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$e^+e^- \to \pi^+\pi^-e^+e^-$: a potential background for $\sigma(e^+e^- \to \pi^+\pi^-)$ measurement via radiative return method *

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A Monte Carlo generator (EKHARA) has been constructed to simulate the reaction $e^+e^- \to \pi^+\pi^-e^+e^-$ based on initial and final state emission of a e^+e^- pair from $e^+e^- \to \pi^+\pi^-$ production diagram. A detailed study of the process, as a potential background for $\sigma(e^+e^- \to \pi^+\pi^-)$ measurement via radiative return method, is presented for Φ- and B- factory energies.

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1. Introduction

The radiative return method [1] (look [2] for a short introduction) is a powerful tool in the measurement of $\sigma(e^+e^- \to$ hadrons), crucial for predictions of the hadronic contributions to a_μ, the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling from its value at low energy up to M_Z (for recent reviews look [3, 4, 5]). Due to a complicated experimental setup, the use of Monte Carlo (MC) event generators [1, 6, 7, 8, 9], which includes various radiative corrections [10, 11] is indispensable. Some more extensive analysis of that subject can be found also in [12, 13, 14]. The most important hadronic mode, i.e. $\pi^+\pi^-$, is currently measured by KLOE [15, 16, 17, 18], and BaBar [19] by means of radiative return method. This measurement can suffer from a background from the process $e^+e^- \to \pi^+\pi^-e^+e^-$, as suggested in [20], for at least two reasons: 1. At present KLOE measures only pions (+ missing momenta) in the final state and for that particular measurement there is

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no difference between photon(s) and pair production. 2. The e^+e^- pair can escape detection, being lost outside a detector, e.g. in the beam pipe, or having energy below a detection threshold. Again a Monte Carlo study is unavoidable, if one likes to know the actual value of the pair production contribution in a given experimental setup, as the analytical, completely inclusive, calculations might lead to misleading results.

2. Monte Carlo program EKHARA and its tests

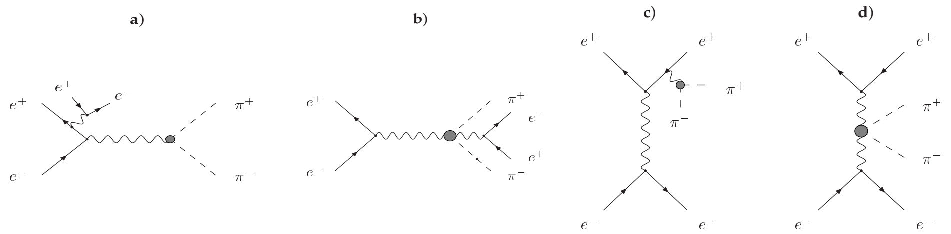

Fig. 1. Diagrams contributing to the process $e^+(p_1)e^-(p_2) \to \pi^+(\pi_1)\pi^-(\pi_2)e^+(q_1)e^-(q_2)$: initial state electron pair emission (a), final state electron pair emission (b), pion pair emission from t–channel Bhabha process (c) and $\gamma^*\gamma^*$ pion pair production (d).

In Fig.1 different types of diagrams contributing to process $e^+e^- \to \pi^+\pi^-e^+e^-$ are shown schematically. In the present version of the Monte Carlo program we include only two gauge invariant sets of diagrams from Fig.1a and 1b. The former represents initial state radiation (ISR), and the latter final state radiation (FSR), of an e^+e^- pair from $e^+e^- \to \pi^+\pi^-$ production diagram. We use scalar QED (sQED) to model FSR e^+e^- pair emission and ρ dominance model for $\gamma^*(\rho^*)\pi\pi$ coupling (see [1] for details). The diagrams from Fig.1c, representing pion pair emission from t–channel Bhabha process, together with s-channel Bhabha pion pair emission (not shown in Fig.1), will be included in the new version [21] of the presented generator, completing the discussion of this paper. The contribution from $\gamma^*\gamma^*$ pion pair production process (Fig.1d) is negligible for DAΦNE energy [17], and as its interference with other diagrams does not contribute to the cross section integrated over charge symmetric cuts, these contributions are not relevant, at least for Φ–factories.

