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QED vertex form factors at two loops

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Abstract

We present the closed analytic expression of the form factors of the two-loop QED vertex amplitude for on-shell electrons of finite mass m and arbitrary momentum transfer $S = -Q^2$. The calculation is carried out within the continuous D-dimensional regularization scheme, with a single continuous parameter D, the dimension of the space-time, which regularizes at the same time UltraViolet (UV) and InfraRed (IR) divergences. The results are expressed in terms of 1-dimensional harmonic polylogarithms of maximum weight 4.

Key words: Feynman diagrams, Multi-loop calculations, Vertex diagrams
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1 Introduction

This paper is devoted to the evaluation of the 2-loop contributions (second order in the expansion in terms of the fine-structure constant) of the form factors of the QED vertex amplitude, for arbitrary momentum transfer $S = -Q^2$ and on-shell external fermion lines, of finite mass m, in the continuous D regularization scheme.

The analytic calculation of the imaginary parts of the form factors at two-loop level for arbitrary momentum transfer, together with the value of the charge slope of the electron, were obtained long ago in [1] using the Pauli-Villars regularization scheme for the ultraviolet (UV) divergences, and giving a small fictitious mass λ to the photon for the regularization of the soft infrared (IR) divergences. The results were given in terms of Nielsen's polylogarithms [2, 3] of maximum weight 3. The properly subtracted dispersion relations for the evaluation of the corresponding real parts were also written, but their explicit analytic evaluation was not carried out in [1], because the results could not be expressed in terms of Nielsen's polylogarithms only.

In [4] that analytic integration of the dispersion relations could at last be performed, expressing the real parts of the 2-loop form factors in terms of the 1-dimensional harmonic polylogarithms (HPLs), introduced in the meanwhile [5, 6], of maximum weight $w = 4$.

In this paper we present the calculation of the real and imaginary parts of the two-loop form factors within the framework of dimensional regularization [7]. Both UV and soft IR divergences are regularized in terms of the same parameter D, the continuous dimension of the space-time.

The Feynman diagrams involved are shown in Fig. 1. The fermion lines carry momenta p_1 and p_2 and are both incoming (as in the kinematical case of electron-positron annihilation), the outgoing photon has momentum $Q = p_1 + p_2$. The electron mass is m and the mass-shell condition for the two fermions is $p_1^2 = p_2^2 = -m^2$.

On Lorentz-invariance grounds, all the vertex diagrams can be expressed in terms of at most three factors, corresponding to the three vectors proportional to the Dirac matrices γ^μ, to $\sigma^{\mu\nu}Q_\nu$, with $\sigma^{\mu\nu} = -\frac{i}{2}[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\nu]$ and to Q^μ (in QED the third form factor is of course vanishing, when the contributions of the various graphs are summed). The on mass-shell form factors are functions of the momentum transfer $S = -Q^2$ and of the mass of the fermions m. Their value, when projected out from the Feynman graph amplitudes in the continuous D-dimensional regularization scheme, is a combination of the scalar integrals associated to the Feynman graphs. In [8] all those integrals were separately expressed in terms of 17 independent scalar integrals, called Master Integrals (MIs), by means of a reduction algorithm based on the integration by part identities (IBPs) [9], Lorentz invariance identities (LI) [10] and general symmetry relations, implemented for the computer language FORM [11]. The MIs were then calculated in [8] by the differential equations method [12, 13, 14].

In this paper we use those results in order to evaluate the explicit analytic value of the form factors for any of the (unrenormalized) diagrams of Fig. 1 and for the

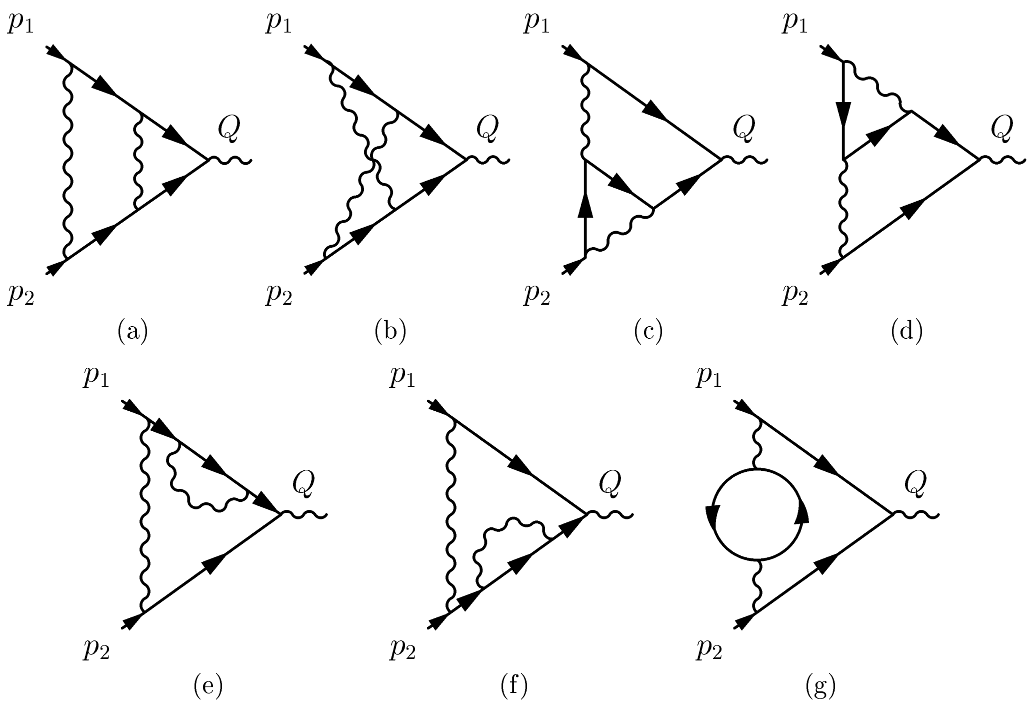


Figure 1: 2-loop vertex diagrams for the QED form factors. The fermionic external lines are on the mass-shell $p_1^2 = p_2^2 = -m^2$; the wavy line on the r.h.s. carries momentum $Q = p_1 + p_2$, with $Q^2 = -S$. The arrows label the flow of the momenta p_1 and p_2.

full renormalized vertex amplitude as well.

The paper is structured as follows.

After a general introduction on the QED form factors, recalled in section 2, in section 3 we give the unrenormalized form factors for each of the 2-loop Feynman diagrams entering in the calculation of the 2-loop QED vertex amplitude, within the D-dimensional regularization scheme. In section 4 the subtractions for the renormalization of UV divergences at the second order in the fine-structure constant are listed. In section 5 we present the full UV-renormalized form factors for the 2-loop QED vertex amplitude in the space-like region $-S = Q^2 > 0$; we also discuss the analytic continuation to the physical region $S = -Q^2 > 4m^2$, presenting the imaginary parts of the form factors. In sections 6 and 7 the behaviours of the form factors for large and small momentum transfer are given (recovering in particular the two loop values of the electron $(g-2)$ and of the charge form factor slope). In the appendix A we list the definition of the propagators used in the explicit calculations, and in appendix B we give the 1-loop contributions to the QED vertex form factors up to first order in $(D-4)$.

2 The QED form factors

Let us call $V^\mu(p_1,p_2)$ the QED vertex amplitude, corresponding to the annihilation of an electron and a positron, of momenta p_1 and p_2, with the two particles on the mass-shell ($p_1^2 = p_2^2 = -m^2$). Let us define the following two vectors:

$$Q^\mu = p_1^\mu + p_2^\mu \, , \qquad \Delta^\mu = p_1^\mu - p_2^\mu \, , \tag{1}$$

such that $Q^2 = -S$, where S is the c.m. energy squared; in the following we will also use the related dimensionless variables

$$q^2 = \frac{Q^2}{m^2} \, , \qquad\qquad s = \frac{S}{m^2} \, . \tag{2}$$

In general $V^\mu(p_1,p_2)$ can be expressed in terms of three dimensionless scalar form factors $F_i(q^2), i = 1,2,3$, depending only on the dimensionless variable q^2 of Eq. (2), as follows:

$$V^\mu(p_1,p_2) = \bar{v}(p_2)\Gamma^\mu(p_1,p_2)u(p_1) \tag{3}$$

$$\Gamma^\mu(p_1,p_2) = \left[F_1(q^2)\,\gamma^\mu + \frac{1}{2m}F_2(q^2)\,\sigma^{\mu\nu}Q_\nu - \frac{i}{m}F_3(q^2)\,Q^\mu \right] \, , \tag{4}$$

where $\bar{v}(p_2), u(p_1)$ are the spinor wave functions of the positron and the electron, $\sigma^{\mu\nu} = -\frac{i}{2}[\gamma^\mu,\gamma^\nu]$. Usually $F_1(q^2)$ is known as the *charge* (Dirac) *form factor* whereas $F_2(q^2)$ as the *magnetic* (Pauli) *form factor*. If we consider single Feynman diagrams, any form factors in Eq. (4) can be in general different from zero. However, if $\Gamma^\mu(p_1,p_2)$ is the full vertex amplitude, the conservation of the electromagnetic current forces the third form factor to vanish, $F_3(q^2) = 0$.

The extraction of each form factors $F_i(q^2)$ from Eq. (4) can be carried out by the following general projector operators $P_\mu^{(i)}$:

$$P_\mu^{(i)}(m,p_1,p_2) = \frac{-i\,\not{p}_1+m}{m}\left[g_1^{(i)}\gamma_\mu + \frac{i}{m}g_2^{(i)}\Delta_\mu - \frac{i}{m}g_3^{(i)}Q_\mu\right]\frac{i\,\not{p}_2+m}{m}, \tag{5}$$

where the constants $g_j^{(i)}$, $j=1,2,3$, are properly chosen to have:

$$\mathrm{Tr}\left(P_\mu^{(i)}(m,p_1,p_2)\Gamma^\mu(p_1,p_2)\right) = F_i(q^2). \tag{6}$$

Let us observe that since we work in a D-dimensional space (to regularize the divergences arising in the computation) the *trace* operation is consistently performed in D dimensions as well.

The explicit values of the constants are:

$$\begin{aligned}
g_1^{(1)} &= -\frac{2}{(D-2)}\,\frac{1}{q^2+4}\,, &(7)\\
g_2^{(1)} &= -\frac{8(D-1)}{(D-2)}\,\frac{1}{(q^2+4)^2}\,, &(8)\\
g_1^{(2)} &= -\frac{8}{(D-2)}\,\frac{1}{q^2(q^2+4)}\,, &(9)\\
g_2^{(2)} &= -\frac{8}{(D-2)}\,\frac{1}{(q^2+4)^2}\left[\frac{4}{q^2}+D-2\right], &(10)\\
g_3^{(3)} &= -\frac{2}{q^2(q^2+4)}\,, &(11)
\end{aligned}$$

and $g_3^{(1)} = g_3^{(2)} = g_1^{(3)} = g_2^{(3)} = 0$.

As the spinor traces are in D-dimensional space-time, in all the above formulas Eqs. (7-11), all the r.h.s., strictly speaking, should be multiplied by the overall constant $(1/4)\ \mathrm{Tr}\mathbf{1}$, where $\mathrm{Tr}\mathbf{1}$ is the trace of the unit Dirac matrix in D-continuous dimensions. The overall constant is in fact undetermined for arbitrary D, except for its limiting value at $D=4$, which is 1. For simplicity, we will therefore omit systematically that overall factor.

In QED, the form factors are given as an expansion in powers of (α/π), $\alpha = e^2/4\pi$ being the fine-structure constant. Showing explicitly the dependence on the regularizing dimension, we write the expansion as

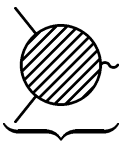
$$\begin{aligned}
F_1(D,q^2) &= 1 + \left(\frac{\alpha}{\pi}\right)F_1^{(1l)}(D,q^2) + \left(\frac{\alpha}{\pi}\right)^2 F_1^{(2l)}(D,q^2) + \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^3\right), &(12)\\
F_2(D,q^2) &= \left(\frac{\alpha}{\pi}\right)F_2^{(1l)}(D,q^2) + \left(\frac{\alpha}{\pi}\right)^2 F_2^{(2l)}(D,q^2) + \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^3\right), &(13)\\
F_3(D,q^2) &= \left(\frac{\alpha}{\pi}\right)F_3^{(1l)}(D,q^2) + \left(\frac{\alpha}{\pi}\right)^2 F_3^{(2l)}(D,q^2) + \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^3\right), &(14)
\end{aligned}$$

where the superscripts "$1l$" and "$2l$" stand for 1- and 2-loop contributions, the first term 1 in $F_1(D,q^2)$ is the tree approximation and $F_3(D,q^2)$ vanishes, as already said, when all the graphs are summed up.

In the following we will focus on the detailed 2-loop computations of $F_1^{(2l)}(D,q^2)$, $F_2^{(2l)}(D,q^2)$ and $F_3^{(2l)}(D,q^2)$. We will present the unrenormalized form factors for each Feynman diagram and we will discuss the renormalization procedure, giving the fully UV-renormalized result for the QED vertex amplitude. The 1-loop contributions to the form factors, $F_1^{(1l)}(D,q^2)$ and $F_2^{(1l)}(D,q^2)$, will be recalled in appendix B for completeness.

3 Unrenormalized contributions

The diagrams which contribute to the order $(\alpha/\pi)^2$ are shown in Fig. 1. Following Eq. (4), we will indicate by $\mathcal{F}_i^{(2l,\mathtt{graph})}(D,q^2)$ $(i=1,2,3)$, $\mathtt{graph} \in \{\mathtt{a},..,\mathtt{g}\}$ the contribution of each unrenormalized graph to the unrenormalized form factors:

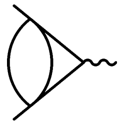


$$\left(\underbrace{\quad}_{\mathtt{graph}\ \in\ \{\mathtt{a},\ldots,\mathtt{g}\}} \right)^{\mu} = \bar{v}(p_2)\left[\mathcal{F}_1^{(2l,\mathtt{graph})}(D,q^2)\,\gamma^\mu + \frac{1}{2m}\mathcal{F}_2^{(2l,\mathtt{graph})}(D,q^2)\,\sigma^{\mu\nu}Q_\nu - \frac{i}{m}\mathcal{F}_3^{(2l,\mathtt{graph})}(D,q^2)\;Q^\mu\right]u(p_1)\ . \tag{15}$$

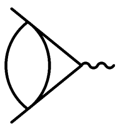
In so doing, each $\mathcal{F}_i^{(2l,\mathtt{graph})}(D,q^2)$ can be extracted by the D-dimensional projection defined in Eq. (6). After the computation of the *trace*, it turns out that each form factor is expressed in terms of several hundreds of scalar integrals. According to [8], one can express all those integrals in terms of only 17 Master Integrals (MIs) *via* integration-by-parts identities, Lorentz invariance and general symmetry relations (exactly in D).

As an example consider the *magnetic* form factor of the diagram (g) in Fig. 1. It can be written as a linear combination of 5 MIs, the coefficients being ratios of simple polynomials in D and q^2, the fermion mass squared m^2 appearing as a dimensional scale factor. (It is to be noted that in the formulas which follow, and which are exact in D, the dimensionless variables q^2 and D are never entangled in a same non factorisable polynomial in the denominators).

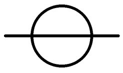
$$\begin{aligned}\mathcal{F}_2^{(2l,\mathtt{g})}(D,q^2) = & \frac{1}{(q^2+4)}\left\{\frac{32(D-4)}{(D-5)}\right. \\ & \left. -\frac{64(D^2-10D+19)}{(D-1)(D-5)}\frac{1}{(q^2+4)}\right\}\end{aligned}$$

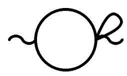


$$-\frac{64(D^3-8D^2+23D-26)}{(D-1)(D-5)}\frac{1}{(q^2+4)^2}\left(\frac{1}{m^2}\ \cdots\ (p_2\cdot k_1)\right)$$



$$-\frac{1}{(q^2+4)}\left\{\frac{4(D^4-14D^3+59D^2-82D+16)}{(D-1)(D-4)(D-6)}\right.$$

$$
+\frac{16(3D-8)(D^2-10D+19)}{(D-1)(D-4)(D-5)}\frac{1}{(q^2+4)}\Bigg\}\Bigg(\frac{1}{m^2}
$$

$$
\Bigg)
$$

$$
-\frac{32(D-2)(D^2-10D+19)}{(D-1)(D-5)}\frac{1}{(q^2+4)^2}\Bigg(\frac{1}{m^2}
$$

$$
\Bigg)
$$

$$
-\frac{2(D-2)^2}{(q^2+4)}\Bigg\{\frac{(3D^4-53D^3+323D^2-795D+642)}{(D-1)(D-3)(D-4)(D-5)(D-6)}
$$

$$
+\frac{4(2D-5)(D^2-10D+19)}{(D-1)(D-3)(D-4)(D-5)}\frac{1}{(q^2+4)}\Bigg\}\Bigg(\frac{1}{m^4}
$$

$$
\Bigg), \tag{16}
$$

where the MIs depicted on the r.h.s. are those of Fig. 7 of [8].

Similar formulas hold for the other form factors and graphs, but are too lengthy to be reported here.

Once the form factors are expressed in terms of MIs, one expands the result around $D = 4$ and inserts the values of the MIs, also given in [8] as an expansion around $D = 4$. In this way one finally obtains the required analytic result where both UV and soft IR divergences, regulated by the same parameter D, appear as poles in $(D-4)$. In the case of Eq. (16), using the Eqs. (88,93,123,125,B.1) of [8], one will get the expression given later in this paper in Eq. (55).

In this section we will give the contributions $\mathcal{F}_i^{(2l,\mathtt{graph})}(D,q^2)$ to the *unrenormalized* form factors from each of the 2-loop still unrenormalized graphs (i.e. where the renormalization of the inserted 1-loop subgraphs has not yet been carried out).

The propagators of the graphs are considered in the Feynman gauge and the corresponding denominators $\mathcal{D}$'s, which will appear in the following formulas, are listed in the appendix A.

The resulting *unrenormalized* form factors are given for space-like Q ($Q^2 > 0$ or $S = -Q^2 < 0$) and are expressed in terms of 1-dimensional harmonic polylogarithms [5, 6] of argument

$$
x = \frac{\sqrt{q^2+4}-\sqrt{q^2}}{\sqrt{q^2+4}+\sqrt{q^2}} = \frac{\sqrt{Q^2+4m^2}-\sqrt{Q^2}}{\sqrt{Q^2+4m^2}+\sqrt{Q^2}} \,. \tag{17}
$$

Let us comment shortly here the normalization of the D-dimensional integrals, or, which is the same, the choice of the loop integration measure in D continuous dimensions. With the natural choice

$$
\mu_0^{(4-D)} \int \frac{d^Dk}{(2\pi)^{(D-2)}} \,, \tag{18}
$$

where μ_0 is the mass scale, one obtains for the simplest loop integral, the 1-loop massive tadpole,

$$
\mu_0^{(4-D)} \int \frac{d^Dk}{(2\pi)^{(D-2)}} \frac{1}{k^2+m^2} = C(D) \left(\frac{m^2}{\mu_0^2}\right)^{\frac{D-4}{2}} \frac{m^2}{(D-2)(D-4)} \,, \tag{19}
$$

where $C(D)$ is the following function of the space-time dimension D:

$$C(D) = (4\pi)^{\frac{(4-D)}{2}} \Gamma\left(3 - \frac{D}{2}\right) , \qquad (20)$$

with the limiting value $C(4) = 1$ for $D = 4$. Note that the explicit form of $C(D)$ is essentially irrelevant, as in any physical quantity, finite for $D = 4$, $C(D)$ can be replaced by 1. The explicit expression in Eq. (20) can matter only in detailed comparisons with calculations using a different integration measure.

To simplify the writing of all the subsequent formulae, we will use the following D-continuous integration measure

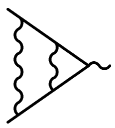
$$\int \mathfrak{D}^D k = \frac{1}{C(D)} \left(\frac{m^2}{\mu_0^2}\right)^{\frac{4-D}{2}} \int \frac{d^D k}{(2\pi)^{(D-2)}} , \qquad (21)$$

using m as mass scale. With that choice, the 1-loop tadpole simply reads

$$\int \mathfrak{D}^D k \, \frac{1}{k^2 + m^2} = \frac{m^2}{(D-2)(D-4)} . \qquad (22)$$

By using for each loop the integration measure $\mathcal{D}^D k$ of Eq. (21) we find the results which follow for the contributions to the unrenormalized form factors from the various 2-loop graphs, with still unrenormalized 1-loop insertions.

