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QCD Light-Cone Sum Rule Estimate of Charming Penguin Contributions in $B \to \pi\pi$

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Abstract

Employing the QCD light-cone sum rule approach we calculate the $B \to \pi\pi$ hadronic matrix element of the current-current operator with c quarks in the penguin topology ("charming penguin"). The dominant contribution to the sum rule is due to the c-quark loop at short distances and is of $O(\alpha_s)$ with respect to the factorizable $B \to \pi\pi$ amplitude. The effects of soft gluons are suppressed at least by $O(\alpha_s m_b^{-2})$. Our result indicates that sizable nonperturbative effects generated by charming penguins at finite m_b are absent. The same is valid for the penguin contractions of the current-current operators with light quarks.

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1. Charmless two-body hadronic B decays, such as $B \to \pi\pi$, are promising sources of information on CP-violation in the b-flavour sector. The task of unfolding CKM phases from the decay observables is challenged by our limited ability to calculate the relevant hadronic interactions. One has to resort to approximate methods based on the expansion in the inverse b-quark mass. The long-distance effects are then parametrized in terms of process-independent characteristics: heavy-light form factors, hadronic decay constants and light-cone distribution amplitudes (DA). For example, in the QCD factorization approach [1] the exclusive B-decay amplitudes in the $m_b \to \infty$ limit are expressed in terms of the factorizable part and calculable $O(\alpha_s)$ nonfactorizable corrections. For phenomenological applications it is important to investigate the subleading effects in the decay amplitudes suppressed by inverse powers of m_b. Especially interesting are "soft" nonfactorizable effects, involving low-virtuality gluons and quarks, not necessarily accompanied by an α_s-suppression.

Quantitative estimates of nonfactorizable contributions, including the power-suppressed ones can be obtained [2] using the method of QCD light-cone sum rules (LCSR) [3]. In particular, the $O(1/m_b)$ soft-gluon corrections have been calculated [2] for the $B \to \pi\pi$ matrix elements with the emission topology. Furthermore, the LCSR estimate of the chromomagnetic dipole operator (gluonic penguin) contribution to $B \to \pi\pi$ was obtained in [4]. In this case the soft-gluon contribution, being suppressed as $1/m_b^2$, at finite m_b is of the magnitude of the $O(\alpha_s)$ hard-gluon part. However, the nonfactorizable corrections are nonuniversal, depending on the effective operators and quark topologies involved in a given B decay channel, therefore these corrections have to be investigated one by one.

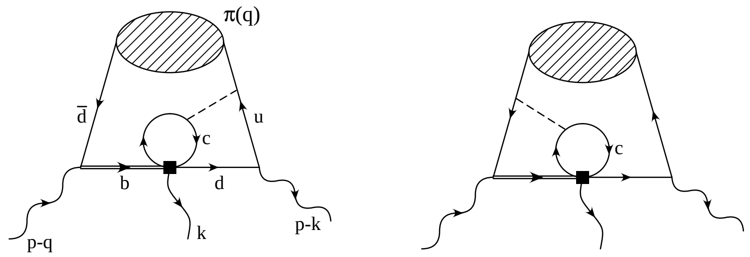
Among the most intriguing effects in charmless B decays are the so called "charming penguins". The c-quark pair emitted in the $b \to c\bar{c}d(s)$ decay propagates in the environment of the light spectator cloud and annihilates to gluons, the latter being absorbed in the final charmless state. In this, so called BSS-mechanism [5] the intermediate $c\bar{c}$ loop generates an imaginary part, contributing to the final-state strong rescattering phase. In QCD factorization approach [1], charming penguins are typically small, being a part of the $O(\alpha_s)$ nonfactorizable correction to the $B \to \pi\pi$ amplitude. On the other hand, fits of two-body charmless B decays do not exclude substantial $O(1/m_b)$ nonperturbative effects of the charming-penguin type [6]. It is therefore important to investigate by independent methods the effects generated by c-quark loops in charmless B decays.

In this letter we report on the LCSR estimate of the penguin topology contributions to the $B \to \pi\pi$ amplitude. We start with calculating the c-quark part of this effect (charming penguin). Later on, we extend the sum rule analysis to the penguin contractions of the u-quark current-current operators.

2. As a study case we choose the $\bar{B}^0 \to \pi^+\pi^-$ channel. The decay amplitude is given by the hadronic matrix element $\langle \pi^+\pi^-|H_{\rm eff}|\bar{B}^0\rangle$ of the effective weak Hamiltonian [7]

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\Bigg\{ \sum_{p=u,c} V_{pb}V_{pd}^*\left(C_1\mathcal{O}_1^p + C_2\mathcal{O}_2^p\right) - V_{tb}V_{td}^*\left(\sum_{i=3}^{10} C_i\mathcal{O}_i + C_{8g}\mathcal{O}_{8g}\right)\Bigg\}, \quad (1)$$

where $\mathcal{O}_1^p = (\bar{d}\Gamma_\mu p)(\bar{p}\Gamma^\mu b)$ and $\mathcal{O}_2^p = (\bar{p}\Gamma_\mu p)(\bar{d}\Gamma^\mu b)$ are the current-current operators ($p = u, c$ and $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$), $\mathcal{O}_{3-10}$ are the penguin operators, and $\mathcal{O}_{8g}$ is the chromomagnetic dipole operator. Each operator entering Eq. (1) contributes to the



