Electroweak corrections to large transverse momentum production of Z bosons and photons at the LHC*

A.Kulesza[1], *S.Pozzorini*[2], *M. Schulze*[1]
[1]Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany
[2]Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, D-80805 München, Germany

Abstract
We investigate the electroweak corrections to the Z-boson and direct photon production at large transverse momentum at the LHC. Analytical results for the high energy next-to-next-to-leading logarithmic approximation of the one-loop weak corrections and the two-loop electroweak corrections in the next-to-leading logarithmic approximation are presented. Numerical results showing the relevance of these corrections are discussed.

1. INTRODUCTION

The Large Hadron Collider (LHC) with its high center-of-mass energy and design luminosity will offer a unique possibility to explore production of gauge bosons with very large transverse momentum (p_{T}). Being embedded in the environment of hadronic collisions, the reaction necessarily involves hadronic physics, like parton distributions, and depends on the strong coupling constant. In turn, the cross section for the Z-boson or direct photon production at large p_{T} is an important means to constrain information on the parton distribution functions (PDFs). For this class of production processes at the LHC, electroweak (EW) corrections from virtual boson exchange become important. This is due to the presence of the logarithmic terms of the form $\alpha^k \log^{2k-i}(\hat{s}/M_W^2)$ (with $i = 0$ for the leading logarithmic (LL) terms, $i = 1$ for the next-to-leading logarithmic (NLL) terms, etc.) at the k-loop level. These corrections, also known as electroweak Sudakov logarithms, may well amount to several tens of per cent. A recent survey of the literature on logarithmic EW corrections can be found in Ref. [**?**]. Specifically, full one-loop weak corrections to the hadronic Z-boson and prompt photon production at large p_{T} have been studied in Refs. [**?**, **?**]. Numerical results for the one-loop corrections to these two processes can be also found in Ref. [**?**]. In the following we briefly discuss the calculation of Refs. [**?**, **?**] and present compact analytic expressions for the Z-boson and direct photon production in the high energy approximation, derived from the exact one-loop results in Refs. [**?**, **?**]. Moreover, we also present the NLL contributions to the two-loop EW corrections, calculated in Refs. [**?**, **?**]. Corresponding numerical results are discussed.

Of course, to achieve reliable predictions at high p_{T}, the QCD corrections in next-to-leading order (NLO) need to be taken into account as they can amount to several tens of per cent correction for both processes. However, in the following we are interested only in the EW corrections to the leading order (LO) in the strong coupling constant α_s cross section for the process $pp \to Vj$ ($V = \gamma, Z;\ j = \mathrm{jet}$).

2. CALCULATION

In the calculation of the one-loop corrections to the hadronic production of Z bosons [**?**] and the hadronic production of photons [**?**] at large p_{T} we consider only weak contributions, i.e. we do not include electromagnetic corrections. The quarks are assumed massless and diagrams involving couplings of quarks to Higgs bosons or would-be-Goldstone bosons are neglected. Moreover, we omit quark-mixing effects in the calculations. The calculation for the $\bar{q}q \to Vg$ ($V = \gamma, Z$) subprocess is performed at the level of matrix elements and allows for full control over polarization effects. Results for the other contributing

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subprocesses, $q\bar{q} \to Vg$, $gq \to Vq$, $qg \to Vq$, $\bar{q}g \to V\bar{q}$ and $g\bar{q} \to V\bar{q}$ are easily obtained from CP symmetry and crossing transformations. The one-loop amplitude is split into two parts: Abelian and non-Abelian, as the structure of the gauge group generators in front of each term contributing to the amplitude can be classified as either Abelian (characteristic for Abelian theories) or non-Abelian (originating from non-commutativity of weak interactions). Tensor loop integrals, appearing in the expressions for one-loop corrections are reduced to scalar integrals by means of Passarino-Veltman technique. The renormalization of Z- and photon-production amplitudes is perfomed in the $\overline{\mathrm{MS}}$ and in the on-shell (OS) scheme, respectively. Further details of the calculation can be found in Refs. [?, ?].

