}_{\pi^-})$), with $\bar{n}_{\pi^+}$ ($\bar{n}_{\pi^-}$) being the unit vector in the direction of $\bar{p}_{\pi^+}$ ($\bar{p}_{\pi^-}$). The symbol $(S_{\Lambda,\bar{\Lambda}} \to \mp\alpha_\Lambda n_{\pi^\mp})$ indicates that S_Λ has to be replaced by $-\alpha_\Lambda n_{\pi^-}$ and $S_{\bar{\Lambda}}$ by $\alpha_\Lambda n_{\pi^+}$.

Let us now present a qualitative discussion of the observables for the radiative return. In [1] it was shown that in the case of single photon emission from the initial state leptons, by choosing as reference frame the hadronic rest frame with the z-axis opposite to the photon direction (we will call that frame RF from now on), one gets (for $Q^2 \ll s$, which is fulfilled at B–factories) the following simple approximate expression for the leptonic tensor

$$L^{ij} \simeq \frac{(4\pi\alpha)^2}{Q^4 y_1 y_2} \frac{(1+\cos^2\theta_\gamma)}{2} \mathrm{diag}(1,1,0) \ , \qquad (17)$$

which is sufficient for a qualitative discussion. The y-axis is chosen in the e^+e^- cms frame as $\bar{e}_y = \bar{e}_{e^+} \times \bar{e}_\gamma$, where $\bar{e}_{e^+}$ and $\bar{e}_\gamma$ are unit vectors pointing the directions of the positron and photon momenta, respectively. It remains unchanged after the transformation from the e^+e^- cms frame to the hadronic rest frame described above. Here $y_{1,2} = \frac{s-Q^2}{2s}(1 \mp \cos\theta_\gamma)$, θ_γ is the photon polar angle in the e^+e^- center of mass frame and $i,j = 1,2,3$ in L^{ij}. Other components of L^{ij} do not contribute to the matrix element. The close similarity with Eq. (10) is obvious. As a result

$$\begin{aligned} L^{ij}H_{ij} \simeq\ & \frac{(4\pi\alpha)^3}{4Q^2 y_1 y_2} \left(1+\cos^2\theta_\gamma\right) \Bigg\{ |G_M|^2 \left(1+\cos^2\theta_{\bar{\Lambda}}\right) + \frac{1}{\tau}|G_E|^2 \sin^2\theta_{\bar{\Lambda}} - \alpha_\Lambda \frac{Im(G_M G_E^*)}{\sqrt{\tau}} \sin(2\theta_{\bar{\Lambda}}) \left(n^y_{\pi^-} - n^y_{\pi^+}\right) \\ & + \alpha_\Lambda^2 \frac{Re(G_M G_E^*)}{\sqrt{\tau}} \sin(2\theta_{\bar{\Lambda}}) \left(n^z_{\pi^-} n^x_{\pi^+} + n^z_{\pi^+} n^x_{\pi^-}\right) - \alpha_\Lambda^2 \left(\frac{1}{\tau}|G_E|^2 + |G_M|^2\right) \sin^2\theta_{\bar{\Lambda}} \ \ n^x_{\pi^+} n^x_{\pi^-} \\ & - \alpha_\Lambda^2 \left(\frac{1}{\tau}|G_E|^2 - |G_M|^2\right) \sin^2\theta_{\bar{\Lambda}} \ \ n^y_{\pi^+} n^y_{\pi^-} + \alpha_\Lambda^2 \left(\frac{1}{\tau}|G_E|^2 \sin^2\theta_{\bar{\Lambda}} - |G_M|^2 \left(1+\cos^2\theta_{\bar{\Lambda}}\right)\right) \ \ n^z_{\pi^+} n^z_{\pi^-} \Bigg\} \ , \qquad (18) \end{aligned}$$

with the angle $\theta_{\bar{\Lambda}}$ being defined now in RF frame and n_{π^+} (n_{π^-}) in the $\bar{\Lambda}$ (Λ) rest frame. Hence after measuring $|G_M|$ and $|G_E|$ with the method proposed in [1] and successfully used by BaBar for the proton form factors [2] (no information on the spin is necessary in this case), one can measure the combination $\alpha_\Lambda \sin(\Delta\phi)$, where $\Delta\phi$ is the relative phase of the two form factors. This can be achieved for example by measuring the asymmetry

$$\mathcal{A}_y^\pm = \frac{d\sigma(a^\pm > 0) - d\sigma(a^\pm < 0)}{d\sigma(a^\pm > 0) + d\sigma(a^\pm < 0)} \ , \qquad (19)$$

where $a^{+(-)} = \sin(2\theta_{\bar{\Lambda}})\ n^y_{\pi^+(\pi^-)}$. Since α_Λ is known with good precision [13], $\sin(\Delta\phi)$, and thus $\Delta\phi$, is determined up to a twofold ambiguity, which can be resolved by measuring the correlations between $n^z_{\pi^-}$ and $n^x_{\pi^+}$. In principle, by studying the double spin correlations one can measure both the sign of $\cos(\Delta\phi)$ and α_Λ. However, the expected number of events at B–factories amounts to a few hundred to a few thousand distributed in the energy range $4m_\Lambda^2 < Q^2 < 10\ \mathrm{GeV}^2$, depending on the accumulated luminosity and the angular cuts in the experimental analysis (see Section IV). The measurement of α_Λ thus cannot be competitive to the existing measurements [13].