• The *Ladder* graph (a) of Fig. 1, defined as



$$= \int \mathfrak{D}^D k_1 \, \mathfrak{D}^D k_2 \, \frac{\mathcal{N}^{\mu}_{(\mathtt{a})}}{\mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_9 \mathcal{D}_{10} \mathcal{D}_{12} \mathcal{D}_{13}} , \qquad (23)$$

where

$$\begin{aligned}
\mathcal{N}^{\mu}_{(\mathtt{a})} \; = \; & \bar{v}(p_2)\gamma_{\sigma}[i(\not{p}_2+\not{k}_1)+m]\gamma_{\lambda}[i(\not{p}_2+\not{k}_1-\not{k}_2)+m]\gamma^{\mu}[-i(\not{p}_1-\not{k}_1+\not{k}_2)+m] \times \\
& \times\gamma^{\lambda}[-i(\not{p}_1-\not{k}_1)+m]\gamma^{\sigma}u(p_1), \qquad (24)
\end{aligned}$$

gives:

$$\begin{aligned}
\mathcal{F}_1^{(2l,\mathtt{a})}(D,q^2) \; = \; & \frac{1}{(D-4)^2}\Biggl\{ \frac{1}{8} + \frac{1}{2}\Bigl[1 - \frac{1}{(1-x)} - \frac{1}{(1+x)}\Bigr] H(0;x) + \Bigl[1 + \frac{1}{(1-x)^2} \\
& - \frac{1}{(1-x)} + \frac{1}{(1+x)^2} - \frac{1}{(1+x)}\Bigr] H(0,0;x) \Biggr\} \\
+ \; & \frac{1}{(D-4)}\Biggl\{ -\frac{5}{32} - \frac{1}{8}\Bigl[1 - \frac{2}{(1-x)}\Bigr] H(0;x) - \frac{1}{2}\Bigl[1 - \frac{2}{(1-x)} \\
& + \frac{2}{(1-x)^2}\Bigr] H(0,0;x) + \frac{1}{4}\Bigl[1 - \frac{1}{(1-x)} - \frac{1}{(1+x)}\Bigr] [\zeta(2) \\
& - H(0;x) - H(0,0;x) + 2H(-1,0;x)] \\
& + \frac{1}{2}\Bigl[1 + \frac{1}{(1-x)^2} - \frac{1}{(1-x)} + \frac{1}{(1+x)^2} - \frac{1}{(1+x)}\Bigr] [\zeta(3)
\end{aligned}$$

$$
\begin{aligned}
&\qquad -2H(0,0;x)-2H(0,0,0;x)+4H(-1,0,0;x)\\
&\qquad +4H(0,1,0;x)]\Big\}\\
&+\frac{55}{128}+\Big[\frac{5}{4(1-x)}-\frac{81}{2(1+x)^4}+\frac{177}{2(1+x)^3}-\frac{123}{2(1+x)^2}\\
&+\frac{55}{4(1+x)}-\frac{23}{8}\Big]\zeta(2)+\Big[\frac{3}{5(1-x)^2}-\frac{31}{80(1-x)}-\frac{51}{2(1+x)^5}\\
&+\frac{255}{4(1+x)^4}-\frac{1071}{20(1+x)^3}+\frac{687}{40(1+x)^2}-\frac{303}{80(1+x)}+\frac{29}{20}\Big]\zeta^2(2)\\
&-\Big[\frac{1}{(1-x)^2}-\frac{5}{4(1-x)}-\frac{60}{(1+x)^4}+\frac{120}{(1+x)^3}-\frac{141}{2(1+x)^2}\\
&+\frac{41}{4(1+x)}-2\zeta(3)\Big]\zeta(3)-\frac{3}{4(1+x)}\Big[1-\frac{1}{(1+x)}\Big]\\
&+\Big[\frac{19}{32}-\Big(\frac{2}{(1-x)^2}-\frac{33}{32(1-x)}+\frac{81}{2(1+x)^5}-\frac{465}{4(1+x)^4}\\
&\quad+\frac{979}{8(1+x)^3}-\frac{865}{16(1+x)^2}+\frac{327}{32(1+x)}-\frac{9}{8}\Big)\zeta(2)\\
&\quad-\Big(\frac{1}{4(1-x)^2}+\frac{1}{4(1-x)}-\frac{60}{(1+x)^5}+\frac{150}{(1+x)^4}-\frac{126)}{(1+x)^3}\\
&\quad+\frac{157}{4(1+x)^2}-\frac{31}{4(1+x)}+\frac{9}{4}\Big)\zeta(3)-\frac{1}{(1-x)}+\frac{9}{4(1+x)^3}\\
&\quad-\frac{27}{8(1+x)^2}+\frac{15}{16(1+x)}\Big]H(0;x)\\
&-\Big[\Big(\frac{1}{4(1-x)}+\frac{1}{4(1+x)}-\frac{1}{4}\Big)\zeta(2)-\Big(1+\frac{1}{(1-x)^2}-\frac{1}{(1-x)}\\
&\quad+\frac{1}{(1+x)^2}-\frac{1}{(1+x)}\Big)\zeta(3)\Big]H(-1;x)\\
&+\Big[\frac{27}{8}+\Big(\frac{7}{4(1-x)^2}-\frac{13}{8(1-x)}-\frac{15}{(1+x)^5}+\frac{75}{2(1+x)^4}-\frac{63}{2(1+x)^3}\\
&\quad+\frac{23}{2(1+x)^2}-\frac{29}{8(1+x)}+\frac{9}{4}\Big)\zeta(2)+\frac{17}{8(1-x)^2}-\frac{27}{8(1-x)}\\
&\quad+\frac{39}{2(1+x)^4}-\frac{93}{2(1+x)^3}+\frac{77}{2(1+x)^2}-\frac{47}{4(1+x)}\Big]H(0,0;x)\\
&+\Big[\Big(\frac{1}{2(1-x)}-\frac{60}{(1+x)^5}+\frac{150}{(1+x)^4}-\frac{126}{(1+x)^3}+\frac{39}{(1+x)^2}\\
&\quad-\frac{15}{2(1+x)}+2\Big)\zeta(2)\Big]H(1,0;x)\\
&+\Big[\Big(1+\frac{1}{2(1-x)^2}-\frac{1}{2(1-x)}+\frac{1}{2(1+x)^2}-\frac{1}{2(1+x)}\Big)\zeta(2)\Big]\times\\
&\quad\times[H(0,1;x)+2H(-1,0;x)+H(0,-1;x)]
\end{aligned}
$$

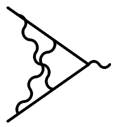
$$
\begin{aligned}
& -\Big[\frac{3}{2}-\frac{5}{2(1-x)}-\frac{15}{(1+x)^3}+\frac{45}{2(1+x)^2}-\frac{8}{(1+x)}\Big]H(-1,0;x) \\
& +\Big[\frac{35}{8}+\frac{1}{(1-x)^2}-\frac{71}{32(1-x)}-\frac{81}{2(1+x)^5}+\frac{525}{4(1+x)^4}-\frac{1219}{8(1+x)^3} \\
& \quad +\frac{1173}{16(1+x)^2}-\frac{471}{32(1+x)}\Big]H(0,0,0;x) \\
& -\Big[\frac{25}{4}+\frac{4}{(1-x)^2}-\frac{17}{4(1-x)}+\frac{60}{(1+x)^4}-\frac{120}{(1+x)^3}+\frac{73}{(1+x)^2} \\
& \quad -\frac{53}{4(1+x)}\Big]H(-1,0,0;x) \\
& +\frac{1}{2}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big]H(-1,-1,0;x) \\
& +\Big[\frac{5}{4}+\frac{1}{4(1-x)}+\frac{30}{(1+x)^4}-\frac{60}{(1+x)^3}+\frac{71}{2(1+x)^2} \\
& \quad -\frac{21}{4(1+x)}\Big]H(0,-1,0;x) \\
& -\Big[\frac{3}{2}+\frac{2}{(1-x)^2}-\frac{2}{(1-x)}+\frac{1}{(1+x)^2}-\frac{1}{(1+x)}\Big]H(0,1,0;x) \\
& +\Big[\frac{3}{2}+\frac{30}{(1+x)^4}-\frac{60}{(1+x)^3}+\frac{71}{2(1+x)^2}-\frac{11}{2(1+x)}\Big]H(1,0,0;x) \\
& +\frac{1}{4}\Big[1+\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{1}{(1+x)^2}-\frac{1}{(1+x)}\Big]\times \\
& \quad \times[5H(0,0,0,0;x)+16H(-1,-1,0,0;x)-4H(-1,0,0,0;x) \\
& \quad +2H(-1,0,1,0;x)-16H(0,-1,-1,0;x)+3H(0,-1,1,0;x) \\
& \quad -12H(0,0,1,0;x)+12H(0,1,-1,0;x)-4H(0,1,1,0;x)] \\
& +\Big[\frac{7}{2}+\frac{3}{2(1-x)^2}-\frac{1}{(1-x)}-\frac{60}{(1+x)^5}+\frac{150}{(1+x)^4}-\frac{126}{(1+x)^3} \\
& \quad +\frac{81}{2(1+x)^2}-\frac{9}{(1+x)}\Big]H(0,-1,0,0;x) \\
& +\Big[\frac{5}{2}+\frac{7}{2(1-x)^2}-\frac{15}{4(1-x)}+\frac{30}{(1+x)^5}-\frac{75}{(1+x)^4}+\frac{63}{(1+x)^3} \\
& \quad -\frac{16}{(1+x)^2}+\frac{1}{4(1+x)}\Big]H(0,0,-1,0;x) \\
& -\Big[3+\frac{2}{(1-x)^2}-\frac{7}{4(1-x)}-\frac{30}{(1+x)^5}+\frac{75}{(1+x)^4}-\frac{63}{(1+x)^3} \\
& \quad +\frac{43}{2(1+x)^2}-\frac{23}{4(1+x)}\Big]H(0,1,0,0;x) \\
& +\Big[2+\frac{1}{2(1-x)}-\frac{60}{(1+x)^5}+\frac{150}{(1+x)^4}-\frac{126}{(1+x)^3}+\frac{39}{(1+x)^2} \\
& \quad -\frac{15}{2(1+x)}\Big]H(1,0,0,0;x) \\
+ \quad & \mathcal{O}(D-4)\,, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_2^{(2l,\mathrm{a})}(D,q^2) \;\; = \;\; & \frac{1}{(D-4)}\Bigg\{\frac{1}{4}\Big[\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big]H(0;x)-\Big[\frac{1}{(1-x)^2}-\frac{1}{(1-x)} \\
& \qquad -\frac{1}{(1+x)^2}+\frac{1}{(1+x)}\Big]H(0,0;x)\Bigg\} \\
& +\Big[\frac{7}{8(1-x)^2}-\frac{5}{4(1-x)}+\frac{81}{2(1+x)^4}-\frac{177}{2(1+x)^3}+\frac{465}{8(1+x)^2} \\
& -\frac{39}{4(1+x)}\Big]\zeta(2)+\Big[\frac{17}{160(1-x)}+\frac{51}{2(1+x)^5}-\frac{255}{4(1+x)^4} \\
& +\frac{2057}{40(1+x)^3}-\frac{1071}{80(1+x)^2}+\frac{17}{160(1+x)}\Big]\zeta^2(2)-\Big[\frac{5}{2(1-x)^2} \\
& -\frac{5}{2(1-x)}+\frac{60}{(1+x)^4}-\frac{120}{(1+x)^3}+\frac{131}{2(1+x)^2}-\frac{11}{2(1+x)}\Big]\zeta(3) \\
& +\frac{3}{4(1+x)}\Big[1-\frac{1}{(1+x)}\Big] \\
& +\Big[\Big(\frac{7}{8(1-x)^3}-\frac{45}{16(1-x)^2}+\frac{87}{32(1-x)}+\frac{81}{2(1+x)^5}-\frac{465}{4(1+x)^4} \\
& \quad +\frac{119}{(1+x)^3}-\frac{191}{4(1+x)^2}+\frac{119}{32(1+x)}\Big)\zeta(2)-\Big(\frac{1}{4(1-x)} \\
& \quad +\frac{60}{(1+x)^5}-\frac{150}{(1+x)^4}+\frac{121}{(1+x)^3}-\frac{63}{2(1+x)^2}+\frac{1}{4(1+x)}\Big)\zeta(3) \\
& \quad -\frac{3}{8(1-x)}-\frac{9}{4(1+x)^3}+\frac{27}{8(1+x)^2}-\frac{3}{4(1+x)}\Big]H(0;x) \\
& +\Big[\Big(\frac{1}{16(1-x)}+\frac{15}{(1+x)^5}-\frac{75}{2(1+x)^4}+\frac{121}{4(1+x)^3}-\frac{63}{8(1+x)^2} \\
& \quad +\frac{1}{16(1+x)}\Big)\zeta(2)+\frac{17}{8(1-x)^2}-\frac{7}{4(1-x)}-\frac{39}{2(1+x)^4} \\
& \quad +\frac{93}{2(1+x)^3}-\frac{295}{8(1+x)^2}+\frac{19}{2(1+x)}\Big]H(0,0;x) \\
& -\Big[\frac{3}{4(1-x)}+\frac{15}{(1+x)^3}-\frac{45}{2(1+x)^2}+\frac{27}{4(1+x)}\Big]H(-1,0;x) \\
& +\Big[\Big(\frac{1}{4(1-x)}+\frac{60}{(1+x)^5}-\frac{150}{(1+x)^4}+\frac{121}{(1+x)^3}-\frac{63}{2(1+x)^2} \\
& \quad +\frac{1}{4(1+x)}\Big)\zeta(2)\Big]H(1,0;x) \\
& +\Big[\frac{7}{8(1-x)^3}-\frac{29}{16(1-x)^2}+\frac{55}{32(1-x)}+\frac{81}{2(1+x)^5}-\frac{525}{4(1+x)^4} \\
& \quad +\frac{149}{(1+x)^3}-\frac{263}{4(1+x)^2}+\frac{215}{32(1+x)}\Big]H(0,0,0;x) \\
& +\Big[\frac{60}{(1+x)^4}-\frac{120}{(1+x)^3}+\frac{68}{(1+x)^2}-\frac{8}{(1+x)}\Big]H(-1,0,0;x) \\
& -\Big[\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{30}{(1+x)^4}-\frac{60}{(1+x)^3}+\frac{33}{(1+x)^2}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3}{(1+x)}\Big][H(0,-1,0;x)+H(1,0,0;x)] \\
& -\Big[\frac{1}{(1-x)^2}-\frac{1}{(1-x)}-\frac{1}{(1+x)^2}+\frac{1}{(1+x)}\Big]H(0,1,0;x) \\
& -\Big[\frac{1}{8(1-x)}+\frac{30}{(1+x)^5}-\frac{75}{(1+x)^4}+\frac{121}{2(1+x)^3}-\frac{63}{4(1+x)^2} \\
& +\frac{1}{8(1+x)}\Big][H(0,0,-1,0;x)+H(0,1,0,0;x) \\
& \qquad -2H(1,0,0,0;x)-2H(0,-1,0,0;x)] \\
& + \ \mathcal{O}(D-4)\,, \qquad (26)
\end{aligned}
$$

$$
\mathcal{F}_3^{(2l,\mathtt{a})}(D,q^2) = 0\,. \qquad (27)
$$

- The *Cross* graph $(\mathtt{b})$ of Fig. 1, defined as



$$
= \int \mathfrak{D}^D k_1\, \mathfrak{D}^D k_2\, \frac{\mathcal{N}^\mu_{(\mathtt{b})}}{\mathcal{D}_1\mathcal{D}_2\mathcal{D}_9\mathcal{D}_{11}\mathcal{D}_{12}\mathcal{D}_{13}}\,, \qquad (28)
$$

where

$$
\begin{aligned}
\mathcal{N}^\mu_{(\mathtt{b})} &= \bar{v}(p_2)\gamma_\sigma[i(\not{p}_2-\not{k}_2)+m]\gamma_\lambda[i(\not{p}_2+\not{k}_1-\not{k}_2)+m]\gamma^\mu[-i(\not{p}_1-\not{k}_1+\not{k}_2)+m]\times \\
&\quad \times\gamma^\sigma[-i(\not{p}_1-\not{k}_1)+m]\gamma^\lambda u(p_1), \qquad (29)
\end{aligned}
$$

gives:

$$
\begin{aligned}
\mathcal{F}_1^{(2l,\mathtt{b})}(D,q^2) &= \frac{1}{(D-4)}\Big\{\frac{1}{4}-\frac{1}{2}\Big[1+\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{1}{(1+x)^2} \\
&\quad -\frac{1}{(1+x)}\Big][\zeta(3)-\zeta(2)H(0;x)+2H(0,0,0;x) \\
&\quad -2H(0,-1,0;x)+2H(0,1,0;x)]\Big\} \\
&\quad -\frac{1}{2}+\frac{1}{2}\Big[\frac{1}{(1-x)}-\frac{45}{(1+x)^4}+\frac{99}{(1+x)^3}-\frac{53}{(1+x)^2} \\
&\quad -\frac{3}{(1+x)}+\frac{27}{4}\Big]\zeta(2)+\Big[\frac{37}{40(1-x)^2}-\frac{971}{320(1-x)}-\frac{321}{20(1+x)^5} \\
&\quad +\frac{321}{8(1+x)^4}-\frac{2299}{80(1+x)^3}+\frac{125}{32(1+x)^2}-\frac{79}{64(1+x)} \\
&\quad +\frac{119}{40}\Big]\zeta^2(2)-\Big[\frac{6}{(1+x)^4}-\frac{12}{(1+x)^3}+\frac{7}{(1+x)^2}-\frac{1}{(1+x)} \\
&\quad +2\Big]\zeta(3)-\frac{3}{4(1+x)}\Big[1-\frac{1}{(1+x)}\Big] \\
&\quad -\Big[\frac{1}{4}-\Big(\frac{25}{32(1-x)}+\frac{45}{2(1+x)^5}-\frac{279}{4(1+x)^4}+\frac{597}{8(1+x)^3}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{463}{16(1+x)^2}+\frac{121}{32(1+x)}-\frac{3}{4}\Big)\zeta(2)-\Big(\frac{1}{4(1-x)^2} \\
& -\frac{7}{8(1-x)}-\frac{6}{(1+x)^5}+\frac{15}{(1+x)^4}-\frac{25}{2(1+x)^3}+\frac{4}{(1+x)^2} \\
& -\frac{47}{8(1+x)}+\frac{13}{4}\Big)\zeta(3)-\frac{5}{8(1-x)}-\frac{9}{4(1+x)^3}+\frac{27}{8(1+x)^2} \\
& -\frac{1}{(1+x)}\Big]H(0;x) \\
+ & \Big[\frac{45}{(1+x)^4}-\frac{90}{(1+x)^3}+\frac{93}{2(1+x)^2}-\frac{3}{2(1+x)}-\frac{3}{2}\Big]\zeta(2)H(-1;x) \\
- & \Big[\frac{13}{4}+\Big(\frac{1}{2(1-x)^2}+\frac{49}{32(1-x)}+\frac{15}{2(1+x)^5}-\frac{75}{4(1+x)^4} \\
& +\frac{105}{8(1+x)^3}-\frac{7}{16(1+x)^2}+\frac{17}{32(1+x)}-\frac{3}{2}\Big)\zeta(2)+\frac{3}{8(1-x)^2} \\
& -\frac{23}{8(1-x)}-\frac{51}{2(1+x)^4}+\frac{129}{2(1+x)^3}-\frac{103}{2(1+x)^2} \\
& +\frac{43}{4(1+x)}\Big]H(0,0;x) \\
- & \Big[\frac{1}{2(1-x)^2}-\frac{23}{16(1-x)}-\frac{45}{(1+x)^5}+\frac{225}{2(1+x)^4}-\frac{351}{4(1+x)^3} \\
& +\frac{157}{8(1+x)^2}-\frac{71}{16(1+x)}+\frac{7}{2}\Big]\zeta(2)H(0,-1;x) \\
+ & \Big[3-\frac{3}{(1-x)}+\frac{21}{(1+x)^3}-\frac{63}{2(1+x)^2}+\frac{15}{2(1+x)}\Big]H(-1,0;x) \\
- & \Big[\frac{5}{4}-\Big(\frac{1}{(1-x)^2}-\frac{5}{2(1-x)}+\frac{12}{(1+x)^5}-\frac{30}{(1+x)^4}+\frac{26}{(1+x)^3} \\
& -\frac{8}{(1+x)^2}+\frac{7}{2(1+x)}\Big)\zeta(2)-\frac{2}{(1-x)}+\frac{6}{(1+x)^3}-\frac{9}{(1+x)^2} \\
& +\frac{5}{2(1+x)}\Big]H(1,0;x) \\
- & \Big[1-\frac{25}{32(1-x)}-\frac{45}{2(1+x)^5}+\frac{255}{4(1+x)^4}-\frac{501}{8(1+x)^3} \\
& +\frac{359}{16(1+x)^2}-\frac{105}{32(1+x)}\Big]H(0,0,0;x) \\
- & \Big[3-\frac{42}{(1+x)^4}+\frac{84}{(1+x)^3}-\frac{41}{(1+x)^2}-\frac{1}{(1+x)}\Big]H(0,-1,0;x) \\
+ & \Big[\frac{5}{2}-\frac{3}{(1+x)^4}+\frac{6}{(1+x)^3}-\frac{3}{2(1+x)^2}-\frac{3}{2(1+x)}\Big]H(-1,0,0;x) \\
+ & \Big[\frac{3}{2}-\frac{12}{(1+x)^4}+\frac{24}{(1+x)^3}-\frac{11}{(1+x)^2}-\frac{1}{(1+x)}\Big]H(0,1,0;x) \\
- & \Big[1-\frac{36}{(1+x)^4}+\frac{72}{(1+x)^3}-\frac{36}{(1+x)^2}\Big]H(1,0,0;x)
\end{aligned}
$$

$$
\begin{aligned}
& +\Big[\frac{11}{2}+\frac{3}{(1-x)^2}-\frac{169}{32(1-x)}-\frac{3}{2(1+x)^5}+\frac{15}{4(1+x)^4} \\
& \quad -\frac{9}{8(1+x)^3}+\frac{15}{16(1+x)^2}-\frac{153}{32(1+x)}\Big]H(0,0,0,0;x) \\
& -\Big[7+\frac{4}{(1-x)^2}-\frac{79}{16(1-x)}+\frac{3}{(1+x)^5}-\frac{15}{2(1+x)^4} \\
& \quad +\frac{17}{4(1+x)^3}+\frac{41}{8(1+x)^2}-\frac{159}{16(1+x)}\Big]H(0,-1,0,0;x) \\
& -\Big[6+\frac{6}{(1-x)^2}-\frac{43}{8(1-x)}-\frac{42}{(1+x)^5}+\frac{105}{(1+x)^4}-\frac{159}{2(1+x)^3} \\
& \quad +\frac{81}{4(1+x)^2}-\frac{35}{8(1+x)}\Big]H(0,0,-1,0;x) \\
& +\Big[5+\frac{4}{(1-x)^2}-\frac{9}{2(1-x)}-\frac{12}{(1+x)^5}+\frac{30}{(1+x)^4}-\frac{22}{(1+x)^3} \\
& \quad +\frac{7}{(1+x)^2}-\frac{9}{2(1+x)}\Big]H(0,0,1,0,x) \\
& +\Big[4+\frac{2}{(1-x)^2}-\frac{17}{4(1-x)}+\frac{36}{(1+x)^5}-\frac{90}{(1+x)^4}+\frac{69}{(1+x)^3} \\
& \quad -\frac{23}{2(1+x)^2}-\frac{21}{4(1+x)}\Big]H(0,1,0,0;x) \\
& +\Big[1+\frac{2}{(1-x)^2}-\frac{7}{2(1-x)}+\frac{12}{(1+x)^5}-\frac{30}{(1+x)^4}+\frac{26}{(1+x)^3} \\
& \quad -\frac{7}{(1+x)^2}+\frac{5}{2(1+x)}\Big]H(1,0,0,0;x) \\
& +\frac{1}{2}\Big[1+\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{1}{(1+x)^2}-\frac{1}{(1+x)}\Big][2\zeta(3)(H(1;x) \\
& \quad -H(-1;x))-2\zeta(2)H(-1,0;x)+\zeta(2)H(0,1;x) \\
& \quad +4H(-1,0,-1,0;x)-4H(-1,0,0,0;x)-4H(-1,0,1,0;x) \\
& \quad +10H(0,-1,-1,0,x)-6H(0,-1,1,0,x)-6H(0,1,-1,0,x) \\
& \quad +2H(0,1,1,0;x)-4H(1,0,-1,0;x)+4H(1,0,1,0;x)]
\end{aligned}
$$