(a) (b)

Figure 1: *Diagrams corresponding to the $O(\alpha_s)$ penguin contractions in the correlation function (2). Only the diagrams contributing to the sum rule are shown. The square denotes the four-quark operator $\tilde{\mathcal{O}}_2^c$.*

$B \to \pi\pi$ decay amplitude with a number of different contractions of the quark lines (topologies) [8]. In what follows we will mainly concentrate on the operator $\mathcal{O}_1^c$. For convenience we decompose this operator $\mathcal{O}_1^c = \frac{1}{3}\mathcal{O}_2^c + 2\tilde{\mathcal{O}}_2^c$, extracting the colour-octet part $\tilde{\mathcal{O}}_2^c = (\bar{c}\Gamma_\mu \frac{\lambda^a}{2} c)(\bar{d}\Gamma^\mu \frac{\lambda^a}{2} b)$.

To derive LCSR for the $B \to \pi\pi$ hadronic matrix element of $\mathcal{O}_1^c$ we follow the procedure which is described in detail in [2, 4]. One starts from introducing the correlation function:

$$F_\alpha^{(\tilde{\mathcal{O}}_2^c)} = i^2 \int d^4x\, e^{-i(p-q)x} \int d^4y\, e^{i(p-k)y} \langle 0| T\{ j_{\alpha 5}^{(\pi)}(y) \tilde{\mathcal{O}}_2^c(0) j_5^{(B)}(x)\} |\pi^-(q)\rangle$$
$$= (p-k)_\alpha F(s_1, s_2, P^2) + \dots , \qquad (2)$$

where $j_{\alpha 5}^{(\pi)} = \overline{u}\gamma_\alpha\gamma_5 d$ and $j_5^{(B)} = i m_b \overline{b}\gamma_5 d$ are the quark currents interpolating pion and B meson, respectively. In the above, ellipses denote the Lorentz-structures which are not used. Only the colour-octet part of $\mathcal{O}_1^c$ needs to be taken into account. The contributions of $\mathcal{O}_2^c$ contain at least a two-gluon annihilation of the color-neutral c-quark pair and have to be considered within higher-order corrections. The correlation function (2) depends on the artificial four-momentum k allowing one to avoid overlaps of the b-flavoured and light-quark states in the dispersion relation. We also choose $p^2 = k^2 = 0$ for simplicity and adopt the chiral limit $q^2 = m_\pi^2 = 0$. Thus, the invariant amplitude F is a function of three variables $s_1 = (p-k)^2$, $s_2 = (p-q)^2$ and $P^2 = (p-q-k)^2$.

The next step is to calculate $F(s_1, s_2, P^2)$ at large spacelike s_1, s_2, P^2 employing the operator product expansion (OPE) near the light-cone. The corresponding diagrams of $O(\alpha_s)$ are shown in Fig. 1. They contain a c-quark loop, which involves a well known function of m_c^2. The divergence of the quark loop is absorbed in the renormalization of the QCD penguin operators $\mathcal{O}_{3-6}$. The finite contribution of the loop is formally of the next-to-leading order and depends on the renormalization scale μ. This dependence is compensated by the Wilson coefficients of the QCD penguin operators so that the remaining μ dependence is weak, since it is of $O(\alpha_s^2 \ln \mu)$. Furthermore, the finite piece of the loop contains scheme-dependent constants; we use the NDR scheme [7].

The sum rule is derived from the dispersion relations for F in the variables s_1 and s_2. Therefore the calculation of the two-loop diagrams in Fig. 1 can be considerably simplified if one starts from their imaginary parts obtained by employing the Cutkosky rule and replacing the propagators by δ-functions. For the diagram in Fig. 1a it is sufficient to calculate its imaginary part in s_1. Furthermore, according to the procedure explained in [2] we have to consider the dispersion relation in s_1 in the quark-hadron duality interval, $0 < s_1 < s_0^\pi \ll m_B^2$ and simultaneously at $|P^2| \sim m_B^2$. Hence, one can safely neglect all $O(s_1/P^2)$ contributions in $\mathrm{Im}_{s_1} F(s_1, s_2, P^2)$. In this approximation we obtain for the diagram in Fig. 1a:

$$
\begin{aligned}
Im_{s_1} F_a^{(\tilde{O}_2^c)}(s_1, s_2, P^2) &= -\frac{\alpha_s C_F f_\pi m_b^2}{8\pi^2} \int\limits_0^1 \frac{du}{m_b^2 - u s_2} \int\limits_0^1 dz\, I(zuP^2, m_c^2) \\
&\times P^2 \Bigg\{ z(1-z)\varphi_\pi(u) + (1-z)\frac{\mu_\pi}{2m_b}\left[\left(2z + \frac{s_2}{P^2}\right) u\varphi_p(u) \right. \\
&\left. + \left(2z - \frac{s_2}{P^2}\right)\left(\frac{\varphi_\sigma(u)}{3} - \frac{u\varphi_\sigma'(u)}{6}\right)\right] \Bigg\} + O\left(\frac{s_1}{P^2}\right). \qquad (3)
\end{aligned}
$$

