Charge asymmetry and radiative ϕ decays ⋆

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Abstract

The study of radiative ϕ decays into scalar mesons, with subsequent decay into $\pi\pi\gamma$, constitutes an important topic at the electron-positron collider DAPHNE. The interference of the respective amplitude with the one for $\pi^+\pi^-\gamma$ production, where the photon originates from initial state radiation, will allow for unambiguous tests of models for the $\phi \to \gamma f_0(\to \pi\pi)$ amplitude. The forward-backward asymmetry of charged pions, which is a clear signal of such an interference, amounts up to 30 % and is at the same time quite sensitive to the details of the various amplitudes. The results for several characteristic cases are presented and their implementation into the Monte Carlo generator PHOKHARA is described.

Key words: radiative return, radiative ϕ decays
PACS: 13.25.Jx, 13.66.Bc, 11.30.Er, 13.40.Gp

1 Introduction

The method of the radiative return, i.e. the study of events with initial state radiation (ISR) at electron-positron colliders, allows to measure the hadronic cross section over a wide range of energies in a single experiment, without changing the beam energy. Originally suggested already long time ago [1], the idea has been revived in [2], after high luminosity electron-positron colliders, specifically ϕ- and B-meson factories, came into operation. Already now the method has been used to study the pion form factor at DAPHNE [3] and the production of $J/\psi(\to \mu^+\mu^-)$ [4] and of $\pi^+\pi^-\pi^0$ at BaBar [5]. It relies on the

⋆ Work supported in part by EC 5-th Framework EURIDICE network project HPRN-CT2002-00311, TARI project RII3-CT-2004-506078 and BMBF/WTZ/POL01/103.

factorization into a part describing the production of the virtual photon from the initial state (including radiative corrections) and a part describing the production of hadrons from the virtual photon, which essentially corresponds to the cross section for $e^+e^- \to$ hadrons. This important feature is affected by contributions from the emission of photons from the final state (FSR). As discussed in [2], suitable kinematic cuts can be chosen, which strongly reduce these effects in $e^+e^- \to \pi^+\pi^-\gamma$. For B-meson factories, configurations with hadrons of low invariant mass in one hemisphere and a hard photon in the opposite hemisphere are completely dominated by ISR, with a small admixture from the "two step" process $e^+e^- \to \gamma\gamma^*(\to \pi^+\pi^-\gamma)$. The situation is more involved at the ϕ-factory DAPHNE. For photons at small angles with respect to the beam direction and centrally produced pions, ISR is again dominant. This is the kinematic region studied in [3] for a measurement of the pion form factor. In general, however, FSR will be more important at these low energies. Unavoidably, the input for any FSR analysis and Monte Carlo simulation will be model dependent and a careful experimental test of the model assumptions is required. The most recent and detailed analysis of this problem [6] has been performed with the Monte Carlo event generator PHOKHARA [7], where FSR for pions is modeled by scalar QED (sQED) (a different ansatz, leading to similar conclusions is presented in [8]). This ansatz is certainly adequate for most of the kinematic regions and for cms energies away from the ϕ resonance. For $\sqrt{s} \simeq 1.02$ GeV, on top of the ϕ-resonance, an additional complication arises: the presence of the narrow ϕ-resonance leads to an enhancement of $\pi\pi\gamma$ final states through radiative ϕ-decays into (dominantly scalar) resonances. This process is important in itself and has, for many years, been the subject of theoretical investigations (see [9–15] and references therein), where the decay chain $\phi \to (f_0(980)+f_0(600))\gamma \to \pi\pi\gamma$ is generally assumed to dominate. First investigations, where this channel has been linked to the radiative return, have been published in [13]. Two distinct features allow to discriminate the radiative ϕ decay from $\pi\pi\gamma$ production through the radiative return on one hand and from FSR from charged pions on the other hand: Radiative ϕ decays to γf_0 will lead to $\pi^+\pi^-\gamma$ and $\pi^0\pi^0\gamma$ states with a relative weight 2:1; this is in contrast to FSR from pions, which, similarly to ISR, leads by construction to $\pi^+\pi^-\gamma$ only. FSR is, furthermore, strongly enhanced for photons collinear to the pions. A second important feature, which will be the main subject of this paper, is the charge asymmetry. It arises from the interference between ISR with $\pi^+\pi^-$ in an odd charge conjugate state, and FSR and radiative ϕ decays, with $\pi^+\pi^-$ in an even charge conjugate state. With the ISR amplitude being already fairly well determined, this interference will allow to pin down size and even the relative phase of the $\phi \to \pi\pi\gamma$ amplitude in a model independent way. As a third possibility the dependence of all this distributions on the cms energy and the energy dependent relative importance of ϕ and continuum amplitude could be used. However, we will not dwell further on this last, fairly obvious, aspect.

