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Two-Loop Matching Coefficients for Heavy Quark Currents

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Abstract

In this paper we consider the matching coefficients up to two loops between Quantum Chromodynamics (QCD) and Non-Relativistic QCD (NRQCD) for the vector, axial-vector, scalar and pseudo-scalar currents. The structure of the effective theory is discussed and analytical results are presented. Particular emphasis is put on the singlet diagrams.

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In the recent years quite a lot of activity has been devoted to the treatment of bound states of two heavy particles both in QED and QCD (for a recent review see, e.g., Ref. [1]). From the theory point of view the calculations have been put onto a solid basis due to the formulation of proper effective theories [2, 3], NRQED and NRQCD, respectively, which provide the possibility to systematically evaluate higher order corrections. The construction of the effective theories consists of essentially two steps: First, the effective operators involving the light degrees of freedom have to be constructed and second the corresponding couplings, the so-called coefficient functions, have to be computed by comparing the full and the effective theories. The latter is also referred to as matching calculation.

The framework which is considered in this letter consists of QCD accompanied by external currents where we allow for vector, axial-vector, scalar and pseudo-scalar couplings. The main results of this letter are the two-loop matching coefficients. Thus, following the prescription outlined above we determine in a first step the effective currents and then perform a matching calculation.

The matching coefficients provided in this paper constitute a building block in all calculations involving the corresponding external currents. This includes in particular production and decay processes of heavy quarkonia or the production of top quark pairs close to threshold. One could also think of the decay of a CP-even or CP-odd Higgs boson (with mass M) into two quarks with $2m \approx M$.

The basic idea behind the construction of the Lagrange density for NRQCD is to expand all terms of the QCD Lagrangian in the limit of a large quark mass. A similar procedure has to be applied to external currents which we define in coordinate space as

$$\begin{aligned} j_v^\mu &= \bar{\psi}\gamma^\mu\psi\,, \\ j_a^\mu &= \bar{\psi}\gamma^\mu\gamma_5\psi\,, \\ j_s &= \bar{\psi}\psi\,, \\ j_p &= \bar{\psi}i\gamma_5\psi\,. \end{aligned} \tag{1}$$

Note that the anomalous dimension of j_v^μ and j_a^μ is zero whereas for the scalar and pseudo-scalar current it is obtained from the renormalization constant $Z_s = Z_p = Z_m$, which is given at the two-loop level in Ref. [4].

In order to perform the transition to the effective theory it is convenient to work in momentum space and to introduce the two-component Pauli-spinors in the form

$$u(\vec{p}) = \sqrt{\frac{E+m}{2E}}\left(\begin{array}{c}\chi \\ \frac{\vec{p}\cdot\vec{\sigma}}{E+m}\chi\end{array}\right)\,,\; v(-\vec{p}) = \sqrt{\frac{E+m}{2E}}\left(\begin{array}{c}\frac{(-\vec{p})\cdot\vec{\sigma}}{E+m}\phi \\ \phi\end{array}\right)\,, \tag{2}$$

where m denotes the heavy quark mass. In Eq. (2) χ is a spinor that annihilates a heavy quark and ϕ correspondingly creates a heavy anti-quark with momentum $\vec{p}$.

In a first step we want to express the currents of Eq. (1) in terms of ϕ and χ and expand in the inverse heavy quark mass. This actually leads to the tree-level matching conditions. When inserting Eq. (2) into (1) it turns out to be convenient to split the time-like and space-like coefficients of the vector and axial-vector currents. This leads to

$$\begin{aligned} j_v^0 &= 0 + \mathcal{O}\left(\frac{1}{m^2}\right)\,, \\ j_v^k &= \tilde{j}_v^k + \mathcal{O}\left(\frac{1}{m^2}\right)\,, \\ j_a^0 &= i\tilde{j}_p + \mathcal{O}\left(\frac{1}{m^2}\right)\,, \\ j_a^k &= \tilde{j}_a^k + \mathcal{O}\left(\frac{1}{m^3}\right)\,, \\ j_s &= \tilde{j}_s + \mathcal{O}\left(\frac{1}{m^3}\right)\,, \\ j_p &= \tilde{j}_p + \mathcal{O}\left(\frac{1}{m^2}\right)\,, \end{aligned} \tag{3}$$

