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$M(\eta_b)$ and α_s from Nonrelativistic Renormalization Group

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We sum up the next-to-leading logarithmic corrections to the heavy-quarkonium hyperfine splitting using the nonrelativistic renormalization group. On the basis of this result, we predict the mass of the η_b meson to be $M(\eta_b) = 9421 \pm 10\,(\mathrm{th})\,^{+9}_{-8}\,(\delta\alpha_s)$ MeV. The experimental measurement of $M(\eta_b)$ with a few MeV error would be sufficient to determine $\alpha_s(M_Z)$ with an accuracy of ± 0.003. The use of the nonrelativistic renormalization group is mandatory to reproduce the experimental value of the hyperfine splitting in charmonium.

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The theoretical study of nonrelativistic heavy-quark-antiquark systems is among the earliest applications of perturbative quantum chromodynamics (QCD) [1] and has by now become a classical problem. Its applications to bottomonium or toponium physics entirely rely on the first principles of QCD. This makes heavy-quark-antiquark systems an ideal laboratory to determine fundamental parameters of QCD, such as the strong-coupling constant α_s and the heavy-quark masses m_q. Besides its phenomenological importance, the heavy-quarkonium system is also very interesting from the theoretical point of view because it possesses a highly sophisticated multiscale dynamics and its study demands the full power of the effective-field-theory approach. The properties of the Υ mesons, the bottom quark-antiquark spin-one bound states, are measured experimentally with great precision, and recent theoretical analysis of the Υ family based on high-order perturbative calculations resulted in determinations of the bottom-quark mass m_b with unprecedent accuracy [2–4].

In contrast to the Υ family, the current experimental situation with the spin-zero η_b meson is rather uncertain: only one candidate event in $\gamma\gamma \to \eta_b$ production has been detected so far, which, however, is consistent with the expected background [5]. Yet, the discovery of the η_b meson is one of the primary goals of the CLEO-c research program [6]. An accurate prediction of its mass $M(\eta_b)$ is thus a big challenge and a test for the QCD theory of heavy quarkonium. Moreover, the hyperfine splitting (HFS) of the bottomonium ground state, $E_{\mathrm{hfs}} = M(\Upsilon(1S)) - M(\eta_b)$, is very sensitive to α_s and, with the advancement of the experimental measurements, could become a competitive source for the determination of the strong coupling constant.

The HFS in quarkonium has been a subject of several theoretical researches [7]. To our knowledge, the next-to-leading order (NLO) $\mathcal{O}(\alpha_s)$ correction is currently known in a closed analytical form only for the ground state HFS [4]. In this letter, we generalize this result to the excited states and present the analytical renormalization-group-improved expression for the heavy-quarkonium HFS in the next-to-leading logarithmic (NLL) approximation, which sums up all the corrections of the form $\alpha_s^n \ln^{n-1} \alpha_s$. We apply it to predict $M(\eta_b)$. The result can be used for extracting $\alpha_s(M_Z)$ from future experimental data on the η_b meson mass.