2 Description of the model

The subsequent analysis will not attempt to study the most general amplitude for the radiative ϕ decay. Instead we will use characteristic models with different resonance admixture and variable phases and demonstrate that these will lead to marked differences in the charge asymmetry.

For definiteness, we have adopted two models describing $\phi \to \pi\pi\gamma$ decays (π stands for both neutral and charged modes), used also in [13]: the "no structure model" of [9] and the K^+K^- model of [10]. The models were extended to include both $f_0(980)$ and $f_0(600)$ (σ) intermediate states in the decay chain $\phi \to (f_0(980) + f_0(600))\gamma \to \pi\pi\gamma$, an extension required to fit experiments [16,17]. More sophisticated version of the K^+K^- model, incorporating also the complete $f_0(980) + f_0(600)$ mixing matrix, can be found in [11,12]. Contributions from the $\phi \to \pi\rho(\to \pi\gamma)$ decay chain are not taken into account. Indeed KLOE data [16] show, that they are negligible for the $\pi^0\pi^0\gamma$ final state. The branching ratios BR($\rho^\pm \to \pi^\pm\gamma$) and BR($\rho^0 \to \pi^0\gamma$) are comparable [18], hence the same applies to the $\pi^+\pi^-\gamma$ final state.

At present this choice of models is not based on definite experimental information, which is still quite poor, but on their simplicity. To discriminate between different models, more experimental work is needed. However, as shown in the next section, a lot can be learned from studies of charge and charge induced asymmetries.

The amplitude for FSR based on sQED, denoted $\mathcal{M}_{\rm sQED}$, is identical to the one used in PHOKHARA and is given by

$$\mathcal{M}_{\rm sQED} = \frac{ie^3}{s} F_{2\pi}(s)\ \bar{v}(p_1)\gamma_\mu u(p_2)\ \Big\{ (q_1 + k - q_2)^\mu \frac{q_1 \cdot \epsilon^*(\gamma)}{q_1 \cdot k} + (q_2 + k - q_1)^\mu \frac{q_2 \cdot \epsilon^*(\gamma)}{q_2 \cdot k} - 2\epsilon^{*\mu}(\gamma) \Big\} \ . \tag{1}$$

The NLO correction to FSR and the complete description of ISR, including NLO corrections, is also identical to the one used in PHOKHARA and described in [7,6,19]. The pion form factor $F_{2\pi}$ is taken from [20].

For the sake of definiteness, we write down explicitely the amplitude $\mathcal{M}_\phi$ for $\phi \to (f_0(980) + f_0(600))\gamma \to \pi\pi\gamma$

$$\mathcal{M}_\phi = -ief_\phi(Q^2)\epsilon_\mu(\phi) d^{\mu\alpha}\epsilon^*_\alpha(\gamma), \qquad d^{\mu\alpha} = (P.k)g^{\mu\alpha} - k^\mu P^\alpha \ , \tag{2}$$

which, when combined with ϕ production, describes the resonant process $e^+e^- \to \phi^* \to (f_0(980) + f_0(600))\gamma \to \pi\pi\gamma$ with the amplitude

$$\mathcal{M}_{e^+e^-}(\phi) = \frac{ie^3}{s}\bar{v}(p_1)\gamma_\mu u(p_2)\ e^{i\alpha_\phi}\ \frac{g_{\phi\gamma}}{M_\phi^2 - s - iM_\phi\Gamma_\phi}\ \ f_\phi(Q^2)\ d^{\mu\alpha}\epsilon^*_\alpha(\gamma)\ . \quad (3)$$

