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FSR at leading and next-to-leading order in the radiative return at meson factories *

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The impact of final-state radiation (FSR) on the radiative return method for the extraction of the e^+e^- hadronic cross section is discussed in detail. Possible experimental tests of the model dependence of FSR are proposed for the $\pi^+\pi^-$ hadronic final state. The importance of the $\pi^+\pi^-\gamma$ final state contribution to the muon anomalous magnetic moment is investigated, and a method based on the radiative return is proposed to extract these contributions from data.

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1. Introduction to the radiative return method

Electron–positron annihilation into hadrons in the low energy region is crucial for predictions of the hadronic contributions to a_μ, the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling from its value at low energy up to M_Z (For reviews see e.g. [1-7] the most recent experimental result for a_μ is presented in [**?**]). Measurements of the hadronic cross section for electron–positron annihilation were traditionally performed by varying the beam energy of the collider. The Φ- and B-meson factories allow to use the radiative return to explore the whole energy region from threshold up to the energy of the collider. Even if the photon radiation from the initial state reduces the cross section by a factor $\mathcal{O}(\alpha/\pi)$, this is easily compensated by the enormous luminosity of these 'factories'. A number of experimental results based on the radiative

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return was already published [9-18] and in the near future one can expect much more data covering large variety of hadronic final states.

The radiative return method [?] (see also [?]), relies on the following factorisation property of the cross section

$$\frac{d\sigma}{dQ^2 d\Omega_\gamma}(e^+e^- \to \mathrm{hadrons} + \gamma) = H(Q^2, \Omega_\gamma)\ \sigma(e^+e^- \to \mathrm{hadrons}, \mathrm{Q}^2)\ , \tag{1}$$

where Q^2 is the invariant mass of the hadronic system, Ω_γ denotes the photon polar and azimuthal angles, and the function $H(Q^2, \Omega_\gamma)$ is given by QED lepton-photon interactions, thus known in principle with any required precision. The formula (**??**) is valid for a photon emitted from initial state leptons (ISR) and what is more important similar factorisation formula applies for the emission of an arbitrary number of photons or even lepton pairs [?] from initial state leptons. Let's forget for a while about FSR. In that case one can, by measuring the Q^2 differential cross section of the process $e^+e^- \to$ hadrons + photons + (*possibly*) lepton pairs (called σ_{RR} from here on) and knowing function the $H(Q^2, ...)$, extract the value of $\sigma(e^+e^- \to \mathrm{hadrons})$. The ... in $H(Q^2, ...)$ stand for the phase space variables of photons and/or lepton pairs. However, in practice the cross section σ_{RR} is measured within a given experimental setup, which corresponds to an integral over a complicated phase space of photons and/or lepton pairs and thus the use of Monte Carlo event generators for extraction of the $\sigma(e^+e^- \to \mathrm{hadrons}, \mathrm{Q}^2)$ become indispensable. Such Monte Carlo programs (EVA [?, ?], PHOKHARA [?, ?, ?]), were and are being developed. The analysis presented in this paper is based on the results obtained by means of the program PHOKHARA 3.0 [?] (For further extensive discussions of various aspects of the radiative return method not covered by this article see [26-31],while for related discussion of the scan method look [?, ?]).

Further complication arises as the photons and/or lepton pairs are emitted also from final state charged hadrons and special care has to be taken when using the radiative return method. That problem is discussed extensively in Sections **??** and **??**, while Section **??** is devoted to NLO contributions to a_μ.