The leading-order (LO) result for the HFS is proportional to the fourth power of α_s, $E_{\mathrm{hfs}}^{LO} = C_F^4 \alpha_s^4(\mu) m_q/(3n^3)$, where $C_F = (N_c^2-1)/(2N_c)$, and suffers from a strong dependence on the renormalization scale μ of $\alpha_s(\mu)$, which essentially limits the numerical accuracy of the approximation. Thus, the proper fixing of μ is mandatory for the HFS phenomenology. The scale dependence of a finite-order result is canceled against the higher-order logarithmic contributions proportional to a power of $\ln(\mu/\bar{\mu})$, where $\bar{\mu}$ is a dynamical scale of the nonrelativistic bound-state problem. The *physical* choice of the scale $\mu = \bar{\mu}$ eliminates these potentially large logarithmic terms and *a priori* minimizes the scale dependence. However, the dynamics of the nonrelativistic bound state is characterized by three well separated scales: the hard scale of the heavy-quark mass m_q, the soft scale of the bound-state momentum vm_q, and the ultrasoft scale of the bound-state energy $v^2 m_q$, where $v \propto \alpha_s$ is the velocity of the heavy quark inside the approximately Coulombic bound state. To make the procedure of scale fixing self-consistent, one has to resum to all orders the large logarithms of the scale ratios characteristic for the nonrelativistic bound-state problem. The resummation of the logarithmic corrections requires an appropriate conceptual framework. The effective field theory [8] is now recognized as a powerful tool for the analysis of multiscale systems, which is at the heart of the recent progress in the perturbative QCD bound-state calculations. The main idea of this method is to decompose the complicated multiscale problem into a sequence of simpler problems, each involving a smaller number of scales. The logarithmic corrections originate from logarithmic integrals over virtual momenta ranging between the scales and reveal themselves as the singularities of the effective-theory couplings. The renormalization of these singularities allows one to derive the equations of the *nonrelativistic renormalization group* (NRG), which describe the running of the effective-theory couplings, *i.e.* their dependence on the effective-theory cutoffs. The solution of these equations sums up the logarithms of the scale ratios.

To derive the NRG equations necessary for the NLL analysis of the HFS, we rely on the method based on the formulation of the nonrelativistic effective theory known as potential NRQCD (pNRQCD) [9]. The method was developed in Ref. [10] where, in particular, the leading logarithmic (LL) result for the HFS has been obtained (see also Ref. [11]). A characteristic feature of the NRG is the correlation of the dynamical scales, which leads to the correlation of the cutoffs [12]. For perturbative calculations within the effective theory, dimensional regularization is used to handle the divergences, and the formal expressions derived from the Feynman rules of the effective theory are understood in the sense of the threshold expansion [13]. This approach [14–17] possesses two crucial virtues: the absence of additional regulator scales and the automatic matching of the contributions from different scales.

Let us give a few details of the NLL analysis. We distinguish the soft, potential, and ultrasoft anomalous dimensions corresponding to the ultraviolet divergences of the soft, potential, and ultrasoft *regions* [13]. The LL approximation is determined by the one-loop soft running of the effective Fermi coupling c_F and the spin-flip four-quark operator [10]. In the NLL approximation, all three types of running contribute. We need the two-loop soft running of c_F, which is known [18], and the two-loop soft running of the spin-flip four-quark operator, which we compute by adopting the technique used in Ref. [14] for the calculation of the two-loop $1/(m_q r^2)$ non-Abelian potential. To compute the potential running, we inspect all operators that lead to spin-dependent ultraviolet divergences in the time-independent perturbation theory contribution with one and two potential loops [10, 19]. They include (i) the $\mathcal{O}(v^2, \alpha_s v)$ operators [2, 3], (ii) the tree $\mathcal{O}(v^4)$ operators, some of which can be checked against the QED analysis [16], and (iii) the one-loop $\mathcal{O}(\alpha_s v^3)$ operators, for which only the Abelian parts are known [16], while the non-Abelian parts are new. In the NLL approximation, we need the LL soft and ultrasoft running of the $\mathcal{O}(v^2)$ and $\mathcal{O}(v^4)$ operators, which enter the two-loop time-independent perturbation theory diagrams, and the NLL soft and ultrasoft running of the $\mathcal{O}(\alpha_s v)$ and $\mathcal{O}(\alpha_s v^3)$ operators, which contribute at one loop. The running of the $\mathcal{O}(v^2, \alpha_s v)$ operators is already known within pNRQCD [10]. The running of the other operators is new. For some of them, it can be obtained using reparameterization invariance [20].