In the above, φ_π and φ_p, φ_σ are the pion DA of twist 2 and 3, respectively, the latter are normalized by $\mu_\pi = m_\pi^2/(m_u + m_d)$ nonvanishing in the chiral limit; we use the same standard definitions as in [4]. Finally, $\varphi_\sigma'(u) = d\varphi_\sigma(u)/du$ and $I(...)$ is the c-quark loop function in the NDR scheme:

$$
I(l^2, m_c^2) = \frac{1}{6}\left(\ln\left(\frac{m_c^2}{\mu^2}\right) + 1\right) + \int\limits_0^1 dx\, x(1-x) \ln\left[1 - \frac{x(1-x)l^2}{m_c^2}\right]. \qquad (4)
$$

Similarly, for the diagram in Fig. 1b it is sufficient to calculate its imaginary part in s_2. The result reads:

$$
\begin{aligned}
Im_{s_2} F_b^{(\tilde{O}_2^c)}(s_1, s_2, P^2) &= \frac{\alpha_s C_F f_\pi m_b^2}{16\pi^2}\left(\frac{s_2 - m_b^2}{s_2}\right)^2 \int\limits_0^1 \frac{du}{\bar{u}P^2 + u s_1} \int\limits_0^1 dz I_c(-uz(s_2 - m_b^2)) \\
&\times \Bigg\{ (P^2 - s_1 - s_2) z\, \varphi_\pi(u) + \frac{\mu_\pi}{2m_b}(P^2 - s_1 + 3s_2)\varphi_p(u) \\
&- \frac{\mu_\pi}{12 m_b}\left[2\left(\frac{P^2 - s_1 - s_2}{u} + 2\frac{P^2 - s_1}{\bar{u}P^2 + u s_1}(P^2 - s_1 - s_2)\right)\varphi_\sigma(u) \right. \\
&\left. + (3P^2 - 3s_1 + s_2)\varphi_\sigma'(u) \right] \Bigg\}. \qquad (5)
\end{aligned}
$$

The remaining diagrams not shown in Fig. 1, with gluons attached to the virtual b and d lines, do not contribute to the sum rule because their double imaginary parts vanish inside the duality regions $0 < s_1 < s_0^\pi$, $m_b^2 < s_2 < s_0^B$.

Eqs. (3) and (5) provide the leading, $O(\alpha_s)$ and twist 2 and 3 answer for the correlation function. The contributions of the higher-twist pion DA's to F are suppressed by inverse powers of large squares of external momenta, reflecting the power counting in the light-cone OPE. Accordingly, in this paper, we neglect small effects of the twist-4 quark-antiquark DA's in the diagrams of Fig. 1.

3. So far we have taken into account the hard-gluon emission off the c-quark loop. The most intriguing and difficult problem concerning charming penguins is the effect

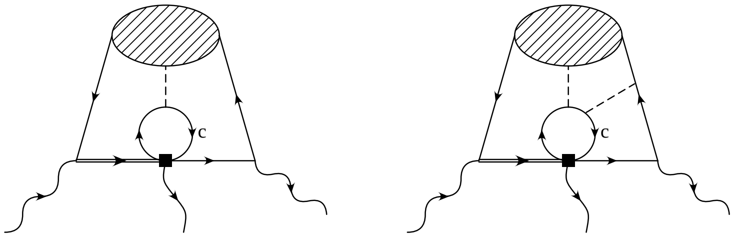
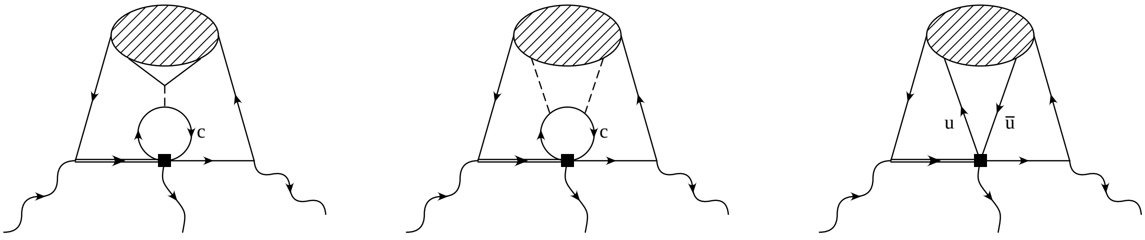


