SSFB/CPP-04-75
TTP04-30

Perturbative QCD and tau-decays*

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The current status of the higher order QCD corrections to the width of the tau is discussed. We report on the result of the first complete calculation of the quadratic quark mass correction to the correlator of the two currents relevant for the strangeness-changing semihadronic tau-decay rate *including its real* part at the four-loop level. We discuss phenomenological implications for determination of m_s.

1. Introduction

Hadronic τ decays present a handle to deal with the correlator of two charged weak currents in an interesting energy region just above the main low-laying resonances. The decay proceeds into vector and axial current induced final states, which can be further separated experimentally into the spin zero and spin one components [1]. In addition to the total rate also various moments of the distribution in s, the invariant mass of the hadronic state, can be considered [2].

With significant number of τ leptons detected in each LEP experiments and CLEO a lot of activity Has been recently devoted to the theoretical analysis of the measured τ structure functions within the QCD frameworks as it was outlined in the pioneering works [3–6].

The main outcome of the study of the total hadronic τ width (normalized to the known leptonic width)

$$R_\tau = \frac{\Gamma(\tau \to \text{hadrons} + \nu_\tau)}{\Gamma(\tau \to l + \bar{\nu}_l + \nu_\tau)} = R_{\tau,NS} + R_{\tau,S} \quad (1)$$

has been a remarkable confirmation of QCD. It was found that the value of the strong coupling constant α_s as obtained from R_τ is in good agreement to those obtained from completely different experiments such as the Z boson decay into hadrons [7–10].

Furthermore, the strangeness changing (Cabbibo-suppressed) part of the decomposition of the decay rate and the moments

$$\begin{aligned} R_\tau^{kl} &\equiv \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{dR_\tau}{ds} \\ &= R_{\tau,NS}^{kl} + R_{\tau,S}^{kl}\,, \qquad (2) \\ R_\tau^{00} &\equiv R_\tau, \quad R_{\tau,NS}^{00} \equiv R_{\tau,NS}, \quad R_{\tau,S}^{00} \equiv R_{\tau,S} \end{aligned}$$

can be used to determine the strange quark mass, one of the fundamental parameters of the Standard Model [11–21]. The moments as introduced above can be experimentally determined from the measured distribution in the invariant mass of the final state hadrons. Hadronic physics is encoded in the quantities r_{ij}^{kl} defined through

$$\begin{aligned} &R_{\tau,NS}^{kl} + R_{\tau,S}^{kl} = \\ &3\, S_{\rm EW} \left(|V_{ud}|^2\, r_{ud}^{kl} + |V_{us}|^2\, r_{us}^{kl}\right), \qquad (3) \end{aligned}$$

where V_{ij} stands for the CKM matrix and S_{EW} for the universal electroweak correction [22,23]. The function r_{ij}^{kl} is directly related to the correlator of the weak charged current[3] $j_\mu(x) =$

*Invited talk given by K.G.Ch. at "TAU-04, 14-17 September 2004, Nara, Japan".

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[3]The combinations (q) and (0) are equivalent to $L+T$ and L in the notation of ref. [16].

$\bar\psi_i\gamma_\mu(1-\gamma_5)\psi_j$

$$i\int \mathrm{d}x\, e^{iqx}\langle T\left[j_\mu(x)j_\nu^\dagger(0)\right]\rangle \quad = \tag{4}$$
$$(-g_{\mu\nu}q^2+q_\mu q_\nu)\Pi_{ij}^{(q)}(q^2) \quad + \quad g_{\mu\nu}q^2\Pi_{ij}^{(0)}.$$

through

$$r_{ij}^{k,l} = r_{ij}^{(q)k,l} + r_{ij}^{(0)k,l} = 6i\pi\oint_{|s|=M_\tau^2}\frac{ds}{M_\tau^2} \tag{5}$$
$$\times \quad \left[w^{(q)k,l}(x)\Pi_{ij}^{(q)}(s) + w^{(0)k,l}(x)\Pi_{ij}^{(0)}(s)\right].$$

