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Charmed Meson Decays and QCD Sum Rules

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The current status of the QCD sum rule predictions for charmed mesons is overviewed.

1 Introduction

Weak decays of charmed mesons provide a useful testing ground for nonperturbative methods, such as QCD sum rules [1]. Contrary to B decays, in D decays the hadronic matrix elements are multiplied by experimentally known CKM parameters: V_{cs} is extracted from charm-tagged W-decays and V_{cd} from neutrino-nucleon production of charm [2]. Therefore, the QCD sum rule predictions for the hadronic parameters of D decays can be directly compared with experimental data. This comparison allows one to gain more confidence in the sum rule results for the B-decay hadronic matrix elements that are used to extract the poorly known CKM parameters.

The outline of the QCD sum rule method and the predictions for B decays are discussed in the context of CKM physics in [3] (in sect. 4.2 of chapter III and in sects. 2.1.3, 2.1.4 of chapter IV), see also [4]. Here I will concentrate on the corresponding results for the charm sector. A detailed review of QCD sum rules can be found in [5].

2 Determination of the c-quark mass

One of the first applications of QCD sum rules was the estimate of the charmed quark mass. According to the original method [6], one employs the correlator of two $\bar{c}\gamma_\mu c$ currents. Due to dispersion relation, the n-th derivative of the correlator in q^2 (the momentum transfer squared) is related to the n-th power moment of the hadronic $e^+e^- \to c\bar{c}$ cross-section σ_c:

$$M_n(q^2) = \frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2) \sim \int\limits_{m^2_{J/\psi}}^{\infty} ds \frac{s\,\sigma_c(s)}{(s-q^2)^{n+1}}\,. \qquad (1)$$

The moments $M_n(q^2)$ are calculated in QCD in a certain range of $\{n,q^2\}$, far off-shell ($q^2 \leq 0$), in terms of the virtual c-quark loops, taking into account the c-quark interactions with perturbative gluons and nonperturbative gluon condensate [1]. The results depend on m_c, α_s and the gluon condensate density. The hadronic cross section in Eq. (1) is saturated by the charmonium resonances $J/\psi, \psi', ...$, and at $\sqrt{s} > 2m_D$ by the open charm-anticharm production.

In Table 1 the recent results for the $\overline{MS}$ mass $\bar{m}_c(\bar{m}_c)$ extracted from the sum rule moments (1) with the $O(\alpha_s^2)$ accuracy are presented and compared with the predictions of other methods.

Table 1.

$\bar{m}_c(\bar{m}_c)$ (GeV)	Ref.	method
1.304(27)	[7]	QCD SR
1.275(15)	[8]	"
1.230(90)	[9]	QCD SR+NRQCD
1.190(110)	[10]	"
1.370(90)	[11]	FESR
1.210(70)(65)(45)	[12]	$m_B - m_D$ + HQET
1.300(40)(200)	[13]	lattice QCD (average)

Despite a reasonable agreement between all four QCD SR predictions within theoretical uncertainties, one has to keep in mind that the quoted estimates of $\bar{m}_c(\bar{m}_c)$ are obtained using M_n with different $\{n,q^2\}$ and making different assumptions. In [7], following the original analysis of [6] the lowest moment $n=1$ at $q^2=0$ is selected, having a little sensitivity to nonperturbative effects but demanding an accurate knowledge of the cross section $\sigma_c(s)$ above the open charm threshold. In the analysis of [8], $n \sim 10$ and $q^2 < 0$ are used, so that the gluon condensate contributions become important. Finally, in [9,10], for the moments with large n, an attempt is made to account for the resummed Coulomb effects that are not accessible in the relativistic calculation of $\Pi(q^2)$. An ansatz for the spectral density of the correlator is adopted, combining the full QCD answer with the resummed NRQCD spectral density at large and

*On leave from Yerevan Physics Institute, 375036 Yerevan, Armenia small c-quark velocities, respectively. This choice emphasizes Coulomb versus gluon-condensate effects in a relatively light $c\bar{c}$ quarkonium, an issue which deserves further studies (for a critical discussion see [8]). In order to further improve the sum rule determinations of m_c, one needs more precise measurements of the $J/\psi,\psi',...$ leptonic widths and of the open charm cross section in e^+e^-.

Furthermore, let me remind that the vector charmonium channel provides a possibility to check the quark-hadron duality approximation, the key element of QCD sum rules. Replacing in the integrand in Eq. (1) the hadronic cross section by the calculated spectral density of the correlator at $s>s_0$, one still successfully fits the moments, even if s_0 is shifted close to the open charm threshold, indicating that the "semi-local" quark-hadron duality also works in this channel.