Figure 2: *Diagrams containing the quark-antiquark-gluon DA's*

of the soft (low-virtuality) gluons coupled to the c-quark loop. Within the sum rule approach this problem has to be addressed in the context of the light-cone OPE of the correlation function. Together with $\bar{u}$ and d quarks, the on-shell gluons emitted at short distances form multiparticle DA's of the pion. Importantly, the contributions of these DA's to the sum rule, being of higher twist, are suppressed [2, 4] by inverse powers of the heavy mass scale with respect to the contributions of 2-particle quark-antiquark DA's of lower twists. Therefore, for a sum rule estimate of the soft-gluon effects it is sufficient to consider diagrams with one "constituent" gluon i.e., diagrams involving quark-antiquark-gluon DA's of the pion. For the gluonic penguin operator, this soft-gluon effect turns out to be important [4], because in the quark-antiquark-gluon term of the sum rule the $1/m_b^2$ suppression is compensated by the absence of α_s. In the correlation function (2) the situation is different. The diagram with one gluon shown in Fig 2a vanishes due to the current conservation in the c-quark loop. Nonvanishing terms with the three-particle DA's emerge from the diagrams, containing at least one hard gluon in addition to the on-shell gluon. One of these diagrams is shown in Fig 2b. Their complete calculation is a difficult task. From the studies of the $b \to s\gamma$ matrix elements of $\mathcal{O}_{1,2}$, where similar diagrams with an on-shell photon and virtual gluon have been calculated [9], we conclude that the contribution of Fig. 2b and similar diagrams not only contain α_s but also have an additional $O(1/m_b^2)$ suppression with respect to the diagrams in Fig. 1.

The next nonvanishing contribution to the expansion of the coloured c-quark loop near the light-cone contains a derivative of the gluon field $D_\nu G^a_{\mu\nu}$ which can be further reduced to the light-quark pair due to QCD equation of motion. The whole effect has a short-distance nature, similar to the formation of the QCD penguin operators. The resulting diagram shown in Fig. 3a has to be calculated in terms of the pion four-quark DA's and is beyond the approximation adopted here and in [4]. Simple dimension counting yields for this diagram a suppression factor of $O(1/m_b^3)$. The c-loop factor in this case reduces to $\ln(m_c^2/\mu^2)$ plus a scheme-dependent constant. The μ- and scheme-dependence are compensated by the contributions of QCD penguin operators, since the c-quark loop momenta larger than μ are included in the short-distance coefficients of these operators.

The diagram with two gluons emitted from the c-quark loop (Fig. 3b) is also not included in our calculation because it contains DA's with multiplicity larger than three. Here both color-neutral and color-octet parts of $\mathcal{O}_1$ (that is, both $\mathcal{O}_2$ and $\tilde{\mathcal{O}}_2$) contribute. For simplicity we concentrate on $\mathcal{O}_2$ where only the axial-vector part of the c-quark current is relevant. Since all light degrees of freedom in this diagram enter DA of the massless pion ($q^2 = 0$), it is clear that the four-momentum flowing



(a) (b) (c)

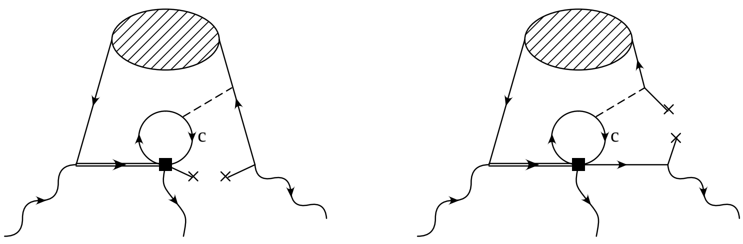
Figure 3: *(a),(b) Diagrams corresponding to the multiparticle DA's of the pion and arising from the expansion of the c-quark loop in the external gluon field; (c) the four-quark "soft" contribution in the case of the operators* $O^u_{1,2}$.

through the c-quark current has to be lightlike, i.e. the c-loop remains far off-shell (with a typical virtuality scale of $2m_c$ independent of m_b). Hence, one can use the local expansion of the c-quark axial-vector current in the gluon field obtained in [10]: $\bar{c}\gamma_\mu\gamma_5 c \to g_s^2/(192\pi^2 m_c^2)\left[\partial_\mu(G^a_{\rho\nu}\widetilde{G}^{a\rho\nu}) - 4(D_\alpha G^{a,\nu\alpha})\widetilde{G}^a_{\mu\nu}\right]$. The suppression factor with respect to the diagrams in Fig. 1 obtained by counting dimensions of the resulting operators (DA's) is for this contribution at least $O(1/(m_c^2 m_b^2))$. The presence of $\ln(m_c)$ and m_c^{-2} in the contributions of Fig.3a and 3b respectively, indicates that at $m_c \to 0$ these terms are divergent. In other words, if c quarks are replaced with the light quarks, e.g., in the case of $\mathcal{O}^u_1$, the light-quark pair propagates at long distances, that is, belongs to the pion four-quark DA. The corresponding diagram is the one shown in Fig. 3c.

As already mentioned, we neglect the contributions of four-quark DA's stemming from the matrix elements of the type $\langle 0 \mid \bar{u}(x_1)\bar{q}(x_2)q(x_3)d(x_4)|\pi\rangle$ (x_i on the light-cone). On the other hand, following [4] we take into account the factorizable parts of the 4-quark vacuum-pion matrix elements, extracting the configurations where one quark-antiquark pair forms the quark vacuum condensate, whereas the other one hadronizes into a twist 2 and 3 pion DA. Such contributions are enhanced by the large parameter μ_π. In the approximation adopted here, only two diagrams shown in Fig. 4 contribute to the sum rule (that is, have a nonvanishing contribution to the double dispersion relation in the duality region). Note that the quark-condensate diagram in Fig. 4b originates from the 4-quark diagram in Fig. 3a. The diagrams in Fig. 4 have only one loop and their calculation is relatively simple yielding the following result:

$$F^{(\tilde{\mathcal{O}}_2^c)}_{\langle q\bar{q}\rangle}(s_1,s_2,P^2) = -\frac{\alpha_s C_F f_\pi m_b \langle q\bar{q}\rangle}{12\pi}\int\limits_0^1 \frac{du}{(m_b^2 - us_2)s_1}\Bigg\{ I(uP^2+\bar{u}s_1, m_c^2)\Bigg[2s_2\varphi_\pi(u)$$
$$+3\mu_\pi m_b\varphi_p(u) + \frac{\mu_\pi}{6m_b}\Bigg(\left[\frac{P^2-s_1}{uP^2+\bar{u}s_1}\left(2+\frac{us_2}{m_b^2-us_2}\right) - \frac{s_2}{m_b^2-us_2}\right]\varphi_\sigma(u) - \varphi'_\sigma(u)\Bigg)\Bigg]$$
$$-I(0,m_c^2)\left[\frac{P^2-3s_2-s_1}{2}\varphi_\pi(u) - \mu_\pi m_b\left(3\varphi_p(u)+\frac{P^2-s_1}{6s_2}\varphi'_\sigma(u)\right)\right]\Bigg\}. \quad (6)$$

Multiparticle contributions which are factorized in the condensates of higher dimension are not taken into account. We also neglect the quark-condensate contributions of the type $\langle\bar{q}q\rangle\langle 0 \mid \bar{u}(x_1)G^a_{\mu\nu}(x_2)d(x_3) \mid \pi\rangle$ arising from the diagram in Fig. 3b after applying



(a) (b)

Figure 4: *Diagram corresponding to the factorizable 4-quark contribution to the correlation function.*

the QCD equation of motion to the derivatives of $G^a_{\mu\nu}$. These terms are suppressed at least by $O(1/m_c^2)$ with respect to the diagrams in Fig. 4.

Summarizing, we do not find significant contributions involving soft gluons in the OPE of the correlation function (2). The dominant effect arises from the c-quark loop annihilation into hard gluons (Fig. 1). In addition, there is the quark-condensate contribution (Fig. 4) which we consider a natural upper limit for all neglected contributions of multiparticle DA's. Importantly, the presence of the new intermediate scale $m_c \ll m_b$ in the correlation function does not noticeably enhance the charming penguin contribution: in the leading terms of OPE m_c enters logarithmically, and the inverse powers of m_c appear only in the subleading suppressed terms.

4. Having calculated the relevant contributions to the correlation function we are now in a position to obtain the sum rule for the hadronic matrix element $A^{(\tilde{\mathcal{O}}_2^c)}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv \langle \pi^-(p)\pi^+(-q)|\tilde{\mathcal{O}}_2^c|\bar{B}_d^0(p-q)\rangle$. The derivation of LCSR explained in detail in [2, 4] includes: use of the dispersion relations for the invariant amplitude $F(s_1, s_2, P^2)$ in the variables s_1 and s_2 ; employing quark-hadron duality in both pion and B meson channels with the threshold parameters s_0^π and s_0^B, respectively; the Borel transformations and the continuation from large spacelike P^2 to large timelike $P^2 = m_B^2$. As a result, LCSR is given by the following expression:

$$f_\pi f_B A^{(\tilde{\mathcal{O}}_2^c)}(\bar{B}_d^0 \to \pi^+\pi^-) e^{-m_B^2/M_2^2}$$
$$= \int\limits_{m_b^2}^{s_0^B} ds_2 e^{-s_2/M_2^2} \left\{ \int\limits_0^{s_0^\pi} ds_1 e^{-s_1/M_1^2} \mathrm{Im}_{s_2} \mathrm{Im}_{s_1} F(s_1, s_2, P^2) \right\}_{P^2 \to m_B^2} , \qquad (7)$$

where M_1 and M_2 are the Borel parameters in the pion and B-meson channels, respectively. Using Eqs. (3),(5) and (6), we obtain from Eq. (7) the "charming penguin" hadronic matrix element at $O(\alpha_s)$ and with twist 2 and 3 accuracy :

$$A^{(\tilde{\mathcal{O}}_2^c)}(\bar{B}_d^0 \to \pi^+\pi^-) = A_{Fig.1} + A_{\langle \bar{q}q \rangle} , \qquad (8)$$

where

$$
\begin{aligned}
A_{Fig.1} &= i\frac{\alpha_s C_F}{\pi} m_B^2 \left(\int\limits_0^{s_0^\pi} \frac{ds e^{-s/M_1^2}}{4\pi^2 f_\pi} \right) \frac{m_b^2 f_\pi}{2m_B^2 f_B} \int\limits_{u_0^B}^1 \frac{du}{u} e^{(m_B^2 - \frac{m_b^2}{u})/M_2^2} \\
&\times \Bigg\{ \int\limits_0^1 dz\, I(z m_B^2 u, m_c^2) \Bigg[z(1-z)\varphi_\pi(u) \\
&+(1-z)\frac{\mu_\pi}{2m_b} \left[u\varphi_p(u) \left(2z + \frac{m_b^2}{u m_B^2} \right) + \left(\frac{\varphi_\sigma(u)}{3} - \frac{u\varphi'_\sigma(u)}{6} \right) \left(2z - \frac{m_b^2}{u m_B^2} \right) \right] \Bigg] \\
&- \frac{m_b \mu_\pi}{4m_B^2} \int\limits_0^1 dz\, I(-z m_b^2 \overline{u}/u) \frac{\overline{u}^2}{u} \left[\varphi_p(1) \left(1 + \frac{3m_b^2}{u m_B^2} \right) + \frac{\varphi'_\sigma(1)}{6} \left(1 - \frac{5m_b^2}{u m_B^2} \right) \right] \Bigg\} \qquad (9)
\end{aligned}
$$