2. Leading order FSR

Leading order (LO) contributions to the process $e^+e^- \to \pi^+\pi^-\gamma$ are schematically (permutations omitted) shown in Fig.**??**. The interference of ISR, which leads to a C-odd (C stands for charge conjugation) configuration of $\pi^+\pi^-$ pair (Fig.1a), with the FSR amplitude from Fig.1b, corresponding

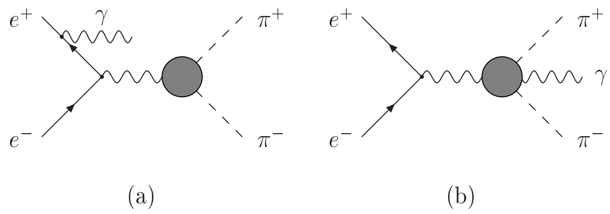

Fig. 1. Leading order contributions to the reaction $e^+e^- \to \pi^+\pi^-\gamma$ from ISR (a) and FSR (b).

to C-even $\pi^+\pi^-$ configuration, vanishes if a charge symmetric event selection is used. It gives rise, however, to charge asymmetries and charge induced forward–backward asymmetries.

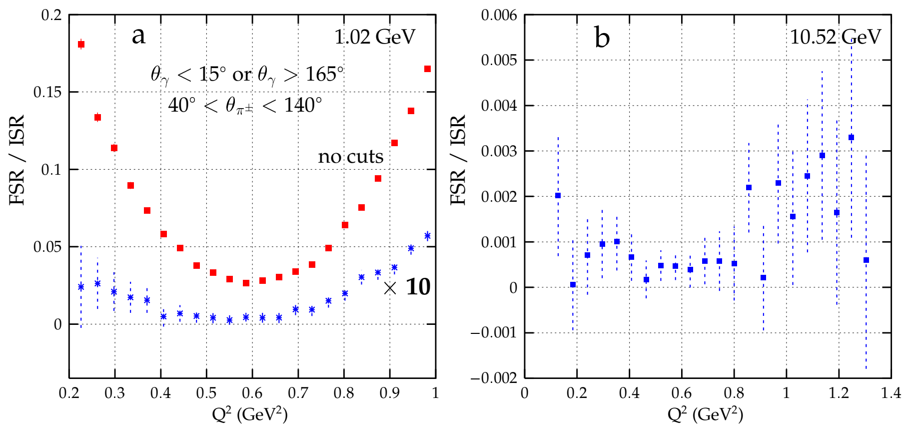

Fig. 2. Relative contribution of FSR with respect to ISR to the inclusive photon spectrum at $\sqrt{s}\ = 1.02$ GeV (without and with cuts (multiplied by a factor 10)) (a) and $\sqrt{s}\ = 10.52$ GeV (b).

The FSR contribution itself can be as big as 20% of the ISR at $\sqrt{s}\ = 1.02$ GeV if no cuts are applied (Fig.**??**a). At $\sqrt{s}\ = 10.52$ GeV however, a very energetic photon has to be radiated to produce a pair of charged pions with invariant mass around the ρ resonance, and its momentum has to be compensated by the momentum of the charged pions. As a result, photon and pions are produced back to back, thus leading to a negligible contribution from LO FSR (Fig.**??**b). At DAΦNE energy FSR has to be controlled through suitable event selection, and an example of a possible event selection is shown in Fig.**??**a (lower curve).

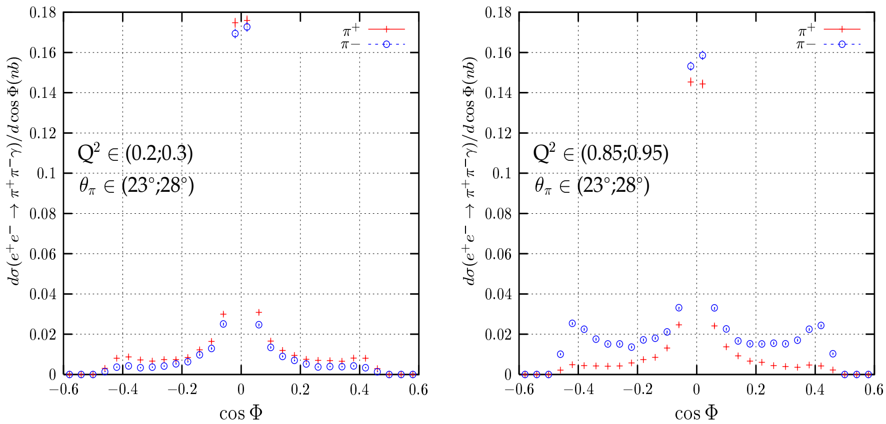

Fig. 3. Charge asymmetric distributions of the pions in Φ (the angle between normal to the production plain and the direction of the initial positron) for different values of Q^2 and fixed pion polar angles.