Here P ($\epsilon(\phi)$) and k ($\epsilon(\gamma)$) are ϕ and photon four momenta (polarization vectors) respectively. $P = p_1 + p_2 = Q + k$ in the case of $e^+(p_1)e^-(p_2)$ annihilation amplitudes, q_1 (q_2) is the π^+ (π^-) four momentum and $Q = q_1 + q_2$. M_ϕ denotes ϕ mass and Γ_ϕ its width. Unless stated otherwise, all numerical values of physical parameters are taken from [18]. The $g_{\phi\gamma}$ coupling, deduced from $\Gamma(\phi \to e^+e^-)$, is $g_{\phi\gamma} = 7.765 \cdot 10^{-2}$ GeV^2. The phase α_ϕ in $M_{e^+e^-}(\phi)$ becomes relevant for the interference between $M_{e^+e^-}(\phi)$ and both $\mathcal{M}_{\mathrm{sQED}}$ and ISR amplitudes and thus can be measured for charged pions. It is kept constant in the 'no structure' model. In the case of the K^+K^- model it is taken from [12]: $\alpha_\phi = b\sqrt{s - 4m_\pi^2}$, with $b = 75^\circ/GeV$ (thus also constant for fixed s).

The function $f_\phi(Q^2)$ reads

$$f_\phi(Q^2) = \frac{g_{\phi f_0\gamma}g_{f_0\pi\pi}}{M_{f_0}^2 - Q^2 - iM_{f_0}\Gamma_{f_0}} + e^{i\alpha_\sigma}\frac{g_{\phi\sigma\gamma}g_{\sigma\pi\pi}}{M_\sigma^2 - Q^2 - iM_\sigma\Gamma_\sigma}\ , \quad (4)$$

in the 'no structure' model, and

$$f_\phi(Q^2) = \frac{g_{\phi K^+K^-}}{2\pi^2 m_K^2}\ \ I\left(\frac{M_\phi^2}{m_K^2}, \frac{Q^2}{m_K^2}\right) \cdot \left[\frac{g_{f_0K^+K^-}g_{f_0\pi\pi}}{M_{f_0}^2 - Q^2 - iM_{f_0}\Gamma_{f_0}} + e^{i\alpha_\sigma}\frac{g_{\sigma K^+K^-}g_{\sigma\pi\pi}}{M_\sigma^2 - Q^2 - iM_\sigma\Gamma_\sigma}\right]\ , \quad (5)$$

in the K^+K^- model. The meaning of the three-particle couplings $g_{m_1m_2m_3}$ is self-explanatory and, assuming isospin symmetry, these couplings are identical for charged and neutral pion modes. M_{f_0} and Γ_{f_0} (M_σ and Γ_σ) are the mass and the width of $f_0(980)$ ($f_0(600)$). Again for simplicity, we use constant widths for both resonances. The phase α_σ has to be substantially different from zero to fit the experimental data [16,17] (see below). $I(\cdot,\cdot)$ is the K-loop function given in [10].

From the amplitude $\mathcal{M}_\phi$ (eq. 2) we have calculated the differential branching ratio $dBr(\phi \to \pi^0\pi^0\gamma)/dM_{\pi\pi}$, as measured in [16,17] ($\sqrt{Q^2} \equiv M_{\pi\pi}$). The parameters obtained in the fits to these data are shown in Table 1. The values of masses and widths obtained in the fits do depend on the choice of the model

Table 1
The values of the parameters obtained by fits to the experimental data [16,17]

	'no structure' model	'K^+K^-' model
M_{f_0} [MeV]	983.9 ± 1.4	1003 ± 8
Γ_{f_0} [MeV]	35.4 ± 3.2	108 ± 7
M_σ [MeV]	588 ± 145	519 ± 23
Γ_σ [MeV]	1653 ± 921	319 ± 65
α_σ	$(101 \pm 11)°$	$(98 \pm 8)°$
$g_{\phi f_0 \gamma} g_{f_0 \pi\pi}$	2.84 ± 0.17	-
$g_{\phi\sigma\gamma} g_{\sigma\pi\pi}$	4.0 ± 1.5	-
$g_{\phi K^+K^-} g_{f_0 K^+K^-} g_{f_0\pi\pi}$ [GeV^2]	-	59.4 ± 5.1
$g_{\phi K^+K^-} g_{\sigma K^+K^-} g_{\sigma\pi\pi}$ [GeV^2]	-	7.9 ± 2.0

and at this point it is difficult to interpret them as physical masses and widths of the resonances.

