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Next-to-leading electroweak logarithms at two loops[†]

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Abstract:
We review a recent calculation of the two-loop next-to-leading logarithmic electroweak corrections to the form factors for massless chiral fermions coupling to an $SU(2) \times U(1)$ singlet gauge boson.

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NEXT-TO-LEADING ELECTROWEAK LOGARITHMS AT TWO LOOPS

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We review a recent calculation of the two-loop next-to-leading logarithmic electroweak corrections to the form factors for massless chiral fermions coupling to an $\mathrm{SU}(2) \times \mathrm{U}(1)$ singlet gauge boson.

1 Introduction

In the energy region $\sqrt{s} \gg M_{\mathrm{W}} \sim M_{\mathrm{Z}}$, the electroweak (EW) radiative corrections are strongly enhanced by logarithms of the form[a] $\alpha^l \log^{2l-j}\left(s/M_{\mathrm{W}}^2\right)$, with $j = 0, 1, \ldots, 2l$. At $\sqrt{s} \sim 1$ TeV, these logarithms yield one-loop corrections of tens of per cent and two-loop corrections of several per cent. These EW corrections will be important for the interpretation of the measurements at the Linear Collider.

At one loop, the EW LLs and NLLs are now well understood[1,2]. Resummation prescriptions for the LL[3] and NLL[4,5] corrections to arbitrary processes exist and the EW logarithmic corrections to 4-fermion processes have been resummed up to the NNLLs[6]. At the TeV scale, the leading and subleading logarithms have similar size[4,6] and these latter must be under control in order to reduce the theoretical error at the few per mille level. The above resummations were obtained by means of the infrared evolution equation[3] (IREE) assuming exact $\mathrm{SU}(2)\times\mathrm{U}(1)$ and $\mathrm{U}(1)_{\mathrm{em}}$ gauge symmetry for the regimes above and below the EW scale, respectively. This approach relies on the assumption that several aspects of symmetry breaking can be neglected in the high-energy limit. In particular, the following aspects are neglected. (i) The couplings proportional to the vacuum expectation value (vev); (ii) The weak-boson mass gap ($M_{\mathrm{W}} = M_{\mathrm{Z}}$ approximation); (iii) The mixing between neutral gauge bosons.

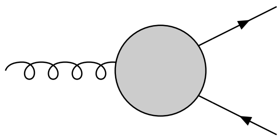
These resummations need to be checked by means of two-loop calculations based on the electroweak Lagrangian, where all effects from symmetry breaking are taken into account. It was alredy confirmed that the LL corrections[7,8,9] as well as the angular-dependent subset of the NLL corrections[10] to arbitrary processes exponentiate as predicted by the IREE. At the NLL level (and beyond) only few two-loop calculations exist[11,12,13]. Here we review a calculation

[a] The terms with $j = 0, 1, 2, \ldots$ are denoted as leading logarithms (LLs), next-to-leading logarithms (NLLs), next-to-next-to-leading logarithms (NNLLs), etc.

of the one- and two-loop virtual EW NLL corrections to the form factors for massless chiral fermions coupling to an $\mathrm{SU}(2) \times \mathrm{U}(1)$ singlet gauge boson[12].

2 Perturbative and asymptotic expansion

The vertex function for an $\mathrm{SU}(2)\times\mathrm{U}(1)$ singlet gauge boson (which might be for instance a gluon) coupled to massless fermions can be written in terms of chiral form factors $F_\pm$ as



$$= \mathrm{i}\bar{u}(p_1)\gamma^\nu \Big(\omega_- F_- + \omega_+ F_+\Big) v(p_2) \qquad (1)$$

with $\omega_\pm = \frac{1}{2}(1 \pm \gamma^5)$ and the perturbative expansion

$$F_\sigma = F_\sigma^{(0)} \left[1 + \sum_{l=1}^{\infty} \left(\frac{\alpha}{4\pi}\right)^l N_\varepsilon^l \delta F_\sigma^{(l)} \right] \quad \text{with} \quad N_\varepsilon = \frac{1}{\Gamma(1-\varepsilon)} \left(\frac{4\pi{\mu_0}^2}{-s}\right)^\varepsilon \qquad (2)$$

in $D = 4 - 2\varepsilon$ dimensions. The one- and two-loop EW corrections, $\delta F_\sigma^{(1)}$ and $\delta F_\sigma^{(2)}$, were evaluated in the asymptotic region $s = (p_1 + p_2)^2 \gg M_\mathrm{W}^2 \sim M_\mathrm{Z}^2$, including mass singular logarithms $L = \log\left(-s/M_\mathrm{W}^2\right)$ originating from massive weak bosons and $1/\varepsilon$ poles from massless photons. Corrections of order M_W^2/s were neglected. The relevant Feynman diagrams were computed to NLL accuracy using an algorithm[15] based on sector decomposition[16,17,18]. The NLL approximation includes all terms of the order $\alpha^l \varepsilon^k \log^{j+k}(s/M_\mathrm{W}^2)$ with $j = 2l, 2l-1$. Contributions depending on $M_\mathrm{Z}/M_\mathrm{W}$ were also included.