Besides the running discussed above, we need the initial conditions for the NRG evolution given by the known one-loop result [7]. With the anomalous dimensions and initial conditions at hand, it is straightforward to solve the system of the nonlinear differential equations for the effective couplings and get the NLL result for the HFS. The corresponding expression for general color (light flavor) number N_c (n_l) and for arbitrary principal quantum number n is too lengthy to be shown in this Letter, so we present the explicit analytical expression only for $N_c = 3$, $n_l = 4$, and $n = 1$, which applies to the bottomonium ground state. It reads

$$\begin{aligned}
E^{NLL}_{\rm hfs} &= \frac{C_F^4 \alpha_s^4(\mu) m_b}{3} \left\{ \frac{27}{14} y^{-1} - \frac{13}{14} y^{-\frac{18}{25}} \right. \\
&+ \frac{\alpha_s(m_b)}{\pi} \left[\left(\frac{1037}{224} + \frac{405086361761\,\pi^2}{25617160800} - \frac{3}{4}\ln 2 \right) y^{-1} \right. \\
&- \frac{1024\,\pi^2}{143} y^{-\frac{39}{50}} - \left(\frac{102973}{26250} + \frac{184336\,\pi^2}{25725} \right) y^{-\frac{18}{25}} \\
&+ \frac{1024\,\pi^2}{675} y^{-\frac{1}{2}} + \frac{671\,\pi^2}{1029} y^{-\frac{11}{25}} - \frac{3\,\pi^2}{23} y^{-\frac{2}{25}} \\
&+ \left(-\frac{13427921}{1260000} + \frac{88057\,\pi^2}{151200} \right) y^{\frac{7}{25}} + \frac{4\,\pi^2}{41} y^{\frac{16}{25}} \\
&+ \frac{1377}{56} - \frac{1253587\,\pi^2}{227500} - \frac{629\,\pi^2}{7500} y \\
&- \frac{2873\,\pi^2}{7182} y^{\frac{32}{25}} {}_2F_1\left(\frac{57}{25}, 1; \frac{82}{25}; \frac{y}{2} \right) \\
&+ \frac{2873\,\pi^2}{3591} y^{-1} {}_2F_1\left(1, 1; \frac{82}{25}; -1 \right) + \left(\frac{675}{28} - \frac{533}{42} y^{\frac{7}{25}} \right) \\
&\times \ln\left(\frac{\mu}{\bar\mu} \right) + \frac{85248\,\pi^2}{30625} y^{-1} \ln y + \left(-\frac{45834}{4375} y^{-1} \right. \\
&\left.\left.\left. + \frac{21216}{4375} - \frac{2873}{1575} y^{\frac{7}{25}} + \frac{243}{1250} y \right) \pi^2 \ln(2-y) \right] \right\}, \qquad (1)
\end{aligned}$$

where α_s is renormalized in the $\overline{\rm MS}$ scheme, $y = \alpha_s(\mu)/\alpha_s(m_b)$, $\bar\mu = C_F \alpha_s(\mu) m_b$, ${}_2F_1(a,b;c;z)$ is the hypergeometric function, and ${}_2F_1(1,1;82/25;-1) = 0.7875078\ldots$. By expanding the resummed expression up to $\mathcal{O}(\alpha_s^2)$, we get

$$\begin{aligned}
E^{NLL}_{\rm hfs} &= E^{LO}_{\rm hfs} \left\{ 1 + \frac{\alpha_s}{\pi} \left[\left(\frac{1}{2} + \frac{5}{4n} - n\Psi_2(n) \right) \beta_0 \right. \right. \\
&+ \left(-\frac{37}{72} - \frac{7}{8n} + \frac{7}{4} \left(\Psi_1(n+1) + \gamma_E + L^n_{\alpha_s} \right) \right) C_A \\
&\left. - \frac{C_F}{2} + \frac{3}{2}(1 - \ln 2) T_F - \frac{5}{9} n_l T_F \right] \\
&+ \left(\frac{\alpha_s}{\pi} \right)^2 L^n_{\alpha_s} \left[\left(\frac{19}{6} C_A^2 - \frac{5}{6} C_A n_l T_F \right) L^n_{\alpha_s} + \frac{\beta_1}{8} \right.
\end{aligned}$$