III. THE IMPLEMENTATION INTO THE EVENT GENERATOR PHOKHARA

To allow for quantitative studies we have implemented the process into the event generator PHOKHARA [1, 3–9]. This implementation will be available within the version PHOKHARA6.0. We use the leading order matrix element only, since the expected cross section is small and correspondingly the expected statistical precision is low. The radiative corrections are expected to be of the order of a few percent [1]. The generation of events is based on the method used in the generator TAUOLA [16], performing first the generation with the unpolarized cross section $d\sigma(e^+e^- \to \Lambda\bar{\Lambda})$ and generating in the second step the proper distributions of the decay products by evaluating the ratio of the cross sections with and without spin effects. Standard tests (see for example [6]), where the generation results were compared with existing analytic formulae, assure the technical accuracy of the implementation at the level of 10^{-4}.

The cross section $\sigma(e^+e^- \to \bar{\Lambda}\Lambda)$ was measured at one energy [14] only, and with poor accuracy. No form factor has been extracted. Thus one has to relay on theoretical assumptions. For the isospin and SU(3) structure of the electromagnetic form factors of the baryon octet we adopt the model ansatz suggested in reference [15]. There all form factors are given in terms of ω-, ϕ- and ρ-dominated contributions. Indeed, the ρ piece does not contribute in the case of $\bar{\Lambda}\Lambda$ production. The adopted model predicts real form factors G_M and G_E and thus zero relative phase $\Delta\phi$. The size of the relative phase between G_M and G_E does not alter the cross section and thus till now no experimental information is available on $\Delta\phi$. To show the effects of a possibly large relative phase we add an ad hoc phase in the analysis presented in the next section.

IV. FEASIBILITY STUDY FOR B- FACTORIES

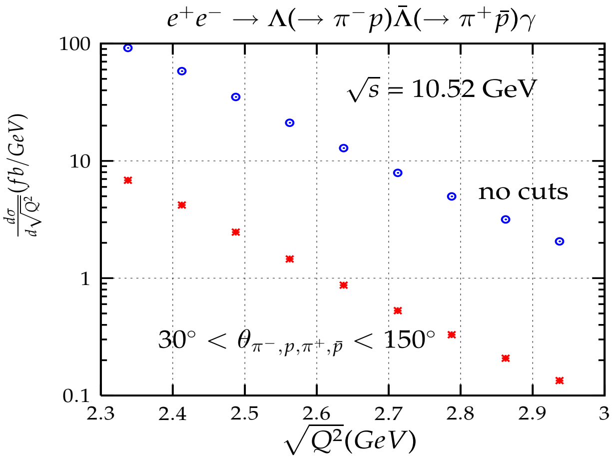

FIG. 2: (color online). The predicted differential cross section of the reaction $e^+e^- \to \Lambda(\to \pi^- p)\bar{\Lambda}(\to \pi^+ \bar{p})\gamma$

The predicted cross sections as functions of Q^2, with no cuts on the final state particles (open circles) and with angular cuts roughly corresponding to the angular range covered by the BaBar detector (filled circles), are shown in Fig. 2. The integrated cross sections, for $\sqrt{Q^2}$ values from threshold up to 3 GeV, are equal to 18 fb when no angular cuts are applied and 1.3 fb, if we restrict the polar angles of the produced charged particles between 30 and 150 degrees. So typically at B-factories we expect about 130 events per 100 fb^{-1} accumulated luminosity. With the already accumulated luminosity at BaBar (390 fb^{-1}) and BELLE (710 fb^{-1}) the Λ form factors can be studied with a decent accuracy.

As already said, the adopted model predicts almost zero asymmetry $\mathcal{A}_y^{\pm}$. However, a priori a much larger relative phase between the G_M and G_E could be present. In Fig.3 the predicted asymmetry is shown for the maximal relative phase between the form factors. It is clear that in this case the measurement of the phase is feasible. Thus the experimental studies of the asymmetry will allow to obtain further constrains on the form factor models supplementing the information obtained by the $|G_M|$ and $|G_E|$ measurement through angular distributions of the