For parametrisation of the phase space we use the following variables: $Q^2 = (\pi_1 + \pi_2)^2$–invariant mass of $\pi^+\pi^-$ system, $k_1^2 = (q_1 + q_2)^2$–invariant mass of e^+e^- system, polar and azimuthal angles of $\vec{Q}$ momentum, defined in the centre-of-mass (CM) frame of initial e^+e^- pair, polar and azimuthal angles of $\vec{\pi}_1$ momentum, defined in Q- rest frame and polar and azimuthal angles of $\vec{q}_1$ momentum, defined in k_1- rest frame. All four vectors are boosted into the initial e^+e^- CM frame after being generated and all necessary cuts can be applied at this stage of the generation. Multi–channel variance reduction method was used to improve efficiency of the generator and all details will be given in a separate publication [21].

We have performed a number of 'standard' tests to ensure that the written FORTRAN code is correct. Gauge invariance of the sum of the amplitudes was checked analytically separately for set of diagrams from Fig.1a and 1b. We use helicity amplitudes in EKHARA to calculate square of the matrix element describing the $e^+e^- \to \pi^+\pi^-e^+e^-$ process, but as a cross check, we have used also the standard trace technique to calculate the square of the matrix element, summed over polarisations of initial and final leptons. Both results were compared numerically scanning the physical phase space, and the biggest relative difference between the two results found was at the level of 10^{-9}. It was necessary to use quadrupole precision of the real numbers for the trace technique result, as one can observe severe cancellations between various terms. The phase space volume calculated by Monte Carlo program was cross checked with the Gauss integration and the relative difference at the level of 10^{-5} was well within the errors of the MC result, which were of the same order.

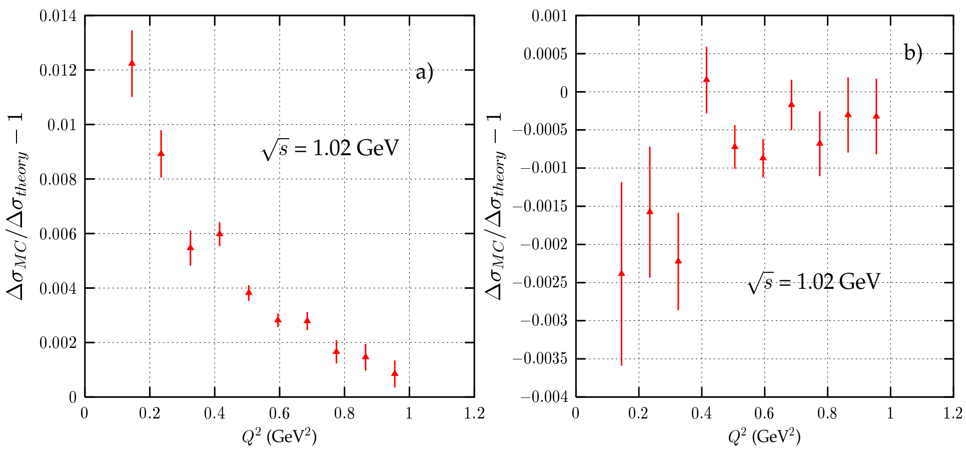

Fig. 2. EKHARA results compared with analytical results of [22] (see text for details). The errors come from MC integration.

Inclusive analytical formulae from [22] provide additional, nontrivial tests of the implementation of the contributions from Fig.1a. Formula (23) from [22] provides Q^2 differential cross section (other variables are integrated over the whole allowed range) valid for large Q^2. In Fig.2a, we compare the values of the integrals, over 10 equally spaced intervals of Q^2, obtained by means of MC program and one-dimensional Gauss integration of the above mentioned analytical formula. The Gauss routine, which we use, guarantees precision of 12 significant digits. One observes a relatively good agreement for values of $Q^2 \sim s$ and worse for Q^2 nearby $\pi^+\pi^-$ production threshold, as expected from the applicability of the analytical formula. EKHARA results agree much better (see Fig.2b) with known analytically doubly differential cross section $\frac{d\sigma}{dQ^2 dk_1^2}$ [23, 22], integrated over the whole allowed range of k_1^2 and 10 equally spaced intervals of Q^2. The exact analytical result was integrated numerically, using recursively one-dimensional 8-point Gauss procedure and dividing the region of integration into pieces small enough to guarantee the overall accuracy of 10 significant digits. From Fig.2b it is clear that a technical precision of EKHARA of the order of 0.1% was achieved.