3 f_D from SVZ Sum Rules

One of the great advantages of the sum rule method is a possibility to easily switch from c to b quark and vice versa, in the analytical expressions. An important example is the calculation of the D-meson decay constant f_D defined as $f_D m_D^2 = m_c\langle 0|\bar{d}i\gamma_5 c|D^+\rangle$. The SVZ sum rule is derived from the correlator of two $\bar{c}i\gamma_5 d$ currents, applying duality approximation for excited D states and Borel transformation. The result has the following schematic form:

$$f_D^2 m_D^4 e^{-m_D^2/M^2} = \\ = \sum_{n=0,3,...}^{n_{max}}\left(\sum_{k=0,1,..}\left(\frac{\alpha_s}{\pi}\right)^k S_{nk}(m_c,M^2,s_0^D,\mu_c)L_n(\mu_c)\right) \quad (2)$$

where S_{nk} are the calculable short-distance coefficients. In particular S_{0k} are given by perturbative heavy-light loops in order α_s^k ($L_0\equiv 1$); $L_{n\geq 3}$ are the universal long-distance parameters (vacuum condensate densities) with dimension n. In the above, $M^2\sim m_c\overline{\Lambda}$ (where $\overline{\Lambda}=m_D-m_c$), is the Borel parameter characterizing the average virtuality of the c-quark in the correlator, s_0^D is the quark-hadron duality threshold , and $\mu_c\sim M$ is the factorization scale. The fact that $L_{n\geq 3}\sim(\Lambda_{QCD})^n$ and $S_{nk}\sim(1/M)^n$ allows one to retain a finite numbers of terms in the sum over n ($n_{max}=6$ is already providing a sufficient accuracy).

The $f_D\to f_B$ transition in the sum rule (2) is realized by replacing

$$m_c\to m_b,\ m_D\to m_B,\ s_0^D\to s_0^B,\ \mu_c\to\mu_b \quad (3)$$

with the scale-dependence given by the relevant renormalization-group factors. The $c\leftrightarrow b$ correspondence does not mean however that both sum rules for f_B and f_D are equally accurate. The numerical hierarchies of corrections are different in the appropriate ranges of Borel parameters, so that generally the D-meson sum rules are less stable. Nevertheless, since f_B is of a crucial importance for B-physics it is very useful to check the method by comparing the sum rule prediction for f_D with experiment.

In Table 2 two recent sum rule determinations of f_D are presented (for a more detailed overview including older results see [5]). Comparison with the lattice QCD results reveals an encouraging agreement. In [14] the perturbative part is taken into account in $O(\alpha_s)$, and the quoted theoretical uncertainty is largely determined by the c-quark mass interval. The one-loop pole mass $m_c^{1loop}=1.3\pm0.1$ GeV was taken, which overlaps with the lower part of the m_c^{1loop} interval obtained from the values of $\bar{m}_c(\bar{m}_c)$ given in Table 1. One possibility to improve the sum rule is to use the recently obtained $O(\alpha_s^2)$ results for the heavy-light correlator [15], providing the coefficient S_{02} in Eq. (2). This was done in [16] in the framework of HQET. In full QCD so far only f_B was recalculated [17] with the $O(\alpha_s^2)$ accuracy.

In addition, to have more confidence in the power expansion of the correlator, it would be useful to calculate the $d=7$ correction proportional to a combination of quark and gluon condensates. A better determination of m_c, inclusion of $O(\alpha_s)^2$ corrections in full QCD, together with a systematical use of $\bar{m}_c(\bar{m}_c)$, are the remaining resources of improvement for the sum rule result for f_D. More difficult is to assess the "systematic" uncertainty related to the quark-hadron duality approximation in the D meson channel.

Table 2.

f_D (MeV)	Ref.	Method
200(20)	[14]	SVZ, $O(\alpha_s)$
195(20)	[16]	SVZ+HQET, $O(\alpha_s^2)$
203(14)	[18]	Lattice QCD average (quench.)
226(15)	[18]	" (unquench.)