where $x = s/M_\tau^2$, and the weight functions

$$w^{(q)k,l} = (1+2x)(1-x)^{k+2}\,x^l,$$

$$w^{(0)k,l} = 2x^{l+1}(1-x)^{k+2}.$$

The behaviour of the perturbative series for the part of the integral arising from $\Pi^{(q)}$ is more stable than the one from $\Pi^{(0)}$. However, the latter can either be modelled theoretically and phenomenologically on the basis of scalar and pseudoscalar resonance physics [14,20,24–26], or, being solely determined by spin zero contribution in the τ decays, determined experimentally through the analysis of angular distribution of the $K\pi$ and $K\pi\pi$, thus separating spin zero and spin one contributions[4].

Since $m_u \ll m_s$ and $m_s \ll M_\tau$, the function $r_{us}^{k,l}$ is well described by setting $m_u = 0$ and keeping only the leading and quadratic contributions to the correlator eq. (4), viz.

$$\Pi_{ij}^{(q)}(q^2,m_s) = \frac{3}{16\pi^2}\left(\Pi_0^{(q)}(q^2) + \frac{m_s^2}{Q^2}\Pi_{2,ij}^{(q)}(q^2)\right) \tag{6}$$

$$\Pi_{ij}^{(0)}(q^2,m_s) = \frac{3}{16\pi^2}\frac{m_s^2}{Q^2}\Pi_{2,ij}^{(0)}(q^2),. \tag{7}$$

Here $Q^2 = -q^2$. In the following, we limit ourselves by the perturbative contributions[5].

[4]A related discussion along these lines, which deals with $J=1$ and $J=0$ spectral functions and the resulting moments separately and the corresponding pQCD predictions can be found in [15].

[5]The small non-perturbative terms, as well as the m_s^4 terms will be effectively included in the phenomenological analysis at the end. For a recent discussion of power-suppressed contributions, see [27].

As a result one has a convenient decomposition

$$r_{ud} = r_0 + \delta_{ud}, \quad r_{us} = r_0 + \delta_{us} \quad . \tag{8}$$

Let us in addition define the difference

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,NS}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = 3\,S_{\mathrm{EW}}\,\delta r^{kl}. \tag{9}$$

which is a usefull combination to probe the $SU(3)$ breaking effects as $\delta r^{kl} \equiv \left(\delta_{ud}^{kl} - \delta_{us}^{kl}\right)$ vanishes in the limit of exact $SU(3)$ flavour symmetry. In the spirit of the decomposition (5) one similarly defines related quantities $\delta R_\tau^{(q)kl}$, $\delta R_\tau^{(0)kl}$, $\delta r^{(q)kl}$ and $\delta r_\tau^{(0)kl}$ such that $\delta R_\tau^{kl} = \delta R_\tau^{(q)kl} + \delta R_\tau^{(0)kl}$, $\delta r^{kl} = \delta r^{(q)kl} + \delta r^{(0)kl}$.

The currently known pQCD predictions for r_0, δ_{us} and δ_{ud} can be shortly summarized as follows[6]

$$r_0^{\mathrm{FOPT}} \quad = \quad \left(1 + \,a_s + 5.202\,a_s^2 + 26.37\,a_s^3\right), \tag{10}$$

$$\delta_{us}^{\mathrm{FOPT}} \quad = \quad \frac{m_s^2}{M_\tau^2}\left(1 + 5.33\,a_s + 46.0\,a_s^2\right), \tag{11}$$

$$\delta_{ud}^{\mathrm{FOPT}} \quad = \quad -0.35\;a_s^2\;\frac{m_s^2}{M_\tau^2}, \tag{12}$$

where $a_s = \alpha_s(M_\tau)/\pi$. Now, with $a_s \approx .1$ one observes that the "apparent convergency" of the series is acceptable for r_0 but should be considered at best as marginal for δ_{us}. Clearly, the next term in (11) is important for the interpretation of the measurements. Partial results for the a_s^4 term in eq. (10) and the a_s^3 term in eq. (11) have been published [30,31]. We will describe below the results of the calculation of the missing term as well as its phenomenological implications.

