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Measuring the Higgs boson parity at a Linear Collider using the τ impact parameter and $\tau \to \rho\nu$ decay

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Abstract

We demonstrate that a measurement of the impact parameter in one-prong τ decay can be useful for the determination of the Higgs boson parity in the $H/A \to \tau^+\tau^-$; $\tau^\pm \to \rho^\pm \bar{\nu}_\tau(\nu_\tau)$ decay chain. We have estimated that for a detection set-up such as TESLA, use of the information from the τ impact parameter can improve the significance of the measurement of the parity of the Standard Model 120 GeV Higgs boson to $\sim 4.5\sigma$, and in general by factor of about 1.5 with respect to the method where this information is not used.

We also show that the variation in the assumption on the precision of the measurement of the impact parameter and/or π's momenta does not affect the sensitivity of the method. This is because the method remains limited by the type of twofold ambiguity in reconstructing the τ momentum.

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1 Introduction

One of the most important goals in the scientific program of a future linear electron–positron collider (LC) is to measure the properties of the Higgs boson precisely. Among them, the parity plays a prominent role. Depending on the mass and the Higgs mechanism, the Higgs parity can be measured by different means [1–5]. In some cases, it can be best determined by investigating the properties of the Higgs boson decay into τ leptons [6, 7]. In [8] we studied the possibility of distinguishing a scalar from a pseudoscalar coupling of a light boson to fermions using its decay to a pair of τ leptons and their subsequent decays $\tau^\pm \to \rho^\pm \bar{\nu}_\tau(\nu_\tau)$ and $\rho^\pm \to \pi^\pm \pi^0$. This decay chain gives an independent test of model and production mechanism. In fact, we have found that the angular distributions of the $\tau^\pm$ decay products, which clearly distinguish the different parity states, are measurable using typical properties of a future detector at an e^+e^- linear collider. A special Monte Carlo technique was developed for that purpose [9]. In the present paper we will extend that analysis, using the information that can be obtained from the measurement of the τ lepton impact parameter.

It turns out that the kinematic configuration of $e^+e^- \to \tau^+\tau^-$ events cannot be fully determined since the momenta of both ν_τ and $\bar{\nu}_\tau$ are unknown. The lost information can be recovered partially by measuring energy and directions of all visible particles [10, 11]. When both τ leptons decay hadronically and all hadron momenta are recovered, the original τ direction can be obtained up to a twofold ambiguity [10, 12]. In the method, the requirement that the two τ leptons be essentially of equal energy and back to back in some known frame, is decisive. The measurement of the tracks of the hadrons, in particular their relative impact parameters, allows us to resolve this twofold ambiguity, see for instance [10, 11, 13].

In the case of linear colliders such as TESLA [14], a sufficiently precise reconstruction of the frame, where τ leptons are expected to be back to back turns out to be impossible, mainly due to beamstrahlung effect. This is a strong limitation for the Higgs boson parity measurement as proposed in Refs. [6,7]. A way around was found in Ref. [8]. Here, we will investigate if the measurement of the τ lepton impact parameter direction, with precision as expected for TESLA, can be used to optimize that method.

This work is organized as follows: in section 2 the geometry of τ production and decay is described, as well as our way to use the τ impact parameter in the measurement of the Higgs boson parity. In section 3 some details of the `TAUOLA` Monte Carlo simulation are given. Section 4 is devoted to the definitions used in our simulation of detector properties. In section 5 definitions of our observable and numerical results are given. The summary, section 6, closes the paper.

2 Description of the method

In the decay of the τ lepton into one charged particle, the τ impact parameter can be defined as the distance of closest approach (in the plane perpendicular to the beam axis) of the charged particle to a reference point, which is assumed to be the production point of the τ (see Fig. 1). A positive sign is given to this quantity if the track crosses the τ direction in the same hemisphere, relative to the τ production point, in which it lays, and negative otherwise. The production point is assumed to lie on the beam line. In our study we will exploit the possibility to determine the direction of the impact parameter, rather than the related distance. In fact we will be interested, where the τ production point is localized with respect to the charged pion track.

Before we address the question of the information available from the impact parameter, let us recall some details of the actual observable proposed in Ref. [8] for the Higgs boson parity measurement. The method relies on measuring the acoplanarity angle of the two planes, spanned on $\rho^{\pm}$ decay products and defined in the $\rho^+\rho^-$ pair rest frame; some additional selection cuts also need to be applied. Let us present the elements of the observable as the following points.