One can easily reduce the FSR contribution to less than 1% and, relying on a MC generator, subtract it from the measured cross section, thus being able to apply the procedure of the hadronic cross section extraction described in the previous section. The FSR contribution is however model dependent and one needs an independent experimental check on the accuracy of the model used. It can be done just by relaxing the cuts and measuring various charge asymmetric distributions. As the actual contribution of FSR to the radiative return cross section, is of the order of 1%, a modest 10% accuracy of the model will lead to an error of 0.1%, sufficient for any high precision measurement. Some of the tests of the model used in EVA and PHOKHARA for FSR (point-like pions and scalar QED (sQED)), proposed in [?], were already done by KLOE [?], where it was shown that the charge asymmetry

$$A(\theta) = \frac{N^{\pi^+}(\theta) - N^{\pi^-}(\theta)}{N^{\pi^+}(\theta) + N^{\pi^-}(\theta)} \quad , \tag{2}$$

agrees well with the EVA MC [?]. However additional tests are needed to assure the accuracy of the model at the required level and comparisons of various charge asymmetric distributions between experimental data and MC are indispensable. If only pions four-momenta are measured, as done in the KLOE experiment at the moment (see [?, ?]), one arrives at distributions as shown in Fig.??. With 500 pb^{-1}, collected till now by KLOE, the 0.1 nb/bin in the plot corresponds to 2000 events per bin. Thus that kind of measurement is feasible and tests can be done with the required precision, provided systematic errors are small enough. The nontrivial Q^2 and polar angle dependence of Φ–distributions provides profound cross checks of the tested model.

3. Next-to-leading order hadronic contributions to muon a_μ

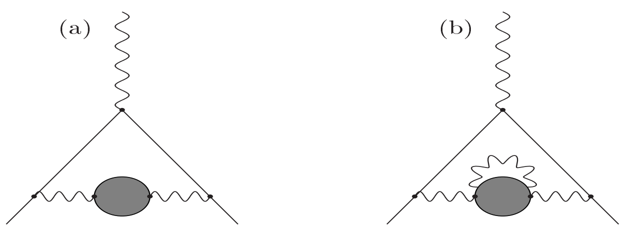

Fig. 4. Hadronic leading order (a) and next-to-leading order (hadrons+photon) (b) contributions to the muon anomalous magnetic moment.

The only reliable method of calculation of the hadronic contribution to the muon anomalous magnetic moment a_μ, available till now in the low energy region, is based on dispersion relations, where the measured e^+e^- hadronic cross section is convoluted with the known kernel function (for definitions and recent reviews see [**?**, **?**, **?**]). The LO hadronic contribution to a_μ enters through the graph(s) shown in Fig.**??**a. One class of contributions at the next-to-leading order (NLO) is shown in Fig.**??**b, where an additional photon is attached to charged hadron line(s). For the two pion plus photon final state one can try to estimate the contribution using point-like pions and sQED or just by inserting a quark loop with photonic corrections. As shown in [**?**], one cannot rely on a theoretical model, as the values obtained vary very much with the chosen model and do depend very strongly on the value of the quark mass. The values obtained are: $\delta a_\mu(\mathrm{quark}, \gamma, m_q = 180\ \mathrm{MeV}) = 1.880 \times 10^{-10}$, $\delta a_\mu(\mathrm{quark}, \gamma, m_q = 66\ \mathrm{MeV}) = 8.577 \times 10^{-10}$ and $\delta a_\mu(\pi^+\pi^-, \gamma) = 4.309 \times 10^{-10}$, where $m_q = 180$ MeV describes well the LO contributions to a_μ , while $m_q = 66$ MeV describes well the lowest order contribution to $\alpha(M_Z)$.