Having obtained a prediction for f_D one is not yet able to compare it with an experimental number, because only f_{D_s} is measured, the latest result [19] being :

$$f_{D_s} = 285\pm19\pm40\,\text{MeV}. \quad (4)$$

Importantly, QCD sum rules also predict the f_{D_s}/f_D ratio in terms of m_s and $\langle\bar{s}s\rangle/\langle\bar{q}q\rangle$, $(q=u,d)$ the ratio of strange and nonstrange quark condensates. The rather old results collected in [5] yield an interval:

$$f_{D_s}/f_D = 1.11\div1.27\,, \quad (5)$$

to be compared with the recent averages $f_{D_s}/f_D = 1.12(2)$ [1.12(4)] of lattice quenched [unquenched] QCD

[18] with a smaller uncertainty (see also [20]). Multiplying the sum rule prediction from Table 2: $f_D = 200 \pm 20$ MeV by the ratio (5) we obtain: $f_{D_s} = 240 \pm 40$ MeV, in the ballpark of the experimental interval (4). Improving the latter and measuring f_D will provide more decisive checks.

4 $D \to \pi, K$ form factors from LCSR

Measuring the semileptonic $D \to \pi l \nu_l$ decay distribution

$$\frac{d\Gamma^{(D\to\pi l\bar{\nu})}}{dq^2} = \frac{G^2|V_{cd}|^2(E_\pi^2 - m_\pi^2)^{3/2}}{24\pi^3}[f^+_{D\pi}(q^2)]^2 + O(m_l^2)\,, \qquad (6)$$

at $m_l^2 < q^2 < (m_D - m_\pi)^2$ and dividing out $|V_{cd}|$ one is able to reproduce the experimental values of the $D \to \pi$ form factor $f^+_{D\pi}(q^2)$. With a sufficiently large statistics, of semileptonic decays with $l = \mu$, the scalar form factor $f^0_{D\pi}(q^2)$ entering the chirally suppressed $O(m_l^2)$ part of the decay distribution can also be extracted. The Cabibbo enhanced $D \to K l \nu_l$ decays provide $D \to K$ form factors with an even better accuracy.

Heavy-to-light form factors are calculated using QCD light-cone sum rules (LCSR). One of the first applications of this sum rule technique was the calculation of $f^+_{D\pi,DK}$ in [21]. The updated results for $f^+_{D\pi,DK}$ including higher twist terms [22] and $O(\alpha_s)$ corrections [23] can be found in [14]. The form factor $f^0_{D\pi}$ was calculated in [24].

The sum rules for $B \to \pi$ are obtained by the same replacement (3). Let me emphasize that this transition is done from one finite mass to the other. Contrary to HQET relations between B and D form factors, no heavy-quark mass approximation is involved. Since $f^+_{B\pi}(q^2)$ is used to extract $|V_{ub}|$ from $B \to \pi l \nu$, a comparison of the sum rule predictions for $D \to \pi$ form factors with experimental data will ensure more confidence in the LCSR method.

The most recent LCSR result $f_{D\pi}(0) = 0.65 \pm 0.11$ obtained in [14] takes into account the twist-2 term in NLO and twist-3,4 contributions in LO (in the expansion of the underlying vacuum-pion correlator in the pion distribution amplitudes with growing twist). From the same correlator, using double dispersion relation one has access to the $D^*D\pi$ coupling $g_{D^*D\pi}$ [22,25], predicting the product $f_{D^*}f_D g_{D^*D\pi}$. Using the SVZ sum rule result for f_D quoted above, the D^*-pole contribution to the $D \to \pi$ form factor is calculated. The two predictions of LCSR are used [14] to fit the $D \to \pi$ form factor in the whole kinematical region $0 < q^2 < (m_D - m_\pi)^2$ to the simple ansatz [26] inspired by dispersion relation:

$$f^+_{D\pi} = \frac{065 \pm 0.11}{(1 - q^2/m_{D^*}^2)(1 - \alpha^{D\pi} q^2/m_{D^*}^2)}\,, \qquad (7)$$

with $\alpha^{D\pi} = 0.01^{+0.11}_{-.07}$. Interestingly, the sum rule results are consistent with the D^*-pole dominance for the form factor. The predicted integrated decay width $\Gamma(D^0 \to \pi^- l^+ \nu_l)/|V_{cd}|^2 = 0.13 \pm 0.05 \quad \mathrm{ps}^{-1}$ is in a reasonable agreement with the experimental number [2] $\Gamma(D^0 \to \pi^- e^+ \nu_e)/|V_{cd}|^2 = 0.174 \pm 0.032 \quad \mathrm{ps}^{-1}$. At the zero momentum transfer the LCSR prediction also agrees with the recent lattice result [27] $f_{D\pi}(0) = 0.57(6)\pm^{0.01}_{0.00}$. On the other hand, the lattice calculation [27] suggests that the contribution of excited D^* states is not small. To assess the reliability of these theoretical predictions, one has to wait until the $D \to \pi l \nu_l$ decay distribution is accurately measured.