3. Monte Carlo Data Analysis

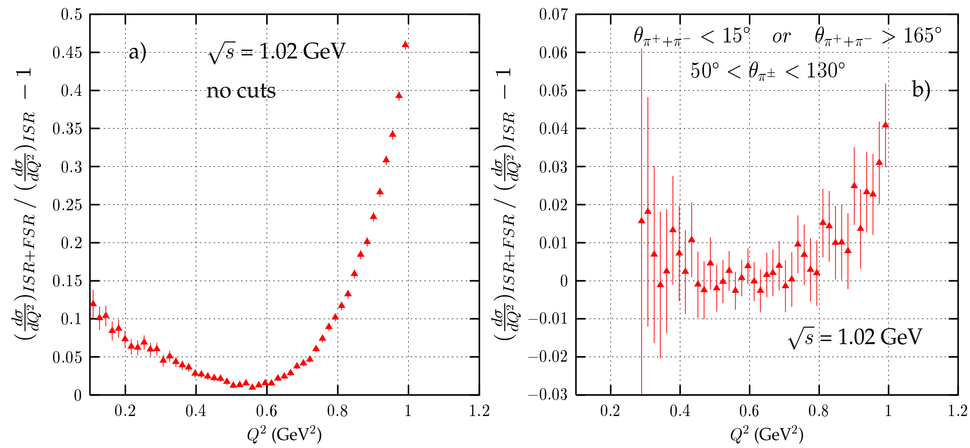

Fig. 3. Comparisons of ISR and FSR contributions to $e^+e^- \to \pi^+\pi^-e^+e^-$ cross section at DAΦNE energy.

For ISR of e^+e^- pair (Fig.1a), a factorisation similar to photon emission holds [22] and adding ISR pair production to ISR photon production results just in a change of the radiator function, thus radiative return method still can be used [2]. On the other hand, FSR of e^+e^- pair (Fig.1b) is model dependent, the same way as it is the emission of a real photon, and the question of its relative, to ISR, contribution to the cross section is as important as for the photon emission. One can observe, that e^+e^- pair emission resembles a lot photon emission, with big contributions of FSR for inclusive configurations (Fig.3a) of a Φ–factory, which can be easily reduced, by suitable cuts, to a negligible level (Fig.3b). Moreover, the cuts which reduce photon FSR reduces as well the e^+e^- pair FSR. In addition, analogously to photon FSR, e^+e^- pair FSR is completely negligible at B–factories.

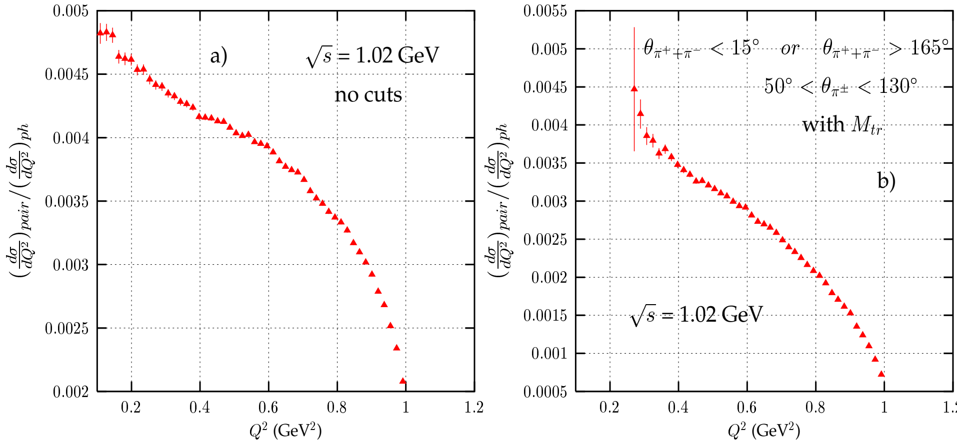

Fig. 4. Ratio of differential cross sections of the process $e^+e^- \to \pi^+\pi^-e^+e^-$ (pair) and $e^+e^- \to \pi^+\pi^- +$ photon(s) (ph).