2.1 Analytical results

The p_{T} distribution for the partonic reaction $\bar{q}q \to Vg$ ($V = \gamma,\ Z$) reads

$$\frac{\mathrm{d}\hat{\sigma}^{\bar{q}q \to Vg}}{\mathrm{d}p_{\mathrm{T}}} = \frac{p_{\mathrm{T}}}{8\pi N_c^2 \hat{s}|\hat{t}-\hat{u}|}\left[\overline{\sum}|\mathcal{M}^{\bar{q}q\to Vg}|^2 + (\hat{t} \leftrightarrow \hat{u})\right], \tag{1}$$

where $N_{\mathrm{c}} = 3$, $\overline{\sum} = \frac{1}{4}\sum_{\mathrm{pol}}\sum_{\mathrm{col}}$, $\hat{s} = (p_{\bar{q}} + p_q)^2$, $\hat{t} = (p_{\bar{q}} - p_V)^2$, $\hat{u} = (p_q - p_V)^2$ and $p_V^2 = M_V^2$. To $\mathcal{O}(\alpha^2\alpha_{\mathrm{S}})$ for the unpolarized squared matrix element we have

$$\begin{aligned}\overline{\sum}|\mathcal{M}_1^{\bar{q}q\to Vg}|^2 = 8\pi^2\alpha\alpha_{\mathrm{S}}(N_{\mathrm{c}}^2-1)\sum_{\lambda=\mathrm{R,L}}\Bigg\{\left(I_{q_\lambda}^V\right)^2\Bigg[H_0^V\left(1+2\delta C_{q_\lambda}^{V,\mathrm{A}}\right)\\ +\frac{\alpha}{2\pi}\sum_{V'=\mathrm{Z,W^\pm}}\left(I^{V'}I^{\bar{V}'}\right)_{q_\lambda}H_1^{V,\mathrm{A}}(M_{V'}^2)\Bigg] + \frac{U_{VW^3}}{s_{\mathrm{W}}}T_{q_\lambda}^3 I_{q_\lambda}^V\left[2H_0^V\delta C_{q_\lambda}^{V,\mathrm{N}} + \frac{\alpha}{2\pi}\frac{1}{{s_{\mathrm{W}}}^2}H_1^{V,\mathrm{N}}(M_W^2)\right]\Bigg\},\end{aligned} \tag{2}$$

where $I_{q_\lambda}^V$ represents the coupling of an electroweak gauge boson V to right-handed ($\lambda = \mathrm{R}$) or left-handed ($\lambda = \mathrm{L}$) quarks. In terms of the electric charge Q_q, the weak isospin $T_{q_\lambda}^3$ and the weak hypercharge Y_{q_λ} we have $I_{q_\lambda}^\gamma = -Q_q$, $I_{q_\lambda}^Z = c_{\mathrm{W}}/s_{\mathrm{W}}T_{q_\lambda}^3 - s_{\mathrm{W}}Y_{q_\lambda}/(2c_{\mathrm{W}})$, with the shorthands $c_{\mathrm{W}} = \cos\theta_{\mathrm{w}}$ and $s_{\mathrm{W}} = \sin\theta_{\mathrm{w}}$ for the weak mixing angle θ_{w}. Moreover the relevant elements of the Weinberg rotation matrix U are given by $U_{\gamma W^3} = -s_{\mathrm{W}}$ and $U_{ZW^3} = c_{\mathrm{W}}$. The term $H_0^V = (\hat{t}^2 + \hat{u}^2 + 2\hat{s}M_V^2)/(\hat{t}\hat{u})$ in Eq. (2) represents the Born contribution. $\delta C_{q_\lambda}^{V,\mathrm{A/N}}$ summarize the counterterms associated with the renormalization of the couplings and the gauge-boson wave function. The contributions resulting from the bare loop diagrams and the fermionic wave-function renormalization correspond to the functions $H_1^{V,\mathrm{A/N}}$. The complete analytic expressions for these functions and for the counterterms, as well as details concerning the choice of the renormalization scheme, can be found in Ref. [?] for the Z-boson production and in Ref. [?] for photon production. In the following we concentrate on the high-energy asymptotic behaviour of the corrections, more precisely the next-to-next-to-leading logarithmic (NNLL) approximation of the full result. Formally we consider the limit where $M_W^2/\hat{s} \to 0$ and $\hat{t}/\hat{s}$, $\hat{u}/\hat{s}$ are constant. In this limit the one-loop weak corrections are strongly enhanced by logarithms of the form $\log(\hat{s}/M_W^2)$. The functions $H_1^{V,\mathrm{A/N}}$ assume the general form

$$H_1^{V,\mathrm{A/N}}(M_{V'}^2) \overset{\mathrm{NNLL}}{=} \mathrm{Re}\left[g_0^{V,\mathrm{A/N}}(M_{V'}^2)\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} + g_1^{V,\mathrm{A/N}}(M_{V'}^2)\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}} + g_2^{V,\mathrm{A/N}}(M_{V'}^2)\right]. \tag{3}$$