Finally, the LCSR result for $D \to K$ form factor [14] is $f_{DK}(0) = 0.78 \pm 0.11$ at $m_s(1\mathrm{GeV}) = 150$ MeV, very sensitive to the strange quark mass. The corresponding integrated width $\Gamma(D^0 \to K^- l^+ \nu_l)/|V_{cs}|^2 = 0.094 \pm 0.036 \quad \mathrm{ps}^{-1}$, is in a good agreement with experiment [2] $\Gamma(D^0 \to K^- l^+ \nu_l)/|V_{cs}|^2 = 0.087 \pm 0.004 \quad \mathrm{ps}^{-1}$.

Concluding this section, let me briefly discuss the important issue of the $D^*D\pi$ coupling (see also [28]). The recent first measurement of the total width of D^* by CLEO collaboration [29]: $\Gamma_{tot}(D^*) = 96 \pm 4 \pm 22$ keV yields for this coupling $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$ (using the definition of Ref. [22]). The LCSR prediction [22,25], $g_{D^*D\pi} = 10 \pm 3.5$, is obtained by dividing the calculated product $f_D f_{D^*} g_{D^*D\pi}$ by the two decay constants, f_D and f_{D^*}, extracted from 2-point SVZ sum rules. Taking into account the estimated theoretical uncertainty, the upper limit of the LCSR prediction is $g_{D^*D\pi} = 13.5$, still 25% lower than the central value of the CLEO number. Meanwhile, the first lattice QCD prediction $g_{D^*D\pi} = 18.8 \pm 2.3^{+1.1}_{-2.0}$ [30] agrees with the CLEO result. If the future measurements (although extremely difficult !) and lattice calculations confirm the large value of this coupling, one has to clarify the status of the LCSR prediction. Having in mind that all other sum rules discussed above use one-variable dispersion relations, one might suspect that certain complications arise in the double dispersion relation used in LCSR for the $D^*D\pi$ coupling. More specifically, the simplest quark-hadron duality ansatz (one resonance plus continuum) in both D and D^* channels may be too crude. One possible scenario was recently discussed in [31]: assuming a partial cancellation between the contributions of excited and ground D, D^* states in the dispersion relation, one gets an increase of the LCSR coupling. Without going into further details, let me only mention that the magnitude of the coupling $g_{D^*D\pi}$ and the shape of the form factor $f^+_{D\pi}(q^2)$ are closely related. Suppose the form factor is dominated by the D^*-pole contribution:

$$f^+_{D\pi}(q^2) = \frac{f_{D^*} g_{D^*D\pi}}{2m_{D^*}(1 - q^2/m_{D^*}^2)}\,. \qquad (8)$$

Taking for $g_{D^*D\pi}$ the CLEO central value and multiplying it with $f_{D^*} = 250$ MeV (within the lattice QCD prediction [32]) one obtains a semileptonic width $\Gamma(D^0 \to \pi^- l^+ \nu_l)/|V_{cd}|^2 = 0.37 \quad \mathrm{ps}^{-1}$ which is about two times larger than the experimental width quoted above [2]. To make the strong coupling measured by CLEO consistent with the semileptonic width, one needs a substantial negative interference between the D^*-pole and excited D^* states in the dispersion relation for the form factor (as also noticed in [31]), resulting in a visible deviation of the $D \to \pi l \nu_l$ decay distribution from the D^*-pole dominance.

5 Rare D decays

Exclusive rare D decays such as $D \to \mu^+\mu^-, 2\gamma$, etc. will become important highlights in the future high-statistics charm physics experiments. Being suppressed in the Standard Model, these decays are promising indicators of new physics. The long-distance amplitudes of rare D decays in the Standard Model still lack a QCD based analysis. I believe, QCD sum rules in both two-pont (SVZ) and light-cone versions can essentially help in solving this problem, One example is the LCSR prediction for weak radiative decays obtained in [33]:

$$\begin{aligned} BR(D^+ \to \rho^+\gamma) &= 2.7 \cdot 10^{-6}, \\ BR(D^0 \to \rho^0\gamma) &= 3.0 \cdot 10^{-6}, \\ BR(D_s \to \rho^+\gamma) &= 2.8 \cdot 10^{-5}, \end{aligned} \tag{9}$$

where all numbers have an $O(50\%)$ accuracy. This analysis can be further improved and extended to the other rare D decay channels.

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