where $k = 1, 2, 3$ and the currents in the effective theory are given by

$$\begin{aligned}
\tilde{j}_v^k &= \phi^\dagger \sigma^k \chi \,, \\
\tilde{j}_a^k &= \frac{1}{2m} \phi^\dagger [\sigma^k, \vec{p}\cdot\vec{\sigma}] \chi \,, \\
\tilde{j}_s &= -\frac{1}{m} \phi^\dagger \vec{p}\cdot\vec{\sigma} \chi \,, \\
\tilde{j}_p &= -i \phi^\dagger \chi \,.
\end{aligned} \tag{4}$$

Note that $\tilde{j}_p$ also appears in the expansion of j_a^0 which means that the corresponding matching coefficients are equal. This will be used as a check of our calculation. Due to the occurrence of the momentum $\vec{p}$ in $\tilde{j}_a^k$ and $\tilde{j}_s$ an expansion in the external momenta has to be performed in order to obtain the loop corrections to the corresponding matching coefficients.

The basic idea to obtain the matching coefficients is to compute vertex corrections induced by the considered current both in the full and the effective theory. In practice it is convenient to consider the renormalized vertex function with two external on-shell quarks and to perform an asymptotic expansion about $s = 4m^2$, where s is the momentum squared of the external current, the so-called threshold expansion [5,6]. Denoting by Γ_x the proper structure of the genuine vertex corrections and by Z_2 and Z_x the renormalization constants due to the quark wave function and the anomalous dimension of the current one obtains the equation

$$Z_2 Z_x \Gamma_x(q_1, q_2) \;=\; c_x \tilde{Z}_2 \tilde{Z}_x^{-1} \tilde{\Gamma}_x + \ldots \,, \tag{5}$$

where $x \in \{v, a, s, p\}$ with the understanding that the axial-vector part is split into time-like and space-like components. The ellipses denote terms suppressed by inverse powers of the heavy quark mass and the quantities in the effective theory are marked by a tilde. c_x is the matching coefficient we are after. In our approximation $\tilde{Z}_2 = 1$. Z_2 to two loops has been computed in Ref. [7]. As far as $\tilde{\Gamma}_x$ is concerned only the tree-level result determined by $\tilde{j}_x$ contributes to Eq. (5). The momenta q_1 and q_2 in Eq. (5) correspond to the outgoing momenta of the quark and anti-quark which are considered on-shell.

Starting from order α_s^2 the matching coefficients c_x exhibit infra-red divergences which are compensated by ultra-violet divergences of the effective theory rendering physical quantities finite. In Eq. (5) the renormalization constant $\tilde{Z}_x$ which generates the anomalous dimension of $\tilde{j}_x$ takes over this part.

The quantities Γ_x are conveniently obtained with the help of projectors which are constructed in such a way that they project on the coefficients of $\tilde{\Gamma}_x$. For the vector case, the zeroth component of the axial-vector and the pseudo-scalar case we can simply identify $q_1^2 = q_2^2 = q^2/4 = m^2$ and use

$$\begin{aligned}
\Gamma_v &= \mathrm{Tr}\left[P_\mu^{(v)} \Gamma^{(v),\mu}\right] \,, \\
\Gamma_p &= \mathrm{Tr}\left[P^{(p)} \Gamma^{(p)}\right] \,, \\
\Gamma_{a,0} &= \mathrm{Tr}\left[P_\mu^{(a,0)} \Gamma^{(a),\mu}\right] \,,
\end{aligned} \tag{6}$$

