SFB/CPP-06-19
TTP06-14
hep-ph/0604194

Top Quark Mediated Higgs Decay into Hadrons to Order α_s^5

P. A. Baikov
Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow 119992, Russia

K. G. Chetyrkin*
Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

We present in analytic form the $\mathcal{O}(\alpha_s^5)$ correction to the $H \to gg$ partial width of the standard-model Higgs boson with intermediate mass $M_H < 2M_t$. Its knowledge is useful because the $\mathcal{O}(\alpha_s^4)$ correction is sizeable (around 20%). For $M_H = 120$ GeV, the resulting QCD correction factor reads $1 + (215/12)\alpha_s^{(5)}(M_H)/\pi + 152.5\left(\alpha_s^{(5)}(M_H)/\pi\right)^2 + 381.5\left(\alpha_s^{(5)}(M_H)/\pi\right)^3 \approx 1 + 0.65 + 0.20 + 0.02$. The new four-loop correction increases the total Higgs-boson hadronic width by a small amount of order 1‰ and stabilizes significantly the residual scale dependence.

PACS numbers: 14.80.Bn 12.38.-t 12.38.Bx

INTRODUCTION

Within the Standard Model (SM) the scalar Higgs boson is responsible for mechanism of the electroweak mass generation. It is the last fundamental particle in the SM which has not yet been directly observed. Its future (non-)discovery will be of primary importance for all the particle physics. The SM Higgs boson mass is constrained from below, $M_H > 114$ GeV, by experiments at LEP and SLC [1, 2]. Indirect constraints from precision electroweak measurements [3] set an upper limit of 200 GeV on M_H.

Adopting the framework of the SM, the coupling of the Higgs boson to gluons is mediated by virtual massive quarks [4] and it is this coupling which plays a crucial rôle in Higgs phenomenology. Indeed, with the Yukawa couplings of the Higgs boson to quarks being proportional to the respective quark masses, the ggH coupling of the SM is essentially generated by the top quark alone. The ggH coupling strength becomes independent of the top-quark mass M_t in the limit $M_H \ll 2M_t$.

The process of the gluon-fusion, $gg \to H$, provides a very important Higgs production mechanism over the all M_H range under consideration. The corresponding cross-section is significantly increased, by approximately 70%, by next-to-leading order (NLO) QCD corrections, available since long [5–8]. The largeness of the correction along with the large residual scheme dependence of the result have motivated the calculations of the NNLO terms [9–11]. Very recently even the leading N^{3}LO corrections to the inclusive cross-section have been computed [12]. As a result of these remarkable theoretical advances the theoretical uncertainty of the production cross section is reduced significantly and is estimated around 20%.

The QCD corrections to the closely-related process—the production cross-section of the Higgs decay into two gluons—are presently known to NNLO [13, 14] only. Recently it was pointed out in work [15] that the ratio of the production cross-section to the decay rate is significantly less (by a factor of 2) sensitive to higher order QCD corrections than the individual observables, since the corresponding K-factors are similar in size and tend to cancel to a significant extent. The work also argues that it is this ratio which presents the theoretical input to analyses of Higgs couplings extractions at the LHC. Thus, the knowledge of the N^{3}LO QCD corrections to the Higgs decay rate into gluons is highly desirable.

In this publication we present in analytic form the four-loop $\mathcal{O}(\alpha_s^5)$ correction to the $H \to gg$ partial width of the standard-model Higgs boson with mass $M_H < 2M_t$.

CALCULATION AND RESULTS

We start by constructing an effective Lagrangian, $\mathcal{L}_{\rm eff}$, by integrating out the top quark [13, 16]. This Lagrangian is a linear combination of certain dimension-four operators acting in QCD with five quark flavours, while all M_t dependence is contained in the coefficient functions. We then renormalize this Lagrangian and compute with its help the $H \to gg$ decay width through $\mathcal{O}(\alpha_s^5)$.

The effective Lagrangian can be written in the form

$$\mathcal{L}_{\rm eff} = -2^{1/4} G_F^{1/2} H C_1 \left[O_1'\right]. \tag{1}$$

Here, $[O_1']$ is the renormalized counterpart of the bare operator $O_1' = G^{0\prime}_{a\mu\nu} G^{0\prime\mu\nu}_a$, where $G_{a\mu\nu}$ is the colour field strength. The superscript 0 denotes bare fields, and primed objects refer to the five-flavour effective theory, C_1 is the corresponding renormalized coefficient function, which carries all M_t dependence.