3 Results

The one-loop corrections read[b]

$$\begin{aligned} e^2 \delta F_\sigma^{(1)} &\overset{\mathrm{NLL}}{=} \left[g_1^2 \left(\frac{Y_\sigma}{2}\right)^2 + g_2^2 C_\sigma \right] I(\varepsilon, M_\mathrm{W}) \\ &+ \left[g_1^2 \left(\frac{Y_\sigma}{2}\right)^2 + g_2^2 (T_\sigma^3)^2 - e^2 Q^2 \right] \Delta I(\varepsilon, M_\mathrm{Z}) + e^2 Q^2 \Delta I(\varepsilon, 0), \end{aligned} \qquad (3)$$

[b] The corrections are expressed in terms of the hypercharge Y_σ, the weak isospin T_σ^3 and the SU(2) Casimir operator C_σ for left-handed ($\sigma = -$) and right-handed ($\sigma = +$) fermions. The electromagnetic charge of the fermions is denoted as Q whereas g_1 and g_2 represent the U(1) and SU(2) coupling constants, respectively.

where[c]

$$
\begin{aligned}
I(\varepsilon, M_{\mathrm{W}}) &\overset{\mathrm{NLL}}{=} -L^2 - \frac{2}{3}L^3\varepsilon - \frac{1}{4}L^4\varepsilon^2 + 3L + \frac{3}{2}L^2\varepsilon + \frac{1}{2}L^3\varepsilon^2 + \mathcal{O}(\varepsilon^3), \\
I(\varepsilon, M_{\mathrm{Z}}) &\overset{\mathrm{NLL}}{=} I(\varepsilon, M_{\mathrm{W}}) + \log\left(\frac{M_{\mathrm{Z}}^2}{M_{\mathrm{W}}^2}\right)\left(2L + 2L^2\varepsilon + L^3\varepsilon^2\right) + \mathcal{O}(\varepsilon^3), \\
I(\varepsilon, 0) &\overset{\mathrm{NLL}}{=} -2\varepsilon^{-2} - 3\varepsilon^{-1},
\end{aligned} \tag{4}
$$

and $\Delta I(\varepsilon, m) = I(\varepsilon, m) - I(\varepsilon, M_{\mathrm{W}})$. The corrections are split into a contribution $I(\varepsilon, M_{\mathrm{W}})$, which corresponds to a symmetric $\mathrm{SU(2)}\times\mathrm{U(1)}$ theory with all gauge boson masses equal to M_{W}, and additional contributions $\Delta I(\varepsilon, M_{\mathrm{Z}})$ and $\Delta I(\varepsilon, 0)$, which result from the Z–W and γ–W mass gaps. It was found that the two-loop corrections can be written in terms of the one-loop functions (4) in a form that is strictly analogous to Catani's formula for two-loop mass singularities in massless QCD[14]. In the on-shell renormalisation scheme with the electromagnetic coupling constant renormalised at the scale M_{W} we have

$$
\begin{aligned}
e^4\delta F_\sigma^{(2)} \overset{\mathrm{NLL}}{=} & \frac{1}{2}\left[e^2\delta F_\sigma^{(1)}\right]^2 + e^2\left[g_1^2 b_1^{(1)}\left(\frac{Y_\sigma}{2}\right)^2 + g_2^2 b_2^{(1)} C_\sigma\right] J(\varepsilon, M_{\mathrm{W}}, M_{\mathrm{W}}) \\
& + e^4 b_{\mathrm{QED}}^{(1)} Q^2 \Delta J(\varepsilon, 0, M_{\mathrm{W}}).
\end{aligned} \tag{5}
$$

The first term on the right-hand side can be regarded as the result of the exponentiation of the one-loop corrections. The additional contributions, which are associated to the one-loop β-function coefficients[12] $b_1^{(1)}$, $b_2^{(1)}$ and $b_{\mathrm{QED}}^{(1)}$, are proportional to the functions J and ΔJ, defined as

$$
J(\varepsilon, m, \mu) = \frac{1}{\varepsilon}\left[I(2\varepsilon, m) - \left(\frac{-s}{\mu^2}\right)^\varepsilon I(\varepsilon, m)\right] \tag{6}
$$

and $\Delta J(\varepsilon, m, \mu) = J(\varepsilon, m, \mu) - J(\varepsilon, M_{\mathrm{W}}, \mu)$. The contributions proportional to $[I(\varepsilon, M_{\mathrm{W}})]^2$ and $J(\varepsilon, M_{\mathrm{W}}, M_{\mathrm{W}})$ correspond to the two-loop corrections within an unbroken $\mathrm{SU(2)}\times\mathrm{U(1)}$ theory where all gauge bosons have mass M_{W}. The only two-loop terms depending on the Z–W mass gap are the NLLs proportional to $\log(M_{\mathrm{Z}}^2/M_{\mathrm{W}}^2)$ that arise from the squared one-loop corrections in (5) via the term $\Delta I(\varepsilon, M_{\mathrm{Z}})$ in (3). This means that such Z–W mass-gap terms exponentiate. The infrared divergent $1/\varepsilon$ poles are isolated in the terms proportional to $Q^2\Delta I(\varepsilon, 0)$ and $Q^2 b_{\mathrm{QED}}^{(1)}\Delta J(\varepsilon, 0, M_{\mathrm{W}})$. Such terms originate from

[c] Note that the one-loop corrections are expanded up to order ε^2. These higher-order terms in the ε-expansion must be taken into account in the relation (5) between one- and two-loop mass singularities.

the γ–W mass gap and correspond to QED corrections with photon mass $\lambda = 0$ subtracted at $\lambda = M_\mathrm{W}$. Apart from the NLLs proportional to $\log(M_\mathrm{Z}^2/M_\mathrm{W}^2)$, the result (5) is independent on the vev, the Z–W mass gap and the weak mixing angle. This simple behaviour of the two-loop form factors results from cancellations between NLLs from different diagrams and is ensured by the relations between the vev, the weak-boson masses and the weak mixing angle[12]. This result confirms the assumptions (i)–(iii) discussed in the introduction and the IREE-approach[3,4,5] to resum the NLL EW corrections.

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