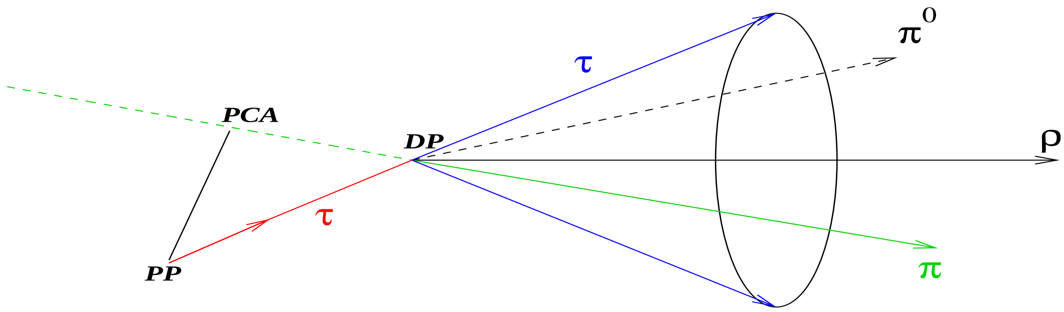

Figure 1: *Schematic view of the τ decay to ρ and ν_τ. PP is the τ production point, DP the τ decay point, and PCA the point of closest approach. Directions and energies of $\pi^{\pm}$, π^0, $\rho^{\pm}$ are directly measurable, the energy of the $\tau^{\pm}$ can be reconstructed from the event topology. The τ momentum must be placed on the cone (drawn on the plot) and the τ impact parameter can be used to determine the actual position.*

1. **Acoplanarity of the ρ^+ and ρ^- decay planes:**

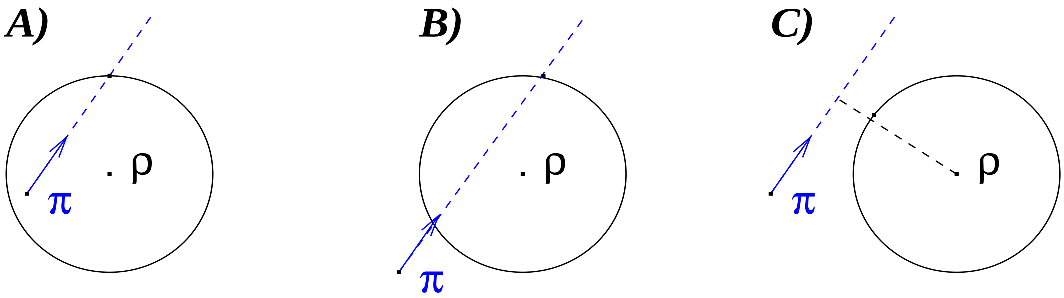

Figure 2: *The circle of candidates for τ momentum. Depending on the possible geometrical configuration, three cases are possible: A) π direction inside the circle: single candidate for τ momentum. B) π direction outside the circle: two candidates for τ momentum. C) π direction outside the circle and no crossing of dashed line with the circle (due to detection ambiguities of measured angles and energies). We take in this case the closest point to the line. Effectively, we obtain a single solution.*

- The reconstructed four-momenta of π^+ and π^0 are combined to yield the ρ^+ four-momentum. The same is done for the ρ^-.
- All reconstructed four-momenta are boosted into the $\rho^+\rho^-$ rest frame.
- The angle φ between the planes of the ρ^+ and ρ^- decay products in this frame is the acoplanarity.

2. **Normalized energy differences:**
   The events are divided into two classes depending on the value of $y_1 y_2$, where

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}} \; ; \qquad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}. \tag{1}$$

   The energies of $\pi^\pm, \pi^0$ are to be taken in the respective $\tau^\pm$ rest frames.

3. **Replacement $\boldsymbol{\tau}$ rest frame:**
   In Ref. [8] the method of reconstruction of *replacement* $\tau^\pm$ *rest frames* was proposed. We will use this method here as well, but we will omit the details. In fact they are rather unimportant. The energies of pions taken directly from the laboratory frame can equally well be used in the above definitions of Eq. (1).

4. **Higgs rest frame:**
   If the information on the beam energies and the energies of all other observed

particles is taken into consideration, the Higgs rest frame can be reconstructed, for instance in the Higgsstrahlung production process, $e^+e^- \to HZ$, when the Z boson decays either into a charged lepton pair or hadronically. We may define the 'reconstructed' Higgs boson momentum as the difference of the sum of beam momenta and momenta of all visible particles, i.e. decay products of Z and all radiative photons of $|\cos\theta| < 0.98$.