As the values are of the order of the present error of the hadronic contribution to a_μ [**?**] it is important to measure also the relevant parts of the $e^+e^- \to \pi^+\pi^-\gamma$ cross section and then, using dispersion relations, get the contribution of the $\pi^+\pi^-\gamma$ intermediate state to a_μ.

This is by no means easy (for more extensive discussions see [**?**]), as the two body cuts of the diagram in Fig.**??**b correspond to the radiative corrections to the $\pi\pi\gamma$ vertex (Fig.**??**b), while the three body cuts correspond to a part of the leading order $e^+e^- \to \pi^+\pi^-\gamma$ process (Fig.**??**c), with the

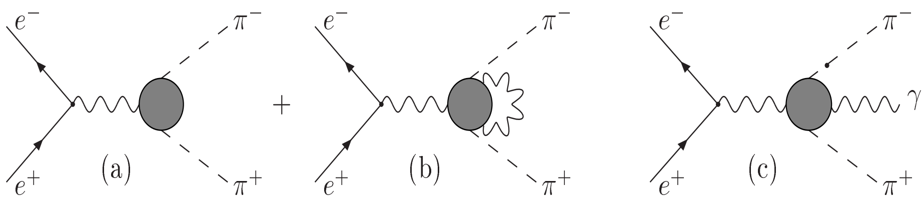

Fig. 5. Leading order (a) and $\pi^+\pi^-\gamma^*$ vertex corrections (b) contributions to the reaction $e^+e^- \to \pi^+\pi^-$; leading order FSR contribution to the reaction $e^+e^- \to \pi^+\pi^-\gamma$ (c).

photon emitted from the final states only. In practice, when measuring the cross section $\sigma(e^+e^- \to \pi^+\pi^-)$, one always measures in combination the contributions from Fig.**??**a and **??**b plus the 'soft' part of the contribution from Fig.**??**c, with the latter depending on the actual experimental setup.

The virtual plus soft corrections to the $\pi^+\pi^-\gamma^*$ vertex are negative. As a result, the actual contribution from the 'hard' part (photon with energy above E_{cut}) can be larger than the inclusive sum of 'virtual'+'soft'+'hard' contributions. This is shown in Fig.**??**a, where differential contributions to $a_\mu^{\mathrm{had},\gamma}$ from $\pi^+\pi^-\gamma$ intermediate states are compared with the contribution from $\pi^+\pi^-$. The contribution to a_μ, integrated over the whole s–spectrum,

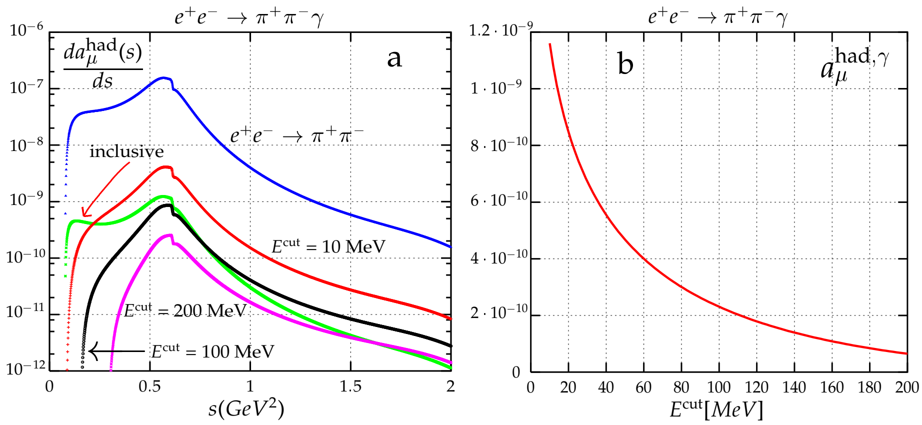

Fig. 6. Differential contribution to $a_\mu^{\mathrm{had},\gamma}$ from $\pi^+\pi^-\gamma$ intermediate states for different cutoff values compared with complete contribution (virtual plus real corrections, labelled 'inclusive') evaluated in scalar QED (FSR) as well as with contribution from $\pi^+\pi^-$ intermediate state (a) and integrated contribution to $a_\mu^{\mathrm{had},\gamma}$ as function of the cutoff E^{cut} (b).