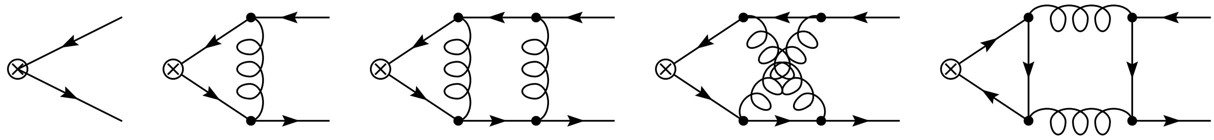

Figure 1: Feynman diagrams contributing to the matching coefficients. In (e) the so-called singlet diagram is shown which does not contribute to c_v. In the closed fermion loop all quark flavours have to be considered.

with

$$
\begin{aligned}
P_\mu^{(v)} &= \frac{1}{8(D-1)m^2}\left(-\frac{\not{q}}{2}+m\right)\gamma_\mu\left(\frac{\not{q}}{2}+m\right) , \\
P^{(p)} &= \frac{1}{8m^2}\left(-\frac{\not{q}}{2}+m\right)\gamma_5\left(\frac{\not{q}}{2}+m\right) , \\
P_\mu^{(a,0)} &= -\frac{1}{8m^2}\left(-\frac{\not{q}}{2}+m\right)\gamma_\mu\gamma_5\left(\frac{\not{q}}{2}+m\right) .
\end{aligned} \tag{7}
$$

As already mentioned above the case $(a,0)$ is used as a check for the pseudo-scalar matching coefficient.

For the axial-vector and scalar cases we have the equations analogous to Eq. (6). However, since the corresponding effective currents have a suppression factor $|\vec{p}\,|/m$ it is necessary to choose $q_1 = q/2 + p$ and $q_2 = q/2 - p$, to expand up to linear order in p and to set afterwards $p = 0$ and $q^2 = 4m^2$. Note that we choose a reference frame where $q \cdot p = 0$ [5,6]. Thus the projectors are more complicated and are given by

$$
\begin{aligned}
P_{(a,i),\mu} &= -\frac{1}{8m^2}\left\{\frac{1}{D-1}\left(-\frac{\not{q}}{2}+m\right)\gamma_\mu\gamma_5\left(-\frac{\not{q}}{2}+m\right)\right. \\
&\qquad \left. -\frac{1}{D-2}\left(-\frac{\not{q}}{2}+m\right)\frac{m}{p^2}\left((D-3)p_\mu+\gamma_\mu\not{p}\right)\gamma_5\left(\frac{\not{q}}{2}+m\right)\right\} , \\
P_{(s)} &= \frac{1}{8m^2}\left\{\left(-\frac{\not{q}}{2}+m\right)\mathbf{1}\left(-\frac{\not{q}}{2}+m\right)+\left(-\frac{\not{q}}{2}+m\right)\frac{m}{p^2}\not{p}\left(\frac{\not{q}}{2}+m\right)\right\} .
\end{aligned} \tag{8}
$$

In Fig. 1 some Feynman diagrams contributing to the matching coefficients are shown. Due to the application of the projectors the corresponding integrals can be reduced to the functions $J_\pm$ and $L_\pm$ as defined in Eqs. (14) and (55) of Ref. [5]. However, due to the expansion in the momentum p the powers of the denominators are higher and a systematic reduction of the scalar integrals to master integrals is necessary. For the current calculation we implemented the method of Ref. [8]. For some of the occuring integrals the program `AIR` [9] is applied. The details will be described elsewhere.