5. **$\boldsymbol{\tau}$ energy:**

   - In the reconstructed Higgs boson rest frame, the τ four-momenta are estimated in a crude way by assuming the direction of the respective ρ's and an energy of $m_H/2$.
   - The τ momenta are boosted back to the laboratory frame and their energies are taken to be the reconstructed energies of τ leptons.

6. **$\boldsymbol{\tau}$ direction:**
   The τ direction in the laboratory system is constrained by two requirements:

   - It has to lie on a cone around the ρ direction with opening angle ψ defined as:

   $$\cos\psi = \frac{2E_\tau E_\rho - m_\tau^2 - m_\rho^2}{2\vec{p}_\tau \vec{p}_\rho}, \tag{2}$$

     where E, $\vec{p}$, m are the energy, momentum and mass, respectively. The four-momentum of ρ is measured as the sum of the four-momenta of its charged and neutral daughters.
   - It has to lie in the plane spanned by the vector pointing from PP to PCA and the π^+ momentum.

   The intersection of the cone and the plane is calculated numerically. There are three cases:

   - One solution: the solution is taken as the τ direction (see Fig. 2 case A).
   - Two solutions: one of the two solutions (see Fig. 2 case B) is taken on a random basis.
   - No solution: the direction on the cone closest to the PCA–π^+ momentum plane is taken (see Fig. 2 case C).

7. **Impact parameter improved** *replacement* $\boldsymbol{\tau}$*-rest frame*:
   With the help of the τ energy and τ momentum defined in points 5 and 6 above, a new impact parameter improved *replacement* τ *rest frame* can be defined. In this way we have an alternative way of estimating the difference of $\pi^\pm$, π^0 energies see Eq. (1) in $\tau^\pm$ rest frames.

3 Monte Carlo set-up

In the following discussion all the Monte Carlo samples have been generated with the `TAUOLA` library [15–17]. For the production of τ lepton pairs, the Monte Carlo program `PYTHIA 6.1` [18] was used[1]. The production process $e^+e^- \to ZH \to \mu^+\mu^-(q\bar{q})H$ has been chosen, with a Higgs boson mass of 120 GeV and a centre-of-mass energy of 350 GeV.

The effects of initial-state bremsstrahlung were included in the `PYTHIA` generation. For the $\tau^\pm$ lepton pair decay (with full spin effects from the $H \to \tau^+\tau^-$, $\tau^\pm \to \rho^\pm\bar{\nu}_\tau(\nu_\tau)$, $\rho^\pm \to \pi^\pm\pi^0$ chain) the interface explained in Ref. [9] was used. It is an extended version of the interface of Refs. [20, 21].

4 Detection parameters

To test the feasibility of the measurement, some assumptions about the detector effects have to be made. We include, as the most critical for our discussion, effects due to inaccuracies in the measurements of the $\pi^\pm, \pi^0$ momenta and $\tau^\pm$ leptons impact parameters. We assume Gaussian spreads of the 'measured' quantities with respect to the generated ones, and we use the following algorithm to reconstruct the energies of π's in their respective $\tau^\pm$ rest frames.

1. **Charged-pion momentum:** We assume a 0.1% spread on its energy and direction.
2. **Neutral-pion momentum:** We assume an energy spread of $\frac{5\%}{\sqrt{E[\mathrm{GeV}]}}$. For the θ and ϕ angular spread we assume $\frac{1}{3}\frac{2\pi}{1800}$. (These neutral pion resolutions can be achieved with a 15% energy error and a $2\pi/1800$ direction error in the gammas resulting from the π^0 decays.) These resolutions have been approximately verified with a `SIMDET` [22], a parametric Monte Carlo program for a TESLA detector [23].

[1] It was shown that the interface can work in the same manner with the `HERWIG` [19] generator.