is shown in Fig.??b. Thus a special care has to be taken when imposing cuts on the photon energy below 50 MeV, as it might lead to significant shift for a_μ.

4. FSR at NLO and tests of the FSR models

The upgrade of the PHOKHARA event generator to version 3.0 [?] consisted in adding the diagrams from Fig.??, where the photon emitted from the initial state is assumed to be 'hard', say with energy above 10 MeV at DAΦNE and 100 MeV at B–factories.

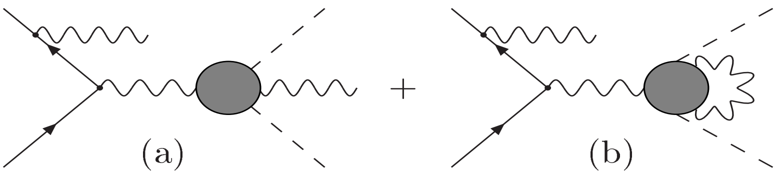

Fig. 7. NLO contributions to the reaction $e^+e^- \to \pi^+\pi^-\gamma$ from real (both soft and hard) FSR emission (ΔIFSNLO(S+H)) (a) and virtual corrections to the $\pi^+\pi^-\gamma^*$ vertex (ΔIFSNLO(V)) (b).

From the analysis of the corresponding corrections to the $e^+e^- \to \pi^+\pi^-$ process [?] in the framework of sQED, this contribution is expected to be of the order of 1%. However, the emission of the initial photon reduces the invariant mass of the $\pi^+\pi^-$ (or $\pi^+\pi^-\gamma$) system to the ρ mass with high probability due to the peak of the pion form factor at the ρ mass. As a result, this contribution is strongly enhanced in the region of invariant mass of the $\pi^+\pi^-$ system below the ρ resonance, as shown in Fig.??a and Fig.??a for KLOE and B–factory energies respectively. Suitably chosen cuts can be applied to suppress these NLO FSR contributions. In Fig.??b one can see that the standard KLOE cuts [?, ?], which consist of the cuts on pion angles, the missing momentum angle and the track mass (M_{tr}), keep the NLO FSR contribution below 2% with respect to the ISR cross section in the whole interesting region of the two–pion invariant mass. Similarly at B–factories, applying the track mass cut only for events with $Q^2 < m_\rho^2$, the NLO FSR contribution is kept at a negligible level (Fig.??b).

Again, as in the case of LO FSR contributions, the main problem consists in the model dependence of FSR. Till now only few tests were performed to verify the model for FSR. However, if one aims at a measurement of the