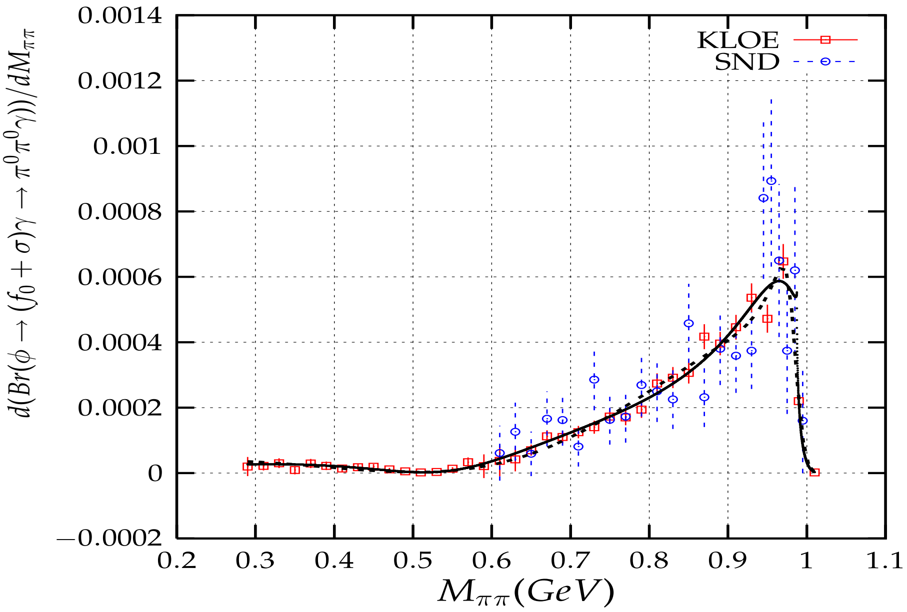

Fig. 1. Fits to the measured [16,17] differential branching rations: K^+K^- model - solid line ($\chi^2 = 29/(53\ d.o.f.)$), 'no structure' model - dashed line ($\chi^2 = 38/(53\ d.o.f.)$).

Both models agree well with the data, as shown in Fig.1, and the destructive interference between $f_0(600)$ and $f_0(980)$ amplitudes is needed in both cases to describe the low $M_{\pi\pi}$ tail of the distribution. As expected, the $\phi \to \pi^0\pi^0\gamma$ data alone cannot differentiate between different models. However, as shown in the next section, charge asymmetries have an enormous analyzing power, and allow to test details of the radiative ϕ decays to charged pions.

3 The analyzing power of charge asymmetries

The importance of charge asymmetries for tests of the FSR model was discussed in details in the context of scalar QED in [2,6]. At the ϕ-factory DAPHNE, the direct radiative ϕ-decays, not included in [2,6], play, however an important role [13]. These a priori small effects are enhanced by the resonant behavior of ϕ and their accurate analysis is compulsory. The interest in these asymmetries is twofold. On one hand they play an important role in the FSR 'background' estimate for the measurement of the pion form factor via the radiative return method. On the other hand, as we will show below, they allow for powerful tests of models describing radiative ϕ-decays. The asymmetry analysis is thus a source of rich experimental information complementary to the cross section measurement.