An important class of diagrams is constituted by the so-called singlet diagrams (cf. Fig. 1(e)) where the external current does not couple to the quark–anti-quark pair of the final state. Due to Furry's theorem there is no contribution to the vector case from these diagrams, however, non-vanishing, finite results are obtained for c_a, c_s and c_p. For the scalar and the pseudo-scalar currents only the heavy quark is running in the closed fermion loop. All other quark flavours are suppressed by the light quark mass. This is different for the axial-vector coupling. Here we consider the effective current formed by the top and bottom quark field

$$j_a^\mu \;=\; \bar{t}\gamma^\mu\gamma_5 t - \bar{b}\gamma^\mu\gamma_5 b\,, \tag{9}$$

which ensures the cancellation of the anomaly-like contributions. For the same reason the contributions from the remaining light quarks cancel.

In the analytical results given below the contributions from the singlet diagrams are marked separately. At this point we only want to mention that in the axial-vector and pseudo-scalar case γ_5 was treated according to the prescription of Ref. [10]. In practice this means that we perform the replacements

$$\begin{aligned} \gamma^\mu\gamma_5 &\rightarrow \frac{i}{3!}\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_\rho\gamma_\sigma\,, \\ \gamma_5 &\rightarrow \frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\,, \end{aligned} \tag{10}$$

strip off the ϵ tensor and deal with the objects with three and four indices, respectively. The corresponding projectors are obtained by performing the replacements of Eq. (10) in Eqs. (7) and (8) which makes them more complicated. However, the very calculation is in close analogy to the non-singlet case. After summing all two-loop contributions one obtains a finite result.

Note that for the non-singlet contributions it is save to use anti-commuting γ_5. Actually, the treatment according to Ref. [10] leads to a wrong result. This is due to the infra-red divergences which are absent in the singlet diagrams.

An alternative method to perform the calculation of the vertex corrections is based on the evaluation of the tensor integrals. Here, we used the T-operator method of Ref. [11] to reduce the tensor integrals to products of the metric tensor and external momenta and scalar integrals with shifted space-time dimension. The resulting Dirac structures were further simplified and rewritten in terms of NRQCD fields.

In addition to the bare two-loop diagrams we have to take into account the one-loop renormalization contribution from the heavy quark mass, which we renormalize on-shell, and the strong coupling renormalized in the $\overline{\mathrm{MS}}$ scheme.

We want to mention that all contributions have been evaluated for general gauge parameter ξ. The final results for the matching coefficients are all independent of ξ which constitutes an important check on our calculation.

Let us in the following present our results and compare with the literature. The two-loop matching coefficient for the vector current has been computed almost ten years ago [12, 13]. We confirmed these results and provide for completeness the analytical expressions

$$
\begin{aligned}
c_v &= 1 - 2\frac{\alpha_s(m)}{\pi}C_F + \left(\frac{\alpha_s(m)}{\pi}\right)^2 \left[C_F T\left(\frac{11}{18}n_l + \frac{22}{9} - \frac{4}{3}\zeta_2\right)\right. \\
&\quad + C_F^2\left(\frac{23}{8} - \frac{79}{6}\zeta_2 + 6\zeta_2\ln 2 - \frac{1}{2}\zeta_3 - \zeta_2\ln\frac{\mu^2}{m^2}\right) \\
&\quad \left. + C_F C_A\left(-\frac{151}{72} + \frac{89}{24}\zeta_2 - 5\zeta_2\ln 2 - \frac{13}{4}\zeta_3 - \frac{3}{2}\zeta_2\ln\frac{\mu^2}{m^2}\right)\right], \qquad (11)
\end{aligned}
$$

where $C_A = N_c$ and $C_F = (N_c^2-1)/(2N_c)$ are the Casimir operators of the adjoint and fundamental representation of $\mathrm{SU}(N_c)$, respectively, $T = 1/2$, and n_l is the number of massless quarks. ζ_n denotes Riemann's zeta-function. The one-loop result can already be found in Ref. [14]. The anomalous dimension of the effective vector current, which is related to $\tilde{Z}_v$ through $\gamma_v = \frac{\mathrm{d}\ln\tilde{Z}_v}{\mathrm{d}\ln\mu}$, reads

$$
\gamma_v = -\left(\frac{\alpha_s}{\pi}\right)^2\left(2C_F^2 + 3C_F C_A\right)\zeta_2\,. \qquad (12)
$$