We find $g_i^{\gamma,\mathrm{A}} = g_i^{Z,\mathrm{A}}$ for $i = 0,1,2$ and $g_j^{\gamma,\mathrm{N}} = g_j^{Z,\mathrm{N}}$ for $j = 1,2$. For $V = \gamma,\ Z$ one has:

$$\begin{aligned}g_0^{V,\mathrm{A}}(M_{V'}^2) &= -\log^2\left(\frac{-\hat{s}}{M_{V'}^2}\right) + 3\log\left(\frac{-\hat{s}}{M_{V'}^2}\right) + \frac{3}{2}\Bigg[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right)\\ &\quad + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right)\Bigg] + \frac{7\pi^2}{3} - \frac{5}{2},\end{aligned} \tag{4}$$

$$
\begin{aligned}
g_1^{V,\mathrm{N}}(M_{V'}^2) &= -g_1^{V,\mathrm{A}}(M_{V'}^2) + \frac{3}{2}\left[\log\left(\frac{\hat u}{\hat s}\right) - \log\left(\frac{\hat t}{\hat s}\right)\right] = \frac{1}{2}\left[\log^2\left(\frac{\hat u}{\hat s}\right) - \log^2\left(\frac{\hat t}{\hat s}\right)\right],\\
g_2^{V,\mathrm{N}}(M_{V'}^2) &= -g_2^{V,\mathrm{A}}(M_{V'}^2) = -2\left[\log^2\left(\frac{\hat t}{\hat s}\right) + \log^2\left(\frac{\hat u}{\hat s}\right) + \log\left(\frac{\hat t}{\hat s}\right) + \log\left(\frac{\hat u}{\hat s}\right)\right] - 4\pi^2.
\end{aligned}
$$

The non-Abelian function $g_0^{V,\mathrm{N}}$ is given by ($\delta_{VV'} = 1$ for $V = V'$, otherwise $\delta_{VV'} = 0$):

$$
\begin{aligned}
g_0^{V,\mathrm{N}}(M_W^2) &= 2\left[\bar\Delta_{\mathrm{UV}} - \delta_{V\gamma}\log\left(\frac{M_W^2}{M_Z^2}\right)\right] + \log^2\left(\frac{-\hat s}{M_W^2}\right) - \log^2\left(\frac{-\hat t}{M_W^2}\right) - \log^2\left(\frac{-\hat u}{M_W^2}\right)\\
&\quad + \log^2\left(\frac{\hat t}{\hat u}\right) - \frac{3}{2}\left[\log^2\left(\frac{\hat t}{\hat s}\right) + \log^2\left(\frac{\hat u}{\hat s}\right)\right] - 2\pi^2 + 2\delta_{VZ}\left(-\frac{\pi^2}{9} - \frac{\pi}{\sqrt{3}} + 2\right). \qquad (5)
\end{aligned}
$$

The ultraviolet singularity, $\bar\Delta_{\mathrm{UV}} = 2/(4-D) - \gamma_{\mathrm{E}} + \log(4\pi\mu^2/M_Z^2)$ is cancelled by the counterterms [**?**, **?**].