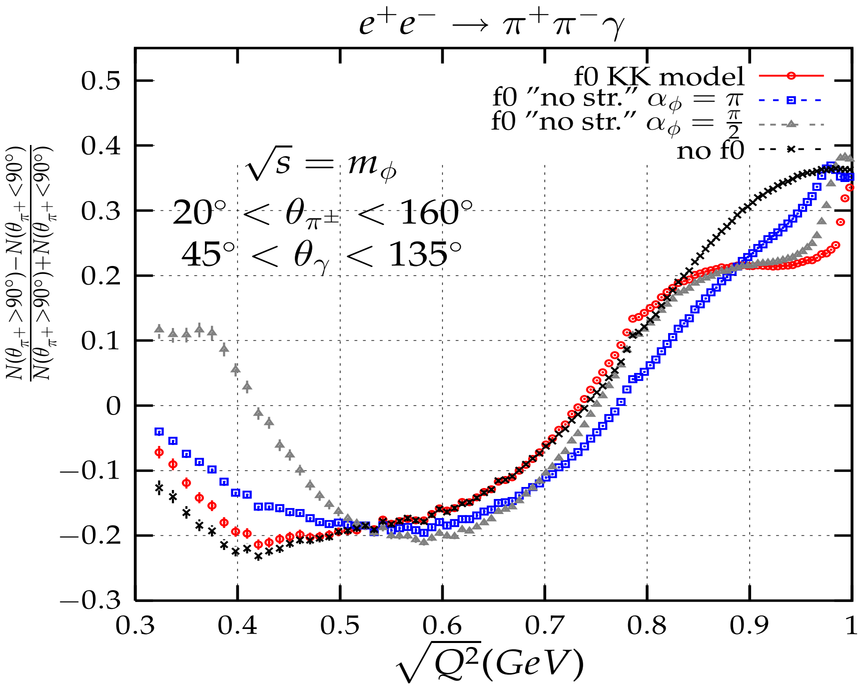

Fig. 2. Forward-backward asymmetry for different radiative ϕ decay models compared with the asymmetry calculated within sQED (no f_0)

Let us start with the forward-backward asymmetry defined for π^+

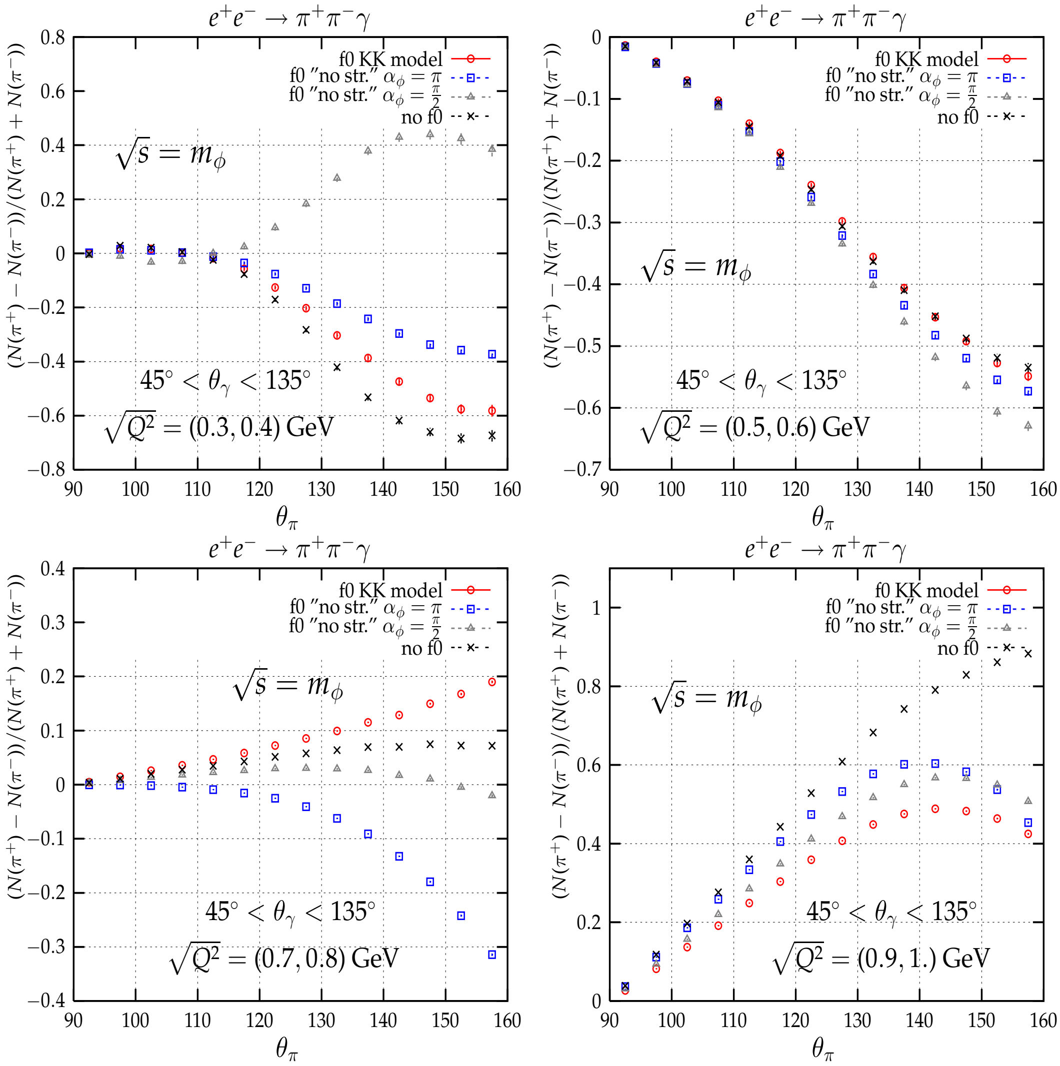

Fig. 3. Angular and Q^2 dependence of charge asymmetries for different radiative ϕ decay models compared with the asymmetry calculated within sQED (no f_0), for centrally produced photons and pions