$$+\left(\left(\frac{169}{144}+\frac{2}{n}+\frac{3}{8}\left(\Psi_1(n+1)+\gamma_E\right)-\frac{7}{4}n\Psi_2(n)\right)C_A - C_F+\frac{3}{2}(1-\ln 2)T_F\right)\beta_0+\left(-\frac{23}{27}-\frac{\pi^2}{6}-\frac{7}{8n} + \frac{7}{4}\left(\Psi_1(n+1)+\gamma_E\right)\right)C_A^2+\left(\frac{7}{4}-\frac{11\pi^2}{8}\right)C_AC_F - \frac{\pi^2}{2}C_F^2-\frac{44}{27}C_AT_Fn_l+\frac{1}{2}C_FT_Fn_l\Bigg]\Bigg\}, \quad (2)$$

where $\alpha_s \equiv \alpha_s(\mu)$, $\mu = \bar{\mu}/n$, $C_A = N_c$, $T_F = 1/2$, β_i is the $(i+1)$-loop coefficient of the QCD β function ($\beta_0 = 11C_A/3 - 4T_Fn_l/3, \ldots$), $L^n_{\alpha_s} = \ln(C_F\alpha_s/n)$, $\Psi_n(x) = d^n \ln\Gamma(x)/dx^n$, $\Gamma(x)$ is Euler's Γ function, and $\gamma_E = 0.577216\ldots$ is Euler's constant. In Eq. (2), we keep the full dependence on N_c, n_l, and n. The $\mathcal{O}(\alpha_s^2\ln^2\alpha_s)$ term is known [10, 11], while the $\mathcal{O}(\alpha_s^2\ln\alpha_s)$ term is new.

For the numerical estimates, we adopt the following strategy. First, we compute the dimensionless ratio $E_{\rm hfs}/M(\Upsilon(1S))$, which only has logarithmic dependence on m_b. At the order of interest, we have $M(\Upsilon(1S)) = 2m_b$. Then, the ratio is converted into $E_{\rm hfs}$ by multiplying it with the physical $\Upsilon(1S)$ meson mass, $M(\Upsilon(1S)) = 9460.30 \pm 0.25$ MeV [26]. In Fig. 1, the HFS for the bottomonium ground state is plotted as a function of μ in the LO, NLO, LL, and NLL approximations. As we see, the LL curve shows a weaker scale dependence compared to the LO one. The scale dependence of the NLO and NLL expressions is further reduced, and, moreover, the NLL approximation remains stable at the physically motivated scale of the inverse Bohr radius, $C_F\alpha_s m_b/2 \sim 1.5$ GeV, where the fixed-order expansion breaks down. At the scale $\mu' \approx 1.3$ GeV, which is close to the inverse Bohr radius, the NLL correction vanishes as one expects from general arguments. Furthermore, at $\mu'' \approx 1.5$ GeV, the result becomes independent of μ; *i.e.*, the NLL curve shows a local maximum. This suggests a nice convergence of the logarithmic expansion despite the presence of the ultrasoft contribution with α_s normalized at the rather low scale $\bar{\mu}^2/m_b \sim 0.8$ GeV. By taking the difference of the NLL and LL results at the local maxima as a conservative estimate of the error due to uncalculated higher-order contributions, we get $E_{\rm hfs} = 39 \pm 8$ MeV.

So far, we restricted the analysis to purely perturbative calculations. The nonperturbative contribution to the HFS can in principle be investigated by the method of vacuum condensate expansion [21]. The resulting series does not converge well and suffers from large numerical uncertainties [22]. A reliable quantitative estimate of the nonperturbative contributions to the HFS can be obtained by comparison with lattice simulations. The HFS in bottomonium was studied on the lattice by several groups. The SESAM Collaboration [23] reported the value $E_{\rm hfs} = 33.4 \pm 1.9$ MeV, while the CP-PACS Collaboration [24] obtained $E_{\rm hfs} = 33.2 \pm 1.0$ MeV (only the statistical errors are quoted). Both groups used unquenched nonrelativistic lattice QCD. The result from quenched relativistic QCD on an anisotropic lattice [25] is $E_{\rm hfs} = 33.8 \pm 3.1$ MeV. The central values are in good agreement and undershoot our NLL result by approximately 6 MeV, which we take as an estimate of the nonperturbative contribution.