The size of the logarithmically enhanced contributions grows with energy and for transverse momenta of hundreds of GeV also the two-loop logarithms become important. At the NLL accuracy, our results for the two-loop corrections include contributions with terms of the form $\alpha^2\mathrm{L}_{\hat s}^4$ and $\alpha^2\mathrm{L}_{\hat s}^3$ where $\mathrm{L}_{\hat r}^k = \log^k(|\hat r|/M_W^2)$. The expressions presented here have been obtained using results of Refs. [**?**, **?**, **?**]. Since at two-loop level the purely weak corrections cannot be isolated from the complete electroweak corrections in a gauge-invariant way, we have to consider the combination of weak and electromagnetic virtual corrections. The latter are regularized by means of a fictitious photon mass $\lambda = M_W$. For the unpolarized squared matrix element we have $\overline{\sum}|\mathcal{M}_2^{\bar q q\to Vg}|^2 - \overline{\sum}|\mathcal{M}_1^{\bar q q\to Vg}|^2 \overset{\mathrm{NLL}}{=} 2\alpha^3\alpha_{\mathrm{S}}(N_{\mathrm{c}}^2-1)H_0^V A^{V,(2)}$ where

$$
\begin{aligned}
A^{V,(2)} &= \sum_{\lambda=\mathrm{L,R}}\Bigg\{\frac{1}{2}\left(I_{q_\lambda}^V C_{q_\lambda}^{\mathrm{ew}} + S_{q_\lambda}^V\right)\Bigg[I_{q_\lambda}^V C_{q_\lambda}^{\mathrm{ew}}\left(\mathrm{L}_{\hat s}^4 - 6\mathrm{L}_{\hat s}^3\right)\\
&\quad + S_{q_\lambda}^V\left(\mathrm{L}_{\hat t}^4 + \mathrm{L}_{\hat u}^4 - \mathrm{L}_{\hat s}^4\right)\Bigg] - (\delta_{VZ} - \delta_{V\gamma})\frac{T_{q_\lambda}^3 Y_{q_\lambda}}{8{s_{\mathrm{W}}}^4}\left(\mathrm{L}_{\hat t}^4 + \mathrm{L}_{\hat u}^4 - \mathrm{L}_{\hat s}^4\right)\\
&\quad + \frac{1}{6}I_{q_\lambda}^V\left[I_{q_\lambda}^V\left(\frac{b_1}{{c_{\mathrm{W}}}^2}\left(\frac{Y_{q_\lambda}}{2}\right)^2 + \frac{b_2}{{s_{\mathrm{W}}}^2}C_{q_\lambda}\right) + S_{q_\lambda}^V b_2\right]\mathrm{L}_{\hat s}^3\Bigg\}. \qquad (6)
\end{aligned}
$$

Here $C_{q_\lambda}^{\mathrm{ew}} = Y_{q_\lambda}^2/(4{c_{\mathrm{W}}}^2) + C_{q_\lambda}/{s_{\mathrm{W}}}^2$ is the electroweak Casimir operator for quarks, with $C_{q_{\mathrm{L}}} = 3/4$ and $C_{q_{\mathrm{R}}} = 0$. $S_{q_\lambda}^V$ is defined as $S_{q_\lambda}^V = C_{VV'}^{\mathrm{ew}} I_{q_\lambda}^{V'}/2$ where $C_{VV'}^{\mathrm{ew}}$ is the electroweak Casimir operator in the adjoint representation [**?**, **?**] and $S_{q_\lambda}^\gamma = -T_{q_\lambda}^3/{s_{\mathrm{W}}}^2$, $S_{q_\lambda}^Z = c_{\mathrm{W}} T_{q_\lambda}^3/{s_{\mathrm{W}}}^3$. The one-loop β-function coefficients read $b_1 = -41/(6{c_{\mathrm{W}}}^2)$ and $b_2 = 19/(6{s_{\mathrm{W}}}^2)$. We stress that although the above one- and two-loop results for the photon and Z-boson production can be put in the same form, their derivation requires separate calculation for each of the processes.

2.2 Numerical results

The hadronic cross sections are obtained using LO MRST2001 PDFs [**?**]. We choose p_T^2 as the factorization scale and, similarly as the scale at which the running strong coupling constant is evaluated. We also adopt the value $\alpha_S(M_Z^2) = 0.13$ and use the one-loop running expression for $\alpha_S(\mu^2)$, in accordance with the LO PDF extraction method of the MRST collaboration. We use the following values of parameters [**?**]: $M_Z = 91.19$ GeV, $M_W = 80.39$ GeV and $\alpha = 1/137.0$, ${s_{\mathrm{W}}}^2 = 1 - {c_{\mathrm{W}}}^2 = 1 - M_W^2/M_Z^2$ for the γ production (OS scheme with α in the Thompson limit) or $\alpha = 1/128.1$, ${s_{\mathrm{W}}}^2 = 0.2314$ for the Z-boson production ($\overline{\mathrm{MS}}$ scheme with M_Z^2 as the renormalization scale).