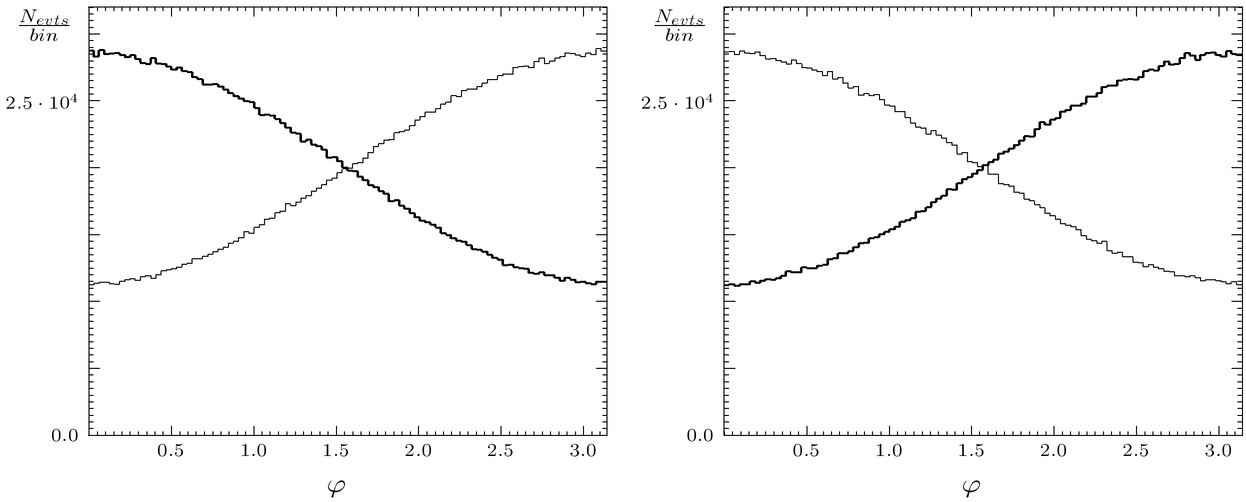

Figure 3: *The acoplanarity distribution angle φ of the $\rho^+\rho^-$ decay products in the rest frame of the $\rho^+\rho^-$ pair. Detector smearing is included. Generator level $\tau^\pm$ rest frames are used. The thick line corresponds to a scalar Higgs boson, the thin line to a pseudoscalar one. The left figure contains events with $y_1y_2 > 0$, the right is for $y_1y_2 < 0$.*

3. **The reconstructed Higgs boson rest frame:** We assume a spread of 2 GeV with respect to the transverse momentum of the Higgs boson, and 5 GeV (because of the beamstrahlung effect) for the longitudinal component.

4. **The impact parameter:**
The angular resolution of the vector pointing from PP to PCA (see Fig. 1) has been simulated for the TESLA-like detector. The simulation is based on the anticipated performance of a 5-layer CCD vertex detector as described in [23]. For Higgsstrahlung events with $H \to \tau^+\tau^-$ and $\tau^\pm \to \rho^\pm\bar{\nu}_\tau(\nu_\tau)$ at $m_H = 120$ GeV and $\sqrt{s} = 350$ GeV, the angular resolution has been found to be approximately 25°. We use this resolution in our simulations.

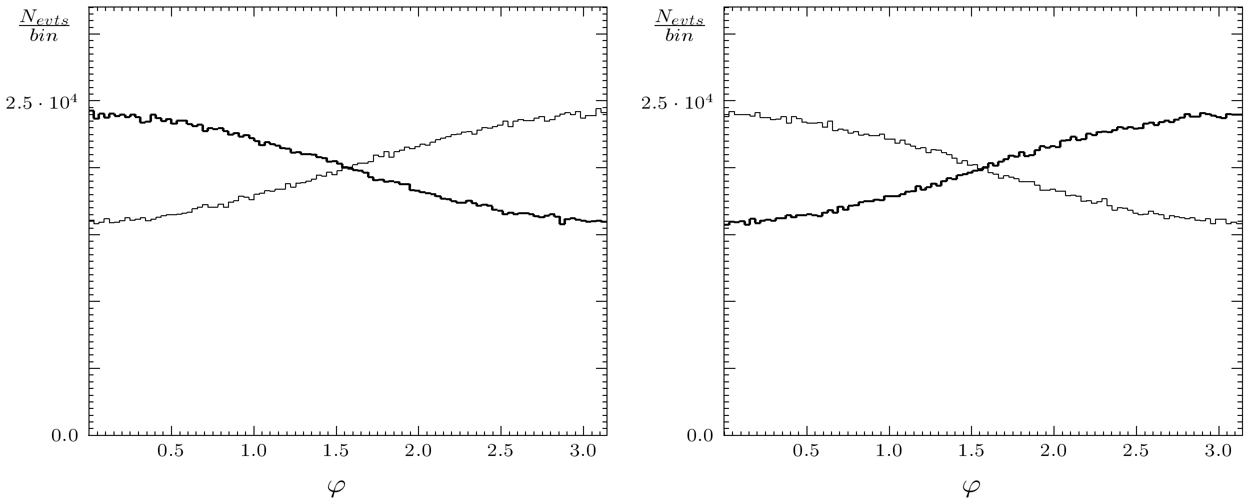

Figure 4: *The acoplanarity distribution angle φ of the $\rho^+\rho^-$ decay products in the rest frame of the $\rho^+\rho^-$ pair. Full smearing is included. Replacement $\tau^\pm$ rest frames defined as in Ref. [8] are used. The thick line corresponds to a scalar Higgs boson, the thin line to a pseudoscalar one. The left figure contains events with $y_1y_2 > 0$, the right is for $y_1y_2 < 0$.*

5 Acoplanarity of the ρ^+ and ρ^- decay products with the help of the impact parameter

For all figures in our paper we will use the same angle φ built from smeared four-momenta of the $\rho^\pm$ decay products. The usefulness of this acoplanarity distribution manifests itself only after selection cuts are applied.