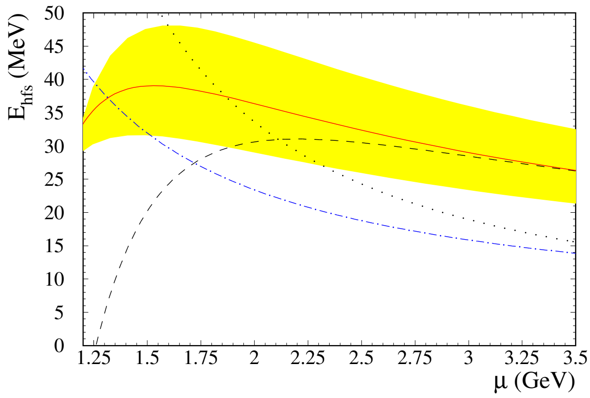

FIG. 1: HFS of 1S bottomonium as a function of the renormalization scale μ in the LO (dotted line), NLO (dashed line), LL (dot-dashed line), and NLL (solid line) approximations. For the NLL result, the band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$.

Another way to estimate the accuracy of our result is by considering the HFS in charmonium, where experimental data are available. The result of our analysis is given in Fig. 2 along with the experimental value 117.7 ± 1.3 MeV [26]. The local maximum of the NLL curve corresponds to $E_{\rm hfs} = 104$ MeV. Taking into account that, in charmonium, the convergence of the perturbative and relativistic expansion is much worse and the nonperturbative effects are much less suppressed in comparison to the bottom-quark case, the good agreement between our perturbative result and the experiment is amazing and supports our error estimates. We should emphasize the crucial role of the resummation to bring the perturbative analysis in agreement with the experimental data. Note that the recent lattice estimates undershoot the experimental value by $20 - 30\%$ [27].

We may also apply our formulae to the $n = 2$ excited states. For bottomonium, a recent study of unquenched nonrelativistic lattice QCD with $1 + 2$ light flavors by the HPQCD and UKQCD collaborations [28] predicts the ratio $E_{\rm hfs}(2S)/E_{\rm hfs}(1S) = 0.32 \pm 0.07$, which is in good agreement with the value 0.25 obtained using our NLL result. For charmonium, our perturbative estimate $E_{\rm hfs}(2S)/E_{\rm hfs}(1S) = 0.37$ also reasonably agrees with the result 0.41 ± 0.03 of the recent experimental measurements [29]. Although one definitely cannot rely on the (even NRG-improved) perturbative analysis of the excited charmonium states, the above agreement suggests

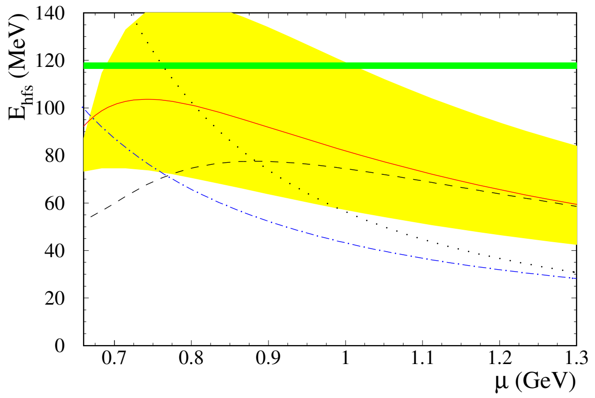

FIG. 2: HFS of 1S charmonium as a function of the renormalization scale μ in the LO (dotted line), NLO (dashed line), LL (dot-dashed line), and NLL (solid line) approximations. For the NLL result, the band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$. The horizontal band gives the experimental value 117.7 ± 1.3 MeV [26].

that the nonperturbative effects are not dominant and well under control at least for the ground state.