$$\mathcal{A}_{FB}(Q^2) = \frac{N(\theta_{\pi^+} > 90^\circ) - N(\theta_{\pi^+} < 90^\circ)}{N(\theta_{\pi^+} > 90^\circ) + N(\theta_{\pi^+} < 90^\circ)}\left(Q^2\right) \ , \tag{6}$$

In this analysis the z- axis is chosen along initial positron direction. The cuts on the pion angles $20^\circ < \theta_{\pi^\pm} < 160^\circ$ are required for pions to be observed in the detector and the photon angular range $45^\circ < \theta_\gamma < 135^\circ$ was chosen to enhance the FSR effects. The results are shown in Fig. 2. The asymmetry is very sensitive to the relative phase between sQED and direct ϕ decay amplitudes (α_ϕ) and predictions of this phase [12] can be definitely tested. The assumption of a Q^2 dependent phase from the complex loop function $I\left(\frac{M_\phi^2}{m_K^2}, \frac{Q^2}{m_K^2}\right)$ can also be tested, since the interference pattern can be measured as a function of

Q^2. The effects depend also on the photon angular range and more detailed studies are possible. Sizable effects are predicted and, as one can qualitatively see from preliminary KLOE results [21] on asymmetries, some of the models we use may already be excluded by the data.

The angular and the Q^2 dependence of the charge asymmetry for different models are shown in Fig. 3. Again large effects are observed, with marked differences between different models. The combination of cross section measurements for both charged and neutral pion final states with asymmetry measurements in the charged mode will allow for the most comprehensive study of the $\phi \to \pi\pi\gamma$ decay.

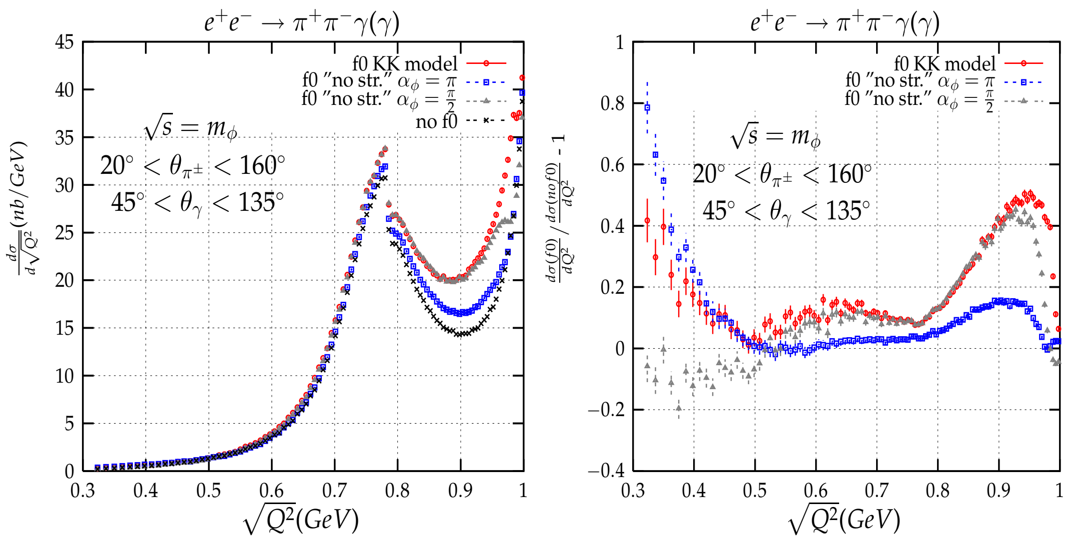

Fig. 4. Differential cross section and for different radiative ϕ decay models compared with sQED (no f_0) cross section (left plot) and their relative difference (right plot).