Eq. (1) directly leads to a general expression for the $H \to gg$ decay width,

$$\Gamma(H \to gg) = \frac{\sqrt{2}G_F}{M_H} C_1^2 \mathrm{Im}\, \Pi^{GG}(q^2 = M_H^2), \qquad (2)$$

where

$$\Pi^{GG}(q^2) = \int e^{iqx} \langle 0|T\left([O_1'](x)\,[O_1'](0)\right)|0\rangle \,\mathrm{d}x \qquad (3)$$

is the vacuum polarization induced by the gluon operator at $q^2 = M_H^2$, with q being the external four-momentum.

It is customary to write eq. (2) in the form

$$\Gamma(H \to gg) = K\,\Gamma_{\mathrm{Born}}(H \to gg), \qquad (4)$$

where (G_F is Fermi's constant)

$$\Gamma_{\mathrm{Born}}(H \to gg) = \frac{G_F M_H^3}{36\pi\sqrt{2}} \left(\frac{\alpha_s^{(n_l)}(M_H)}{\pi}\right)^2, \qquad (5)$$

and so-called K-factor reads:

$$K = \frac{72\,\pi^3}{M_H^4}\, \frac{C_1^2\ \mathrm{Im}\, \Pi^{GG}(q^2 = M_H^2)}{\left(\alpha_s^{(n_l)}(M_H)\right)^2} = 1 + \ldots \qquad (6)$$

The coefficient function C_1 is known in N^{3}LO [17–19] and reads

$$\begin{aligned}
C_1^{\mathrm{OS}} =& -\frac{1}{12}\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\Bigg\{1 + \frac{\alpha_s^{(n_f)}(\mu)}{\pi}\left(\frac{11}{4} - \frac{1}{6}\ell_{\mu t}\right) + \left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^2 \left[\frac{2693}{288} - \frac{25}{48}\ell_{\mu t} + \frac{1}{36}\ell_{\mu t}^2 + n_l\left(-\frac{67}{96} + \frac{1}{3}\ell_{\mu t}\right)\right] \\
&+ \left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^3 \Bigg[-\frac{4271255}{62208} - \frac{2}{3}\zeta(2)\left(1 + \frac{\ln 2}{3}\right) + \frac{1306661}{13824}\zeta(3) - \frac{4937}{864}\ell_{\mu t} + \frac{385}{144}\ell_{\mu t}^2 - \frac{1}{216}\ell_{\mu t}^3 \\
&+ n_l\left(\frac{181127}{62208} + \frac{1}{9}\zeta(2) - \frac{110779}{13824}\zeta(3) + \frac{109}{48}\ell_{\mu t} + \frac{53}{96}\ell_{\mu t}^2\right) + n_l^2\left(-\frac{6865}{31104} + \frac{77}{1728}\ell_{\mu t} - \frac{1}{18}\ell_{\mu t}^2\right)\Bigg]\Bigg\} \\
\approx& -\frac{1}{12}\frac{\alpha_s^{(n_f)}(M_t)}{\pi}\left[1 + 2.75000\frac{\alpha_s^{(n_f)}(M_t)}{\pi} + 5.86111\left(\frac{\alpha_s^{(n_f)}(M_t)}{\pi}\right)^2 + 5.39967\left(\frac{\alpha_s^{(n_f)}(M_t)}{\pi}\right)^3\right]. \qquad (7)
\end{aligned}$$

Here $\ell_{\mu t} = \ln\frac{\mu^2}{M_t^2}$, with M_t being the on-shell top quark mass; we have displayed for generality the result with the effective number of light quark flavors denoted as n_l. In the numerical evaluation we have set $\mu = M_t$ and $n_l = n_f - 1 = 5$. $\zeta_n \equiv \zeta(n)$ is the Riemann's Zeta-function.

Thus we are left with the calculation of the last factor in Eq. (2), namely the absorptive part Π^{GG} in N^{3}LO, that is to α_s^3. In fact it turned out to be more convenient to calculate the correlator (3) per se and take its absorptive part subsequently. Since Π^{GG} starts in the leading order from a one-loop diagram, the $\mathcal{O}(\alpha_s^3)$ calculation faces as many as 10240 *four loop* diagrams (at NNLO [13] the number was 403).

The overall strategy of our calculations was identical to that used by us before in works [20, 21]. First we generate the contributing diagrams with the package QGRAF [22]. Second, using the criterion of irreducibility of Feynman integrals [23, 24], the set of irreducible integrals involved in the problem was constructed. Third, the coefficients multiplying these integrals were calculated as series in the $1/D \to 0$ expansion with the help of an auxiliary integral representation [25]. Fourth, the exact answer, i.e. a rational function of D, was reconstructed from this expansion. The major part of the calculations was performed on the SGI Altix 3700 computer (32 Itanium-2 1.3 GHz processors) using the parallel version of FORM [26–28] and took about two weeks of calendar time in total.