In principle, our result can also be applied to the analysis of the difference between the resonance energies in $e^+e^- \to t\bar{t}$ and $\gamma\gamma \to t\bar{t}$ threshold production [30], which is beyond the scope of this Letter.

To conclude, we have computed the heavy-quarkonium HFS in the NLL approximation by summing up the subleading logarithms of α_s to all orders in the perturbative expansion. The use of the NRG extends the range of μ where the perturbative result is stable to the physical scale of the inverse Bohr radius. We found the resummation of logarithms to be crucial to reach agreement between our perturbative estimate and the experimental data on the HFS in charmonium despite *a priori* unsuppressed nonperturbative effects. Our results further indicate that the properties of the physical charmonium and bottomonium ground states are dictated by perturbative dynamics. As an application of the result to the bottomonium spectrum, we predict the mass of the as-yet undiscovered η_b meson to be

$$M(\eta_b) = 9421 \pm 10\,(\mathrm{th})\,{}^{+9}_{-8}\,(\delta\alpha_s)\ \mathrm{MeV}\,, \qquad (3)$$

where the errors due to the high-order perturbative corrections and the nonperturbative effects are added up in quadrature in "th", whereas "$\delta\alpha_s$" stands for the uncertainty in $\alpha_s(M_Z) = 0.118 \pm 0.003$. If the experimental error in future measurements of $M(\eta_b)$ will not exceed a few MeV, the bottomonium HFS will become a competitive source of $\alpha_s(M_Z)$ with an estimated accuracy of ± 0.003, as can be seen from Fig. 1.

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[1] T. Appelquist and H.D. Politzer, Phys. Rev. Lett. **34**, 43 (1975).

[2] J.H. Kühn, A.A. Penin, and A.A. Pivovarov, Nucl. Phys. **B534**, 356 (1998); A.A. Penin and A.A. Pivovarov, Phys. Lett. B **435**, 413 (1998); Nucl. Phys. **B549**, 217 (1999); K. Melnikov and A. Yelkhovsky, Phys. Rev. D **59**, 114009 (1999); M. Beneke and A. Signer, Phys. Lett. B **471**, 233 (1999); J.H. Kühn and M. Steinhauser, Nucl. Phys. **B619**, 588 (2001); **B640**, 415(E) (2002); A.H. Hoang, Report No. CERN-TH-2000-227 and hep-ph/0008102.

[3] A. Pineda and F.J. Yndurain, Phys. Rev. D **58**, 094022 (1998); A. Pineda, JHEP **0106**, 022 (2001); N. Brambilla, Y. Sumino, and A. Vairo, Phys. Rev. D **65**, 034001 (2002).

[4] A.A. Penin and M. Steinhauser, Phys. Lett. B **538**, 335 (2002).

[5] ALEPH Collaboration, A. Heister *et al.*, Phys. Lett. B **530**, 56 (2002).

[6] H. Stöck, hep-ex/0310021.

[7] W. Buchmüller, Y.J. Ng, and S.H.H. Tye, Phys. Rev. D **24**, 3003 (1981); S.N. Gupta, S.F. Radford, and W.W. Repko Phys. Rev. D **26**, 3305 (1982); J. Pantaleone, S.H.H. Tye, and Y.J. Ng, Phys. Rev. D **33**, 777 (1986); S. Titard and F.J. Yndurain, Phys. Rev. D **49**, 6007 (1994).

[8] W.E. Caswell and G.P. Lepage, Phys. Lett. B **167**, 437 (1986); G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).

[9] A. Pineda and J. Soto, Nucl. Phys. B (Proc. Suppl.) **64**, 428 (1998).

[10] A. Pineda, Phys. Rev. D **65**, 074007 (2002); **66**, 054022 (2002).