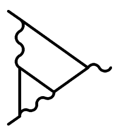
$$
+ \quad \mathcal{O}(D-4)\,, \tag{30}
$$

$$
\begin{aligned}
\mathcal{F}_2^{(2l,\mathrm{b})}(D,q^2) \quad = \quad & -\Big[\frac{1}{8(1-x)^2}+\frac{5}{4(1-x)}-\frac{45}{2(1+x)^4}+\frac{99}{2(1+x)^3} \\
& -\frac{197}{8(1+x)^2}-\frac{15}{4(1+x)}\Big]\zeta(2)+\Big[\frac{39}{80(1-x)^3}-\frac{117}{160(1-x)^2} \\
& -\frac{337}{320(1-x)}+\frac{321}{20(1+x)^5}-\frac{321}{8(1+x)^4}+\frac{137}{5(1+x)^3} \\
& -\frac{39}{40(1+x)^2}-\frac{337}{320(1+x)}\Big]\zeta^2(2)+\Big[\frac{1}{2(1-x)^2}-\frac{1}{2(1-x)} \\
& +\frac{6}{(1+x)^4}-\frac{12}{(1+x)^3}+\frac{13}{2(1+x)^2}-\frac{1}{2(1+x)}\Big]\zeta(3)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{3}{4(1+x)^2}+\frac{3}{4(1+x)}\\
&-\Big[\Big(\frac{1}{8(1-x)^3}-\frac{13}{16(1-x)^2}+\frac{19}{32(1-x)}+\frac{45}{2(1+x)^5}-\frac{279}{4(1+x)^4}\\
&\quad+\frac{291}{4(1+x)^3}-\frac{25}{(1+x)^2}-\frac{13}{32(1+x)}\Big)\zeta(2)-\Big(\frac{1}{2(1-x)^3}\\
&\quad-\frac{3}{4(1-x)^2}+\frac{1}{8(1-x)}+\frac{6}{(1+x)^5}-\frac{15}{(1+x)^4}+\frac{12}{(1+x)^3}\\
&\quad-\frac{3}{(1+x)^2}+\frac{1}{8(1+x)}\Big)\zeta(3)+\frac{5}{16(1-x)}+\frac{9}{4(1+x)^3}\\
&\quad-\frac{27}{8(1+x)^2}+\frac{13}{16(1+x)}\Big]H(0;x)\\
&-\Big[\frac{3}{4(1-x)^2}-\frac{3}{4(1-x)}+\frac{45}{(1+x)^4}-\frac{90}{(1+x)^3}+\frac{171}{4(1+x)^2}\\
&\quad+\frac{9}{4(1+x)}\Big]\zeta(2)H(-1;x)\\
&+\Big[\Big(\frac{5}{8(1-x)^3}-\frac{15}{16(1-x)^2}-\frac{15}{32(1-x)}+\frac{15}{2(1+x)^5}-\frac{75}{4(1+x)^4}\\
&\quad+\frac{25}{2(1+x)^3}-\frac{15}{32(1+x)}\Big)\zeta(2)-\frac{19}{8(1-x)^2}+\frac{21}{4(1-x)}\\
&\quad-\frac{51}{2(1+x)^4}+\frac{129}{2(1+x)^3}-\frac{395}{8(1+x)^2}+\frac{15}{2(1+x)}\Big]H(0,0;x)\\
&-\Big[\Big(\frac{1}{4(1-x)}+\frac{12}{(1+x)^5}-\frac{30}{(1+x)^4}+\frac{25}{(1+x)^3}-\frac{15}{2(1+x)^2}\\
&\quad+\frac{1}{4(1+x)}\Big)\zeta(2)-\frac{1}{(1-x)}-\frac{6}{(1+x)^3}+\frac{9}{(1+x)^2}\\
&\quad-\frac{2}{(1+x)}\Big]H(1,0;x)\\
&-\Big[\frac{19}{4(1-x)}+\frac{21}{(1+x)^3}-\frac{63}{2(1+x)^2}+\frac{23}{4(1+x)}\Big]H(-1,0;x)\\
&-\Big[\frac{3}{4(1-x)^3}-\frac{9}{8(1-x)^2}-\frac{21}{16(1-x)}+\frac{45}{(1+x)^5}-\frac{225}{2(1+x)^4}\\
&\quad+\frac{84}{(1+x)^3}-\frac{27}{2(1+x)^2}-\frac{21}{16(1+x)}\Big]\zeta(2)H(0,-1;x)\\
&-\Big[\frac{1}{8(1-x)^3}-\frac{13}{16(1-x)^2}+\frac{19}{32(1-x)}+\frac{45}{2(1+x)^5}\\
&\quad-\frac{255}{4(1+x)^4}+\frac{243}{4(1+x)^3}-\frac{19}{(1+x)^2}-\frac{13}{32(1+x)}\Big]H(0,0,0;x)\\
&+\Big[\frac{12}{(1+x)^4}-\frac{24}{(1+x)^3}+\frac{10}{(1+x)^2}+\frac{2}{(1+x)}\Big]H(0,1,0;x)\\
&+\Big[\frac{1}{(1-x)^2}-\frac{1}{(1-x)}-\frac{36}{(1+x)^4}+\frac{72}{(1+x)^3}-\frac{33}{(1+x)^2}
\end{aligned}
$$

$$
\begin{aligned}
&\quad -\frac{3}{(1+x)}\Big]H(1,0,0;x) \\
&\quad -\Big[\frac{3}{4(1-x)^2}-\frac{3}{4(1-x)}-\frac{3}{(1+x)^4}+\frac{6}{(1+x)^3}-\frac{5}{4(1+x)^2} \\
&\qquad -\frac{7}{4(1+x)}\Big]H(-1,0,0;x) \\
&\quad +\Big[\frac{1}{2(1-x)^2}-\frac{1}{2(1-x)}-\frac{42}{(1+x)^4}+\frac{84}{(1+x)^3}-\frac{75}{2(1+x)^2} \\
&\quad -\frac{9}{2(1+x)}\Big]H(0,-1,0;x) \\
&\quad +\Big[\frac{5}{8(1-x)^3}-\frac{15}{16(1-x)^2}-\frac{11}{32(1-x)}+\frac{3}{2(1+x)^5}-\frac{15}{4(1+x)^4} \\
&\qquad +\frac{1}{(1+x)^3}+\frac{9}{4(1+x)^2}-\frac{11}{32(1+x)}\Big]H(0,0,0,0;x) \\
&\quad -\Big[\frac{3}{4(1-x)^3}-\frac{9}{8(1-x)^2}+\frac{11}{16(1-x)}-\frac{3}{(1+x)^5}+\frac{15}{2(1+x)^4} \\
&\qquad -\frac{4}{(1+x)^3}-\frac{3}{2(1+x)^2}+\frac{11}{16(1+x)}\Big]H(0,-1,0,0;x) \\
&\quad +\Big[\frac{1}{2(1-x)^3}-\frac{3}{4(1-x)^2}+\frac{17}{8(1-x)}-\frac{42}{(1+x)^5}+\frac{105}{(1+x)^4} \\
&\qquad -\frac{76}{(1+x)^3}+\frac{9}{(1+x)^2}+\frac{17}{8(1+x)}\Big]H(0,0,-1,0;x) \\
&\quad -\Big[\frac{3}{4(1-x)}-\frac{12}{(1+x)^5}+\frac{30}{(1+x)^4}-\frac{21}{(1+x)^3}+\frac{3}{2(1+x)^2} \\
&\qquad +\frac{3}{4(1+x)}\Big]H(0,0,1,0;x) \\
&\quad +\Big[\frac{1}{(1-x)^3}-\frac{3}{2(1-x)^2}+\frac{7}{4(1-x)}-\frac{36}{(1+x)^5}+\frac{90}{(1+x)^4} \\
&\qquad -\frac{66}{(1+x)^3}+\frac{9}{(1+x)^2}+\frac{7}{4(1+x)}\Big]H(0,1,0,0;x) \\
&\quad -\Big[\frac{1}{4(1-x)}+\frac{12}{(1+x)^5}-\frac{30}{(1+x)^4}+\frac{25}{(1+x)^3}-\frac{15}{2(1+x)^2} \\
&\qquad +\frac{1}{4(1+x)}\Big]H(1,0,0,0;x) \\
&+ \quad \mathcal{O}(D-4)\,, \qquad (31) \\
\mathcal{F}_3^{(2l,\mathtt{b})}(D,q^2) &= 0\,.
\end{aligned}
$$

- The *Down-Corner* graph (c) of Fig. 1, defined as



$$
= \int \mathfrak{D}^D k_1\, \mathfrak{D}^D k_2\, \frac{\mathcal{N}^{\mu}_{(\mathtt{c})}}{\mathcal{D}_1\mathcal{D}_5\mathcal{D}_7\mathcal{D}_8\mathcal{D}_9\mathcal{D}_{10}}\,, \qquad (32)
$$

where

$$
\begin{aligned}
\mathcal{N}^{\mu}_{(\mathsf{c})} &= \bar{v}(p_2)\gamma_\sigma[i(\not{k}_2)+m]\gamma_\lambda[i(\not{k}_1+\not{k}_2)+m]\gamma_\sigma[i(\not{p}_2+\not{k}_1)+m]\gamma^\mu \times \\
&\quad \times[-i(\not{p}_1-\not{k}_1)+m]\gamma^\lambda u(p_1), \qquad (33)
\end{aligned}
$$

gives:

$$
\begin{aligned}
\mathcal{F}_1^{(2l,\mathsf{c})}(D,q^2) &= \frac{1}{(D-4)^2}\Bigg\{\frac{1}{8}+\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big]H(0;x)\Bigg\} \\
&- \frac{1}{(D-4)}\Bigg\{\frac{3}{32}-\frac{7\zeta(2)}{2}-\frac{1}{8}\Big[1-\frac{2}{(1+x)}\Big]H(0;x) \\
&\qquad -\frac{1}{4}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big][2\zeta(2)-4H(0;x) \\
&\qquad\qquad +H(0,0;x)+2H(1,0;x)]\Bigg\} \\
&+\frac{17}{128}+\Big[\frac{23}{16}+\frac{1}{2(1-x)}-\frac{15}{4(1+x)^4}+\frac{15}{2(1+x)^3}-\frac{23}{8(1+x)^2} \\
&-\frac{1}{2(1+x)}\Big]\zeta(2)-\Big[\frac{9}{(1+x)^2}-\frac{9}{(1+x)}+\frac{9}{2}\Big]\zeta(2)\ln 2+\Big[\frac{1}{20} \\
&-\frac{31}{160(1-x)}-\frac{3}{5(1+x)^5}+\frac{3}{2(1+x)^4}-\frac{71}{40(1+x)^3}+\frac{93}{80(1+x)^2} \\
&-\frac{31}{160(1+x)}\Big]\zeta^2(2)-\Big[\frac{5}{8(1-x)}-\frac{6}{(1+x)^4}+\frac{12}{(1+x)^3}-\frac{8}{(1+x)^2} \\
&+\frac{11}{8(1+x)}+\frac{1}{8}\Big]\zeta(3)+\frac{13}{8(1+x)}\Big[1-\frac{1}{(1+x)}\Big] \\
&+\Big[\frac{49}{32}-\Big(\frac{85}{64(1-x)}+\frac{15}{4(1+x)^5}-\frac{75}{8(1+x)^4}+\frac{117}{16(1+x)^3} \\
&\quad +\frac{117}{32(1+x)^2}-\frac{331}{64(1+x)}+\frac{3}{8}\Big)\zeta(2)-\Big(\frac{5}{16(1-x)}+\frac{6}{(1+x)^5} \\
&\quad -\frac{15}{(1+x)^4}+\frac{45}{4(1+x)^3}-\frac{15}{8(1+x)^2}+\frac{5}{16(1+x)}-\frac{1}{2}\Big)\zeta(3) \\
&\quad -\frac{21}{16(1-x)}-\frac{39}{8(1+x)^3}+\frac{117}{16(1+x)^2}-\frac{67}{16(1+x)}\Big]H(0;x) \\
&-\Big[\frac{1}{4(1-x)}+\frac{1}{4(1+x)}-\frac{1}{4}\Big]\zeta(2)H(1;x) \\
&+\Big[\frac{21}{2(1+x)^2}-\frac{21}{2(1+x)}+\frac{9}{4}\Big]\zeta(2)H(-1;x) \\
&+\Big[\frac{35}{8}-\Big(\frac{1}{16(1-x)}+\frac{3}{(1+x)^5}-\frac{15}{2(1+x)^4}+\frac{19}{4(1+x)^3} \\
&\quad +\frac{3}{8(1+x)^2}-\frac{1}{16(1+x)}-\frac{1}{4}\Big)\zeta(2)+\frac{3}{16(1-x)^2}-\frac{47}{16(1-x)} \\
&\quad -\frac{15}{4(1+x)^4}+\frac{3}{(1+x)^3}+\frac{31}{8(1+x)^2}-\frac{43}{8(1+x)}\Big]H(0,0;x)
\end{aligned}
$$

$$
\begin{aligned}
&-\Big[\frac{29}{8}-\frac{4}{(1-x)}-\frac{6}{(1+x)^3}+\frac{9}{(1+x)^2}-\frac{25}{4(1+x)}\Big]H(-1,0;x)\\
&+\frac{1}{9}\Big[\frac{3}{(1-x)}+\frac{12}{(1+x)^3}-\frac{18}{(1+x)^2}+\frac{3}{(1+x)}\Big]\zeta(2)H(0,-1;x)\\
&+\Big[\frac{11}{8}+\Big(\frac{1}{4(1-x)}+\frac{6}{(1+x)^5}-\frac{15}{(1+x)^4}+\frac{11}{(1+x)^3}-\frac{3}{2(1+x)^2}\\
&\quad+\frac{1}{4(1+x)}-\frac{1}{2}\Big)\zeta(2)-\frac{3}{2(1-x)}-\frac{3}{(1+x)^3}+\frac{9}{2(1+x)^2}\\
&\quad-\frac{11}{4(1+x)}\Big]H(1,0;x)\\
&-\Big[\frac{13}{8}+\frac{21}{64(1-x)}+\frac{15}{4(1+x)^5}-\frac{51}{8(1+x)^4}+\frac{21}{16(1+x)^3}\\
&\quad+\frac{197}{32(1+x)^2}-\frac{363}{64(1+x)}\Big]H(0,0,0;x)\\
&-\frac{1}{2}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big][H(1,1,0;x)+6H(-1,-1,0;x)\\
&\quad-3H(-1,1,0;x)-3H(1,-1,0;x)]\\
&+\Big[3-\frac{9}{4(1-x)}+\frac{7}{2(1+x)^2}-\frac{23}{4(1+x)}\Big]H(-1,0,0;x)\\
&+\Big[\frac{19}{4}-\frac{9}{4(1-x)}+\frac{12}{(1+x)^4}-\frac{24}{(1+x)^3}+\frac{17}{(1+x)^2}\\
&\quad-\frac{29}{4(1+x)}\Big]H(0,-1,0;x)\\
&-\Big[\frac{9}{4}-\frac{1}{(1-x)}+\frac{6}{(1+x)^4}-\frac{12}{(1+x)^3}+\frac{17}{2(1+x)^2}\\
&\quad-\frac{7}{2(1+x)}\Big]H(0,1,0;x)\\
&+\Big[\frac{3}{4}+\frac{1}{(1-x)}+\frac{12}{(1+x)^4}-\frac{24}{(1+x)^3}+\frac{27}{2(1+x)^2}\\
&\quad-\frac{1}{2(1+x)}\Big]H(1,0,0;x)\\
&-\frac{1}{8}\Big[\frac{1}{(1-x)}+\frac{4}{(1+x)^3}-\frac{6}{(1+x)^2}+\frac{1}{(1+x)}\Big][H(0,0,0,0;x)\\
&\quad-H(0,-1,0,0;x)]\\
&-\Big[1-\frac{1}{2(1-x)}-\frac{12}{(1+x)^5}+\frac{30}{(1+x)^4}-\frac{22}{(1+x)^3}+\frac{3}{(1+x)^2}\\
&\quad-\frac{1}{2(1+x)}\Big]H(0,0,-1,0;x)\\
&+\Big[\frac{1}{2}-\frac{1}{4(1-x)}-\frac{6}{(1+x)^5}+\frac{15}{(1+x)^4}-\frac{11}{(1+x)^3}+\frac{3}{2(1+x)^2}\\
&\quad-\frac{1}{4(1+x)}\Big]H(0,0,1,0;x)
\end{aligned}
$$

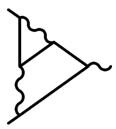
$$
\begin{aligned}
& -\Big[1-\frac{3}{8(1-x)}-\frac{12}{(1+x)^5}+\frac{30}{(1+x)^4}-\frac{43}{2(1+x)^3}+\frac{9}{4(1+x)^2} \\
& \quad -\frac{3}{8(1+x)}\Big]H(0,1,0,0;x) \\
& -\Big[\frac{1}{2}-\frac{1}{4(1-x)}-\frac{6}{(1+x)^5}+\frac{15}{(1+x)^4}-\frac{11}{(1+x)^3}+\frac{3}{2(1+x)^2} \\
& -\frac{1}{4(1+x)}\Big]H(1,0,0,0;x) \\
+ \quad & \mathcal{O}(D-4)\,, \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_2^{(2l,\mathrm{c})}(D,q^2) \quad = \quad & \frac{1}{(D-4)}\Big\{\frac{1}{4}\Big[\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big]H(0;x)\Big\} \\
& +\Big[\frac{1}{16(1-x)^2}+\frac{1}{16(1-x)}+\frac{15}{4(1+x)^4}-\frac{15}{2(1+x)^3}+\frac{89}{16(1+x)^2} \\
& -\frac{31}{16(1+x)}\Big]\zeta(2)-\frac{6\zeta(2)\ln 2}{(1+x)}\Big[1-\frac{1}{(1+x)}\Big]-\Big[\frac{27}{40(1-x)^3} \\
& -\frac{81}{80(1-x)^2}+\frac{3}{80(1-x)}-\frac{3}{5(1+x)^5}+\frac{3}{2(1+x)^4}-\frac{69}{40(1+x)^3} \\
& +\frac{87}{80(1+x)^2}+\frac{3}{80(1+x)}\Big]\zeta^2(2)-\Big[\frac{1}{4(1-x)^2}-\frac{1}{4(1-x)}-\frac{6}{(1+x)^4} \\
& +\frac{12}{(1+x)^3}-\frac{27}{4(1+x)^2}+\frac{3}{4(1+x)}\Big]\zeta(3)-\frac{13}{8(1+x)}\Big[1-\frac{1}{(1+x)}\Big] \\
& +\Big[\Big(\frac{1}{16(1-x)^3}-\frac{27}{32(1-x)^2}+\frac{57}{64(1-x)}+\frac{15}{4(1+x)^5}-\frac{75}{8(1+x)^4} \\
& \quad +\frac{7}{(1+x)^3}+\frac{21}{8(1+x)^2}-\frac{263}{64(1+x)}\Big)\zeta(2)-\Big(\frac{1}{4(1-x)^3} \\
& \quad +\frac{3}{8(1-x)^2}-\frac{3}{8(1-x)}+\frac{6}{(1+x)^5}-\frac{15}{(1+x)^4}+\frac{43}{4(1+x)^3} \\
& \quad -\frac{9}{8(1+x)^2}-\frac{3}{8(1+x)}\Big)\zeta(3)-\frac{27}{32(1-x)}+\frac{39}{8(1+x)^3} \\
& \quad -\frac{117}{16(1+x)^2}+\frac{105}{32(1+x)}\Big]H(0;x) \\
& +\frac{3}{2}\Big[\frac{1}{(1-x)^2}-\frac{1}{(1-x)}-\frac{5}{(1+x)^2}+\frac{5}{(1+x)}\Big]\zeta(2)H(-1;x) \\
& -\Big[\Big(\frac{3}{4(1-x)^3}-\frac{9}{8(1-x)^2}-\frac{3}{16(1-x)}+\frac{3}{(1+x)^5}-\frac{15}{2(1+x)^4} \\
& \quad +\frac{9}{2(1+x)^3}+\frac{3}{4(1+x)^2}-\frac{3}{16(1+x)}\Big)\zeta(2)+\frac{13}{16(1-x)^2} \\
& \quad +\frac{1}{16(1-x)}-\frac{15}{4(1+x)^4}+\frac{3}{(1+x)^3}+\frac{51}{16(1+x)^2} \\
& \quad -\frac{53}{16(1+x)}\Big]H(0,0;x)
\end{aligned}
$$