After renormalization, our result reads

$$\mathrm{Im}\, \Pi^{GG}(q^2) = q^4\, \frac{2}{\pi}\left\{1 + \sum_{i=1}^{\infty} g_i\,(a_s')^i\right\}, \qquad (8)$$

$$g_1 = n_l\left[-\frac{7}{6}\right]+\frac{73}{4}, \tag{9}$$

$$g_2 = n_l^2\left[\frac{127}{108}-\frac{1}{36}\pi^2\right]+n_l\left[-\frac{7189}{144}+\frac{11}{12}\pi^2+\frac{5}{4}\zeta_3\right] + \frac{37631}{96}-\frac{121}{16}\pi^2-\frac{495}{8}\zeta_3, \tag{10}$$

$$\begin{aligned} g_3 &= n_l^3\left[-\frac{7127}{5832}+\frac{7}{108}\pi^2+\frac{1}{27}\zeta_3\right] \\ &+ n_l^2\left[\frac{115207}{1296}-\frac{1609}{432}\pi^2-\frac{113}{24}\zeta_3\right] \\ &+ n_l\left[-\frac{368203}{216}+\frac{18761}{288}\pi^2+\frac{11677}{48}\zeta_3-\frac{95}{36}\zeta_5\right] \\ &+ \left[\frac{15420961}{1728}-352\,\pi^2-\frac{44539}{16}\zeta_3+\frac{3465}{8}\zeta_5\right], \end{aligned} \tag{11}$$

where a'_s stands for $\alpha_s^{(n_l)}/\pi$.

In eqs. (9-11) we have set $\mu^2 = q^2$, the full μ dependence can be easily recovered with the standard RG techniques from the fact the product

$$\left(\beta^{(n_l)}(a'_s)\right)^2\, \Pi^{GG}(a'_s, \mu/q^2)$$

is scale independent ([16]). In numerical form $\mathrm{Im}\,\Pi^{GG}$ reads (we set $n_l = 5$)

$$\frac{2\,q^4}{\pi}\,\mathrm{Im}\,\Pi^{GG} = 1 + 12.4167\,a_s + 68.6482\,a_s^2 - 212.447\,a_s^3. \tag{12}$$

In order to better understand the structure of the α_s^3 term in (12) it is instructive to separate the genuine four-loop contributions from the function $\Pi^{GG}(q^2)$ from essentially "kinematical", so-called π^2-terms originating from the analytic continuation. For a given order in α_s these extra contributions are completely predictable from the standard evolution equations applied to the "more leading" terms in $\Pi^{GG}(q^2)$ proportional to some smaller powers of α_s. The corresponding expression for $\mathrm{Im}\,\Pi^{GG}$ assumes the form

$$\begin{aligned} \frac{2\,q^4}{\pi}\,\mathrm{Im}\,\Pi^{GG} &= 1 + 12.4167\,a_s \\ &+ (104.905 - \underline{36.257})\,a_s^2 + (886.037 - \underline{1098.48})\,a_s^3, \end{aligned} \tag{13}$$

where we have underlined the contributions coming from analytic continuation. Thus, the present calculation confirms the pattern first observed on the example of the scalar correlator in work [21]: the "kinematical" π^2 terms tend to neatly cancel the genuine higher order contributions.

We are now in a position to find the $\mathcal{O}(\alpha_s^3)$ term of the K factor in Eq. (6). To this end, we first multiply C_1^2 by $R^G(q^2 = M_H^2)$, then eliminate $\alpha_s^{(6)}(\mu)$ in favour of $\alpha_s^{(n_l)}(\mu)$ [17, 29, 30] and, finally, choose the $\mu = M_H$ (to get a compact expression). The result reads:

$$K = 1 + 17.9167\,a'_s + (156.81 - 5.7083\ln\frac{M_t^2}{M_H^2})(a'_s)^2 + (467.68 - 122.44\ln\frac{M_t^2}{M_H^2} + 10.94\ln^2\frac{M_t^2}{M_H^2})\;(a'_s)^3. \tag{14}$$

If we also use $M_t = 175$ GeV, $M_H = 120$ GeV and $\alpha_s^{(5)}(M_H)/\pi = 0.0363$ we arrive at

$$\begin{aligned} K &= 1 + 17.9167\,a'_s + 152.5(a'_s)^2 + 381.5(a'_s)^3 \\ &= 1 + 0.65038 + 0.20095 + 0.01825. \end{aligned} \tag{15}$$

Thus, we observe that, unlike the NLO and NNLO cases, the $\mathcal{O}(\alpha_s^3)$ correction, being by a factor more than ten less than the previous α_s^2 one, has quite a moderate size. The welcome stability of the perturbation theory is also confirmed by testing the scale dependence of the K-factor. By changing the scale μ from $M_H/2$ to $2\,M_H$ we find that the maximal deviation of the K-factor from its value at $\mu = M_H$ decreases from 24% (LO) to 22% (NLO), 10% (NNLO) and, finally, to only 3% at NNNLO!