Our results for the two-loop matching coefficients c_a, c_s and c_p are given by

$$
\begin{aligned}
c_a &= 1 - \frac{\alpha_s(m)}{\pi}C_F + \left(\frac{\alpha_s(m)}{\pi}\right)^2\left[C_F T\left(\frac{7}{18}n_l + \frac{20}{9} - \frac{4}{3}\zeta_2\right)\right. \\
&\quad + C_F^2\left(\frac{23}{24} - \frac{27}{4}\zeta_2 + \frac{19}{4}\zeta_2\ln 2 - \frac{27}{16}\zeta_3 - \frac{5}{4}\zeta_2\ln\frac{\mu^2}{m^2}\right) \\
&\quad \left. + C_F C_A\left(-\frac{101}{72} + \frac{35}{24}\zeta_2 - \frac{7}{2}\zeta_2\ln 2 - \frac{9}{8}\zeta_3 - \frac{1}{2}\zeta_2\ln\frac{\mu^2}{m^2}\right) + C_F T X_{\mathrm{sing}}^{(a)}\right], \\
c_s &= 1 - \frac{1}{2}\frac{\alpha_s(m)}{\pi}C_F + \left(\frac{\alpha_s(m)}{\pi}\right)^2\left[C_F T\left(-\frac{5}{36}n_l + \frac{121}{36} - 2\zeta_2\right)\right. \\
&\quad + C_F^2\left(\frac{5}{16} - \frac{37}{8}\zeta_2 + 3\zeta_2\ln 2 - \frac{11}{4}\zeta_3 - 2\zeta_2\ln\frac{\mu^2}{m^2}\right) \\
&\quad \left. + C_F C_A\left(\frac{49}{144} + \frac{1}{8}\zeta_2 - 3\zeta_2\ln 2 - \frac{5}{4}\zeta_3 - \frac{1}{2}\zeta_2\ln\frac{\mu^2}{m^2}\right) + C_F T X_{\mathrm{sing}}^{(s)}\right], \\
c_p &= 1 - \frac{3}{2}\frac{\alpha_s(m)}{\pi}C_F + \left(\frac{\alpha_s(m)}{\pi}\right)^2\left[C_F T\left(\frac{1}{12}n_l + \frac{43}{12} - 2\zeta_2\right)\right. \\
&\quad + C_F^2\left(\frac{29}{16} - \frac{79}{8}\zeta_2 + 6\zeta_2\ln 2 - \frac{9}{2}\zeta_3 - 3\zeta_2\ln\frac{\mu^2}{m^2}\right) \\
&\quad \left. + C_F C_A\left(-\frac{17}{48} + \frac{17}{8}\zeta_2 - 6\zeta_2\ln 2 - 3\zeta_3 - \frac{3}{2}\zeta_2\ln\frac{\mu^2}{m^2}\right) + C_F T X_{\mathrm{sing}}^{(p)}\right]. \qquad (13)
\end{aligned}
$$

The one-loop result for c_p can already be found in Ref. [15]; the two-loop coefficients of Eq. (13) are new. They constitute our main result. The one-loop coefficients can be easily obtained from the one-loop on-shell vertex corrections with arbitrary momentum squared of the external current, s. In the analytic expressions it is straightforward to perform the limit where the velocity of the produced quarks is small. After subtracting the leading term, which corresponds to the Coulomb singularity, one remains with the result for the matching coefficients [16]. At two loops this simple trick does not work any more and the calculation has to be performed from scratch as has been done in this letter.