$$
\begin{aligned}
&+\Big[\frac{5}{4(1-x)}-\frac{6}{(1+x)^3}+\frac{9}{(1+x)^2}-\frac{17}{4(1+x)}\Big]H(-1,0;x)\\
&+\Big[\frac{3\zeta(2)}{8(1-x)}-\frac{6\zeta(2)}{(1+x)^5}+\frac{15\zeta(2)}{(1+x)^4}-\frac{21\zeta(2)}{2(1+x)^3}+\frac{3\zeta(2)}{4(1+x)^2}\\
&\quad+\frac{3\zeta(2)}{8(1+x)}-\frac{1}{2(1-x)}+\frac{3}{(1+x)^3}-\frac{9}{2(1+x)^2}\\
&\quad+\frac{2}{(1+x)}\Big]H(1,0;x)\\
&+\Big[\frac{1}{16(1-x)^3}-\frac{19}{32(1-x)^2}+\frac{41}{64(1-x)}+\frac{15}{4(1+x)^5}-\frac{51}{8(1+x)^4}\\
&\quad+\frac{1}{(1+x)^3}+\frac{39}{8(1+x)^2}-\frac{215}{64(1+x)}\Big]H(0,0,0;x)\\
&+\Big[\frac{6}{(1+x)^4}-\frac{12}{(1+x)^3}+\frac{7}{(1+x)^2}-\frac{1}{(1+x)}\Big]H(0,1,0;x)\\
&+\frac{1}{2}\Big[\frac{1}{(1-x)^2}-\frac{1}{(1-x)}-\frac{5}{(1+x)^2}+\frac{5}{(1+x)}\Big]H(-1,0,0;x)\\
&-\Big[\frac{12}{(1+x)^4}-\frac{24}{(1+x)^3}+\frac{14}{(1+x)^2}-\frac{2}{(1+x)}\Big]H(0,-1,0;x)\\
&-\Big[\frac{1}{2(1-x)^2}-\frac{1}{2(1-x)}+\frac{12}{(1+x)^4}-\frac{24}{(1+x)^3}+\frac{23}{2(1+x)^2}\\
&\quad+\frac{1}{2(1+x)}\Big]H(1,0,0;x)\\
&+\frac{1}{2}\Big[\frac{1}{(1-x)^3}-\frac{3}{2(1-x)^2}-\frac{1}{(1+x)^3}+\frac{3}{2(1+x)^2}\Big]\times\\
&\qquad\times[3\zeta(2)H(0,-1;x)-H(0,0,0,0;x)+H(0,-1,0,0;x)]\\
&+\Big[\frac{3}{4(1-x)}-\frac{12}{(1+x)^5}+\frac{30}{(1+x)^4}-\frac{21}{(1+x)^3}+\frac{3}{2(1+x)^2}\\
&\quad+\frac{3}{4(1+x)}\Big]H(0,0,-1,0;x)\\
&-\Big[\frac{3}{8(1-x)}-\frac{6}{(1+x)^5}+\frac{15}{(1+x)^4}-\frac{21}{2(1+x)^3}+\frac{3}{4(1+x)^2}\\
&\quad+\frac{3}{8(1+x)}\Big]H(0,0,1,0;x)\\
&-\Big[\frac{1}{2(1-x)^3}-\frac{3}{4(1-x)^2}-\frac{3}{4(1-x)}+\frac{12}{(1+x)^5}-\frac{30}{(1+x)^4}\\
&\quad+\frac{41}{2(1+x)^3}-\frac{3}{4(1+x)^2}-\frac{3}{4(1+x)}\Big]H(0,1,0,0;x)\\
&+\Big[\frac{3}{8(1-x)}-\frac{6}{(1+x)^5}+\frac{15}{(1+x)^4}-\frac{21}{2(1+x)^3}+\frac{3}{4(1+x)^2}\\
&\quad+\frac{3}{8(1+x)}\Big]H(1,0,0,0;x)\\
+\quad&\mathcal{O}(D-4), \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_3^{(2l,\mathrm{c})}(D,q^2) \;=\;& -\Big[\frac{3}{16(1-x)^2} - \frac{3}{16(1-x)} + \frac{21}{16(1+x)^2} - \frac{21}{16(1+x)}\Big]\zeta(2) \\
& -\frac{3\zeta(2)\ln 2}{(1-x)}\Big[1 - \frac{1}{(1-x)}\Big] + \Big[\frac{27}{80(1-x)^3} - \frac{81}{160(1-x)^2} \\
& -\frac{27}{80(1+x)^3} + \frac{81}{160(1+x)^2}\Big]\zeta^2(2) - \Big[\frac{1}{8(1-x)^2} - \frac{1}{8(1-x)} \\
& +\frac{1}{8(1+x)^2} - \frac{1}{8(1+x)}\Big]\zeta(3) - \frac{1}{16(1-x)^2} + \frac{1}{16(1-x)} \\
& -\Big[\Big(\frac{3}{16(1-x)^3} - \frac{69}{32(1-x)^2} + \frac{33}{16(1-x)} + \frac{9}{16(1+x)^3} \\
& \quad -\frac{15}{32(1+x)^2} - \frac{3}{16(1+x)}\Big)\zeta(2) - \Big(\frac{1}{8(1-x)^3} + \frac{3}{16(1-x)^2} \\
& \quad +\frac{1}{8(1+x)^3} - \frac{3}{16(1+x)^2}\Big)\zeta(3) + \frac{1}{8(1-x)^3} - \frac{3}{16(1-x)^2} \\
& \quad -\frac{11}{32(1-x)} + \frac{13}{32(1+x)}\Big]H(0;x) \\
& +\Big[\Big(\frac{3}{8(1-x)^3} - \frac{9}{16(1-x)^2} - \frac{3}{8(1+x)^3} + \frac{9}{16(1+x)^2}\Big)\zeta(2) \\
& \quad -\frac{1}{8(1-x)^4} + \frac{1}{4(1-x)^3} + \frac{5}{8(1-x)^2} - \frac{3}{4(1-x)} - \frac{13}{16(1+x)^2} \\
& \quad +\frac{13}{16(1+x)}\Big]H(0,0;x) \\
& -\Big[\frac{3}{16(1-x)^3} - \frac{49}{32(1-x)^2} + \frac{23}{16(1-x)} + \frac{9}{16(1+x)^3} \\
& \quad -\frac{19}{32(1+x)^2} - \frac{1}{16(1+x)}\Big]H(0,0,0;x) \\
& -\frac{1}{4}\Big[\frac{5}{(1-x)^2} - \frac{5}{(1-x)} - \frac{1}{(1+x)^2} + \frac{1}{(1+x)}\Big][3\zeta(2)H(-1;x) \\
& \quad +H(-1,0,0;x) - H(1,0,0;x)] \\
& -\frac{1}{4}\Big[\frac{1}{(1-x)^3} - \frac{3}{2(1-x)^2} - \frac{1}{(1+x)^3} + \frac{3}{2(1+x)^2}\Big]\times \\
& \quad \times[3\zeta(2)H(0,-1;x) + H(0,0,0,0;x) - H(0,-1,0,0;x) \\
& \qquad +H(0,1,0,0;x)] \\
+\;& \mathcal{O}(D-4)\,.
\end{aligned}
\tag{36}
$$

- The *Up-Corner* graph $(\mathtt{d})$ of Fig. 1, defined as



$$
= \int \mathfrak{D}^D k_1\, \mathfrak{D}^D k_2\, \frac{\mathcal{N}^{\mu}_{(\mathtt{d})}}{\mathcal{D}_3\mathcal{D}_4\mathcal{D}_6\mathcal{D}_7\mathcal{D}_{12}\mathcal{D}_{13}}\,, \tag{37}
$$

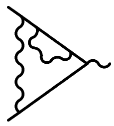
where

$$
\begin{aligned}
\mathcal{N}^{\mu}_{(\mathtt{d})} \;=\;& \bar{v}(p_2)\gamma_\sigma[i(\not{p}_2 + \not{k}_1 - \not{k}_2) + m]\gamma^\mu[-i(\not{p}_1 - \not{k}_1 + \not{k}_2) + m]\gamma_\lambda \times \\
& \times[-i(\not{k}_2) + m]\gamma_\sigma[-i(\not{k}_1) + m]\gamma^\lambda u(p_1),
\end{aligned}
\tag{38}
$$

due to reasons of symmetry gives:

$$\begin{aligned}
\mathcal{F}_1^{(2l,\mathtt{d})}(D,q^2) &= \mathcal{F}_1^{(2l,\mathtt{c})}(D,q^2)\,, &(39)\\
\mathcal{F}_2^{(2l,\mathtt{d})}(D,q^2) &= \mathcal{F}_2^{(2l,\mathtt{c})}(D,q^2)\,, &(40)\\
\mathcal{F}_3^{(2l,\mathtt{d})}(D,q^2) &= -\mathcal{F}_3^{(2l,\mathtt{c})}(D,q^2)\,. &(41)
\end{aligned}$$

- The *Up-Self-Mass-insertion* graph `(e)` of Fig. 1, defined as



$$= \int \mathfrak{D}^D k_1\, \mathfrak{D}^D k_2\, \frac{\mathcal{N}^{\mu}_{(\mathtt{e})}}{\mathcal{D}_1\mathcal{D}_2\mathcal{D}_9^2\mathcal{D}_{10}\mathcal{D}_{12}} \qquad (42)$$

where

$$\begin{aligned}
\mathcal{N}^{\mu}_{(\mathtt{e})} &= \bar{v}(p_2)\gamma_\sigma[i(\not p_2+\not k_1)+m]\gamma_\mu[-i(\not p_1-\not k_1)+m]\gamma_\lambda[-i(\not p_1-\not k_1+\not k_2)+m]\gamma^\lambda\times\\
&\quad\times[-i(\not p_1-\not k_1)+m]\gamma^\sigma u(p_1), &(43)
\end{aligned}$$

gives:

$$\begin{aligned}
\mathcal{F}_1^{(2l,\mathtt{e})}(D,q^2) &= \frac{1}{(D-4)^2}\Biggl\{\frac{11}{8}+\frac{3}{(1+x)^2}-\frac{3}{(1+x)}-\Bigl[1+\frac{1}{2(1-x)}\\
&\qquad -\frac{3}{(1+x)^3}+\frac{9}{2(1+x)^2}-\frac{4}{(1+x)}\Bigr]H(0;x)\Biggr\}\\
&+\frac{1}{(D-4)}\Biggl\{\frac{19}{32}-\frac{2}{(1+x)^2}+\frac{2}{(1+x)}-\Bigl(\frac{1}{4(1-x)}+\frac{3}{2(1+x)^3}\\
&\qquad -\frac{9}{4(1+x)^2}+\frac{2}{(1+x)}-\frac{1}{2}\Bigr)\zeta(2)\\
&\qquad +\Bigl[\frac{7}{8}+\frac{3}{4(1-x)}-\frac{5}{(1+x)^3}+\frac{15}{2(1+x)^2}\\
&\qquad -\frac{5}{(1+x)}\Bigr]H(0;x)\\
&\qquad -\Bigl[\frac{1}{4}-\frac{1}{(1-x)}+\frac{3}{2(1+x)^3}-\frac{9}{4(1+x)^2}\\
&\qquad +\frac{5}{4(1+x)}\Bigr]H(0,0;x)\\
&\qquad -\frac{3}{2}\Bigl[\frac{1}{(1-x)}-\frac{2}{(1+x)^3}+\frac{3}{(1+x)^2}-\frac{2}{(1+x)}\Bigr]H(-1,0,x)\\
&\qquad -\Bigl[\frac{1}{2}-\frac{1}{2(1-x)}-\frac{1}{2(1+x)}\Bigr]H(1,0;x)\Biggr\}\\
&+\frac{179}{128}+\Bigl[\frac{3}{8(1-x)}+\frac{21}{4(1+x)^4}-\frac{13}{(1+x)^3}+\frac{105}{8(1+x)^2}\\
&-\frac{53}{8(1+x)}+\frac{31}{16}\Bigr]\zeta(2)+\Bigl[\frac{11}{8(1-x)}-\frac{3}{2(1+x)^3}+\frac{9}{4(1+x)^2}\\
&-\frac{7}{8(1+x)}-\frac{5}{8}\Bigr]\zeta(3)+\frac{59}{8(1+x)^2}-\frac{59}{8(1+x)}
\end{aligned}$$

$$
\begin{aligned}
& -\Big[\frac{45}{32}+\Big(\frac{17}{8}-\frac{75}{64(1-x)}-\frac{21}{4(1+x)^5}+\frac{105}{8(1+x)^4}-\frac{215}{16(1+x)^3} \\
&\quad +\frac{225}{32(1+x)^2}-\frac{291}{64(1+x)}\Big)\zeta(2)+\frac{21}{16(1-x)}-\frac{89}{8(1+x)^3} \\
&\quad +\frac{267}{16(1+x)^2}-\frac{155}{16(1+x)}\Big]H(0;x) \\
& -\Big[\frac{3}{4(1-x)}-\frac{3}{2(1+x)^3}+\frac{9}{4(1+x)^2}-\frac{3}{2(1+x)}\Big]\zeta(2)H(-1;x) \\
& -\frac{1}{4}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big]\zeta(2)H(1;x) \\
& +\Big[\frac{13}{4}+\frac{15}{16(1-x)^2}-\frac{57}{16(1-x)}+\frac{21}{4(1+x)^4}-\frac{8}{(1+x)^3} \\
&\quad +\frac{45}{8(1+x)^2}-\frac{25}{8(1+x)}\Big]H(0,0;x) \\
& -\Big[\frac{13}{8}-\frac{15}{4(1-x)}+\frac{5}{(1+x)^3}-\frac{15}{2(1+x)^2}+\frac{3}{(1+x)}\Big]H(-1,0;x) \\
& +\Big[\frac{5}{4}-\frac{3}{2(1-x)}-\frac{1}{(1+x)}\Big]H(1,0;x) \\
& -\Big[\frac{9}{8}+\frac{37}{64(1-x)}-\frac{21}{4(1+x)^5}+\frac{105}{8(1+x)^4}-\frac{239}{16(1+x)^3} \\
&\quad +\frac{297}{32(1+x)^2}-\frac{323}{64(1+x)}\Big]H(0,0,0;x) \\
& +\Big[3-\frac{9}{2(1-x)}+\frac{3}{(1+x)^3}-\frac{9}{2(1+x)^2}\Big]H(-1,-1,0;x) \\
& -\Big[\frac{9}{4}-\frac{3}{(1-x)}+\frac{3}{2(1+x)^3}-\frac{9}{4(1+x)^2}-\frac{3}{4(1+x)}\Big]H(-1,0,0;x) \\
& -\Big[\frac{9}{4}-\frac{3}{(1-x)}+\frac{3}{2(1+x)^3}-\frac{9}{4(1+x)^2}-\frac{3}{4(1+x)}\Big]H(0,-1,0;x) \\
& -\frac{1}{4}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big][\zeta(2)H(1;x)+6H(-1,1,0;x) \\
&\qquad -4H(0,1,0;x)+6H(1,-1,0;x)-4H(1,0,0;x) \\
&\qquad -4H(1,1,0;x)]
\end{aligned}
$$

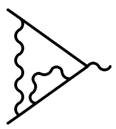
$$
\begin{aligned}
& + \ \mathcal{O}(D-4)\,, \qquad (44) \\
\mathcal{F}_2^{(2l,\mathrm{e})}(D,q^2) \ = \ & \frac{1}{(D-4)^2}\Big\{\frac{3}{(1+x)}\Big[1-\frac{1}{(1+x)}\Big]+\frac{3}{4}\Big[\frac{1}{(1-x)}-\frac{4}{(1+x)^3} \\
&\quad +\frac{6}{(1+x)^2}-\frac{3}{(1+x)}\Big]H(0;x)\Big\} \\
+ \ & \frac{1}{(D-4)}\Big\{-\frac{5}{(1+x)}\Big[1-\frac{1}{(1+x)}\Big] \\
&\quad -\Big[\frac{3}{4(1-x)}-\frac{8}{(1+x)^3}+\frac{12}{(1+x)^2}-\frac{19}{4(1+x)}\Big]H(0;x)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{3}{8}\Big[\frac{1}{(1-x)}-\frac{4}{(1+x)^3}+\frac{6}{(1+x)^2}-\frac{3}{(1+x)}\Big][\zeta(2) \\
& \qquad -H(0,0;x)+2H(-1,0;x)]\Big\} \\
& -\Big[\frac{7}{16(1-x)^2}-\frac{1}{16(1-x)}+\frac{21}{4(1+x)^4}-\frac{29}{2(1+x)^3} \\
& +\frac{239}{16(1+x)^2}-\frac{97}{16(1+x)}\Big]\zeta(2)-\frac{51}{8(1+x)^2}+\frac{51}{8(1+x)} \\
& -\Big[\Big(\frac{7}{16(1-x)^3}-\frac{21}{32(1-x)^2}-\frac{25}{64(1-x)}+\frac{21}{4(1+x)^5} \\
& \quad -\frac{105}{8(1+x)^4}+\frac{13}{(1+x)^3}-\frac{51}{8(1+x)^2}+\frac{119}{64(1+x)}\Big)\zeta(2) \\
& \quad -\frac{53}{32(1-x)}+\frac{105}{8(1+x)^3}-\frac{315}{16(1+x)^2}+\frac{263}{32(1+x)}\Big]H(0;x) \\
& +\Big[\frac{19}{16(1-x)^2}-\frac{25}{16(1-x)}-\frac{21}{4(1+x)^4}+\frac{13}{2(1+x)^3} \\
& \quad -\frac{47}{16(1+x)^2}+\frac{33}{16(1+x)}\Big]H(0,0;x) \\
& -\frac{1}{2}\Big[\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big]H(1,0;x) \\
& +\Big[\frac{1}{4(1-x)}+\frac{8}{(1+x)^3}-\frac{12}{(1+x)^2}+\frac{15}{4(1+x)}\Big]H(-1,0;x) \\
& -\Big[\frac{7}{16(1-x)^3}-\frac{21}{32(1-x)^2}-\frac{49}{64(1-x)}+\frac{21}{4(1+x)^5}-\frac{105}{8(1+x)^4} \\
& \quad +\frac{29}{2(1+x)^3}-\frac{69}{8(1+x)^2}+\frac{191}{64(1+x)}\Big]H(0,0,0;x) \\
& -\frac{3}{8}\Big[\frac{1}{(1-x)}-\frac{12}{(1+x)^3}+\frac{6}{(1+x)^2}-\frac{3}{(1+x)}\Big][\zeta(3)-H(-1;x) \\
& \qquad -2H(-1,-1,0;x)+H(-1,0,0;x)+H(0,-1,0;x)] \\
+ \;& \mathcal{O}(D-4)\,, \qquad (45)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_3^{(2l,\mathsf{e})}(D,q^2) \;=\; & \frac{1}{(D-4)^2}\Big\{-\frac{3}{2(1-x)}\Big[1-\frac{1}{(1-x)}\Big]+\frac{3}{2}\Big[\frac{1}{(1-x)^3}-\frac{3}{2(1-x)^2} \\
& \qquad +\frac{3}{4(1-x)}-\frac{1}{4(1+x)}\Big]H(0;x)\Big\} \\
+ \;& \frac{1}{(D-4)}\Big\{-\frac{1}{2(1-x)}\Big[1-\frac{1}{(1-x)}\Big]+\frac{3}{4}\Big[\frac{1}{(1-x)^3}-\frac{3}{2(1-x)^2} \\
& \qquad +\frac{3}{4(1-x)}-\frac{1}{4(1+x)}\Big]\Big[\zeta(2)+\frac{2}{3}H(0;x)-H(0,0;x) \\
& \qquad +2H(-1,0;x)\Big]\Big\} \\
& +\Big[\frac{1}{4(1-x)^3}+\frac{9}{16(1-x)^2}-\frac{9}{8(1-x)}-\frac{7}{16(1+x)^2}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{3}{4(1+x)}\Big]\zeta(2)+\frac{19}{16(1-x)}\Big[1-\frac{1}{(1-x)}\Big] \\
& +\Big[\Big(\frac{9}{16(1-x)^3}-\frac{27}{32(1-x)^2}+\frac{19}{32(1-x)}-\frac{7}{16(1+x)^3} \\
& \quad +\frac{21}{32(1+x)^2}-\frac{17}{32(1+x)}\Big)\zeta(2)-\frac{13}{8(1-x)^3}+\frac{39}{16(1-x)^2} \\
& \quad +\frac{21}{32(1+x)^2}+\frac{9}{32(1-x)}-\frac{35}{32(1+x)}\Big]H(0;x) \\
& -\Big[\frac{7}{8(1-x)^4}-\frac{3}{2(1-x)^3}+\frac{3}{8(1-x)^2}-\frac{1}{16(1-x)} \\
& \quad +\frac{7}{16(1+x)^2}-\frac{1}{8(1+x)}\Big]H(0,0;x) \\
& +\frac{1}{2}\Big[\frac{1}{(1-x)^3}-\frac{3}{2(1-x)^2}-\frac{3}{4(1-x)}+\frac{5}{4(1+x)}\Big]H(-1,0;x) \\
& +\Big[\frac{21}{16(1-x)^3}-\frac{63}{32(1-x)^2}+\frac{37}{32(1-x)}-\frac{7}{16(1+x)^3} \\
& \quad +\frac{21}{32(1+x)^2}-\frac{23}{32(1+x)}\Big]H(0,0,0;x) \\
& -\frac{3}{4}\Big[\frac{1}{(1-x)^3}-\frac{3}{2(1-x)^2}+\frac{3}{4(1-x)}-\frac{1}{4(1+x)}\Big]\big[\zeta(3) \\
& \quad -\zeta(2)H(-1;x)-2H(-1,-1,0;x)+H(-1,0,0;x) \\
& \quad +H(0,-1,0;x)\big] \\
+ \quad & \mathcal{O}(D-4)\,. \qquad (46)
\end{aligned}
$$

- The *Down-Self-Mass-insertion* graph (f) of Fig. 1, defined as



$$
= \int \mathfrak{D}^D k_1 \, \mathfrak{D}^D k_2 \, \frac{\mathcal{N}^{\mu}_{(\mathtt{f})}}{\mathcal{D}_1\mathcal{D}_2\mathcal{D}_9\mathcal{D}_{10}^2\mathcal{D}_{13}}\,, \qquad (47)
$$

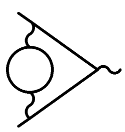
where

$$
\begin{aligned}
\mathcal{N}^{\mu}_{(\mathtt{f})} \;=\; & \bar{v}(p_2)\gamma_\sigma[i(\not{p}_2+\not{k}_1)+m]\gamma_\lambda[i(\not{p}_2+\not{k}_1-\not{k}_2)+m]\gamma^\lambda[i(\not{p}_2+\not{k}_1)+m]\gamma_\mu\times \\
& \times[-i(\not{p}_1-\not{k}_1)+m]\gamma^\sigma u(p_1), \qquad (48)
\end{aligned}
$$

due to reasons of symmetry, gives:

$$
\begin{aligned}
\mathcal{F}_1^{(2l,\mathtt{f})}(D,q^2) &= \mathcal{F}_1^{(2l,\mathtt{e})}(D,q^2)\,, & (49)\\
\mathcal{F}_2^{(2l,\mathtt{f})}(D,q^2) &= \mathcal{F}_2^{(2l,\mathtt{e})}(D,q^2)\,, & (50)\\
\mathcal{F}_3^{(2l,\mathtt{f})}(D,q^2) &= -\mathcal{F}_3^{(2l,\mathtt{e})}(D,q^2)\,. & (51)
\end{aligned}
$$