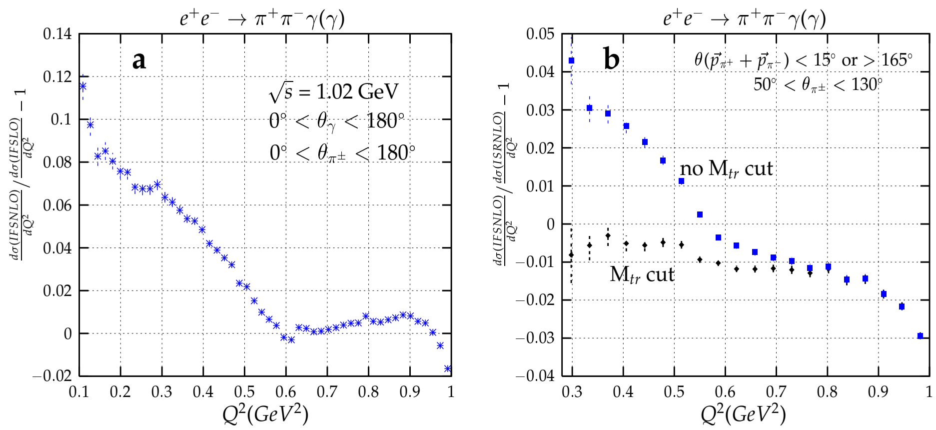

Fig. 8. Comparison of the Q^2 differential cross sections for $\sqrt{s} = 1.02$ GeV: IFSNLO contains the complete NLO contribution, while IFSLO has FSR at LO only. The pion and photon(s) angles are not restricted in (a). In (b) cuts are imposed on the missing momentum direction and the track mass (see text for description).

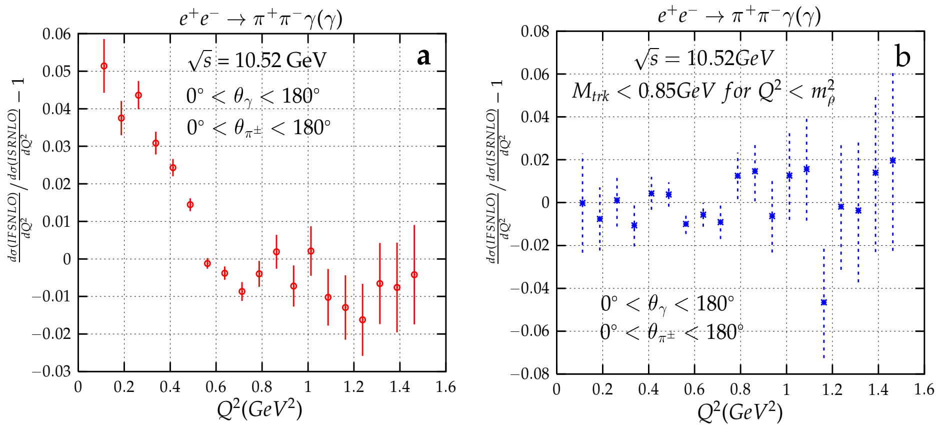

Fig. 9. Comparison of the Q^2 differential cross sections for $\sqrt{s} = 10.52$ GeV: IFSNLO contains the complete NLO contribution, while IFSLO has FSR at LO only. The pion and photon(s) angles are not restricted in (a). In (b) cuts are imposed on the track mass for $Q^2 < m_\rho^2$.

accuracy below 1% such tests become indispensable.

In the present KLOE experimental setup, where only four momenta of the pions are measured, a possibility to test the hard part of the NLO FSR (ΔIFSNLO(H)) contribution is to look at the dependence of the cross section on the missing invariant mass. Completely different effect of the cut on missing invariant mass on ISR (ISRNLO) and FSR at NLO, as shown in Fig.**??**a, provides a powerful tool for testing the hard part of the IFSNLO contributions. A measurement of this few percent effect, depending on the two–pion invariant mass Q^2 is within reach of the KLOE experiment.