is the contribution of the diagrams in Fig. 1 and

$$
\begin{aligned}
A_{\langle q\bar{q}\rangle} &= i\frac{\alpha_s C_F}{6\pi} m_b^2 \left(-\frac{\langle q\bar{q}\rangle}{f_\pi m_b} \right) \left(\frac{m_b^2 f_\pi}{2m_B^2 f_B} \right) \int\limits_{u_0^B}^1 \frac{du}{u^2} e^{(m_B^2 - \frac{m_b^2}{u})/M_2^2} \Bigg\{ I(u m_B^2, m_c^2) \\
&\times \left[2\varphi_\pi(u) + \frac{\mu_\pi}{m_b} \left(3u\varphi_p(u) + \frac{\varphi_\sigma(u)}{3} - \frac{u\varphi'_\sigma(u)}{6} \right) \right] \\
&+ I(0, m_c^2) \left[\frac{\varphi_\pi}{2} \left(3 - \frac{m_B^2 u}{m_b^2} \right) + \frac{\mu_\pi}{m_b} \left(3u\varphi_p(u) + \frac{m_B^2 u^2}{6 m_b^2} \varphi'_\sigma(u) \right) \right] \Bigg\} \qquad (10)
\end{aligned}
$$

is the factorizable 4-quark contribution. In the above, $u_0^B = m_b^2/s_0^B$. The sum rule (10) is obtained at finite m_b but we neglect numerically very small corrections of order s_0^π/m_B^2. Note that the c-quark loop factor $I(z m_B^2 u, m_c^2)$ in the sum rule originating from the Fig. 1a diagram has a complex phase which is generated by the analytic continuation in P^2 and has to be associated with the strong-interaction phase (in the quark-hadron duality approximation). The BSS mechanism [5] is thus recovered in LCSR.

To compare our result with QCD factorization we investigate the heavy-quark mass limit of the sum rule (8), expanding all heavy-mass dependent quantities in powers of m_b. At $m_b \to \infty$ only the diagram of Fig. 1a contributes. In this limit duality interval of the u-integration in Eq. (9) reduces to the end-point region allowing one to put $u \to 1$ in the c-quark loop factor. After that, the integral over u multiplying this factor reproduces LCSR for the $B \to \pi$ form factor [11]. To see that one has to use the relation [12] following from the QCD equation of motion : $u\varphi_p(u) + \varphi_\sigma(u)/3 - u\varphi'_\sigma(u)/6 = 0$ (neglecting the quark-antiquark gluon DA's). After these transformations we obtain :

$$
A^{(\tilde{\mathcal{O}}_2^c)}(\bar{B}_d^0 \to \pi^+\pi^-)\Big|_{m_b\to\infty} = \frac{\alpha_s C_F}{\pi} \left(\int\limits_0^1 dz z(1-z) I(z m_b^2, m_c^2) \right) \left(i\sqrt{m_b} f_\pi^{(SVZ)} \hat{f}^+_{B\pi}(0)^{(LCSR)} \right). \qquad (11)
$$

In the above, we have denoted the integral over s in Eq. (9) by $f_\pi^{(SVZ)}$ indicating that it coincides with the leading-order SVZ sum rule for f_π [13]. Furthermore we have replaced the LCSR $B \to \pi$ form factor by an m_b-independent effective form factor $f^+_{B\pi}(0)^{(LCSR)} = \hat{f}^+_{B\pi}(0)^{(LCSR)}/m_b^{3/2}$. Thus, the charming penguin matrix element at $m_b \to \infty$ factorizes into the c-loop integral and the factorizable $B \to \pi\pi$ amplitude equal to the matrix element of $\mathcal{O}^u_1$ in the emission topology: $A_E^{(\mathcal{O}^u_1)}(\bar{B}^0_d \to \pi^+\pi^-)_{m_b\to\infty} = i\sqrt{m_b} f_\pi \hat{f}^+_{B\pi}(0)$. Furthermore, since the asymptotic twist-2 pion DA has the form $\varphi_\pi(z) = 6z(1-z)$, the limiting expression (11) reproduces the c-quark penguin-loop contribution to the $B \to \pi\pi$ amplitude in the QCD factorization [1] at the twist 2 level. For brevity, we omit a more detailed discussion of various $1/m_b$ terms which at the end lead to a numerical deviation of the LCSR (8) from the $m_b \to \infty$ limit. We only note that the contribution of the diagram in Fig. 1b which does not have a counterpart in QCD factorization is suppressed as $1/m_b^2$. Strictly speaking this diagram turns out to be beyond the adopted accuracy since in LCSR we have neglected other small $O(1/m_b^2)$ terms.