The contributions from the singlet diagrams correspond to

$$\begin{aligned}
X_{\text{sing}}^{(a)} &= -\frac{23}{12}\zeta_2 + 4\zeta_2 \ln 2 - 2\ln 2 + \frac{2}{3}\ln^2 2 + i\pi\left(1 - \frac{2}{3}\ln 2\right) , \\
X_{\text{sing}}^{(s)} &= \frac{2}{3} - \frac{29}{12}\zeta_2 + 4\zeta_2 \ln 2 - \ln 2 + i\frac{\pi}{2} , \\
X_{\text{sing}}^{(p)} &= \frac{5}{4}\zeta_2 + 3\zeta_2 \ln 2 - \frac{21}{8}\zeta_3 + i\pi\frac{3}{4}\zeta_2 . \qquad (14)
\end{aligned}$$

$X_{\text{sing}}^{(s)}$ and $X_{\text{sing}}^{(p)}$ receive only contributions from diagrams which are finite and contain only the heavy quark. The corresponding result holds both for top and bottom quarks. This is different in the case of $X_{\text{sing}}^{(a)}$. Actually, the result in Eq. (14) corresponds to the case where top quarks are considered in the final state. Note that $X_{\text{sing}}^{(a)}$ receives contributions from diagrams with top and bottom quarks in the closed triangle loop (cf. Fig. 1(e)). Taken separately they are divergent, however, the sum is finite. If one considers bottom quarks in the final state one still has to consider top and bottom quarks in the closed triangle loop. Again only the sum of all diagrams is finite with the result

$$X_{\text{sing}}^{(a)} = \frac{55}{24} + \frac{19}{12}\zeta_2 - 4\zeta_2 \ln 2 - \frac{3}{4}\ln\frac{m_b^2}{m_t^2} + \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) . \qquad (15)$$

The diagram in Fig. 1(e) for axial-vector coupling and with bottom in the final state was also considered in Ref. [17, 18], for abritrary values of s and m_t, but for $m_b = 0$ so that no direct comparison with Eq. (15) is possible.

As mentioned above, it is possible to extract the result for c_p from the zero component of the axial-vector current. This is quite evident in the non-singlet case. However, for the singlet contribution this check is highly non-trivial since in this approach c_p is obtained from diagrams both with top and bottom quarks in the closed triangle whereas in the direct calculation only one type of quarks appears.

The singlet results of Eqs. (14) and (15) and the fermionic contributions of Eq. (13) are in agreement with Ref. [19–21] where the off-shell contributions have been considered. Since they do not develop an infrared singularity the limit $s \to 4m^2$ can be performed. This is different in the case of the non-fermionic contributions where due to the infrared divergence the off-shell results [20–22] cannot be used in order to obtain the matching coefficients.

For completeness we also provide the result for the anomalous dimensions corresponding to Eq. (13) which read

$$\begin{aligned}
\gamma_a &= -\left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{5}{2}C_F^2 + C_F C_A\right)\zeta_2\,,\\
\gamma_s &= -\left(\frac{\alpha_s}{\pi}\right)^2 \left(4C_F^2 + C_F C_A\right)\zeta_2\,,\\
\gamma_p &= -\left(\frac{\alpha_s}{\pi}\right)^2 \left(6C_F^2 + 3C_F C_A\right)\zeta_2\,.
\end{aligned} \tag{16}$$

The result for γ_p agrees with the one extracted from Ref. [23].

We want to mention that the coefficient c_p has been considered in Ref. [24] in the context of the B_c meson. The latter consists of two heavy quarks, however, with different masses. This makes the calculation significantly more difficult since two instead of one mass scale appear in the integrals. In Ref. [24] the reduction to master integrals has been performed exactly whereas the latter have been evaluated in the limit $m_c \ll m_b$ so that a comparison with the present analysis is not possible.

To summarize, in this paper we computed the two-loop matching coefficients between QCD and NRQCD for an axial-vector, scalar and pseudo-scalar current. Furthermore, we performed an independent check of the matching coefficient in the vector case. The latter contributes to the second order result of the threshold production of top quark pairs. The result for the axial-vector current only contributes to the fourth-order analysis which is currently still out of reach.

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