2. Calculation

At first sight, with the first $\mathcal{O}(\alpha_s^3)$ calculations of the absorptive part of two-point correlators being completed in the beginning of 90-ties [32], it should have been not very difficult to compute the $\mathcal{O}(\alpha_s^3)$ contribution to $\delta_2^{(q)}$ and $\delta_2^{(0)}$ long ago. This is indeed the case for $\delta^{(0)}$ [13]. However, it does not apply for $\delta^{(q)}$, as a consequence of

[6]Displayed below are so called Fixed Order Perturbation Theory results. A procedure of "improving" of pQCD predictions by a partial resummation of higher order terms [28,29] will be discussed later.

the (purely kinematical) appearance of the factor $1/Q^2$ in eq. (6). Indeed, in performing the contour integration over s in eq. (5) one may freely ignore any s-independent term in the function $\Pi_2^{(0)}$, however, *not* in $\Pi_2^{(q)}$: in the latter case the $1/Q^2$ factor produces a non-zero contribution from a constant term in $\Pi_2^{(q)}$. One could rephrase this as follows: the function δr^{kl} depends on the absorptive part of $\Pi_2^{(g)}(q^2)$ only but on *both* absorptive *and* real parts of the function $\Pi_2^{(q)}(q^2)$.

Now, the $\mathcal{O}(\alpha_s^3)$ contributions to eq. (5) are represented by four-loop propagators. The calculation of the absorptive part of (massless) four-loop propagators can be reduced to the evaluation of a properly constructed set of *three*-loop massless propagator via so-called R^* operation [33,34]. The resulting three-loop diagrams can be conveniently computed with the help of the algebraic program MINCER [35] written in FORM [36], which implements the traditional integration-by-parts algorithm to evaluate massless three loop propagators. Naturally, the absorptive part of five-loop propagators (which correspond to the $\mathcal{O}(\alpha_s^4)$ contribution to r_0) can be also reduced to the calculation of the real parts of the four-loop propagator, and so on.

However, any direct generalization of the MINCER algorithm to the four loop case is far from being trivial, if possible at all. Instead we have used a new method to reduce a four-loop propagator to the master integrals, namely an expansion of the corresponding coefficient functions in the limit of large space-time dimension D [37]. The method has demonstrated its feasibility for a restricted set of topologies of four-loop propagators in [30,31,38]. The present calculation deals, for the first time, with a complete problem including all possible topologies.

To compute the real part of $\Pi_2^{(q)}(q^2)$ we proceed as follows. First, using the criterion of irreducibility of Feynman integrals [39], the set of irreducible integrals involved in the problem was constructed. Second, the coefficients multiplying these integrals were calculated as series in the $1/D \to 0$ expansion. Third, the exact answer, i.e. a rational function of D, was reconstructed from this expansion. And, finally, extensive testing of the answer was performed.

3. Results

Our results for the functions $\Pi_{2,us}^{(q)}$ and $\Pi_{2,ud}^{(q)}$ read:

$$
\begin{aligned}
\Pi_{2,us}^{(q)} &= -4 - \frac{28}{3}\, a_s \qquad (13)\\
+ \quad & a_s^2 \left\{ -\frac{13981}{108} - \frac{646}{27}\,\zeta_3 + \frac{2080}{27}\,\zeta_5 \right\} \\
+ \quad & a_s^3 \left\{ -\frac{2092745}{1296} - \frac{14713}{162}\,\zeta_3 - 122\,\zeta_3^2 + 10\,\zeta_4 \right. \\
& \left. + \frac{41065}{27}\,\zeta_5 - \frac{79835}{162}\,\zeta_7 \right\} \\
&= -4\left(1 + 2.333\, a_s + 19.58\ a_s^2 + 202.309\ a_s^3\right),
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{2,ud}^{(q)} &= a_s^2 \left\{ \frac{128}{9} - \frac{32}{3}\,\zeta_3 \right\} \qquad (14)\\
+ \quad & a_s^3 \left\{ \frac{6392}{27} - \frac{4496}{27}\,\zeta_3 - 16\,\zeta_3^2 + \frac{320}{27}\,\zeta_5 \right\} \\
= \quad & -4\left(-0.35\, a_s^2 - 6.437\ a_s^3\right).
\end{aligned}
$$

where we have set the normalization scale $\mu^2 = -q^2 = Q^2$ and $a_s = a_s(Q^2)$. The results for generic values of μ can be easily recovered with the standard RG techniques.