- The *Vacuum-Polarization-insertion* graph (g) of Fig. 1, defined as



$$
= \int \mathfrak{D}^D k_1 \, \mathfrak{D}^D k_2 \, \frac{\mathcal{N}^{\mu}_{(\mathtt{g})}}{\mathcal{D}_1^2\mathcal{D}_7\mathcal{D}_8\mathcal{D}_9\mathcal{D}_{10}}\,, \qquad (52)
$$

where

$$\mathcal{N}^{\mu}_{(\mathrm{g})} = -\bar{v}(p_2)\gamma_\sigma[i(\not{p}_2+\not{k}_1)+m]\gamma_\mu[-i(\not{p}_1-\not{k}_1)+m]\gamma_\lambda\gamma_\lambda[i(\not{k}_1+\not{k}_2)+m] \times \times\gamma^\sigma[i(\not{k}_2)+m]u(p_1), \tag{53}$$

gives:

$$\begin{aligned}
\mathcal{F}_1^{(2l,\mathrm{g})}(D,q^2) &= \frac{1}{(D-4)^2}\Big\{-\frac{1}{3}-\frac{2}{3}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big]H(0;x)\Big\} \\
&+ \frac{1}{(D-4)}\Big\{\frac{1}{8}+\frac{1}{6}\Big[1-\frac{2}{(1-x)}\Big]H(0;x) \\
&\qquad -\frac{1}{3}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big][\zeta(2)-H(0;x) \\
&\qquad -H(0,0;x)+2H(-1,0;x)]\Big\} \\
&\quad +\frac{223}{864}+\Big[2-\frac{1}{3(1-x)}+\frac{98}{3(1+x)^4}-\frac{196}{3(1+x)^3}+\frac{229}{6(1+x)^2} \\
&\quad -\frac{17}{3(1+x)}\Big]\zeta(2)-\frac{49}{9(1+x)}\Big[1-\frac{1}{(1+x)}\Big] \\
&\quad -\Big[\frac{409}{216}-\Big(\frac{1}{4(1-x)}+\frac{6}{(1+x)^5}-\frac{15}{(1+x)^4}+\frac{11}{(1+x)^3}-\frac{3}{2(1+x)^2} \\
&\qquad -\frac{1}{4(1+x)}\Big)\zeta(2)-\frac{77}{27(1-x)}-\frac{89}{9(1+x)^3}+\frac{89}{6(1+x)^2} \\
&\qquad -\frac{635}{108(1+x)}\Big]H(0;x) \\
&\quad +\Big[\frac{5}{18}+\frac{1}{3(1-x)}+\frac{62}{9(1+x)^4}-\frac{124}{9(1+x)^3}+\frac{163}{18(1+x)^2} \\
&\qquad -\frac{2}{(1+x)}\Big]H(0,0;x) \\
&\quad +\frac{1}{6}\Big[1-\frac{2}{(1-x)}\Big]H(-1,0;x) \\
&\quad -\Big[\frac{1}{3}-\frac{7}{12(1-x)}+\frac{6}{(1+x)^5}-\frac{15}{(1+x)^4}+\frac{11}{(1+x)^3} \\
&\qquad -\frac{3}{2(1+x)^2}-\frac{7}{12(1+x)}\Big]H(0,0,0;x) \\
&\quad +\frac{1}{3}\Big[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Big][\zeta(3)-\zeta(2)H(-1;x)+H(-1,0;x) \\
&\qquad -2H(-1,-1,0;x)+H(-1,0,0;x)+H(0,-1,0;x)] \\
&+ \mathcal{O}(D-4)\,, \qquad\qquad (54) \\
\mathcal{F}_2^{(2l,\mathrm{g})}(D,q^2) &= \frac{1}{(D-4)}\Big\{\frac{1}{3}\Big[\frac{1}{(1+x)}-\frac{1}{(1-x)}\Big]H(0;x)\Big\} \\
&\quad -\Big[\frac{1}{6(1-x)}-\frac{34}{(1+x)^4}+\frac{68}{(1+x)^3}-\frac{33}{(1+x)^2}-\frac{5}{6(1+x)}\Big]\zeta(2)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{17}{3(1+x)}\Big[1-\frac{1}{(1+x)}\Big] \\
&-\Big[\Big(\frac{3}{8(1-x)}-\frac{6}{(1+x)^5}+\frac{15}{(1+x)^4}-\frac{21}{2(1+x)^3}+\frac{3}{4(1+x)^2} \\
&\quad+\frac{3}{8(1+x)}\Big)\zeta(2)+\frac{11}{9(1-x)}+\frac{31}{3(1+x)^3}-\frac{31}{2(1+x)^2} \\
&\quad+\frac{71}{18(1+x)}\Big]H(0;x) \\
&+\Big[\frac{1}{6(1-x)}-\frac{22}{3(1+x)^4}+\frac{44}{3(1+x)^3}-\frac{23}{3(1+x)^2} \\
&\quad+\frac{1}{6(1+x)}\Big]H(0,0;x) \\
&+\frac{1}{3}\Big[\frac{1}{(1+x)}-\frac{1}{(1-x)}\Big]H(-1,0;x) \\
&-\Big[\frac{3}{8(1-x)}-\frac{6}{(1+x)^5}+\frac{15}{(1+x)^4}-\frac{21}{2(1+x)^3}+\frac{3}{4(1+x)^2} \\
&\quad+\frac{3}{8(1+x)}\Big]H(0,0,0;x) \\
&+ \; \mathcal{O}(D-4)\,, \qquad (55)
\end{aligned}
$$

$$
\mathcal{F}_3^{(2l,\mathtt{g})}(D,q^2) \;=\; 0\,. \qquad (56)
$$

Note the structure of the contribution to the third *unrenormalized* form factor $\mathcal{F}_3^{(2l)}(D,x)$ of the seven diagrams given above: the contributions of the *Ladder*, *Cross* and *Vacuum-Polarization-insertion* vertices, the diagrams (a), (b) and (g) of Fig. 1, vanish separately, while those of the *Corner* and *Self-energy-insertion* diagrams, Fig. 1 (c), (d), (e) and (f), cancel pairwise. That leads to the vanishing of the third form factor

$$
\mathcal{F}_3^{(2l)}(D,q^2) = \sum_{\mathtt{graph}} \mathcal{F}_3^{(2l,\mathtt{graph})}(D,x) = 0\ , \qquad (57)
$$

as expected (and already repeatedly anticipated) from the conservation of the electromagnetic current.

4 Renormalization subtractions

As a next step we have to renormalize the 1-loop insertions in the above 2-loop graphs, when present (i.e. in all the graphs, with the exception of the *Cross* graph $\mathtt{b}$ of Fig. 1). That will be done by subtracting from each graphs suitable contributions, which will be specified in the next subsection, proportional to the 1-loop renormalization constants $Z_1^{(1l)}(D)$ (charge), $Z_2^{(1l)}(D)$ (electron wave function; in QED one has $Z_2^{(1l)}(D) = -Z_1^{(1l)}(D)$, due to the Ward identity), $Z_3^{(1l)}(D)$ (photon wave function) and $\delta m^{(1l)}(D,m)$ (electron mass).

Those subtractions are sufficient to obtain the 2-loop renormalized form factor $F_2^{(2l)}(D,q^2)$. To obtain also the renormalized value of $F_1^{(2l)}(D,q^2)$, we have

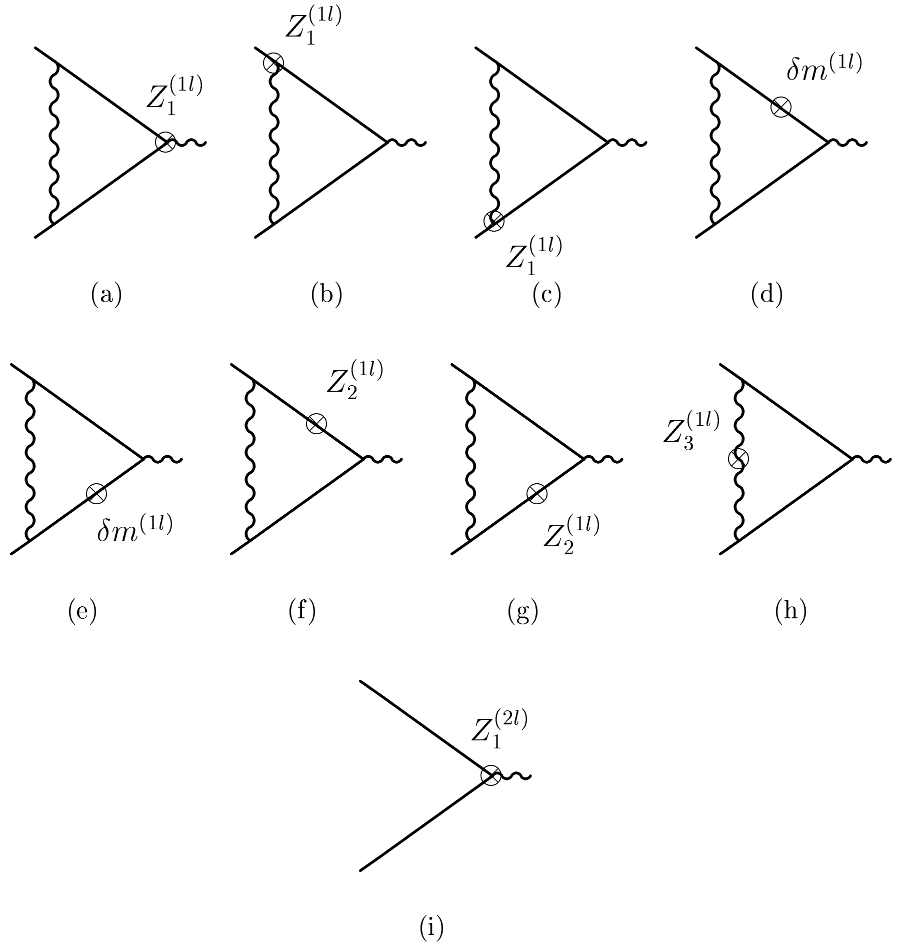


Figure 2: Subtraction terms for the renormalization at 2 loops.

to subtract from the unrenormalized charge form factor (obtained after the subtractions due to the renormalization of the 1-loop insertions) its value at $q^2 = 0$, $Z_1^{(2l)}(D) = \mathcal{F}_1^{(2l)}(D, q^2 = 0)$.

4.1 1-loop renormalization constants times 1-loop subdiagrams

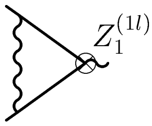
We give here the values of the subtractions to the 2-loop graphs due to the renormalization of the 1-loop insertions shown in Fig. 2 `(a-h)`.

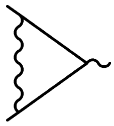
The 1-loop renormalization constants, $Z_1^{(1l)}(D)$, $Z_2^{(1l)}(D)$, $Z_3^{(1l)}(D)$ and $\delta m^{(1l)}(D, m)$, calculated according to the *On-Shell* renormalization prescription, have the following expressions exact in D:

$$
\begin{aligned}
Z_1^{(1l)}(D) &= -\frac{1}{2}\frac{(D-1)}{(D-3)(D-4)} , && (58) \\
Z_2^{(1l)}(D) &= -Z_1^{(1l)}(D) \qquad \text{(Ward identity)} , && (59) \\
Z_3^{(1l)}(D) &= \frac{2}{3(D-4)} , && (60) \\
\delta m^{(1l)}(D, m) &= m \, \frac{1}{2}\frac{(D-1)}{(D-3)(D-4)} , && (61)
\end{aligned}
$$

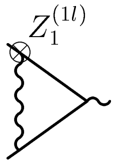
The subtractions graphs of Fig. 2 `(a-h)` are defined by the following relations:

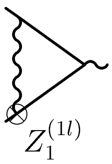
• graph (a) in Fig. (2)

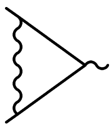


$\stackrel{def}{=} Z_1^{(1l)}(D) \times$  ; (62)

• graphs (b) and (c) in Fig. (2)

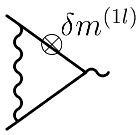


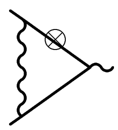
$=$ 

$\stackrel{def}{=} Z_1^{(1l)}(D) \times$  ; (63)

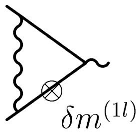
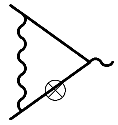
equal to the graph (a) of Fig. (2).

• graph (d) in Fig. (2)

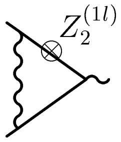
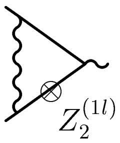
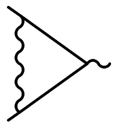


$\stackrel{def}{=} \dfrac{\delta m^{(1l)}(D, m)}{m} \times \left(m \right.$  $\left. \right)$; (64)

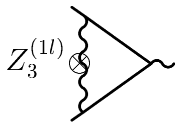
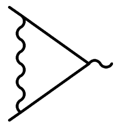
• graph (e) in Fig. (2)

 $\stackrel{def}{=} \dfrac{\delta m^{(1l)}(D,m)}{m} \times \Bigg(m$  $\Bigg)$; (65)

- graphs (f) and (g) in Fig. (2)

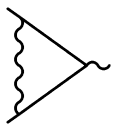
 $=$  $\stackrel{def}{=} Z_2^{(1l)}(D) \times$  ; (66)

- graph (h) in Fig. (2)

 $\stackrel{def}{=} Z_3^{(1l)}(D) \times$  . (67)

From the *r.h.s* of Eqs. (62-67), it turns out that only three 1-loop vertex subdiagrams appear in the calculation, two of them being in fact equal for symmetry reasons. Each of the subdiagrams can be written in terms of its vertex form factors by using an expression analogous to Eq. (4) and Eq. (15).

The first subdiagram, occurring in Eqs. (62,63,66,67), is exactly the 1-loop QED vertex:

 $= \displaystyle\int \mathfrak{D}^D k_1 \, \frac{\mathcal{N}^\mu}{\mathcal{D}_1 \mathcal{D}_9 \mathcal{D}_{10}} \, ,$ (68)

where

$$\mathcal{N}^\mu = \bar{v}(p_2)\gamma_\sigma[i(\not{p}_2+\not{k}_1)+m]\gamma^\mu[-i(\not{p}_1-\not{k}_1)+m][-i(\not{p}_1-\not{k}_1)+m]\gamma_\sigma u(p_1) \, . \quad (69)$$

The corresponding form factors are

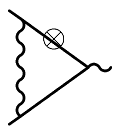
$$\begin{aligned}
\mathcal{F}_1^{(1l)}(D,q^2) &= \frac{1}{(D-4)}\left\{-\frac{1}{2} - \left[1 - \frac{1}{(1+x)} - \frac{1}{(1-x)}\right]H(0;x)\right\} \\
&\quad +\frac{1}{4}\left[1 - \frac{2}{(1-x)}\right]H(0;x) \\
&\quad -\frac{1}{2}\left[1 - \frac{1}{(1+x)} - \frac{1}{(1-x)}\right][\zeta(2) - H(0;x) - H(0,0;x) \\
&\qquad +2H(-1,0;x)] \\
&+ (D-4)\left\{\frac{1}{8}\left[1 - \frac{2}{(1-x)}\right][\zeta(2) - H(0,0;x) + 2H(-1,0;x)]\right. \\
&\qquad +\frac{1}{4}\left[1 - \frac{1}{(1+x)} - \frac{1}{(1-x)}\right][\zeta(2) + 2\zeta(3) \\
&\qquad\quad -(4-\zeta(2))H(0;x) - 2\zeta(2)H(-1;x) - H(0,0;x)
\end{aligned}$$

$$
\begin{aligned}
& \qquad\qquad +2H(-1,0;x) - H(0,0,0;x) + 2H(-1,0,0;x) \\
& \qquad\qquad +2H(0,-1,0;x) - 4H(-1,-1,0;x)] \Big\} \\
&+ \; \mathcal{O}\left((D-4)^2\right) \, , \qquad (70)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_2^{(1l)}(D,q^2) &= -\frac{1}{2}\Big[\frac{1}{(1-x)} - \frac{1}{(1+x)}\Big] H(0;x) \\
&- (D-4)\Big\{\frac{1}{4}\Big[\frac{1}{(1-x)} - \frac{1}{(1+x)}\Big][\zeta(2) - 4H(0;x) - H(0,0;x) \\
&\qquad +2H(-1,0;x)]\Big\} \\
&+ \mathcal{O}\left((D-4)^2\right) \, , \qquad (71) \\
\mathcal{F}_3^{(1l)}(D,q^2) &= 0 \, . \qquad (72)
\end{aligned}
$$

Let us observe that these are exactly the 1-loop *unrenormalized* form factors, $\mathcal{F}_i^{(1l)}(D,q^2)$ $(i = 1,2,3)$. As in the 2-loop counterterms they are multiplied by the 1-loop renormalization constants Eqs. (58-61), which behave for $D \to 4$ as $1/(D-4)$, the 1-loop form factors must be evaluated up to the first order term in $(D-4)$ included; similarly, as the 1-loop unrenormalized form factors can develop poles in $(D-4)$ (that is the case of the charge form factor), the 1-loop renormalization constants are also needed up to first order in $(D-4)$ (a requirement trivially fulfilled by Eqs. (58,60,61), which are exact in D).

The second subdiagram occurring in the *r.h.s.* of Eq. (64) is:

m  $$= m \int \mathfrak{D}^D k_1 \, \frac{\mathcal{U}^\mu}{\mathcal{D}_1 \mathcal{D}_9^2 \mathcal{D}_{10}} \, , \qquad (73)$$

where

$$
\mathcal{U}^\mu = \bar{v}(p_2)\gamma_\sigma[i(\not{p}_2+\not{k}_1)+m]\gamma^\mu[-i(\not{p}_1-\not{k}_1)+m]i[-i(\not{p}_1-\not{k}_1)+m]\gamma_\sigma u(p_1) \; ; \qquad (74)
$$

the corresponding (dimensionless) form factors are:

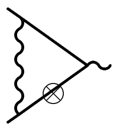
$$
\begin{aligned}
\mathcal{F}_1^{(\otimes,up)}(D,q^2) &= \frac{1}{(D-4)}\Big\{1 - \frac{2}{(1+x)} - \frac{2}{(1+x)^2} - \Big[\frac{1}{(1-x)} - \frac{2}{(1+x)} \\
&\qquad + \frac{3}{(1+x)^2} - \frac{2}{(1+x)^3}\Big] H(0;x)\Big\} \\
&\quad +1 - \Big[\frac{1}{2(1+x)} - \frac{3}{2(1+x)^2} + \frac{1}{(1+x)^3}\Big] H(0;x) \\
&\quad -\frac{1}{2}\Big[\frac{1}{(1-x)} - \frac{2}{(1+x)} + \frac{3}{(1+x)^2} - \frac{2}{(1+x)^3}\Big][\zeta(2) - H(0;x) \\
&\qquad -H(0,0;x) + 2H(-1,0;x)] \\
&+ (D-4)\Big\{1 - \frac{3}{(1+x)} + \frac{3}{(1+x)^2}
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \Big[\frac{1}{4}\Big(\frac{1}{(1-x)} - \frac{2}{(1+x)} + \frac{3}{(1+x)^2} - \frac{2}{(1+x)^3}\Big)\zeta(2) \\
&\qquad - \frac{1}{2(1-x)} + \frac{3}{(1+x)} - \frac{9}{2(1+x)^2} \\
&\qquad + \frac{1}{(1+x)^3}\Big] H(0;x) \\
&\quad + \Big[\frac{1}{2(1-x)} - \frac{1}{(1+x)} + \frac{3}{2(1+x)^2} - \frac{1}{(1+x)^3}\Big][\zeta(3) \\
&\qquad - H(-1;x)] \\
&\quad + \frac{1}{4}\Big[\frac{1}{(1-x)} - \frac{3}{(1+x)} + \frac{6}{(1+x)^2} - \frac{4}{(1+x)^3}\Big][\zeta(2) \\
&\qquad - H(0,0;x) + 2H(-1,0;x)] \\
&\quad - \frac{1}{4}\Big[\frac{1}{(1-x)} - \frac{2}{(1+x)} + \frac{3}{(1+x)^2} - \frac{2}{(1+x)^3}\Big] \times \\
&\qquad \times [H(0,0,0;x) + 4H(-1,-1,0;x) \\
&\qquad - 2H(-1,0,0;x) - 2H(0,-1,0;x)]\Big\} \\
&+ \ \mathcal{O}\left((D-4)^2\right) , \qquad (75)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_2^{(\otimes,up)}(D,q^2) \ = \ & \frac{1}{(D-4)}\Big\{\frac{2}{(1+x)}\Big[1 - \frac{1}{(1+x)}\Big] + \frac{1}{2}\Big[\frac{1}{(1-x)} - \frac{3}{(1+x)} \\
& \qquad + \frac{6}{(1+x)^2} - \frac{4}{(1+x)^3}\Big] H(0;x)\Big\} \\
& - \frac{2}{(1+x)}\Big[1 - \frac{1}{(1+x)}\Big] \\
& + \frac{2}{(1+x)}\Big[1 - \frac{3}{(1+x)} + \frac{2}{(1+x)^2}\Big] H(0;x) \\
& + \frac{1}{4}\Big[\frac{1}{(1-x)} - \frac{3}{(1+x)} + \frac{6}{(1+x)^2} - \frac{4}{(1+x)^3}\Big][\zeta(2) \\
& \quad - H(0,0;x) + 2H(-1,0;x)] \\
+ \ & (D-4)\Big\{\frac{1}{(1+x)}\Big[1 - \frac{1}{(1+x)}\Big] \\
& \qquad + \frac{1}{2(1+x)}\Big[1 - \frac{3}{(1+x)} + \frac{2}{(1+x)}\Big][2\zeta(2) \\
& \qquad - 3H(0;x) - 2H(0,0;x) + 4H(-1,0;x)] \\
& \qquad - \frac{1}{8}\Big[\frac{1}{(1-x)} - \frac{3}{(1+x)} + \frac{6}{(1+x)^2} - \frac{4}{(1+x)^3}\Big][2\zeta(3) \\
& \qquad + \zeta(2)(H(0;x) - 2H(-1;x)) - H(0,0,0;x) \\
& \qquad - 4H(-1,-1,0;x) + 2H(-1,0,0;x) \\
& \qquad + 2H(0,-1,0;x)]\Big\} \\
+ \ & \mathcal{O}\left((D-4)^2\right) , \qquad (76)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_3^{(\otimes,up)}(D,q^2) &= \frac{1}{(D-4)}\Big\{-\frac{1}{(1-x)}\Big[1-\frac{1}{(1-x)}\Big]-\frac{1}{4}\Big[\frac{1}{(1+x)}-\frac{3}{(1-x)} \\
&\qquad +\frac{6}{(1-x)^2}-\frac{4}{(1-x)^3}\Big]H(0;x)\Big\} \\
&\quad -\frac{1}{(1-x)}\Big[1-\frac{1}{(1-x)}\Big] \\
&\quad +\frac{1}{4}\Big[\frac{1}{(1+x)}+\frac{1}{(1-x)}-\frac{6}{(1-x)^2}+\frac{4}{(1-x)^3}\Big]H(0;x) \\
&\quad -\frac{1}{8}\Big[\frac{1}{(1-x)}-\frac{3}{(1+x)}+\frac{6}{(1+x)^2}-\frac{4}{(1+x)^3}\Big][\zeta(2) \\
&\qquad -H(0,0;x)+2H(-1,0;x)] \\
&+ (D-4)\Big\{\frac{1}{2(1-x)}\Big[1-\frac{1}{(1-x)}\Big] \\
&\qquad -\frac{1}{4}\Big[\frac{1}{(1+x)}-\frac{3}{(1-x)^2}+\frac{2}{(1-x)^3}\Big]H(0;x) \\
&\qquad +\frac{1}{8}\Big[\frac{1}{(1+x)}+\frac{1}{(1-x)}-\frac{6}{(1-x)^2}+\frac{4}{(1-x)^3}\Big][\zeta(2) \\
&\qquad\quad -H(0,0;x)+2H(-1,0;x)] \\
&\qquad +\frac{1}{16}\Big[\frac{1}{(1+x)}-\frac{3}{(1-x)}+\frac{6}{(1-x)^2}-\frac{4}{(1-x)^3}\Big]\times \\
&\qquad\quad \times[2\zeta(3)+\zeta(2)(H(0;x)-2H(-1;x)) \\
&\qquad\qquad -H(0,0,0;x)-4H(-1,-1,0;x) \\
&\qquad\qquad +2H(-1,0,0;x)+2H(0,-1,0;x)]\Big\} \\
&+ \mathcal{O}\left((D-4)^2\right) . \qquad (77)
\end{aligned}
$$