The effects of the direct radiative ϕ decay on the differential cross section are shown in Fig. 4 for centrally produced photons and pions. They are big and the question arise if they can affect the KLOE [3] analysis of the pion form factor. The answer can be found in Fig. 5. With the models used in this paper one predicts effects up to 1% in the $e^+e^- \to \pi^+\pi^-\gamma$ cross section in the "KK-model" and the "no- structure" model, if $\alpha_\phi = \pi/2$ is adopted, but significantly smaller values for a different choice of the phase α_ϕ (an event selection close to the one used by KLOE has been adopted). All models are compatible with KLOE data on $\phi \to \pi^0\pi^0\gamma$ decay and thus the simultaneous analysis of $\phi \to \pi^0\pi^0\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$ data is highly desirable. It will also allow for a full control of the FSR contributions to the $e^+e^- \to \pi^+\pi^-\gamma$ cross section, hopefully reducing the error from FSR in $\sigma(e^+e^- \to \pi^+\pi^-)$, as measured via the radiative return, to a negligible level.

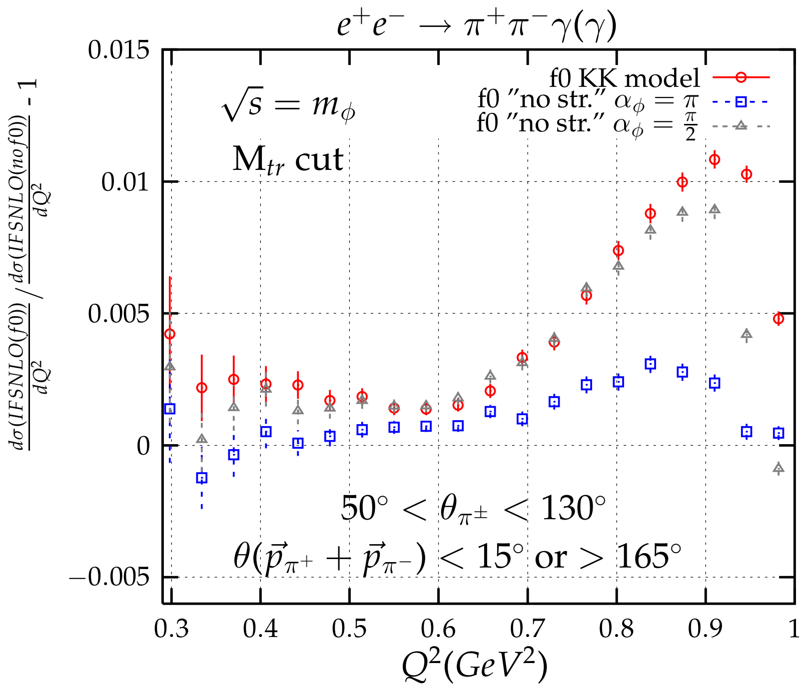

Fig. 5. The relative size of the direct radiative ϕ decay on the differential cross section for an event selection close to the one used by KLOE [3].

4 Summary and Conclusions

The interplay between radiative corrections to pion pair production in electron-positron annihilation on one hand and $\pi\pi\gamma$ production through radiative ϕ-decays on the other hand has been investigated. The charge asymmetry is shown to be a unique signal for the interference between the amplitude for $\pi\pi$ production through the radiative return and $\pi\pi$ production from the radiative ϕ-decay. This asymmetry can be studied as a function of the invariant mass of the $\pi\pi$-system and is particularly sensitive to the model for $\phi \to \pi\pi\gamma$. This has been demonstrated by implementing model amplitudes for $e^+e^- \to \phi \to \pi\pi\gamma$ into the PHOKHARA event generator and by studying the resulting distributions. Indeed different models, which lead to seemingly indistinguishable $\pi\pi$ mass distributions in ϕ-decay, give rise to strikingly different charge asymmetries. We conclude with a discussion of the impact of radiative ϕ-decays on the measurement of the pion form factor and demonstrate, that their effect can be kept well under control, if suitable kinematic regions are selected.

5 Acknowledgments

We would like to thank A. Denig, W. Kluge, D. Leone and S. Müller for discussions of the experimental aspects of our analysis. H.C. and A.G. are grateful for the support and the kind hospitality of the Institut für Theoretische Teilchenphysik of the Universität Karlsruhe.

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