First, we study the behaviour of the one-loop EW corrections to the transverse momentum distributions of photons and Z bosons produced at the LHC, see Fig. 1. The contribution provided by the

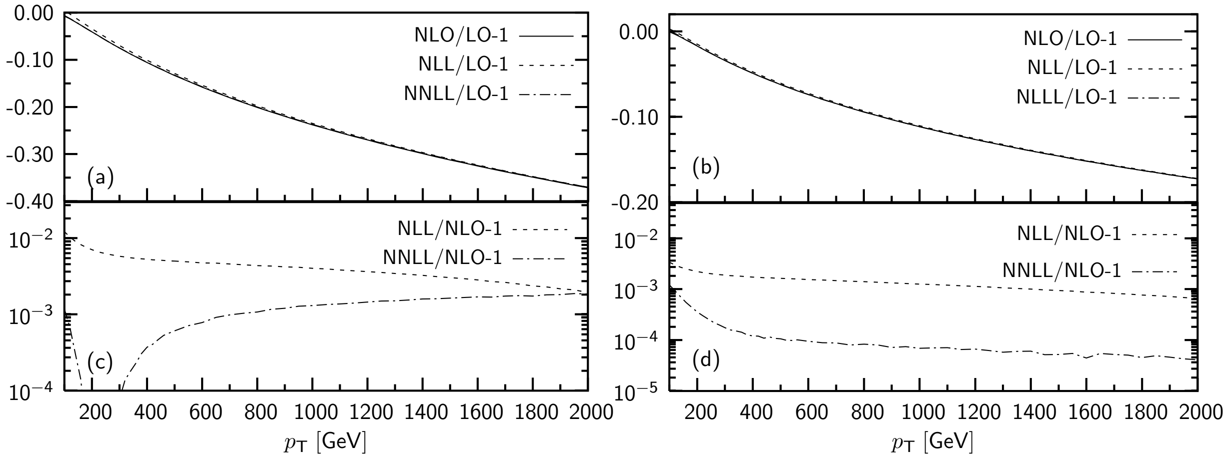

Fig. 1: Upper plots: relative NLO (solid), NNL (dotted) and NNLL (dot-dashed) weak corrections wrt. the LO p_{T} distribution for the process (a) $pp \to Zj$ and (b) $pp \to \gamma j$ at $\sqrt{s} = 14$ TeV. Lower plots: NLL (dotted) and NNLL (dot-dashed) approximation compared to the full NLO result for (c) $pp \to Zj$ and (d) $pp \to \gamma j$ at $\sqrt{s} = 14$ TeV.

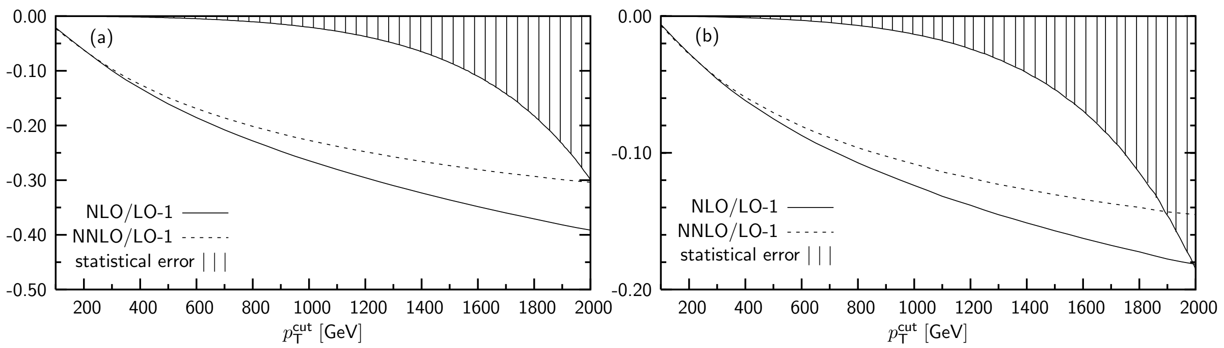

Fig. 2: Relative NLO (solid) and NNLO (dotted) corrections wrt. the LO prediction and statistical error (shaded area) for the unpolarized integrated cross section for (a) $pp \to Zj$ and (b) $pp \to \gamma j$ at $\sqrt{s} = 14$ TeV as a function of p_T^{cut}.