The most relevant information, how big is the contribution of the process $e^+e^- \to \pi^+\pi^-e^+e^-$ in comparison to the main process used in the radiative return method, mainly $e^+e^- \to \pi^+\pi^- +$ photon(s), is presented in Fig.4 for DAΦNE energy, both without any cuts (Fig.4a), and with cuts resembling KLOE event selection [16, 18] (Fig.4b). The results of $e^+e^- \to \pi^+\pi^- +$ photon(s) cross section were obtained using PHOKHARA 3.0 MC generator [9] and in the following, whenever we refer to $e^+e^- \to \pi^+\pi^- +$ photon(s) cross section we mean cross section obtained using PHOKHARA 3.0. As one can see from Fig.4, the contribution of the e^+e^- pair production is below 0.5%, independently on the cuts. It is Q^2 dependant, being big for low Q^2 values. Even if it is small, this 0.5% contribution can become important, when aiming at the precision below, or of the order of 1%, for the $e^+e^- \to \pi^+\pi^-$ cross section measurement. At B–factories, the relative contribution of the pair production might be as big as 0.9% (Fig.5a) and it is again Q^2 and cut dependent (Fig.5b).

As stated already, ISR of electron pairs can be treated in a similar way as ISR of photons resulting in the change of radiator function in the

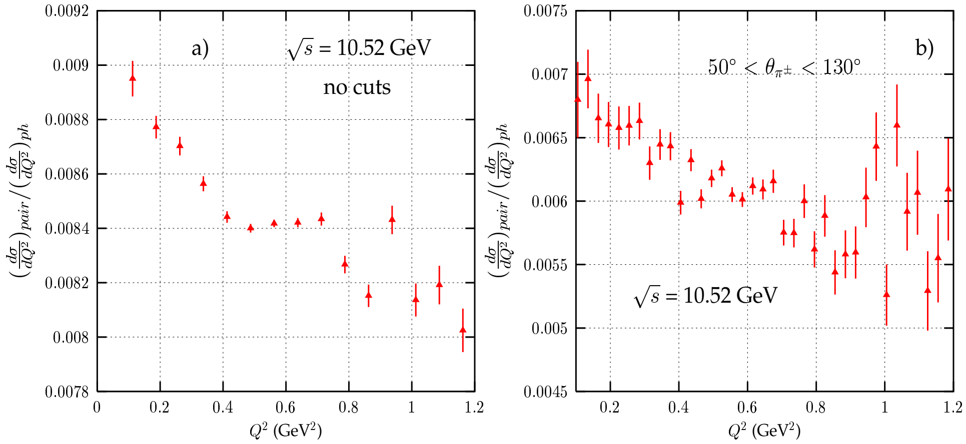

Fig. 5. Ratio of differential cross sections of the process $e^+e^- \to \pi^+\pi^-e^+e^-$ (pair) and $e^+e^- \to \pi^+\pi^-$ + photon(s) (ph).

radiative return method. However, one can alternatively try to treat it as a background to the process with photon(s) emission. In this case, the most natural way of reducing that background is to veto the electron (positron) in the final state. In Fig.6 we show an example of such a procedure performed for Φ–factory energy. We assume here that an electron or positron can be