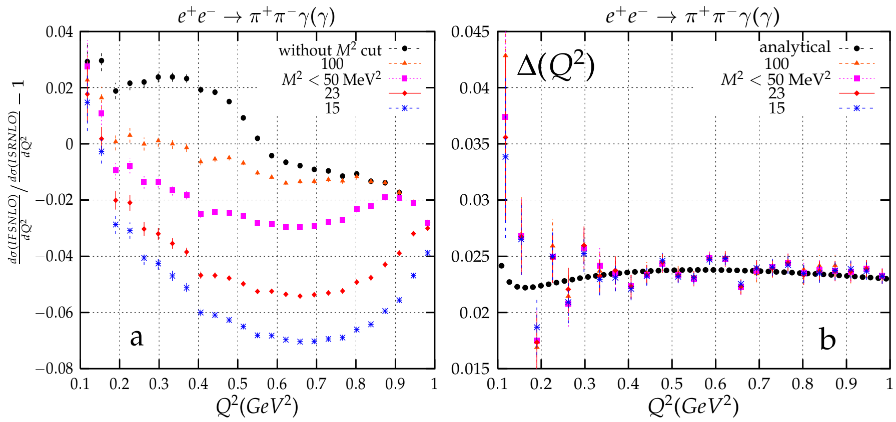

Fig. 10. Dependance of the relative IFSNLO contribution on the cut on missing invariant mass M^2 (a) and radiative correction factor Δ extracted from MC data (see text for explanation) (b).

The tests proposed in section **??** for the FSR at LO and the above tests of the hard part of the IFSNLO contributions can be further extended to the test of the virtual radiative corrections to the FSR of Fig.**??**b following the procedure outlined below.

The pion form factor parameters can be fitted to the data using the measured radiative return cross section. The cross section can be splited into four parts

$$\frac{d\sigma_{RR}}{dQ^2} = \mathrm{ISRNLO} + \mathrm{FSRLO} + \Delta\mathrm{IFSNLO(V+S)} + \Delta\mathrm{IFSNLO(H)} \ . \quad (3)$$

Having the form factor parameters, one can subtract from the data, relying on the Monte Carlo simulation: 1. the ISR contributions, which involve QED corrections only, 2. the FSRLO contributions tested via charge asymmetries, so well known, and 3. the hard part of the IFSNLO corrections (ΔIFSNLO(H)), tested as described above, so known with required accuracy. The left over virtual plus soft photon corrections to the FSR (ΔIFSNLO(V+S)) can be written in the following way

$$\Delta\mathrm{IFSNLO(V+S)} = \frac{d\sigma^{\mathrm{Born}}}{dQ^2}\left(\ln w \cdot f + \Delta(Q^2)\right) \ , \quad (4)$$

where $\sigma^{\rm Born}$ stands for the LO $e^+e^- \to \pi^+\pi^-\gamma$ cross section with photons emitted from the initial state only and the $\ln w$ is the standard soft photon logarithm with a known function f as a coefficient. It is clear that one can now extract the function $\Delta(Q^2)$ from data and compare it with the analytical value of the tested model. The function $\Delta(Q^2)$ can be extracted separately in the same way for each cut on invariant mass. Moreover, for self consistency of the tested model the values of $\Delta(Q^2)$ obtained for each invariant mass cut should be the same. It provides a nontrivial check of the model, as the relative contribution of the ΔIFSNLO(V+S) part to the total cross section does depend on the invariant mass cut. The results of that procedure applied to the Monte Carlo data obtained by PHOKHARA 3.0 are shown in Fig.**??**b.

Having all separate ingredients of the cross section one can extract not only the pion form factor but also calculate the NLO corrections to the muon magnetic moment coming from the $\pi^+\pi^-\gamma$ intermediate state.

5. Conclusions

Extensive discussion of possible experimental tests of the model dependence of FSR for the $\pi^+\pi^-$ hadronic final state has been made. The importance of the $\pi^+\pi^-\gamma$ final state contribution to the muon anomalous magnetic moment has been emphasised and a method has been proposed to extract that contributions from the data using radiative return.

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REFERENCES

[1] S. Eidelman and F. Jegerlehner, Z. Phys. **C67** (1995) 585 [hep-ph/9502298].

[2] F. Jegerlehner, J. Phys. **G29** (2003)101 [hep-ph/0104304], hep-ph/0310234.

[3] K. Melnikov, Int. J. Mod. Phys. A **16** (2001) 4591 [hep-ph/0105267].

[4] M. Davier, S. Eidelman, A. Höcker and Z. Zhang Eur. Phys. J. **C 27** (2003) 497 [hep-ph/0208177].

[5] K. Hagiwara, A.D. Martin, Daisuke Nomura and T. Teubner, Phys. Lett. **B 557** (2003) 69 [hep-ph/0209187].