We conclude this section with a comment on the penguin contractions of the current-current operator $\mathcal{O}^u_1$. The corresponding sum rule for the hadronic matrix element $A_P^{(\tilde{\mathcal{O}}^u_2)}(\bar{B}^0_d \to \pi^+\pi^-) \equiv \langle \pi^-\pi^+|\tilde{\mathcal{O}}^u_2|\bar{B}^0_d\rangle_P$ is easily obtained from LCSR (8) by putting $m_c \to 0$ everywhere except in the second term of the quark condensate contribution (10). As we already discussed this term emerges from the factorization of the diagram in Fig. 3a and in the case of the light quarks it has to be absorbed in the four-quark DA's (Fig. 3c). This four-quark DA contribution does not contain any α_s suppression. However, counting the dimensions we find that it is power-suppressed, at least by $O(1/m_b^3)$.

5. For a numerical estimate of the charming penguin in $B \to \pi\pi$ decay we calculate the ratio of the sum rule (8) to the factorizable amplitude $A_E^{(\mathcal{O}^u_1)}(\bar{B}^0_d \to \pi^+\pi^-) = i m_B^2 f_\pi f^+_{B\pi}(0)$:

$$r^{(\mathcal{O}^c_1)}(\bar{B}^0_d \to \pi^+\pi^-) \equiv \frac{A^{(\mathcal{O}^c_1)}(\bar{B}^0_d \to \pi^+\pi^-)}{A_E^{(\mathcal{O}^u_1)}(\bar{B}^0_d \to \pi^+\pi^-)} \simeq \frac{2A^{(\tilde{\mathcal{O}}^c_2)}(\bar{B}^0_d \to \pi^+\pi^-)}{i m_B^2 f_\pi f^+_{B\pi}(0)^{(LCSR)}}\,. \tag{12}$$

We use the same input as in [4] and in the numerical analysis of the LCSR for the $B \to \pi$ form factor [11]. The following parameters are taken for the pion channel: $f_\pi = 132$ MeV, $s_0^\pi = 0.7$ GeV^2, $M_1^2 = 0.5$-1.5 GeV^2; and for the B channel: $m_b = 4.7 \pm 0.1$ GeV (the one-loop pole mass), $s_0^B = 35 \mp 2$ GeV^2, $M_2^2 = 8$-12 GeV^2. The normalization scale adopted for the pion DA's and α_s is $\mu_b = \sqrt{m_B^2 - m_b^2} \simeq 2.4$ GeV. For the c quark mass we take $m_c = 1.3 \pm 0.1$ GeV, and for the quark condensate density $\langle \bar{q}q\rangle(1\text{GeV}) = -(240 \pm 10 \text{ MeV})^3$, or, equivalently, $\mu_\pi(1\text{GeV}) = 1.59 \pm 0.2$ GeV. All pion light-cone DA's are taken in the asymptotic form: $\varphi_\pi(u) = \varphi_\sigma(u) = 6u(1-u)$, $\varphi_p(u) = 1$. To get a feeling how the nonasymptotic form of the pion DA influences our result we also recalculate Eq. (12) using a model for $\varphi_\pi(u)$ with a nonzero second Gegenbauer coefficient $a_2(1\text{GeV}) = 0.4$. With the above input and adding the uncertainties caused by the variation of all parameters linearly, we get the following range:

$$r^{(\mathcal{O}^c_1)}(\bar{B}^0_d \to \pi^+\pi^-) = [-(0.29 \div 0.56) - (1.3 \div 1.6)i] \cdot 10^{-2}\,. \tag{13}$$

The corresponding estimate for the penguin contraction of the operator $\mathcal{O}^u_1$ is:

$$r_P^{(\mathcal{O}^u_1)}(\bar{B}^0_d \to \pi^+\pi^-) = [(0.09 \div 0.21) - (1.6 \pm 2.1)i] \cdot 10^{-2}\,. \tag{14}$$

The penguin-topology contributions turn out to be very small, not larger than the other nonfactorizable corrections in $B \to \pi\pi$, however, as we shall see in the next section, they add up to a noticeable effect in the CP asymmetry. The contribution of the quark condensate term in the sum rule to the real parts of Eqs. (13) and (14) is relatively small, but constitutes about 50% of the imaginary parts.

6. It is well known that the penguin contributions play a key role in the direct and mixing-induced CP violation in $B \to \pi\pi$. The time-dependent CP-asymmetry is given by

$$\mathcal{A}_{\rm CP}(B_d^0 \to \pi^+\pi^-)(t) = a_{\rm CP}^{\rm dir}\cos(\Delta M_d\, t) + a_{\rm CP}^{\rm mix}\sin(\Delta M_d\, t)\,, \tag{15}$$

where $a_{\rm CP}^{\rm dir} \equiv (1-|\xi|^2)/(1+|\xi|^2)$ and $a_{\rm CP}^{\rm mix} \equiv (2\,{\rm Im}\,\xi)/(1+|\xi|^2)$, with $\xi = e^{-2i(\beta+\gamma)}(1+R\,e^{i\gamma})/(1+R\,e^{-i\gamma})$ and $R \equiv -P/(R_bT)$. Here T is the contribution to the $B \to \pi\pi$ amplitude proportional to $V_{ub}V_{ud}^* = |V_{ub}V_{ud}^*|e^{-i\gamma}$ and contains also the penguin contraction of the current-current operator $\mathcal{O}_1^u$. The remaining amplitude P is proportional to $V_{cb}V_{cd}^*$. The factor $R_b = |V_{ub}||V_{ud}|/(|V_{cb}||V_{cd}|)$ as usual defines one side of the unitarity triangle. We take $R_b = 0.39 \pm 0.04$.