It is instructive to compare the exact result for the $\mathcal{O}(\alpha_s^3)$ contribution to (13) with the recently obtained predictions ([31]) based on the optimization schemes as PMS and FAC:

$$k_{2,us}^{(q)3} = 202.309\,(\text{exact}),\ 201\,(\text{PMS}),\ 199\,(\text{FAC}).$$

This astonishingly good agreement (a similar phenomenon has been observed for the prediction of the $O(\alpha_s^3)$ term for the correlator of the diagonal currents [40] can be considered as a strong argument to repeat the procedure and predict, starting from the now completely known $k_2^{(q)3}$, the corresponding result for one loop more, that is for $k_2^{(q)4}$. To be definite, we use the PMS predictions (again for $n_f = 3$; the FAC result is very similar)

$$k_{2,us}^{(q)4} = 2276 \pm 200 \qquad (15)$$

Table 1
Contributions of successive orders of PT to $\delta\, r^{(q)kl}$ for the value of $\alpha_s(M_{tau}) = .334$, and normalized to the value of $\delta\, r^{(q)kl}$ in the Born approximation. First five lines: fixed order PT, second five lines: contour improved version, with RG improvement for the Adler $D^{(q)}$; last five lines: contour improved version with RG summation made for the function $\Pi^{(q)}$.

(kl)	Perturbative series	
(0,0)	1 + 0.425 + 0.283 + 0.178 + 0.0857	= 1.97
(1,0)	1 + 0.532 + 0.458 + 0.423 + 0.424	= 2.84
(2,0)	1 + 0.606 + 0.589 + 0.623 + 0.721	= 3.54
(3,0)	1 + 0.663 + 0.695 + 0.793 + 0.987	= 4.14
(4,0)	1 + 0.708 + 0.784 + 0.943 + 1.23	= 4.67
(0,0)	0.753 + 0.214 + 0.065 - 0.0611 - 0.213	= 0.75
(1,0)	0.912 + 0.334 + 0.192 + 0.0675 - 0.0969	= 1.41
(2,0)	1.05 + 0.451 + 0.33 + 0.228 + 0.0802	= 2.14
(3,0)	1.19 + 0.571 + 0.484 + 0.425 + 0.33	= 3.
(4,0)	1.32 + 0.697 + 0.657 + 0.665 + 0.664	= 4.01
(0,0)	0.857 + 0.122 - 0.00901 - 0.158 - 0.34	= 0.473
(1,0)	1.11 + 0.232 + 0.11 - 0.0417 - 0.268	= 1.14
(2,0)	1.35 + 0.347 + 0.251 + 0.124 - 0.115	= 1.95
(3,0)	1.59 + 0.471 + 0.42 + 0.35 + 0.139	= 2.97
(4,0)	1.83 + 0.61 + 0.623 + 0.648 + 0.521	= 4.23

and

$$k_{2,us}^{(q)4} - k_{2,ud}^{(q)4} = 2378 \pm 200. \tag{16}$$

It is, of course, difficult to estimate uncertainty in the above predictions; however, the simple comparison with eq. (3) clearly demonstrates that an error of about 10% should be considered quite conservative.

4. Phenomenology

Apart of evaluations using fixed order perturbation theory two formally equivalent versions of the contour improved procedure can be found in the literature. The first [15] is based directly on the integration of the polarization function $\Pi_2^{(q)}$, the second [16] is based on the integration of the Adler function

$$D_2^{(q)} \equiv s\frac{d}{ds}\Pi_2^{(q)}$$

and is obtained from the first one by partial integration. Partial integration and renormalization group improvement do not commute as long as finite orders are considered. Since the second procedure moves part of the lower order input to higher orders (contrary to the spirit of CIPT) and since, furthermore, the first procedure leads to a somewhat more stable perturbation series [7], we consider the first of the two choices as preferable. The three options for the perturbative series are displayed in Table 1. As a consequence of the large value of α_s and the rapidly growing coefficients of the perturbative series it seems at first

[7]At least this is true for small values of a_s; for realistic higher value $a_s = .334$ both series start to oscillate.