[11] A.H. Hoang, A.V. Manohar, and I.W. Stewart, Phys. Rev. D **64**, 014033 (2001); A.H. Hoang and I.W. Stewart, Phys. Rev. D **67**, 014020 (2003).

[12] M.E. Luke, A.V. Manohar, and I.Z. Rothstein, Phys. Rev. D **61**, 074025 (2000).

[13] M. Beneke and V.A. Smirnov, Nucl. Phys. **B522**, 321 (1998); V.A. Smirnov, *Applied Asymptotic Expansions in Momenta and Masses* (Springer-Verlag, Heidelberg, 2001).

[14] B.A. Kniehl, A.A. Penin, V.A. Smirnov, and M. Steinhauser, Phys. Rev. D **65**, 091503(R) (2002); Nucl. Phys. **B635**, 357 (2002); Phys. Rev. Lett. **90**, 212001 (2003); **91**, 139903(E) (2003).

[15] A. Pineda and J. Soto, Phys. Lett. B **420**, 391 (1998); Phys. Rev. D **59**, 016005 (1999).

[16] A. Czarnecki, K. Melnikov, and A. Yelkhovsky, Phys. Rev. A **59**, 4316 (1999).

[17] B.A. Kniehl and A.A. Penin, Nucl. Phys. **B563**, 200

(1999); N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Phys. Lett. B **470**, 215 (1999).

[18] G. Amorós, M. Beneke, and M. Neubert, Phys. Lett. B **401**, 81 (1997).

[19] B.A. Kniehl and A.A. Penin, Nucl. Phys. **B577**, 197 (2000).

[20] A.V. Manohar, Phys. Rev. D **56**, 230 (1997).

[21] M.B. Voloshin, Nucl. Phys. **B154**, 365 (1979); Yad. Fiz. **36**, 247 (1982) [Sov. J. Nucl. Phys. **36**, 143 (1982)]; H. Leutwyler, Phys. Lett. B **98**, 447 (1981).

[22] S. Titard and F.J. Yndurain, Phys. Rev. D **51**, 6348 (1995); A. Pineda, Nucl. Phys. **B494**, 213 (1997).

[23] SESAM Collaboration, N. Eicker *et al.*, Phys. Rev. D **57**, 4080 (1998).

[24] CP-PACS Collaboration, T. Manke *et al.*, Phys. Rev. D **62**, 114508 (2000). We quote the result of the lattice simulation which fits the experimental values of the $\Upsilon(1S)$ mass and the ratio of the $2^3S_1 - 1^3S_1$ and $^1P_1 - 1^3S_1$ splittings.

[25] X. Liao and T. Manke, Phys. Rev. D **65**, 074508 (2002). We quote the result of the lattice simulation which fits the experimental value of the $\Upsilon(1S)$ mass.

[26] K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[27] QCD-TARO Collaboration, S. Choe *et al.*, JHEP **0308**, 022 (2003); MILC Collaboration, M. di Pierro *et al.*, hep-lat/0310042.

[28] HPQCD Collaboration, A. Gray *et al.*, Nucl. Phys. B (Proc.Suppl.) **119**, 592 (2003). The quoted value is extracted from Fig. 3.

[29] We take the average of the results reported in BELLE Collaboration, S.-K. Choi *et al.*, Phys. Rev. Lett. **89**, 102001 (2002); **89**, 129901(E) (2002); BELLE Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **89**, 142001 (2002); CLEO Collaboration, J. Ernst *et al.*, Report No. CLEO-CONF-03-05 and hep-ex/0306060; BABAR Collaboration, B. Aubert *et al.*, Report No. BABAR-PUB-03/023 and hep-ex/0311038.

[30] A.A. Penin and A.A. Pivovarov, Nucl. Phys. **B550**, 375 (1999); Yad. Fiz. **64**, 323 (2001) [Phys. Atom. Nucl. **64**, 275 (2001)].