NLO correction is negative and increases in size with p_{T}, reaching -37% (-17%) for the Z-boson (photon) production at $p_{\mathrm{T}} = 2$ TeV. We also conclude that the NLL approximation works good, at a per cent (or better) level of accuracy both for the photon and Z-boson production. In comparison, the quality of the NNLL approximation is excellent, at the level of accuracy of 10^{-3} or better in the entire p_{T} range for both processes.

To demonstrate the relevance of the EW effects for the transverse momentum distributions of the gauge bosons produced at the LHC, in Fig. 2 we present the relative NLO and next-to-next-to-leading order (NNLO) corrections for the cross section, integrated over p_{T} starting from $p_T = p_T^{\mathrm{cut}}$, as a function of p_T^{cut}. This is compared with the statistical error, defined as $\Delta\sigma_{\mathrm{stat}}/\sigma = 1/\sqrt{N}$ with $N = \mathcal{L} \times \sigma_{\mathrm{LO}}$. We assume a total integrated luminosity $\mathcal{L} = 300\ \mathrm{fb}^{-1}$ for the LHC. It is clear from Fig. 2, that the size of the one-loop (two-loop logarithmic) corrections is much bigger than (comparable to) the statistical error for both the Z-boson and the γ production.

In Fig. 3, we plot the ratio of the p_{T} distribution for the γ production to the p_{T} distribution for the Z-boson production. Such ratio is expected to be less sensitive to theoretical errors than the distributions themselves, since many uncertainties such as the scale at which α_{S} is calculated or the choice of PDFs cancel to a large extend in the ratio. Moreover, due to a similar cancellation mechanism, the ratio should remain stable against QCD corrections. From Fig. 3 we observe that the EW corrections modify the

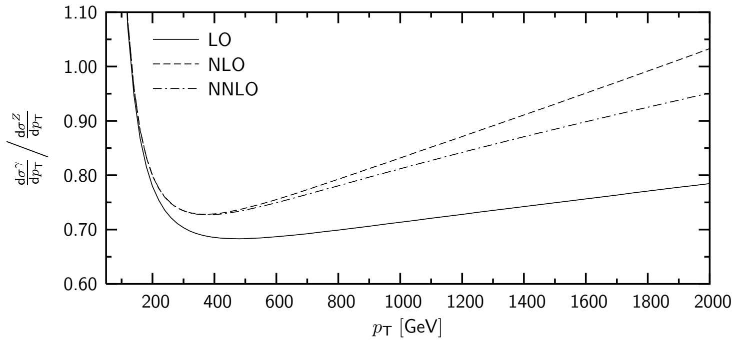

Fig. 3: Ratio of the transverse momentum distributions for the processes $pp \to \gamma j$ and $pp \to Zj$ at $\sqrt{s} = 14$ TeV for the LO (solid), NLO (dashed) and NNLO (dotted) predictions.

production ratio considerably. The effect is the strongest at high $p_{\rm T}$. In this region, the LO photon cross section is smaller than the cross section for Z-boson production by about 25%. The relatively large NLO corrections for Z-boson production, as compared to γ production, cause the full NLO production rates to become equal at the highest $p_{\rm T}$ considered here, i.e. $p_T \sim 2$ TeV. The two-loop corrections modify the ratio and lead to a few per cent decrease at high $p_{\rm T}$.

CONCLUSIONS

At the LHC, the transverse momentum of Z bosons or photons produced in the process $pp \to Vj$ ($V = \gamma, Z$) will reach the TeV scale. In this $p_{\rm T}$ regime one-loop weak corrections are negative and large, of the order of tens of per cent. In the high-energy limit these corrections are approximated with an excellent accuracy by the compact analytical expressions presented here. We also present expressions for the dominant logarithmic part of the two-loop EW corrections and point out that these corrections are important for the correct interpretation of the measurements. Moreover, we find that the EW corrections modify significantly the ratio of the Z-boson and γ transverse momentum distributions at high $p_{\rm T}$.

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