Table 2
Result for m_s, derived from different levels of approximation, based on contour improvement from [16], and a list of different conributions to the associated error.

Parameter	Value	(2,0)	(3,0)	(4,0)
$m_s(\mathcal{O}(a_s^3), \text{est.})$		124.	107.	94.5
$\mathcal{O}(a_s^3)$	$2\times\mathcal{O}(a_s^3)$ no $\mathcal{O}(a_s^3)$	-5.4 6.2	-6.6 8.1	-7.2 9.3
ξ	1.5 1	3.3 0	6.2 0	8.3 0
$\alpha_s(M_\tau)$	$.334 \pm 0.022$	0.99 0.14	-0.94 1.8	-2.2 3.
others [12,26]		$+22.6$ $-26.$	$+17.6$ -19.7	$+14.9$ -16.5
Total		$+23.7$ -26.5	$+20.4$ -20.8	$+19.6$ -18.1
$m_s(\mathcal{O}(a_s^3), \text{exact})$		123.	106.	92.8
$\mathcal{O}(a_s^3)$	$2\times\mathcal{O}(a_s^3)$ no $\mathcal{O}(a_s^3)$	-6.3 7.4	-7.5 9.6	$-8.$ $11.$
ξ	1.5 1	3.6 0	6.7 0	8.8 0
$\alpha_s(M_\tau)$	$.334 \pm 0.022$	1.1 0.1	-0.93 1.9	-2.2 3.
others [12,26]		$+22.4$ -25.7	$+17.4$ -19.5	$+14.8$ -16.3
Total		$+23.8$ -26.5	$+21.$ -20.9	$+20.6$ -18.3
$m_s(\mathcal{O}(a_s^4), \text{PMS})$		121.	99.9	84.7
$\mathcal{O}(a_s^4)$	$2\times\mathcal{O}(a_s^4)$ no $\mathcal{O}(a_s^4)$	-2.2 2.3	-5.1 $6.$	-6.3 $8.$
ξ	1.5 1	$-2.$ 0	3.7 0	6.8 0
$\alpha_s(M_\tau)$	$.334 \pm 0.022$	4. -1.4	0.26 1.4	-1.8 3.
others [12,26,41]		$+22.$ -25.3	$+16.7$ -18.7	$+14.$ -15.5
Total		$+22.5$ -25.5	$+18.1$ -19.4	$+17.8$ -16.8

Table 3
The same as Table 2 but with the contour improvement done according to [31].

Parameter	Value	(2,0)	(3,0)	(4,0)
$m_s(\mathcal{O}(a_s^3), \text{exact})$		123.	103.	88.
$\mathcal{O}(a_s^3)$	$2\times\mathcal{O}(a_s^3)$ no $\mathcal{O}(a_s^3)$	-3.5 3.9	-5.8 $7.$	-6.8 8.9
ξ	1.5 1	-1.4 20.	4.5 9.1	7.5 2.7
$\alpha_s(M_\tau)$	$.334 \pm 0.022$	3.8 -1.4	0.44 1.2	-1.5 2.8
others [12,26]		$+22.3$ -25.7	$+17.$ -19.1	$+14.3$ -15.8
Total		$+23.$ $-26.$	$+19.$ $-20.$	$+18.6$ -17.3
$m_s(\mathcal{O}(a_s^4), \text{PMS})$		126.	100.	82.4
$\mathcal{O}(a_s^4)$	$2\times\mathcal{O}(a_s^4)$ no $\mathcal{O}(a_s^4)$	3.9 -3.6	-2.3 2.4	-4.6 5.6
ξ	1.5 0	-8.9 0	$-2.$ 0	4.3 0
$\alpha_s(M_\tau)$	$.334 \pm 0.022$	13. -5.4	4.2 -0.65	0.24 2.1
others [12,26,41]		$+22.9$ -26.4	$+16.7$ -18.8	$+13.8$ -15.3
Total		$+26.4$ -28.6	$+17.4$ $-19.$	$+15.7$ $-16.$