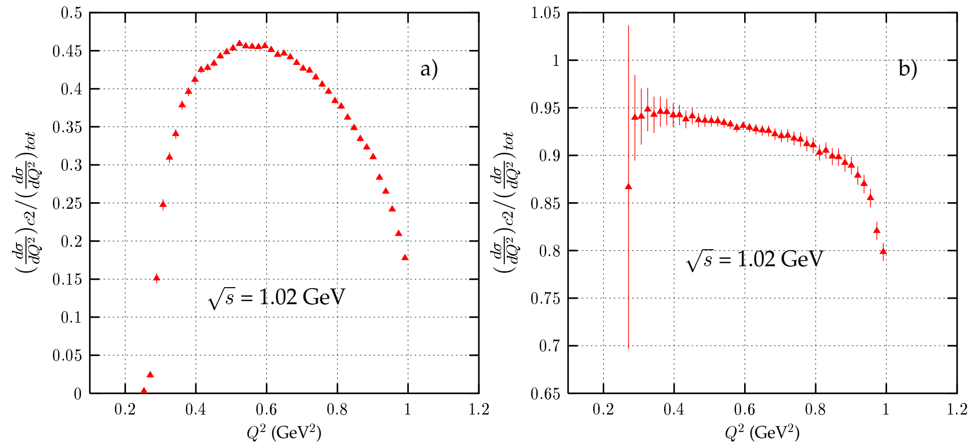

Fig. 6. Non-reducible pair production background at DAΦNE energy : (a) no cuts in $(\frac{d\sigma}{dQ^2})_{tot}$; (b) for both $(\frac{d\sigma}{dQ^2})_{c2}$ and $(\frac{d\sigma}{dQ^2})_{tot}$ the following cuts are imposed: $50° < \theta_{\pi^\pm} < 130°$, $\theta_{\pi^+ + \pi^-} < 15°$ or $\theta_{\pi^+ + \pi^-} > 165°$ and M_{tr} cut. For (a) and (b) c2 denotes additional cuts: ($\theta_{e^+} < 20°$ or $\theta_{e^+} > 160°$) and ($\theta_{e^-} < 20°$ or $\theta_{e^-} > 160°$).

seen, and the event rejected, if its angle with respect to the beam axis is bigger then 20°. Fig.6a shows that up to 50% of the events pass the rejection procedure, when no other cuts are applied. However, in the case of KLOE event selection, which requires that the e^+e^- pair is emitted along the beam axis, one rejects only a small fraction of these events (Fig.6b).

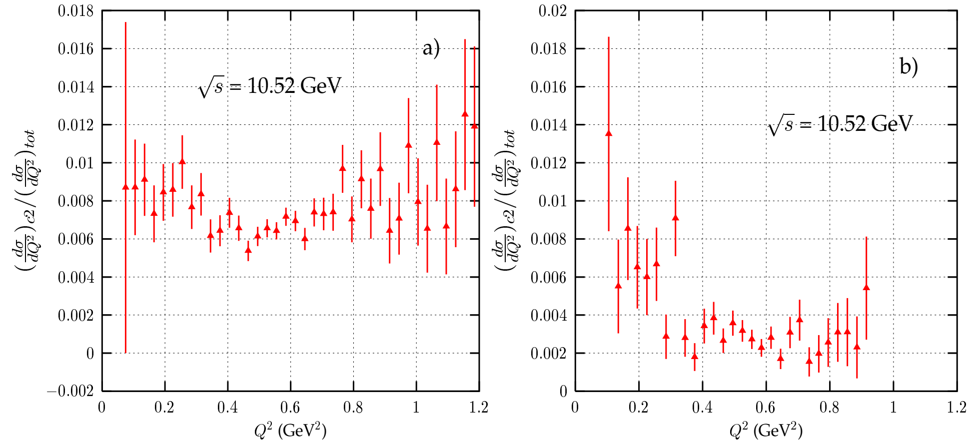

Fig. 7. Non-reducible pair production background at B–factory energy : (a) no cuts in $(\frac{d\sigma}{dQ^2})_{tot}$; (b) for both $(\frac{d\sigma}{dQ^2})_{c2}$ and $(\frac{d\sigma}{dQ^2})_{tot}$ cuts on pion angles are imposed: $50° < \theta_{\pi^\pm} < 130°$. For (a) and (b) c2 denotes additional cuts: ($\theta_{e^+} < 20°$ or $\theta_{e^+} > 160°$) and ($\theta_{e^-} < 20°$ or $\theta_{e^-} > 160°$).

The situation is completely different at B–factories, where one can almost completely reduce the background coming from e^+e^- pairs (Fig.7), by rejecting the events with at least one charged lepton, electron or positron, in the detector.

4. Conclusions

We have constructed the Monte Carlo generator EKHARA, which simulates the reaction $e^+e^- \to \pi^+\pi^-e^+e^-$ based on initial and final state emission of a e^+e^- pair from $e^+e^- \to \pi^+\pi^-$ production diagram. A detailed study of this process, as a potential background for $\sigma(e^+e^- \to \pi^+\pi^-)$ measurement via radiative return method, shows that it can become important, when the experimental precision will reach 1%, or better.

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