Strong phases originate from both T and P; thus we have $T = |T|e^{i\delta_T}$ and $P = |P|e^{i\delta_P}$ and

$$a_{\rm CP}^{\rm dir} = \frac{-2|R|\,\sin(\delta_P-\delta_T)\,\sin\gamma}{1-2|R|\,\cos(\delta_P-\delta_T)\,\cos\gamma+|R|^2}\,. \tag{16}$$

In the previous sections we have calculated the contributions arising from the penguin contractions of the operators $\mathcal{O}_1^c$ and $\mathcal{O}_1^u$. In addition to these we have also to take into account the tree and penguin contributions of the penguin operators $\mathcal{O}_{3-6}$ and the gluonic penguin contribution of the dipole operator $\mathcal{O}_{8g}$. For the latter we use the LCSR result from [4]. The electroweak penguin contributions to $B \to \pi\pi$ are color-suppressed and very likely negligible.

Apart from the penguin contractions one also has to include the hard nonfactorizable $O(\alpha_s)$ and soft $1/m_b$ corrections (in the emission topology) to both T and P, obtained, respectively in the QCD factorization approach [1] and from LCSR [2]. It turns out that their contributions to the CP-asymmetry are negligibly small. To this end, we can numerically study $a_{\rm CP}^{\rm dir}$ using the penguin contributions calculated from LCSR at finite m_b and compare the result to the infinite-mass limit that agrees with the QCD factorization prediction [1]. The result for $a_{\rm CP}^{\rm dir}$ as a function of γ is shown in Fig. 5. In this calculation the Wilson coefficients in H_{eff} are taken at the same scale $\mu_b \simeq m_b/2$ as used in LCSR. Let us emphasize that this is not yet the final prediction because the annihilation effects in $B \to \pi\pi$ are not included in this analysis. The main aim of this numerical illustration is to demonstrate that there is a difference between the finite m_b and the $m_b \to \infty$ result and that this difference could be sizable (within the uncertainty of the LCSR prediction).

7. Concluding, we estimated the $B \to \pi\pi$ hadronic matrix elements with penguin topology for the current-current operators with c and u quarks using the LCSR approach. The main contribution to the sum rule stems from the $O(\alpha_s)$ quark loop annihilating to a hard gluon. This contribution determines the strong phase of the hadronic matrix element justifying the use of the (perturbative) BSS mechanism. The soft-gluon effects, which in the sum rule approach correspond to multiparticle pion DA's, are suppressed, at least by $O(\alpha_s/m_b^2)$. In $m_b \to \infty$ limit our result agrees with the QCD factorization prediction for the penguin contractions. In both approaches a small value of the direct CP asymmetry in $B_d^0 \to \pi^+\pi^-$ is expected, due to the fact that the strong phase is generated perturbatively. At finite m_b we predict an even smaller numerical value for this effect, indicating that $O(\alpha_s/m_b)$ corrections are important. Our result does not support

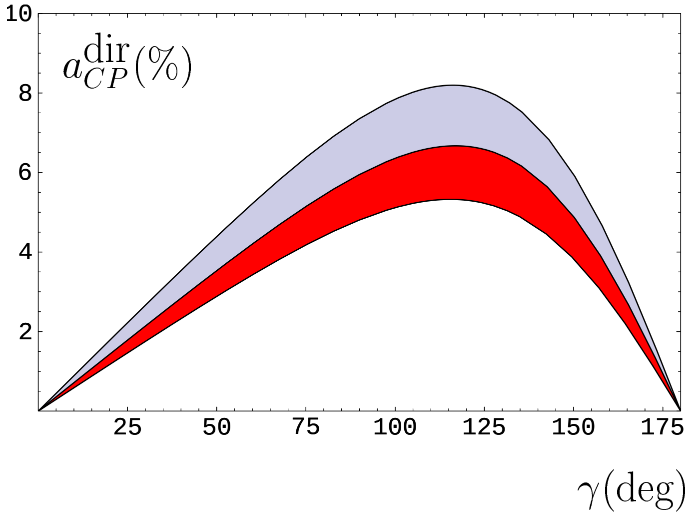


Figure 5: *Direct CP asymmetry in* $B \to \pi\pi$ *as a function of the CKM angle* γ. *The upper curve is the result obtained for* $m_b \to \infty$. *The dark region is the LCSR result, with all uncertainties from the method included (uncertainties in the CKM matrix elements are not taken into account). The light region shows the deviation from the* $m_b \to \infty$ *limit result.*

models employing hadronic dispersion relations with intermediate charmed meson pairs, predicting sizable charming-penguin effects [14]. The method and the results of this paper can be used for analysing penguin effects in other charmless two-body B decays. The remaining problem for these decays is the estimate of the annihilation effects which is however a very challenging and technically difficult task even for the LCSR method.

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