glance difficult to consider any of the theoretical predictions as truly preferable. Nevertheless, the results for moments $(2,0)$, $(3,0)$ and $(4,0)$ are at least in plausible agreement among the three methods and exhibit acceptably decreasing subsequent terms. Since these moments are also relatively most precise, as far as experiment is concerned, they will be used in the subsequent analysis. The phenomenological analysis will be based on the most recent evaluation [41] of $|V_{us}|$. For the phenomenological description of the contribution due to $r^{(0)kl}$ we adopt the analysis presented in [26] where the contour improvent for $D_2^{(q)}$ was employed.

The results for m_s, derived from different moments and different ways of implementing the contour improvement procedure are shown in Table 3. (Details about the error estimates and the corresponding analysis will be given elsewhere.) The main difference, compared to the previous analysis, is a downward shift of m_s by about 5 MeV from the inclusion of the α_s^4 terms and an upward shift by as much as 20 MeV from the new input for $|V_{us}|$. The ambiguity in the determined value of m_s, including the newly computed α_s^3 term, and the estimate for the α_s^4 term is shown in Table 3. The renormalzation scale $\mu = \xi M_\tau$ is allowed to vary between $\xi = 1 - 1.5$. Values of μ lower than M_τ lead to a blow up of α_s and destabilize the result. By "others" we mean all uncertainties (added in quadrature) of the input parameters different from the $\mathcal{O}(a_s^3)$ (or $\mathcal{O}(a_s^4)$) terms in the perturbative contribution. They include experimental errors in the moments as reported in [12], errors in $|V_{us}| = 0.2259(23)$ from [41] as well as uncertainties in the parameters related to construction of the subtracted longitudinal part. We have computed the uncertainties using numbers from [26].

From Table 1 we find that the inclusion of the α_s^4 term leads for the third and the fourth moments to a better agreement between predictions based on two different methods of implementing of the "contour improvement" approach. Since the third moment also exhibits a smaller theoretical error we choose it to derive our final result for m_s which will be given below.

In total we find

$$m_s(M_\tau) = 100 + \left({}^{+4.8}_{-3.1}\right)_{theo} \quad + \left({}^{+17.}_{-19.}\right)_{rest} . \quad (17)$$

If one compares (17) to the $\mathcal{O}(\alpha_s^3)$ result of [26] (corrected for a different $|V_{us}|$),namely $m_s(M_\tau) = 106$ (with larger theoretical and identical remaning errors) one sees an essential but still not too large sensitivity to the α_s^4 contribution. In fact, for the third moment the shift in m_s due inclusion of the α_s^4 term (-2.5 MeV) is about a third of of the corresponding change (-7 MeV) due to the α_s^3 contribution. Thus, the purely theoretical uncertainty from not yet computed higher orders could be estimated as roughly 3 MeV. Unfortunately, one can hardly hope that the error from not yet computed higher orders in α_s could be reduced further by means of a direct calculation in any foreseeable future.

5. Acknowledgments

The authors are grateful to A. Pivovarov and K. Maltman for very useful discussions. This work was supported by the the Deutsche Forschungsgemeinschaft in the Sonderforschungsbereich/Transregio SFB/TR-9 "Computational Particle Physics", by INTAS (grant 03-51-4007), by RFBR (grant 03-02-17177) and by Volkswagen Foundation (grant I/77788).

REFERENCES

1. J.H. Kuhn and E. Mirkes, Z. Phys. C56 (1992) 661.
2. F. Le Diberder and A. Pich, Phys. Lett. B289 (1992) 165.
3. K. Schilcher and M.D. Tran, Phys. Rev. D29 (1984) 570.
4. E. Braaten, Phys. Rev. D39 (1989) 1458.
5. S. Narison and A. Pich, Phys. Lett. B211 (1988) 183.
6. E. Braaten, S. Narison and A. Pich, Nucl. Phys. B373 (1992) 581.
7. ALEPH, R. Barate et al., Eur. Phys. J. C4 (1998) 409.
8. OPAL, K. Ackerstaff et al., Eur. Phys. J. C7 (1999) 571, hep-ex/9808019.