The last subdiagram, occurring in the *r.h.s.* of Eq. (65) is:

m  $= m\int \mathfrak{D}^D k_1 \, \frac{\mathcal{V}^\mu}{\mathcal{D}_1\mathcal{D}_9\mathcal{D}_{10}^2} \,, \qquad (78)$

where

$$
\mathcal{V}^\mu = \bar{v}(p_2)\gamma_\sigma[i(\not{p}_2+\not{k}_1)+m]i[i(\not{p}_2+\not{k}_1)+m]\gamma^\mu[-i(\not{p}_1-\not{k}_1)+m]\gamma_\sigma u(p_1) \; ; \qquad (79)
$$

its form factors are:

$$
\begin{aligned}
\mathcal{F}_1^{(\otimes,down)}(D,q^2) &= \mathcal{F}_1^{(\otimes,up)}(D,q^2) && (80) \\
\mathcal{F}_2^{(\otimes,down)}(D,q^2) &= \mathcal{F}_2^{(\otimes,up)}(D,q^2) && (81) \\
\mathcal{F}_3^{(\otimes,down)}(D,q^2) &= -\mathcal{F}_3^{(\otimes,up)}(D,q^2) . && (82)
\end{aligned}
$$

We will write the contributions to the 2-loop form factors from the subtraction graphs with the 1-loop renormalization counter-terms of Fig. 2 as $\mathcal{F}_i^{(C,\mathtt{cnt})}(D,q^2)$

($i = 1, 2, 3$), where the C in the superscript stands for "counter-term" and the `cnt` (`cnt` $\in \{$`a`, ..., `e`$\}$) refers to the corresponding graphs of Fig. 2. According to Eqs. (62-66), we have:

$$
\begin{aligned}
\mathcal{F}_i^{(C,\mathtt{a})}(D,q^2) &= Z_1^{(1l)}(D) \times \mathcal{F}_i^{(1l)}(D,q^2)\,, && (83)\\
\mathcal{F}_i^{(C,\mathtt{b})}(D,q^2) &= \mathcal{F}_i^{(C,\mathtt{c})}(D,q^2) \;=\; Z_1^{(1l)}(D) \times \mathcal{F}_i^{(1l)}(D,q^2)\,, && (84)\\
\mathcal{F}_i^{(C,\mathtt{d})}(D,q^2) &= \frac{1}{m}\,\delta m^{(1l)}(D,m) \times \mathcal{F}_i^{(\otimes,up)}(D,q^2)\,, && (85)\\
\mathcal{F}_i^{(C,\mathtt{e})}(D,q^2) &= \frac{1}{m}\,\delta m^{(1l)}(D,m) \times \mathcal{F}_i^{(\otimes,down)}(D,q^2)\,, && (86)\\
\mathcal{F}_i^{(C,\mathtt{f})}(D,q^2) &= \mathcal{F}_i^{(C,\mathtt{g})}(D,q^2) \;=\; Z_2^{(1l)}(D) \times \mathcal{F}_i^{(1l)}(D,q^2)\,, && (87)\\
\mathcal{F}_i^{(C,\mathtt{h})}(D,q^2) &= Z_3^{(1l)}(D) \times \mathcal{F}_i^{(1l)}(D,q^2)\,, && (88)
\end{aligned}
$$

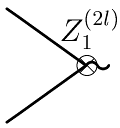
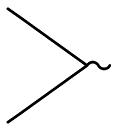
where $i = 1, 2, 3$.

As can be seen from Eqs. (72,77,82), the total contribution to the third form factor vanishes:

$$
\mathcal{F}_3^{(C)}(D,q^2) = \sum_{\mathtt{cnt}} \mathcal{F}_3^{(C,\mathtt{cnt})}(D,q^2) = 0 \,. \tag{89}
$$

4.2 2-loop charge renormalization constant.

The only subtraction from the 2-loop renormalization counter-terms, in our case, is given by the product of the charge renormalization constant at 2-loop times the tree-level vertex, corresponding to the diagram `(i)` of Fig. 2:

$Z_1^{(2l)}$

$\stackrel{def}{=} Z_1^{(2l)}(D) \times$

. (90)

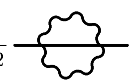
The 2-loop charge renormalization constant $Z_1^{(2l)}(D)$ is given by the value at $q^2 = 0$ of the unrenormalized charge form factor at two loops,

$$
\mathcal{F}_1^{(2l)}(D,q^2) = \sum_{\mathtt{graph}} \mathcal{F}_1^{(2l,\mathtt{graph})}(D,q^2) - \sum_{\mathtt{cnt}} \mathcal{F}_1^{(C,\mathtt{cnt})}(D,q^2)\,, \tag{91}
$$

where `graph` $\in \{$`a`, ..., `g`$\}$ runs over the diagrams of Fig. 1 and `cnt` $\in \{$`a`, ..., `h`$\}$ runs over the subtraction graphs of Fig. 2.

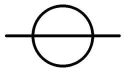
As the evaluation of the value at $q^2 = 0$ is somewhat simpler than the value for arbitrary q^2, we give here its expression exact in D:

$$
\begin{aligned}
Z_1^{(2l)}(D) &= \mathcal{F}_1^{(2l)}(D,q^2=0)\\
&= \frac{(D-6)}{4(D-5)(D-4)(D-3)}\Big(360 - 650D + 470D^2\\
&\qquad\qquad -167D^3 + 29D^4 - 2D^5\Big)\left(\frac{1}{m^2}\right.
\end{aligned}
$$



$\left.\right)$

$$+ \frac{(D+4)}{8(D-6)(D-4)^2}\Big(736 - 1348D + 800D^2 -207D^3 + 24D^4 - D^5\Big)\left(\frac{1}{m^2}\right.$$



$$\left.\right) + \frac{(D-2)}{48(D-7)(D-6)(D-5)^2(D-4)^2(D-3)^2} \times \Big(7131744 - 9801144D + 1271956D^2 + 5512286D^3 -4884843D^4 + 2058126D^5 - 514065D^6 + 79836D^7 -7567D^8 + 400D^9 - 9D^{10}\Big)\left(\frac{1}{m^4}\right.$$

$$\left.\right), \tag{92}$$

where the MIs depicted in the r.h.s. are those of Fig. 7 of [8].

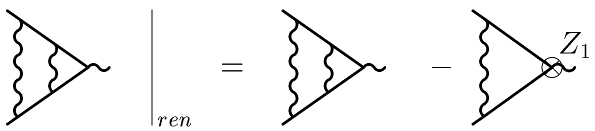
The corresponding expansion in $(D-4)$ is:

$$Z_1^{(2l)}(D) = -\frac{9}{8}\frac{1}{(D-4)^2} + \frac{55}{32}\frac{1}{(D-4)} - \frac{7685}{1152} + \frac{55}{8}\zeta(2) - 6\zeta(2)\ln 2 + \frac{3}{2}\zeta(3) + \mathcal{O}(D-4). \tag{93}$$

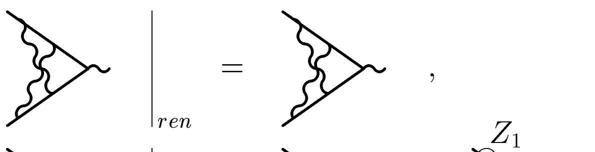
Let us recall once more that this counter-term is required for the renormalization of the *charge* form factor $\mathcal{F}_1^{(2l)}(D, q^2)$ only.

5 The renormalized form factors

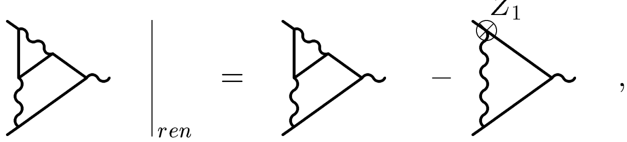
As a first step the renormalization procedure requires the proper subtraction of the 8 graphs with the counter-terms at one loop of Fig. 2 from the 7 unrenormalized 2-loop graphs of section 3. That is carried out according to the following scheme:



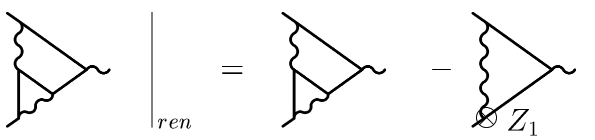
(94)



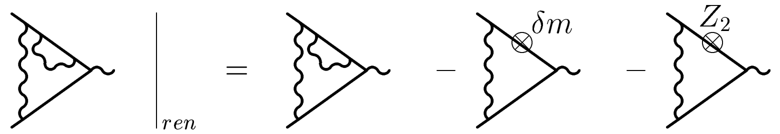
(95)



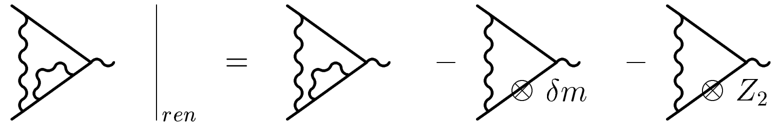
(96)



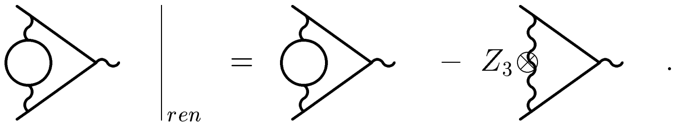
(97)



(98)



(99)



(100)

As already anticipated in the previous section, we will indicate by $\mathcal{F}_i^{(2l)}(D,q^2)$ the sum of the contributions to the form factors from all the above graphs:

$$\mathcal{F}_i^{(2l)}(D,q^2) = \sum_{\texttt{graph}} \mathcal{F}_i^{(2l,\texttt{graph})}(D,q^2) - \sum_{\texttt{cnt}} \mathcal{F}_i^{(C,\texttt{cnt})}(D,q^2)\,, \tag{101}$$

where $\texttt{graph} \in \{\texttt{a},...,\texttt{g}\}$ runs over the diagrams of Fig. 1 and $\texttt{cnt} \in \{\texttt{a},...,\texttt{h}\}$ runs over the counter-terms of Fig. 2. Note that, as $Z_2^{(1l)}(D) = -Z_1^{(1l)}(D)$ (the Ward identity), all the terms with $Z_2^{(1l)}(D)$ are canceled by corresponding terms with $Z_1^{(1l)}(D)$, so that only a single term proportional to $Z_1^{(1l)}(D)$ remains in the sum of the counter-terms.

For $i=3$ the renormalized form factor vanishes, as expected

$$F_3^{(2l)}(D,q^2) = 0\,. \tag{102}$$

To obtain the 2-loop fully renormalized form factors $F_i^{(2l)}(D,q^2)$ we have to subtract from the first form factor $\mathcal{F}_1^{(2l)}(D,q^2)$ its value at $q^2=0$ (2-loop charge renormalization); no renormalization is needed for the second form factor, and we have finally

$$\begin{aligned} F_1^{(2l)}(D,q^2) &= \mathcal{F}_1^{(2l)}(D,q^2) - \mathcal{F}_1^{(2l)}(D,0) \\ &= \mathcal{F}_1^{(2l)}(D,q^2) - Z_1^{(2l)}(D)\,, \qquad (103) \\ F_2^{(2l)}(D,q^2) &= \mathcal{F}_2^{(2l)}(D,q^2)\,. \qquad (104) \end{aligned}$$

We report now the analytic expression of the UV-renormalized form factors $F_i^{(2l)}(D,q^2)$ in the space-like region $-s = q^2 > 0$, in terms of HPLs of the variable x already introduced in Eq. (17),

$$x = \frac{\sqrt{q^2+4}-\sqrt{q^2}}{\sqrt{q^2+4}+\sqrt{q^2}}\,,$$

up to the finite term in the expansion in $(D-4)$:

$$F_1^{(2l)}(D,q^2) \;=\; \frac{1}{(D-4)^2}\Bigg\{\frac{1}{2} - \left[1 - \frac{1}{(1-x)} - \frac{1}{(1+x)}\right] H(0;x)$$

$$
\begin{aligned}
& +\left[1+\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{1}{(1+x)^2}\right.\\
& \left.\left.-\frac{1}{(1+x)}\right]H(0,0;x)\right\}\\
- \quad & \frac{1}{(D-4)}\left\{1+\frac{1}{2}\left[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\right][\zeta(2)+2H(-1,0;x)]\right.\\
& -\left[\frac{7}{4}-\frac{2}{(1-x)}-\frac{3}{2(1+x)}\right]H(0;x)\\
& +\left[1+\frac{2}{(1-x)^2}-\frac{3}{2(1-x)}+\frac{1}{(1+x)^2}\right.\\
& \left.-\frac{1}{2(1+x)}\right]H(0,0;x)\\
& -\frac{1}{2}\left[1+\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{1}{(1+x)^2}-\frac{1}{(1+x)}\right]\times\\
& \times[H(0;x)\zeta(2)+4H(-1,0,0;x)+H(0,-1,0;x)\\
& \left.-3H(0,0,0;x)]\right\}\\
& +\frac{1387}{216}-\frac{49}{9(1+x)}\left[1-\frac{1}{(1+x)}\right]+\left[\frac{51}{16}+\frac{1}{2(1-x)}-\frac{82}{3(1+x)^4}\right.\\
& \left.+\frac{200}{3(1+x)^3}-\frac{221}{6(1+x)^2}-\frac{33}{8(1+x)}\right]\zeta(2)-\left[3+\frac{18}{(1+x)^2}\right.\\
& \left.-\frac{18}{(1+x)}\right]\zeta(2)\ln 2-\left[\frac{7}{4}+\frac{1}{(1-x)^2}-\frac{1}{2(1-x)}-\frac{42}{(1+x)^4}\right.\\
& \left.+\frac{84}{(1+x)^3}-\frac{95}{2(1+x)^2}+\frac{6}{(1+x)}\right]\zeta(3)+\left[\frac{181}{40}+\frac{61}{40(1-x)^2}\right.\\
& -\frac{1219}{320(1-x)}-\frac{171}{4(1+x)^5}+\frac{855}{8(1+x)^4}-\frac{6867}{80(1+x)^3}+\frac{749}{32(1+x)^2}\\
& \left.-\frac{1731}{320(1+x)}\right]\zeta^2(2)\\
& -\left[\frac{3355}{864}-\frac{985}{216(1-x)}-\frac{89}{9(1+x)^3}+\frac{89}{9(1+x)^2}-\frac{3521}{432(1+x)}\right.\\
& \quad+\left(\frac{53}{12}+\frac{2}{(1-x)^2}-\frac{19}{24(1-x)}+\frac{21}{(1+x)^5}-\frac{54}{(1+x)^4}\right.\\
& \quad\left.+\frac{45}{(1+x)^3}-\frac{3}{(1+x)^2}-\frac{349}{24(1+x)}\right)\zeta(2)-\left(2-\frac{7}{4(1-x)}\right.\\
& \quad+\frac{42}{(1+x)^5}-\frac{105}{(1+x)^4}+\frac{91}{(1+x)^3}-\frac{63}{2(1+x)^2}\\
& \quad\left.\left.+\frac{5}{4(1+x)}\right)\zeta(3)\right]H(0;x)
\end{aligned}
$$

$$+\left[\frac{5}{2}+\frac{1}{2(1-x)}+\frac{45}{(1+x)^4}-\frac{90}{(1+x)^3}+\frac{135}{2(1+x)^2}\right.$$
$$\left.-\frac{22}{(1+x)}\right]\zeta(2)H(-1;x)$$
$$+\left[\frac{2137}{144}+\frac{4}{(1-x)^2}-\frac{21}{2(1-x)}+\frac{494}{9(1+x)^4}-\frac{1258}{9(1+x)^3}-\frac{1258}{9(1+x)^3}\right.$$
$$+\frac{1130}{9(1+x)^2}-\frac{1081}{24(1+x)}+\left(\frac{13}{4}+\frac{5}{4(1-x)^2}-\frac{105}{32(1-x)}\right.$$
$$-\frac{33}{2(1+x)^5}+\frac{165}{4(1+x)^4}-\frac{281}{8(1+x)^3}+\frac{203}{16(1+x)^2}$$
$$\left.\left.-\frac{137}{32(1+x)}\right)\zeta(2)\right]H(0,0;x)$$
$$-\left[\frac{55}{8}-\frac{9}{(1-x)}-\frac{48}{(1+x)^3}+\frac{72}{(1+x)^2}-\frac{115}{4(1+x)}\right]H(-1,0;x)$$
$$-\left[\frac{5}{2}-\frac{1}{2(1-x)^2}-\frac{19}{16(1-x)}-\frac{45}{(1+x)^5}+\frac{225}{2(1+x)^4}-\frac{363}{4(1+x)^3}\right.$$
$$\left.+\frac{185}{8(1+x)^2}-\frac{67}{16(1+x)}\right]\zeta(2)H(0,-1;x)$$
$$+\left[4-\frac{4}{(1-x)}-\frac{12}{(1+x)^3}+\frac{18}{(1+x)^2}-\frac{10}{(1+x)}+\left(1+\frac{1}{(1-x)^2}\right.\right.$$
$$-\frac{3}{2(1-x)}-\frac{36}{(1+x)^5}+\frac{90}{(1+x)^4}-\frac{78}{(1+x)^3}+\frac{28}{(1+x)^2}$$
$$\left.\left.-\frac{7}{2(1+x)}\right)\zeta(2)\right]H(1,0;x)$$
$$-\left[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\right]H(-1,-1,0;x)$$
$$-\left[\frac{3}{2}+\frac{4}{(1-x)^2}-\frac{7}{2(1-x)}+\frac{63}{(1+x)^4}-\frac{126}{(1+x)^3}+\frac{135}{2(1+x)^2}\right.$$
$$\left.-\frac{4}{(1+x)}\right]H(-1,0,0;x)$$
$$+\left[4-\frac{1}{2(1-x)}+\frac{96}{(1+x)^4}-\frac{192}{(1+x)^3}+\frac{221}{2(1+x)^2}\right.$$
$$\left.-\frac{15}{(1+x)}\right]H(0,-1,0;x)$$
$$-\left[\frac{8}{3}-\frac{1}{(1-x)^2}+\frac{41}{24(1-x)}+\frac{21}{(1+x)^5}-\frac{69}{(1+x)^4}+\frac{75}{(1+x)^3}\right.$$
$$\left.-\frac{95}{4(1+x)^2}-\frac{223}{24(1+x)}\right]H(0,0,0;x)$$
$$-\left[\frac{5}{2}+\frac{2}{(1-x)^2}-\frac{2}{(1-x)}+\frac{24}{(1+x)^4}-\frac{49}{(1+x)^3}+\frac{29}{(1+x)^2}\right.$$