CONCLUSION

We have computed the $\mathrm{N^3LO}$ correction of order α_s^5 to the $H \to gg$ partial width of the standard-model Higgs boson with intermediate mass $M_H < 2M_t$. Our calculation significantly reduces the theoretical uncertainty of the QCD prediction for this important process.

We are grateful to J.H. Kühn for his interest in the work, numerous discussions and careful reading the manuscript. We want to thank M. Steinhauser for his timely reminding the importance of the calculation.

This work was supported by the Deutsche Forschungsgemeinschaft in the Sonderforschungsbereich/Transregio SFB/TR-9 "Computational Particle Physics", by INTAS (grant 03-51-4007) and by RFBR (grant 05-02-17645).

---

* Permanent address: Institute for Nuclear Research, Russian Academy of Sciences, Moscow 117312, Russia.

[1] R. Barate et al. (ALEPH), Phys. Lett. **B565**, 61 (2003), hep-ex/0306033.
[2] [LEP Collaborations] (2004), hep-ex/0412015.
[3] M. Carena and H. E. Haber, Prog. Part. Nucl. Phys. **50**, 63 (2003), hep-ph/0208209.
[4] F. Wilczek, Phys. Rev. Lett. **39**, 1304 (1977).
[5] S. Dawson, Nucl. Phys. **B359**, 283 (1991).
[6] D. Graudenz, M. Spira, and P. M. Zerwas, Phys. Rev. Lett. **70**, 1372 (1993).
[7] M. Spira, A. Djouadi, D. Graudenz, and P. M. Zerwas, Nucl. Phys. **B453**, 17 (1995), hep-ph/9504378.
[8] A. Djouadi, M. Spira, and P. M. Zerwas, Phys. Lett. **B264**, 440 (1991).
[9] R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88**, 201801 (2002), hep-ph/0201206.
[10] C. Anastasiou and K. Melnikov, Nucl. Phys. **B646**, 220 (2002), hep-ph/0207004.
[11] V. Ravindran, J. Smith, and W. L. van Neerven, Nucl. Phys. **B665**, 325 (2003), hep-ph/0302135.
[12] S. Moch and A. Vogt, Phys. Lett. **B631**, 48 (2005), hep-ph/0508265.
[13] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Phys. Rev. Lett. **79**, 353 (1997), hep-ph/9705240.
[14] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Nucl. Phys. **B510**, 61 (1998), hep-ph/9708255.
[15] C. Anastasiou, K. Melnikov, and F. Petriello, Phys. Rev. **D72**, 097302 (2005), hep-ph/0509014.
[16] T. Inami, T. Kubota, and Y. Okada, Z. Phys. **C18**, 69 (1983).
[17] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Nucl. Phys. **B510**, 61 (1998), hep-ph/9708255.
[18] Y. Schroder and M. Steinhauser, JHEP **01**, 051 (2006), hep-ph/0512058.
[19] K. G. Chetyrkin, J. H. Kühn, and C. Sturm (2005), hep-ph/0512060.
[20] P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Phys. Rev. Lett. **95**, 012003 (2005), hep-ph/0412350.
[21] P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Phys. Rev. Lett. **96**, 012003 (2006), hep-ph/0511063.
[22] P. Nogueira, J. Comput. Phys. **105**, 279 (1993).
[23] P. A. Baikov, Phys. Lett. **B474**, 385 (2000), hep-ph/9912421.
[24] P. A. Baikov, Phys. Lett. **B634**, 325 (2006), hep-ph/0507053.
[25] P. A. Baikov, Phys. Lett. **B385**, 404 (1996), hep-ph/9603267.
[26] J. A. M. Vermaseren (2000), math-ph/0010025.
[27] D. Fliegner, A. Retey, and J. A. M. Vermaseren (1999), hep-ph/9906426.
[28] D. Fliegner, A. Retey, and J. A. M. Vermaseren (2000), hep-ph/0007221.
[29] W. Bernreuther and W. Wetzel, Nucl. Phys. **B197**, 228 (1982).
[30] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Phys. Rev. Lett. **79**, 2184 (1997), hep-ph/9706430.