9. K.G. Chetyrkin, J.H. Kuhn and A. Kwiatkowski, Phys. Rept. 277 (1996) 189.
10. A combination of preliminary electroweak measurements and constraints on the standard model, LEP, ALEPH, DELPHI, L3, OPAL Collaborations, LEP- and SLD Electroweak Group, (2003), hep-ex/0312023.
11. ALEPH, R. Barate et al., Eur. Phys. J. C11 (1999) 599, hep-ex/9903015.
12. OPAL, G. Abbiendi et al., Eur. Phys. J. C35 (2004) 437, hep-ex/0406007.
13. K.G. Chetyrkin and A. Kwiatkowski, Z. Phys. C59 (1993) 525, hep-ph/9805232.
14. K. Maltman, Phys. Rev. D58 (1998) 093015, hep-ph/9804298.
15. K.G. Chetyrkin, J.H. Kühn and A.A. Pivovarov, Nucl. Phys. B533 (1998) 473, hep-ph/9805335.
16. A. Pich and J. Prades, JHEP 06 (1998) 013, hep-ph/9804462.
17. A. Pich and J. Prades, JHEP 10 (1999) 004, hep-ph/9909244.
18. J.G. Korner, F. Krajewski and A.A. Pivovarov, Eur. Phys. J. C20 (2001) 259, hep-ph/0003165.
19. S. Chen et al., Eur. Phys. J. C22 (2001) 31, hep-ph/0105253.
20. J. Kambor and K. Maltman, Phys. Rev. D62 (2000) 093023, hep-ph/0005156.
21. K. Maltman and J. Kambor, Nucl. Phys. A680 (2000) 155, hep-ph/0006272.
22. W.J. Marciano and A. Sirlin, Phys. Rev. Lett. 61 (1988) 1815.
23. E. Braaten and C.S. Li, Phys. Rev. D42 (1990) 3888.
24. K. Maltman and J. Kambor, Phys. Rev. D64 (2001) 093014, hep-ph/0107187.
25. E. Gamiz et al., JHEP 01 (2003) 060, hep-ph/0212230.
26. E. Gamiz et al., (2004), hep-ph/0408044.
27. D. S. Gorbunov and A. A. Pivovarov, (2004), hep-ph/0410196.
28. A.A. Pivovarov, Z. Phys. C53 (1992) 461, hep-ph/0302003.
29. F. Le Diberder and A. Pich, Phys. Lett. B286 (1992) 147.
30. P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, Phys. Rev. Lett. 88 (2002) 012001, hep-ph/0108197.
31. P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, Phys. Rev. D67 (2003) 074026, hep-ph/0212299.
32. S.G. Gorishnii, A.L. Kataev and S.A. Larin, Phys. Lett. B259 (1991) 144.
33. K.G. Chetyrkin and V.A. Smirnov, Phys. Lett. B144 (1984) 419.
34. K.G. Chetyrkin, Phys. Lett. B391 (1997) 402, hep-ph/9608480.
35. S.A. Larin, F.V. Tkachov and J.A.M. Vermaseren, (1991), NIKHEF-H-91-18.
36. J.A.M. Vermaseren, Symbolic Manipulation with FORM, Version 2 (CAN, Amsterdam, 1991).
37. P.A. Baikov, Nucl. Phys. Proc. Suppl. 116 (2003) 378.
38. P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, Phys. Lett. B559 (2003) 245, hep-ph/0212303.
39. P.A. Baikov, Phys. Lett. B474 (2000) 385, hep-ph/9912421.
40. P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, Nucl. Phys. Proc. Suppl. 116 (2004) 243.
41. A. Czarnecki, W.J. Marciano and A. Sirlin, (2004), hep-ph/0406324.