$$
\begin{aligned}
& -\frac{5}{(1+x)}\Bigg]H(0,1,0;x) \\
& +\Bigg[4+\frac{90}{(1+x)^4}-\frac{180}{(1+x)^3}+\frac{197}{2(1+x)^2}-\frac{17}{2(1+x)}\Bigg]H(1,0,0;x) \\
& -\Bigg[\frac{7}{2}+\frac{5}{2(1-x)^2}-\frac{67}{16(1-x)}+\frac{63}{(1+x)^5}-\frac{315}{2(1+x)^4}+\frac{517}{4(1+x)^3} \\
& \quad -\frac{271}{8(1+x)^2}-\frac{19}{16(1+x)}\Bigg]H(0,-1,0,0;x) \\
& -\Bigg[\frac{11}{2}+\frac{5}{2(1-x)^2}-\frac{21}{8(1-x)}-\frac{96}{(1+x)^5}+\frac{240}{(1+x)^4}-\frac{373}{2(1+x)^3} \\
& \quad +\frac{169}{4(1+x)^2}-\frac{45}{8(1+x)}\Bigg]H(0,0,-1,0;x) \\
& +\Bigg[\frac{27}{4}+\frac{17}{4(1-x)^2}-\frac{217}{32(1-x)}-\frac{3}{2(1+x)^5}+\frac{15}{4(1+x)^4}-\frac{17}{8(1+x)^3} \\
& \quad +\frac{59}{16(1+x)^2}-\frac{201}{32(1+x)}\Bigg]H(0,0,0,0;x) \\
& +\Bigg[3+\frac{1}{(1-x)^2}-\frac{2}{(1-x)}-\frac{24}{(1+x)^5}+\frac{60}{(1+x)^4}-\frac{44}{(1+x)^3} \\
& \quad +\frac{7}{(1+x)^2}-\frac{2}{(1+x)}\Bigg]H(0,0,1,0;x) \\
& -\Bigg[1+\frac{7}{4(1-x)}-\frac{90}{(1+x)^5}+\frac{225}{(1+x)^4}-\frac{175}{(1+x)^3}+\frac{75}{2(1+x)^2} \\
& \quad -\frac{5}{4(1+x)}\Bigg]H(0,1,0,0;x) \\
& +\Bigg[2+\frac{2}{(1-x)^2}-\frac{5}{2(1-x)}-\frac{36}{(1+x)^5}+\frac{90}{(1+x)^4}-\frac{78}{(1+x)^3} \\
& \quad +\frac{29}{(1+x)^2}-\frac{9}{2(1+x)}\Bigg]H(1,0,0,0;x) \\
& +\Bigg[1+\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{1}{(1+x)^2}-\frac{1}{(1+x)}\Bigg][\zeta(3)H(1;x) \\
& \quad +\zeta(2)H(-1,0;x)+4H(-1,-1,0,0;x)+2H(-1,0,-1,0;x) \\
& \quad -3H(-1,0,0,0;x)+H(0,-1,-1,0;x)-2H(1,0,-1,0;x) \\
& \quad +2H(1,0,1,0;x)] \\
+ \quad & \mathcal{O}(D-4)\,, \qquad (105)
\end{aligned}
$$

$$
\begin{aligned}
F_2^{(2l)}(D,q^2) \quad = \quad & \frac{1}{(D-4)}\Bigg\{\frac{1}{2}\left[\frac{1}{(1+x)}-\frac{1}{(1-x)}\right]H(0;x)+\Bigg[\frac{1}{(1+x)^2} \\
& \quad -\frac{1}{(1+x)}-\frac{1}{(1-x)^2}+\frac{1}{(1-x)}\Bigg]H(0,0;x)\Bigg\} \\
& -\Bigg[\frac{17}{8(1-x)}-\frac{26}{(1+x)^4}+\frac{64}{(1+x)^3}-\frac{43}{(1+x)^2}+\frac{23}{8(1+x)}\Bigg]\zeta(2)
\end{aligned}
$$

$$
\begin{aligned}
&+\left[\frac{12}{(1+x)^2}-\frac{12}{(1+x)}\right]\zeta(2)\ln 2-\left[\frac{69}{80(1-x)^3}-\frac{207}{160(1-x)^2}\right.\\
&+\frac{327}{320(1-x)}-\frac{171}{4(1+x)^5}+\frac{855}{8(1+x)^4}-\frac{3291}{40(1+x)^3}\\
&\left.+\frac{1323}{80(1+x)^2}+\frac{327}{320(1+x)}\right]\zeta^2(2)-\left[\frac{5}{2(1-x)^2}-\frac{5}{2(1-x)}\right.\\
&\left.+\frac{42}{(1+x)^4}-\frac{84}{(1+x)^3}+\frac{91}{2(1+x)^2}-\frac{7}{2(1+x)}\right]\zeta(3)\\
&+\frac{17}{3(1+x)}\left[1-\frac{1}{(1+x)}\right]\\
&-\left[\left(\frac{19}{8(1-x)^2}-\frac{75}{16(1-x)}-\frac{21}{(1+x)^5}+\frac{54}{(1+x)^4}-\frac{173}{4(1+x)^3}\right.\right.\\
&\quad\left.+\frac{13}{4(1+x)^2}+\frac{149}{16(1+x)}\right)\zeta(2)+\left(\frac{7}{8(1-x)}+\frac{42}{(1+x)^5}-\frac{105}{(1+x)^4}\right.\\
&\quad\left.+\frac{175}{2(1+x)^3}-\frac{105}{4(1+x)^2}+\frac{7}{8(1+x)}\right)\zeta(3)-\frac{7}{144(1-x)}\\
&\quad\left.+\frac{31}{3(1+x)^3}-\frac{31}{2(1+x)^2}+\frac{751}{144(1+x)}\right]H(0;x)\\
&+\left[\frac{9}{4(1-x)^2}-\frac{9}{4(1-x)}-\frac{45}{(1+x)^4}+\frac{90}{(1+x)^3}-\frac{231}{4(1+x)^2}\right.\\
&\quad\left.+\frac{51}{4(1+x)}\right]\zeta(2)H(-1;x)\\
&-\left[\frac{9}{4(1-x)}+\frac{48}{(1+x)^3}-\frac{72}{(1+x)^2}+\frac{87}{4(1+x)}\right]H(-1,0;x)\\
&+\left[\frac{9}{4(1-x)^3}-\frac{27}{8(1-x)^2}+\frac{21}{16(1-x)}-\frac{45}{(1+x)^5}+\frac{225}{2(1+x)^4}\right.\\
&\quad\left.-\frac{87}{(1+x)^3}+\frac{18}{(1+x)^2}+\frac{21}{16(1+x)}\right]\zeta(2)H(0,-1;x)\\
&-\left[\left(\frac{7}{8(1-x)^3}-\frac{21}{16(1-x)^2}+\frac{1}{32(1-x)}-\frac{33}{2(1+x)^5}+\frac{165}{4(1+x)^4}\right.\right.\\
&\quad\left.-\frac{135}{4(1+x)^3}+\frac{75}{8(1+x)^2}+\frac{1}{32(1+x)}\right)\zeta(2)-\frac{1}{2(1-x)^2}\\
&\quad-\frac{1}{8(1-x)}+\frac{166}{3(1+x)^4}-\frac{422}{3(1+x)^3}+\frac{709}{6(1+x)^2}\\
&\quad\left.-\frac{773}{24(1+x)}\right]H(0,0;x)\\
&+\left[\left(\frac{3}{4(1-x)}+\frac{36}{(1+x)^5}-\frac{90}{(1+x)^4}+\frac{75}{(1+x)^3}-\frac{45}{2(1+x)^2}\right.\right.\\
&\quad\left.+\frac{3}{4(1+x)}\right)\zeta(2)-\frac{1}{(1-x)}+\frac{12}{(1+x)^3}-\frac{18}{(1+x)^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{7}{(1+x)}\Bigg]H(1,0;x)\\
&+\Bigg[\frac{1}{4(1-x)^2}-\frac{1}{4(1-x)}+\frac{63}{(1+x)^4}-\frac{126}{(1+x)^3}+\frac{257}{4(1+x)^2}\\
&-\frac{5}{4(1+x)}\Bigg]H(-1,0,0;x)\\
&-\Bigg[\frac{1}{2(1-x)^2}-\frac{1}{2(1-x)}+\frac{96}{(1+x)^4}-\frac{192}{(1+x)^3}+\frac{197}{2(1+x)^2}\\
&-\frac{5}{2(1+x)}\Bigg]H(0,-1,0;x)\\
&-\Bigg[\frac{7}{8(1-x)^2}-\frac{51}{16(1-x)}-\frac{21}{(1+x)^5}+\frac{69}{(1+x)^4}-\frac{293}{4(1+x)^3}\\
&+\frac{91}{4(1+x)^2}+\frac{77}{16(1+x)}\Bigg]H(0,0,0;x)\\
&-\Bigg[\frac{1}{(1-x)^2}-\frac{1}{(1-x)}-\frac{24}{(1+x)^4}+\frac{48}{(1+x)^3}-\frac{25}{(1+x)^2}\\
&+\frac{1}{(1+x)}\Bigg]H(0,1,0;x)\\
&-\Bigg[\frac{1}{(1-x)^2}-\frac{1}{(1-x)}+\frac{90}{(1+x)^4}-\frac{180}{(1+x)^3}+\frac{89}{(1+x)^2}\\
&+\frac{1}{(1+x)}\Bigg]H(1,0,0;x)\\
&+\Bigg[\frac{1}{4(1-x)^3}-\frac{3}{8(1-x)^2}-\frac{7}{16(1-x)}+\frac{63}{(1+x)^5}-\frac{315}{2(1+x)^4}\\
&+\frac{124}{(1+x)^3}-\frac{57}{2(1+x)^2}-\frac{7}{16(1+x)}\Bigg]H(0,-1,0,0;x)\\
&+\Bigg[\frac{1}{2(1-x)^3}-\frac{3}{4(1-x)^2}+\frac{7}{2(1-x)}-\frac{96}{(1+x)^5}+\frac{240}{(1+x)^4}\\
&-\frac{357}{2(1+x)^3}+\frac{111}{4(1+x)^2}+\frac{7}{2(1+x)}\Bigg]H(0,0,-1,0;x)\\
&-\Bigg[\frac{3}{8(1-x)^3}-\frac{9}{16(1-x)^2}+\frac{11}{32(1-x)}-\frac{3}{2(1+x)^5}+\frac{15}{4(1+x)^4}\\
&-\frac{2}{(1+x)^3}-\frac{3}{4(1+x)^2}+\frac{11}{32(1+x)}\Bigg]H(0,0,0,0;x)\\
&-\Bigg[\frac{3}{2(1-x)}-\frac{24}{(1+x)^5}+\frac{60}{(1+x)^4}-\frac{42}{(1+x)^3}+\frac{3}{(1+x)^2}\\
&+\frac{3}{2(1+x)}\Bigg]H(0,0,1,0;x)\\
&+\Bigg[\frac{25}{8(1-x)}-\frac{90}{(1+x)^5}+\frac{225}{(1+x)^4}-\frac{335}{2(1+x)^3}+\frac{105}{4(1+x)^2}
\end{aligned}
$$

$$
\begin{aligned}
& \quad + \frac{25}{8(1+x)} \Bigg] H(0,1,0,0;x) \\
& + \left[\frac{3}{4(1-x)} + \frac{36}{(1+x)^5} - \frac{90}{(1+x)^4} + \frac{75}{(1+x)^3} - \frac{45}{2(1+x)^2} \right. \\
& \quad \left. + \frac{3}{4(1+x)} \right] H(1,0,0,0;x) \\
+ \quad & \mathcal{O}(D-4) \, . \qquad (106)
\end{aligned}
$$

Even after the full renormalization has been carried out, the on-shell renormalized form factors still develop polar singularities in $(D-4)$, due to soft IR divergences. These divergences are not physical and are removed in any physical process by the corresponding divergences due to soft real emission.

As it is easy to check explicitly, the IR divergences are the same as in [1, 4], provided the following formal replacement is done

$$
\log\left(\frac{\lambda}{m}\right) = -\frac{1}{(D-4)} \, , \qquad (107)
$$

so that they follow the general structure already pointed out in [15]. The finite parts are however different in the D-dimensional and in the λ-mass regularization schemes.

5.1 Continuation to time-like momentum transfer and imaginary parts

The expressions of the UV-renormalized form factors given in the previous section, Eqs. (105,106), can be analytically continued to the time-like region, $S = -Q^2 > 0$, and in particular above the physical threshold $S > 4m^2$, where the form factors develop an imaginary part.

The analytic continuation to $S + i\epsilon$ for $S > 4m^2$ is performed with the substitution

$$
x = -y + i\epsilon \, , \qquad (108)
$$

where now:

$$
y = \frac{\sqrt{s} - \sqrt{s-4}}{\sqrt{s} + \sqrt{s-4}} = \frac{\sqrt{S} - \sqrt{S-4m^2}}{\sqrt{S} + \sqrt{S-4m^2}} \, , \qquad (109)
$$

with $S = m^2 s$ according to Eq. (2).

The imaginary part of the form factors is originated by the imaginary parts developed by the polylogarithms with rightmost index equal to 0 (for details see [4, 5, 6]).

In the kinematical region above threshold the form factors can be written as

$$
\begin{aligned}
F_1^{(2l)}(D,-s-i\epsilon) &= \Re F_1^{(2l)}(D,-s) + i\pi \, \Im F_1^{(2l)}(D,-s) \, , \qquad (110) \\
F_2^{(2l)}(D,-s-i\epsilon) &= \Re F_2^{(2l)}(D,-s) + i\pi \, \Im F_2^{(2l)}(D,-s) \, ; \qquad (111)
\end{aligned}
$$

for short, we give the explicit expressions of the imaginary parts only:

$$
\begin{aligned}
\Im F_1^{(2l)}(D,-s) &= \frac{1}{(D-4)^2}\Bigg\{ -1+\frac{1}{(1-y)}+\frac{1}{(1+y)}+\Bigg[1+\frac{1}{(1-y)^2} \\
&\qquad -\frac{1}{(1-y)}+\frac{1}{(1+y)^2}-\frac{1}{(1+y)}\Bigg]H(0;y)\Bigg\} \\
&+ \frac{1}{(D-4)}\Bigg\{\frac{7}{4}-\frac{3}{2(1-y)}-\frac{2}{(1+y)}-\Bigg[1+\frac{1}{(1-y)^2} \\
&\qquad -\frac{1}{2(1-y)}+\frac{2}{(1+y)^2}-\frac{3}{2(1+y)}\Bigg]H(0;y) \\
&\qquad +\Bigg[1-\frac{1}{(1-y)}-\frac{1}{(1+y)}\Bigg]H(1;y) \\
&\qquad +\frac{1}{2}\Bigg[1+\frac{1}{(1-y)^2}-\frac{1}{(1-y)}+\frac{1}{(1+y)^2}-\frac{1}{(1+y)}\Bigg]\times \\
&\qquad\quad \times[4\zeta(2)-3H(0,0;y)-2H(0,1;y) \\
&\qquad\quad -4H(1,0;y)]\Bigg\} \\
&-\frac{3355}{864}+\frac{89}{9(1-y)^3}-\frac{89}{6(1-y)^2}+\frac{3521}{432(1-y)}+\frac{985}{216(1+y)} \\
&-\Bigg(\frac{7}{4}+\frac{15}{(1-y)^4}-\frac{30}{(1-y)^3}+\frac{83}{4(1-y)^2}-\frac{21}{4(1-y)}+\frac{3}{(1+y)^2} \\
&-\frac{5}{2(1+y)}\Bigg)\zeta(2)+\Bigg(2+\frac{42}{(1-y)^5}-\frac{105}{(1-y)^4}+\frac{91}{(1-y)^3} \\
&-\frac{63}{2(1-y)^2}+\frac{5}{4(1-y)}-\frac{7}{4(1+y)}\Bigg)\zeta(3) \\
&+\Bigg[\frac{2137}{144}+\frac{494}{9(1-y)^4}-\frac{1258}{9(1-y)^3}+\frac{1130}{9(1-y)^2}-\frac{1081}{24(1-y)} \\
&\quad +\frac{4}{(1+y)^2}-\frac{21}{2(1+y)}-\Bigg(\frac{7}{2}+\frac{15}{(1-y)^5}-\frac{75}{2(1-y)^4} \\
&\quad +\frac{33}{(1-y)^3}-\frac{9}{(1-y)^2}-\frac{2}{(1-y)}+\frac{3}{(1+y)^2} \\
&\quad -\frac{7}{2(1+y)}\Bigg)\zeta(2)\Bigg]H(0;y) \\
&-\Bigg[4-\frac{12}{(1-y)^3}+\frac{18}{(1-y)^2}-\frac{10}{(1-y)}-\frac{4}{(1+y)}\Bigg]H(-1;y) \\
&+\Bigg[\frac{55}{8}-\frac{48}{(1-y)^3}+\frac{72}{(1-y)^2}-\frac{115}{4(1-y)}-\frac{9}{(1+y)}\Bigg]H(1;y) \\
&-\Bigg[4+\frac{90}{(1-y)^4}-\frac{180}{(1-y)^3}+\frac{197}{2(1-y)^2}-\frac{17}{2(1-y)}\Bigg]H(-1,0;y) \\
&+\Bigg[\frac{5}{2}+\frac{24}{(1-y)^4}-\frac{48}{(1-y)^3}+\frac{29}{(1-y)^2}-\frac{5}{(1-y)}+\frac{2}{(1+y)^2}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{2}{(1+y)}\Bigg]H(0,-1;y)\\
&-\Bigg[\frac{8}{3}+\frac{21}{(1-y)^5}-\frac{69}{(1-y)^4}+\frac{75}{(1-y)^3}-\frac{95}{4(1-y)^2}-\frac{223}{24(1-y)}\\
&\quad-\frac{1}{(1+y)^2}+\frac{41}{24(1+y)}\Bigg]H(0,0;y)\\
&-\Bigg[4+\frac{96}{(1-y)^4}-\frac{192}{(1-y)^3}+\frac{221}{2(1-y)^2}-\frac{15}{(1-y)}\\
&\quad-\frac{1}{2(1+y)}\Bigg]H(0,1;y)\\
&+\Bigg[\frac{3}{2}+\frac{63}{(1-y)^4}-\frac{126}{(1-y)^3}+\frac{135}{2(1-y)^2}-\frac{4}{(1-y)}+\frac{4}{(1+y)^2}\\
&\quad-\frac{7}{2(1+y)}\Bigg]H(1,0;y)\\
&-\Bigg[1-\frac{1}{(1-y)}-\frac{1}{(1+y)}\Bigg]H(1,1;y)\\
&-\Bigg[2-\frac{36}{(1-y)^5}+\frac{90}{(1-y)^4}-\frac{78}{(1-y)^3}+\frac{29}{(1-y)^2}-\frac{9}{2(1-y)}\\
&\quad+\frac{2}{(1+y)^2}-\frac{5}{2(1+y)}\Bigg]H(-1,0,0;y)\\
&+\Bigg[1-\frac{90}{(1-y)^5}+\frac{225}{(1-y)^4}-\frac{175}{(1-y)^3}+\frac{75}{2(1-y)^2}\\
&\quad-\frac{5}{4(1-y)}+\frac{7}{4(1+y)}\Bigg]H(0,-1,0;y)\\
&-\Bigg[3-\frac{24}{(1-y)^5}+\frac{60}{(1-y)^4}-\frac{44}{(1-y)^3}+\frac{7}{(1-y)^2}-\frac{2}{(1-y)}\\
&\quad+\frac{1}{(1+y)^2}-\frac{2}{(1+y)}\Bigg]H(0,0,-1;y)\\
&+\Bigg[\frac{27}{4}-\frac{3}{2(1-y)^5}+\frac{15}{4(1-y)^4}-\frac{17}{8(1-y)^3}+\frac{59}{16(1-y)^2}\\
&\quad-\frac{201}{32(1-y)}+\frac{17}{4(1+y)^2}-\frac{217}{32(1+y)}\Bigg]H(0,0,0;y)\\
&+\Bigg[\frac{11}{2}-\frac{96}{(1-y)^5}+\frac{240}{(1-y)^4}-\frac{373}{2(1-y)^3}+\frac{169}{4(1-y)^2}\\
&\quad-\frac{45}{8(1-y)}+\frac{5}{2(1+y)^2}-\frac{21}{8(1+y)}\Bigg]H(0,0,1;y)\\
&+\Bigg[\frac{7}{2}+\frac{63}{(1-y)^5}-\frac{315}{2(1-y)^4}+\frac{517}{4(1-y)^3}-\frac{271}{8(1-y)^2}\\
&\quad-\frac{19}{16(1-y)}+\frac{5}{2(1+y)^2}-\frac{67}{16(1+y)}\Bigg]H(0,1,0;y)
\end{aligned}
$$