[6] M. Davier, S. Eidelman, A. Höcker and Z. Zhang, hep-ph/0308213.

[7] A. Nyffeler, hep-ph/0305135 and these proceedings.

[8] G.W.Bennett *et al.* [Muon $g-2$ Collaboration], Phys. Rev. Lett. **89** (2002) 101804; Erratum, ibid. **89** (2002) 129903, [hep-ex/0208001].
[9] A. Aloisio *et al.* [KLOE Collaboration], hep-ex/0107023.
[10] A. Denig *et al.* [KLOE Collaboration], eConf **C010430** (2001) T07 [hep-ex/0106100].
[11] E. P. Solodov [BABAR collaboration], eConf **C010430** (2001) T03 [hep-ex/0107027].
[12] B. Valeriani *et al.* [KLOE Collaboration], hep-ex/0205046.
[13] N. Berger, eConf **C020620** (2002) THAP10, [hep-ex/0209062].
[14] G. Venanzoni *et al.* [KLOE Collaboration], eConf **C0209101** (2002) WE07, [hep-ex/0210013]; hep-ex/0211005.
[15] A. Denig *et al.* [KLOE Collaboration], Nucl. Phys. Proc. Suppl. **116** (2003)243 [hep-ex/0211024].
[16] A. Aloisio *et al.* [KLOE Collaboration], hep-ex/0307051.
[17] A. Blinov, talk at International Conference "New trends in high-energy physics" Alushta, Crimea (May 2003) http://www.slac.stanford.edu/ ablinov/
[18] S. di Falco (KLOE), these proceedings.
[19] S. Binner, J. H. Kühn and K. Melnikov, Phys. Lett. B **459** (1999) 279 [hep-ph/9902399].
[20] Min-Shih Chen and P. M. Zerwas, Phys. Rev. D **11** (1975) 58.
[21] B.A. Kniehl, M. Krawczyk, J.H. Kühn, R.G. Stuart, Phys. Lett. **B209** (1988) 337.
[22] H. Czyż and J. H. Kühn, Eur. Phys. J. C **18** (2001) 497 [hep-ph/0008262].
[23] G. Rodrigo, H. Czyż, J.H. Kühn and M. Szopa, Eur. Phys. J. C **24** (2002) 71 [hep-ph/0112184].
[24] H. Czyż, A. Grzelińska, J. H. Kühn and G. Rodrigo, Eur. Phys. J. C **27** (2003) 563 [arXiv:hep-ph/0212225].
[25] H. Czyż, A. Grzelińska, J. H. Kühn and G. Rodrigo, hep-ph/0308312.
[26] J. H. Kühn Nucl. Phys. Proc. Suppl. **98** (2001) 289 [hep-ph/0101100].
[27] G. Rodrigo, A. Gehrmann-De Ridder, M. Guilleaume and J. H. Kühn, Eur. Phys. J. C **22** (2001) 81 [hep-ph/0106132].
[28] G. Rodrigo, Acta Phys. Polon. B **32** (2001) 3833 [hep-ph/0111151].
[29] J. H. Kühn and G. Rodrigo, Eur. Phys. J. C **25** (2002) 215 [hep-ph/0204283].
[30] G. Rodrigo, H. Czyż and J. H. Kühn, hep-ph/0205097; Nucl. Phys. Proc. Suppl.123(2003)167 [hep-ph/0210287]; Nucl. Phys. Proc. Suppl. **116** (2003) 249 [hep-ph/0211186].
[31] J. Gluza, A. Hoefer, S. Jadach and F. Jegerlehner, Eur. Phys. J. **C28** (2003) 261, [hep-ph/0212386] .
[32] A. Hoefer, J. Gluza and F. Jegerlehner, Eur. Phys. J. **C24** (2002) 51 [hep-ph/0107154].
[33] J. S. Schwinger, *Particles, Sources, And Fields. Vol. 3*, Redwood City, USA: Addison-Wesley (1989) p. 99.