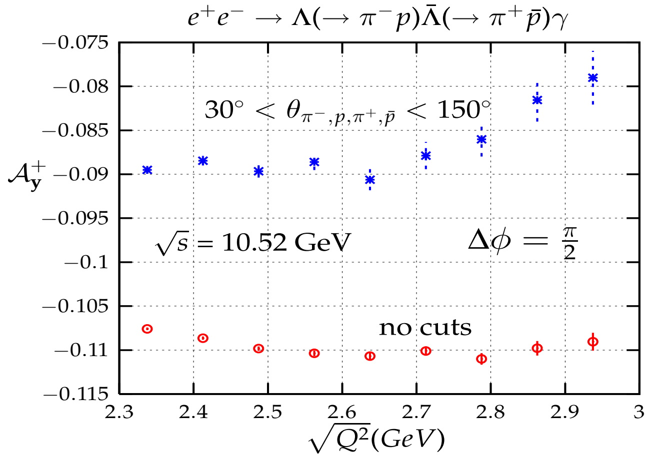

FIG. 3: (color online). The asymmetry $\mathcal{A}_y^+$ (see Eq.(19) for definition) for the maximal relative phase ($\Delta\phi = \pi/2$) between the G_M and G_E form factors.

Λs. More refined strategies of this measurement are conceivable if the asymmetry is studied as a function of θ_Λ. Furthermore, the sample is doubled by considering Λ and $\bar{\Lambda}$ decays.

The combination $Re(G_M G_E^*)$ (or $\cos(\Delta\phi)$) can be extracted by measuring the asymmetry

$$\mathcal{A}_{xz} = \frac{d\sigma(\tilde{a} > 0) - d\sigma(\tilde{a} < 0)}{d\sigma(\tilde{a} > 0) + d\sigma(\tilde{a} < 0)} \; ,$$
$$\tilde{a} = \sin(2\theta_{\bar{\Lambda}})\,(n^z_{\pi^-} n^x_{\pi^+} + n^z_{\pi^+} n^x_{\pi^-}) \; . \qquad (20)$$

Knowing both $\sin(\Delta\phi)$ and $\cos(\Delta\phi)$ the relative phase is fixed. The predicted asymmetry $\mathcal{A}_{xz}$ for $\Delta\phi = \pi$ and a B- factory energy is shown in Fig. 4. Measuring other spin correlations or respective asymmetries can in principle allow for the measurement of α_Λ and independent measurements of $|G_M|$ and $|G_E|$. On the one hand, due to the limited statistics, correlations studies alone will not be competitive with the existing measurement of α_Λ [13] or with the extraction of $|G_M|$ and $|G_E|$ by means of the method proposed in [1]. On the other hand, for each value of Q^2 the distribution, which is differential in $\cos\theta_\Lambda$ and in the variables describing the decay distributions of Λ and $\bar{\Lambda}$, depends on the three quantities $|G_M|$, $|G_E|$ and $\Delta\phi$ only. A more refined experimental investigation of this distribution, using the generator PHOKHARA, may well lead to a reasonably precise determination of these three numbers.

V. SUMMARY

A compact formula has been presented for the spin dependent baryon production through e^+e^- annihilation

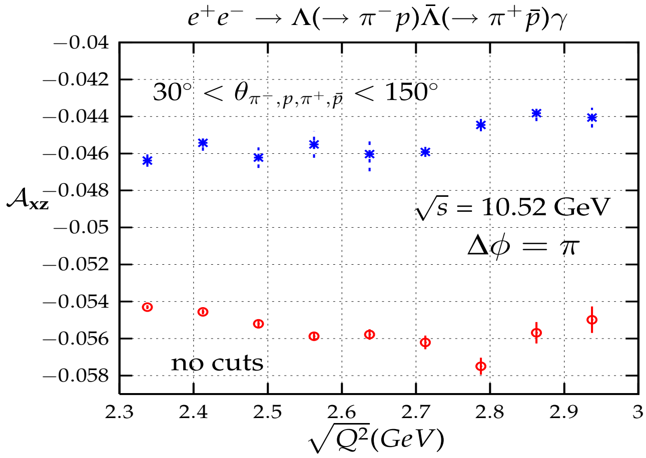

FIG. 4: (color online). The asymmetry $\mathcal{A}_{xz}$ (see Eq.(20) for definition) for the maximal relative phase ($\Delta\phi = \pi$) between the G_M and G_E form factors.

and through the radiative return. It describes the dependence on a single spin as well as spin-spin correlations. Single spin asymmetries can be used to determine the relative phase between the electric and the magnetic form factor up to a twofold ambiguity, which can be resolved by analyzing spin–spin correlations. The construction of an extension of the Monte Carlo event generator PHOKHARA allowed us to demonstrate that the spin dependent decay distribution of Λ (and $\bar{\Lambda}$) can be used to perform this analysis under realistic experimental conditions. Given enough data, the study could be extended in a straightforward way to $\bar{\Sigma}\Sigma$, $\bar{\Xi}\Xi$ production with the subsequent decays $\Sigma^+ \to n\pi^+, p\pi^0$, $\Sigma^- \to n\pi^-$, $\Xi^0 \to \Lambda\pi^0$ and $\Xi^- \to \Lambda\pi^-$ as typical examples.

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