$$
\begin{aligned}
& & +\left[1+\frac{1}{(1-y)^2}-\frac{1}{(1-y)}+\frac{1}{(1+y)^2}-\frac{1}{(1+y)}\right]\times \\
& & \times[\zeta(2)H(-1;y)-4\zeta(2)H(1;y)+2H(-1,0,-1;y) \\
& & -2H(-1,0,1;y)+H(0,1,1;y)+3H(1,0,0;y) \\
& & +2H(1,0,1;y)+4H(1,1,0;y)] \\
& + & \mathcal{O}(D-4)\,, \qquad (112) \\
\Im F_2^{(2l)}(D,-s) & = & \frac{1}{(D-4)}\Bigg\{\frac{1}{2(1-y)}-\frac{1}{2(1+y)}+\Bigg[\frac{1}{(1-y)^2}-\frac{1}{(1-y)} \\
& & -\frac{1}{(1+y)^2}+\frac{1}{(1+y)}\Bigg]H(0;y)\Bigg\} \\
& & -\frac{31}{3(1-y)^3}+\frac{31}{2(1-y)^2}-\frac{751}{144(1-y)}+\frac{7}{144(1+y)} \\
& & +\Bigg[\frac{15}{(1-y)^4}-\frac{30}{(1-y)^3}+\frac{39}{2(1-y)^2}-\frac{9}{2(1-y)}-\frac{3}{2(1+y)^2} \\
& & +\frac{3}{2(1+y)}\Bigg]\zeta(2)-\Bigg[\frac{42}{(1-y)^5}-\frac{105}{(1-y)^4}+\frac{175}{2(1-y)^3} \\
& & -\frac{105}{4(1-y)^2}+\frac{7}{8(1-y)}+\frac{7}{8(1+y)}\Bigg]\zeta(3) \\
& & -\Bigg[\frac{12}{(1-y)^3}-\frac{18}{(1-y)^2}+\frac{7}{(1-y)}-\frac{1}{(1+y)}\Bigg]H(-1;y) \\
& & -\Bigg[\frac{166}{3(1-y)^4}-\frac{422}{3(1-y)^3}+\frac{709}{6(1-y)^2}-\frac{773}{24(1-y)}-\frac{1}{2(1+y)^2} \\
& & -\frac{1}{8(1+y)}-\Bigg(\frac{15}{(1-y)^5}-\frac{75}{2(1-y)^4}+\frac{127}{4(1-y)^3} \\
& & -\frac{81}{8(1-y)^2}+\frac{5}{16(1-y)}-\frac{1}{2(1+y)^3}+\frac{3}{4(1+y)^2} \\
& & +\frac{5}{16(1+y)}\Bigg)\zeta(2)\Bigg]H(0;y) \\
& & +\Bigg[\frac{48}{(1-y)^3}-\frac{72}{(1-y)^2}+\frac{87}{4(1-y)}+\frac{9}{4(1+y)}\Bigg]H(1;y) \\
& & +\Bigg[\frac{90}{(1-y)^4}-\frac{180}{(1-y)^3}+\frac{89}{(1-y)^2}+\frac{1}{(1-y)}+\frac{1}{(1+y)^2} \\
& & -\frac{1}{(1+y)}\Bigg]H(-1,0;y) \\
& & -\Bigg[\frac{24}{(1-y)^4}-\frac{48}{(1-y)^3}+\frac{25}{(1-y)^2}-\frac{1}{(1-y)}-\frac{1}{(1+y)^2} \\
& & +\frac{1}{(1+y)}\Bigg]H(0,-1;y) \\
& & +\Bigg[\frac{21}{(1-y)^5}-\frac{69}{(1-y)^4}+\frac{293}{4(1-y)^3}-\frac{91}{4(1-y)^2}-\frac{77}{16(1-y)}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{7}{8(1+y)^2}+\frac{51}{16(1+y)}\Bigg]H(0,0;y) \\
& +\Bigg[\frac{96}{(1-y)^4}-\frac{192}{(1-y)^3}+\frac{197}{2(1-y)^2}-\frac{5}{2(1-y)}+\frac{1}{2(1+y)^2} \\
& -\frac{1}{2(1+y)}\Bigg]H(0,1;y) \\
& -\Bigg[\frac{63}{(1-y)^4}-\frac{126}{(1-y)^3}+\frac{257}{4(1-y)^2}-\frac{5}{4(1-y)}+\frac{1}{4(1+y)^2} \\
& -\frac{1}{4(1+y)}\Bigg]H(1,0;y) \\
& -\Bigg[\frac{36}{(1-y)^5}-\frac{90}{(1-y)^4}+\frac{75}{(1-y)^3}-\frac{45}{2(1-y)^2}+\frac{3}{4(1-y)} \\
& +\frac{3}{4(1+y)}\Bigg]H(-1,0,0;y) \\
& +\Bigg[\frac{90}{(1-y)^5}-\frac{225}{(1-y)^4}+\frac{335}{2(1-y)^3}-\frac{105}{4(1-y)^2}-\frac{25}{8(1-y)} \\
& -\frac{25}{8(1+y)}\Bigg]H(0,-1,0;y) \\
& -\Bigg[\frac{24}{(1-y)^5}-\frac{60}{(1-y)^4}+\frac{42}{(1-y)^3}-\frac{3}{(1-y)^2}-\frac{3}{2(1-y)} \\
& -\frac{3}{2(1+y)}\Bigg]H(0,0,-1;y) \\
& +\Bigg[\frac{3}{2(1-y)^5}-\frac{15}{4(1-y)^4}+\frac{2}{(1-y)^3}+\frac{3}{4(1-y)^2}-\frac{11}{32(1-y)} \\
& -\frac{3}{8(1+y)^3}+\frac{9}{16(1+y)^2}-\frac{11}{32(1+y)}\Bigg]H(0,0,0;y) \\
& +\Bigg[\frac{96}{(1-y)^5}-\frac{240}{(1-y)^4}+\frac{357}{2(1-y)^3}-\frac{111}{4(1-y)^2}-\frac{7}{2(1-y)} \\
& -\frac{1}{2(1+y)^3}+\frac{3}{4(1+y)^2}-\frac{7}{2(1+y)}\Bigg]H(0,0,1;y) \\
& -\Bigg[\frac{63}{(1-y)^5}-\frac{315}{2(1-y)^4}+\frac{124}{(1-y)^3}-\frac{57}{2(1-y)^2}-\frac{7}{16(1-y)} \\
& +\frac{1}{4(1+y)^3}-\frac{3}{8(1+y)^2}-\frac{7}{16(1+y)}\Bigg]H(0,1,0;y) \\
+ \quad & \mathcal{O}(D-4)\,. \qquad (113)
\end{aligned}
$$

6 Expansion for $Q^2 \gg m^2$

We present in this section the asymptotic limit $Q^2 \gg m^2$ ($x \to 0$) of the space-like UV-renormalized form factors, given in section 5, putting for brevity $L =$

$\log(Q^2/m^2) = \log(q^2)$. The analytic continuation for large time-like $S = -Q^2$ can be carried out with the replacement $Q^2 = -(S + i\epsilon)$, generating an imaginary parts in the logarithm L (which becomes $L = \log(-s - i\epsilon) = \log(s) - i\pi$,).

$$
\begin{aligned}
F_1^{(2l)}(D, q^2) \;=\; & \frac{1}{(D-4)^2}\Bigg\{\frac{1}{2} - L + \frac{1}{2}L^2 + \frac{1}{q^2}\Bigg(-2 + 2L\Bigg) \\
& \quad + \frac{1}{q^4}\Bigg(5 - 5L + 2L^2\Bigg) \\
& \quad + \frac{1}{q^6}\Bigg(-\frac{50}{3} + \frac{68}{3}L - 8L^2\Bigg)\Bigg\} \\
& + \frac{1}{(D-4)}\Bigg\{-1 + \frac{1}{2}\zeta(2) + \frac{7}{4}L - \frac{1}{2}\zeta(2)L - L^2 + \frac{1}{4}L^3 \\
& \quad + \frac{1}{q^2}\Bigg(\frac{5}{2} - \zeta(2) - \frac{7}{2}L + 2L^2\Bigg) \\
& \quad + \frac{1}{q^4}\Bigg(-6 + \frac{5}{2}\zeta(2) + \frac{55}{4}L - 2\zeta(2)L \\
& \quad - \frac{29}{4}L^2 + L^3\Bigg) + \frac{1}{q^6}\Bigg(\frac{212}{9} - \frac{34}{3}\zeta(2) \\
& \quad - \frac{500}{9}L + 8\zeta(2)L + \frac{97}{3}L^2 - 4L^3\Bigg)\Bigg\} \\
& + \frac{1387}{216} + \frac{33}{16}\zeta(2) - 3\zeta(2)\ln 2 - \frac{11}{4}\zeta(3) - \frac{59}{40}\zeta^2(2) \\
& - \frac{3355}{864}L + \frac{1}{12}\zeta(2)L + 2\zeta(3)L + \frac{571}{288}L^2 - \frac{3}{8}\zeta(2)L^2 \\
& - \frac{4}{9}L^3 + \frac{7}{96}L^4 \\
& + \frac{1}{q^2}\Bigg(-\frac{6301}{432} - \frac{173}{24}\zeta(2) + 18\zeta(2)\ln 2 - \frac{3}{2}\zeta(3) + \frac{8}{5}\zeta^2(2) \\
& \quad + \frac{487}{48}L - \frac{15}{4}\zeta(2)L + 3\zeta(3)L - \frac{97}{16}L^2 + \frac{1}{2}\zeta(2)L^2 \\
& \quad + \frac{7}{6}L^3 - \frac{1}{48}L^4\Bigg) \\
& + \frac{1}{q^4}\Bigg(\frac{12871}{144} - \frac{109}{12}\zeta(2) - 72\zeta(2)\ln 2 + \frac{123}{2}\zeta(3) \\
& \quad - \frac{503}{20}\zeta^2(2) - \frac{7487}{216}L + \frac{401}{12}\zeta(2)L - 37\zeta(3)L \\
& \quad + \frac{1261}{36}L^2 - \frac{27}{4}\zeta(2)L^2 - \frac{295}{72}L^3 + \frac{7}{12}L^4\Bigg) \\
& + \frac{1}{q^6}\Bigg(-\frac{94525}{144} + \frac{5317}{18}\zeta(2) + 288\zeta(2)\ln 2 - \frac{1447}{3}\zeta(3) \\
& \quad + 227\zeta^2(2) + \frac{11401}{216}L - \frac{1175}{6}\zeta(2)L + 296\zeta(3)L
\end{aligned}
$$

$$
\begin{aligned}
& \qquad -\frac{4451}{24}L^2 + 55\zeta(2)L^2 + \frac{403}{18}L^3 - \frac{9}{4}L^4\Big) \\
& + \ \mathcal{O}\Big(\frac{1}{q^8}\Big) \ , \qquad (114)
\end{aligned}
$$

$$
\begin{aligned}
F_2^{(2l)}(D,q^2) \ &= \ \frac{1}{(D-4)}\Bigg\{\frac{1}{q^2}\Big(L - L^2\Big) + \frac{1}{q^4}\Big(2 - 6L + 2L^2\Big) \\
& \qquad + \frac{1}{q^6}\Big(- 11 + 20L - 8L^2\Big)\Bigg\} \\
& \quad + \frac{1}{q^2}\Big(\frac{17}{3} + \frac{11}{4}\zeta(2) - 12\zeta(2)\ln 2 \\
& \qquad + \zeta(3) - \frac{379}{72}L + 3\zeta(2)L + \frac{55}{24}L^2 - \frac{1}{2}L^3\Big) \\
& \quad + \frac{1}{q^4}\Big(- \frac{1717}{36} + \frac{31}{2}\zeta(2) + 48\zeta(2)\ln 2 - 56\zeta(3) \\
& \qquad + \zeta(3) + \frac{66}{5}\zeta^2(2) + \frac{931}{36}L - 19\zeta(2)L + 28\zeta(3)L \\
& \qquad - \frac{289}{12}L^2 + 4\zeta(2)L^2 + \frac{7}{3}L^3 - \frac{1}{12}L^4\Big) \\
& \quad + \frac{1}{q^6}\Big(+ \frac{34471}{72} - 254\zeta(2) - 192\zeta(2)\ln 2 + 448\zeta(3) \\
& \qquad - \frac{858}{5}\zeta^2(2) - \frac{295}{6}L + 129\zeta(2)L - 252\zeta(3)L \\
& \qquad + 147L^2 - 44\zeta(2)L^2 - \frac{83}{6}L^3 + \frac{1}{4}L^4\Big) \\
& + \ \mathcal{O}\Big(\frac{1}{q^8}\Big) . \qquad (115)
\end{aligned}
$$

7 Expansion for $Q^2 \ll m^2$

We present in this section the expansion in $Q^2 = m^2q^2$ around $Q^2 = 0$, valid for $Q^2 \ll m^2$ ($x \to 1$), of the form factors. Since we are in the analyticity region, the expansions are valid for space-like as well as time-like values of Q^2. The expansion reads

$$
\begin{aligned}
F_1^{(2l)}(D,q^2) \ &= \ \frac{q^4}{(D-4)^2}\Bigg\{\frac{1}{18} - q^2\Big(\frac{1}{60}\Big)\Bigg\} \\
& - \ \frac{q^4}{(D-4)}\Bigg\{\frac{1}{24} - q^2\Big(\frac{31}{1440}\Big)\Bigg\} \\
& \quad + q^2\Big(\frac{4819}{5184} + \frac{49}{72}\zeta(2) - 3\zeta(2)\ln 2 + \frac{3}{4}\zeta(3)\Big) \\
& \quad - q^4\Big(\frac{1349}{6480} + \frac{8731}{28800}\zeta(2) - \frac{11}{10}\zeta(2)\ln 2 + \frac{11}{40}\zeta(3)\Big)
\end{aligned}
$$

$$
\begin{aligned}
& \quad + q^6 \Big(\frac{109069}{1814400} + \frac{723901}{5644800}\zeta(2) - \frac{113}{280}\zeta(2)\ln 2 + \frac{113}{1120}\zeta(3) \Big) \\
&+ \quad \mathcal{O}(q^8) \, , \qquad (116)
\end{aligned}
$$

$$
\begin{aligned}
F_2^{(2l)}(D, q^2) \;=\; & -\frac{q^2}{(D-4)} \left\{ \frac{1}{6} - q^2 \left(\frac{19}{360} \right) + q^4 \left(\frac{73}{5040} \right) \right\} \\
& + \frac{197}{144} + \frac{1}{2}\zeta(2) - 3\zeta(2)\ln 2 + \frac{3}{4}\zeta(3) \\
& - q^2 \Big(\frac{1031}{2160} + \frac{13}{20}\zeta(2) - \frac{23}{10}\zeta(2)\ln 2 + \frac{23}{40}\zeta(3) \Big) \\
& + q^4 \Big(\frac{2551}{15120} + \frac{187}{525}\zeta(2) - \frac{15}{14}\zeta(2)\ln 2 + \frac{15}{56}\zeta(3) \Big) \\
& - q^6 \Big(\frac{69019}{1209600} + \frac{140951}{940800}\zeta(2) - \frac{29}{70}\zeta(2)\ln 2 + \frac{29}{280}\zeta(3) \Big) \\
+ \quad & \mathcal{O}(q^8) \, . \qquad (117)
\end{aligned}
$$

It is to be noted that the absence of the first order term in q^2 from the (infrared) singular part of $F_1^{(2l)}(D, q^2)$ and the absence of the zeroth order term from the (infrared) singular part of $F_2^{(2l)}(D, q^2)$ guarantee the finiteness of the 2-loop value for the charge slope of the electron ($-F_1^{(2l)\prime}(0)$ in the current notation) and of the magnetic anomaly $F_2^{(2l)}(0)$. The corresponding values are of course in agreement with the well known results in the literature (see for example [1]).

8 Summary

We carried out the analytic calculation of the renormalized on shell form factors of the electron vertex at two loops in perturbative QED. We gave the full results, for arbitrary momentum transfer S and on-shell fermionic external lines with finite mass m, in the space-like region, $S < 0$, and the imaginary parts in the time-like region above the threshold, $S > 4m^2$. The results are expressed in term of 1-dimensional HPLs of maximum weight 4.

We gave also the expansions of the form factors in the two kinematical regions of large momentum transfer, $S \to \infty$, and small momentum transfer, $S \to 0$, recovering in particular the known results for the charge slope and the anomalous magnetic moment of the electron.

For the calculation we used the MIs calculated in a previous paper, regularized in the dimensional regularization scheme. Both UV and soft IR divergences are regularized with the parameter D, dimension of the space-time. In the final results, after the renormalization of the UV divergences, the form factors still possess polar singularities in $(D-4)$, because of the soft IR divergences (the poles in $1/(D-4)$ cancel out from any physical observable when soft real photons are also accounted for).

9 Acknowledgment

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We wish to thank A. Ferroglia for useful discussions.

A Propagators

We list here the denominators of the integral expressions appeared in the paper.

$$
\begin{aligned}
\mathcal{D}_1 &= k_1^2\,, &(118)\\
\mathcal{D}_2 &= k_2^2\,, &(119)\\
\mathcal{D}_3 &= (k_1-k_2)^2\,, &(120)\\
\mathcal{D}_4 &= (p_1-k_1)^2\,, &(121)\\
\mathcal{D}_5 &= (p_2-k_2)^2\,, &(122)\\
\mathcal{D}_6 &= [k_1^2+m^2]\,, &(123)\\
\mathcal{D}_7 &= [k_2^2+m^2]\,, &(124)\\
\mathcal{D}_8 &= [(k_1+k_2)^2+m^2]\,, &(125)\\
\mathcal{D}_9 &= [(p_1-k_1)^2+m^2]\,, &(126)\\
\mathcal{D}_{10} &= [(p_2+k_1)^2+m^2]\,, &(127)\\
\mathcal{D}_{11} &= [(p_2-k_2)^2+m^2]\,, &(128)\\
\mathcal{D}_{12} &= [(p_1-k_1+k_2)^2+m^2]\,, &(129)\\
\mathcal{D}_{13} &= [(p_2+k_1-k_2)^2+m^2]\,. &(130)
\end{aligned}
$$

B One-loop results

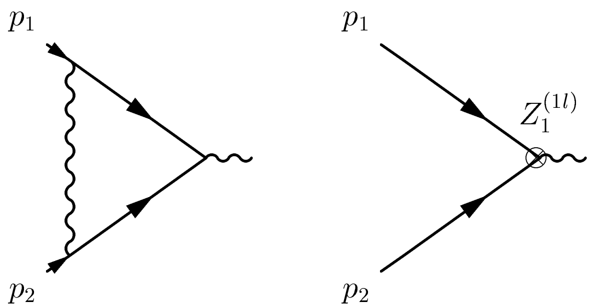
For completeness we give in this appendix the 1-loop form factors, corresponding to the first order in α/π in Eqs. (12-14), expanded up to the first order in $(D-4)$.

The Feynman diagrams involved are those in Fig. 3, where the order α/π Feynman diagram and the relative subtraction for the renormalization of the charge form factor are shown.

The unrenormalized form factors $\mathcal{F}_1^{(1l)}(D,q^2)$, $\mathcal{F}_2^{(1l)}(D,q^2)$ and $\mathcal{F}_3^{(1l)}(D,q^2)$ for the diagram (a) in Fig. 3 were already given in Eqs. (70–72), while $Z_1^{(1l)}(D)$ was given in Eq. (58). The counter-term in Fig. 3 (b) contributes only to the renormalization of the *charge* form factor, being the product of the renormalization constant $Z_1^{(1l)}(D)$ at 1-loop level, Eq. (58), and the tree-level vertex.

The UV-renormalized form factors at 1-loop level, in the space-like region $-S = Q^2 = m^2q^2 > 0$, up to the first order in $(D-4)$, are the following:

$$
F_1^{(1l)}(D,q^2) \;=\; \frac{1}{(D-4)}\Biggl\{1-\Bigl[1-\frac{1}{(1-x)}-\frac{1}{(1+x)}\Bigr]H(0;x)\Biggr\}
$$



(a) (b)

Figure 3: 1-loop vertex diagrams for the QED form factor.

$$
\begin{aligned}
& \quad -1 + \frac{1}{2}\Big[\frac{1}{2} - \frac{1}{(1+x)}\Big] H(0;x) \\
& \quad -\frac{1}{2}\Big[1 - \frac{1}{(1-x)} - \frac{1}{(1+x)}\Big][4\zeta(2) - 2H(0;x) - H(0,0;x) \\
& \quad -2H(1,0;x)] \\
& + \quad (D-4)\Big\{1 - \frac{1}{8}\Big[1 - \frac{2}{(1+x)}\Big][\zeta(2) - H(0,0;x) - 2H(-1,0;x)] \\
& \qquad +\frac{1}{4}\Big[1 - \frac{1}{(1-x)} - \frac{1}{(1+x)}\Big][2(\zeta(2)+\zeta(3)) \\
& \qquad +(\zeta(2)-4)H(0;x) - 2\zeta(2)H(1;x) - 2H(0,0;x) \\
& \qquad +2H(-1,0;x) - H(0,0,0;x) - 4H(-1,-1,0;x) \\
& \qquad +2H(-1,0,0;x) + 2H(0,-1,0;x)]\Big\} \\
& + \quad \mathcal{O}\left((D-4)^2\right) , \qquad (131)
\end{aligned}
$$

$$
\begin{aligned}
F_2^{(1l)}(D,q^2) &= -\frac{1}{2}\left[\frac{1}{(1-x)} - \frac{1}{(1+x)}\right] H(0;x) \\
& - \quad (D-4)\left\{\frac{1}{4}\left[\frac{1}{(1-x)} - \frac{1}{(1+x)}\right][\zeta(2) - 4H(0;x) \right. \\
& \qquad \left. -4H(0,0;x) + 2H(-1,0;x)]\right\} \\
& + \quad \mathcal{O}\left((D-4)^2\right) , \qquad (132) \\
F_3^{(1l)}(D,q^2) &= 0 \, . \qquad (133)
\end{aligned}
$$

where $F_2^{(2l)}(D,q^2)$ is exactly equal to $\mathcal{F}_2(D,q^2)$, given in Eq. (71) and $F_1^{(1l)}(D,q^2)$ is obtained by subtracting $Z_1^{(1l)}$ of Eq. (58) from $\mathcal{F}_1(D,q^2)$ given in Eq. (70).

Eqs. (131,132) can be analytically continued in the time-like region $S = -Q^2 > 0$ and in particular above the physical threshold $S > 4m^2$, where an imaginary part appears. Using the substitution of Eq. (108) and writing ($s = S/m^2$)

$$
\begin{aligned}
F_1^{(1l)}(D,-s-i\epsilon) &= \Re F_1^{(1l)}(D,-s) + i\pi \Im F_1^{(1l)}(D,-s)\,, &(134)\\
F_2^{(1l)}(D,-s-i\epsilon) &= \Re F_2^{(1l)}(D,-s) + i\pi \Im F_2^{(1l)}(D,-s)\,, &(135)
\end{aligned}
$$

the imaginary parts have the following expressions:

$$
\begin{aligned}
\Im F_1^{(1l)}(D,-s) &= -\frac{1}{(D-4)}\left\{1-\frac{1}{(1-y)}-\frac{1}{(1+y)}\right\}\\
&\quad +\frac{1}{4}-\frac{1}{2(1+y)}+\frac{1}{2}\left[1-\frac{1}{(1-y)}-\frac{1}{(1+y)}\right][1+H(0;y)\\
&\quad +2H(1;y)]\\
&\quad -(D-4)\left\{\frac{1}{8}\left[1-\frac{2}{(1+y)}\right][H(0;y)+2H(1;y)]\right.\\
&\qquad +\frac{1}{4}\left[1-\frac{1}{(1-y)}-\frac{1}{(1+y)}\right][4-2\zeta(2)+H(0;y)\\
&\qquad +2H(1;y)+H(0,0;y)+2H(0,1;y)\\
&\qquad \left.+2H(1,0;y)+4H(1,1;y)]\right\}\\
&\quad +\mathcal{O}\left((D-4)^2\right)\,, &(136)\\
\Im F_2^{(1l)}(D,-s) &= \frac{1}{2}\left[\frac{1}{(1-y)}-\frac{1}{(1+y)}\right]\\
&\quad -(D-4)\left\{\frac{1}{4}\left[\frac{1}{(1-y)}-\frac{1}{(1+y)}\right][\zeta(2)-4H(0;y)\right.\\
&\qquad \left.-H(0,0;y)+2H(-1,0;y)]\right\}\\
&\quad +\mathcal{O}\left((D-4)^2\right)\,. &(137)
\end{aligned}
$$

The presence of $1/(D-4)$ singularities even in the on-shell renormalized form factors is due to the fact that soft IR divergences are still present. As already recalled, in any physical quantity they will cancel against similar divergences due to soft real photons.

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