In Fig. 3, our $\rho^+\rho^-$ decay products acoplanarity distribution angle φ, defined in the rest frame of the reconstructed $\rho^+\rho^-$ pair is shown. Unobservable generator-level $\tau^\pm$ rest frames are used for the calculation of selection cuts (Eq. (1)). The two plots represent events selected by the differences of $\pi^\pm$ π^0 energies defined in these respective $\tau^\pm$ rest frames. On the left plot, the signs are required to be the same $y_1y_2 > 0$, whereas on the right one, events are taken with $y_1y_2 < 0$. This figure quantifies the size of the parity effect in an idealized condition, which we will attempt to approach with realistic ones. The size of the effect was substantially diminished when the detector set-up was included for $\tau^\pm$ rest frames reconstruction as well, see Fig. 4, following exactly the method of Ref. [8].

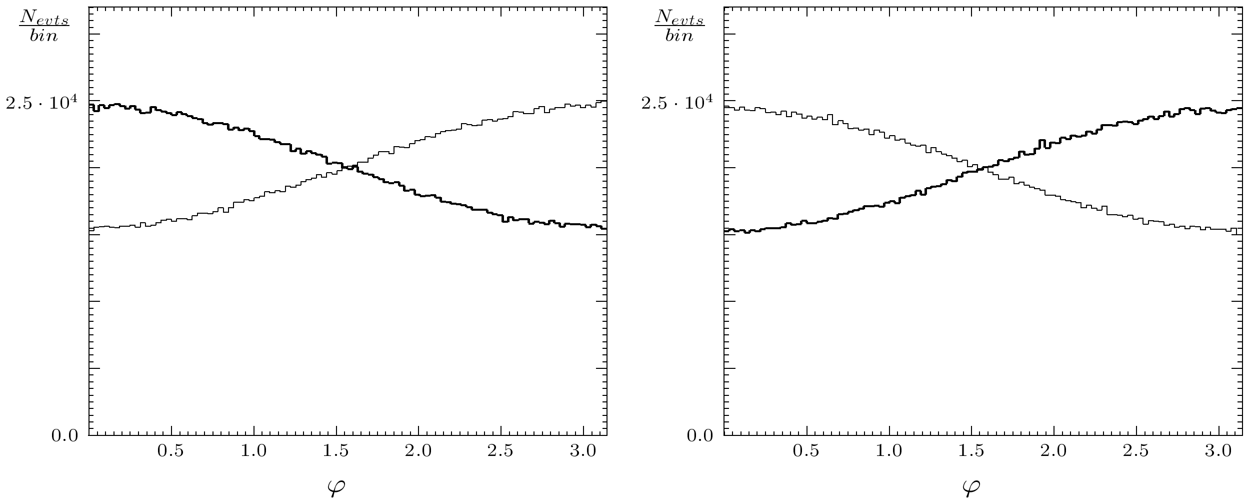

Figure 5: *The $\rho^+\rho^-$ decay products' acoplanarity distribution angle, φ, in the rest frame of the $\rho^+\rho^-$ pair. Full smearing is included. The τ lepton impact parameter is used in the reconstruction of the $\tau^\pm$ rest frames. The thick line corresponds to a scalar Higgs boson, the thin line to a pseudoscalar one. The left figure contains events with $y_1 y_2 > 0$, the right is for $y_1 y_2 < 0$.*

Let us use this result later, as a reference point to quantify the size of our improvements.

It turns out that a slight improvement is visible, when the impact parameter is taken for the $\tau^\pm$ rest frames determination, as explained in section 4. In fact, the maximum relative difference between the scalar and pseudoscalar cases is larger by $\sim 12\%$ (compare Figs. 4 and 5) if a Gaussian spread of the τ impact parameter $\sigma_{IP} = 25^\circ$ is used. We have also checked that the precision of the τ lepton impact parameter measurement is not critical in this case. The twofold ambiguities as explained in Fig. 2 are the main reason for the sensitivity to be much lower than in Fig. 3.

At the cost of introducing cuts and thus reducing the number of accepted events, we can go one step further in the improvement of the method. We first start by requiring that the signs of the energy differences y_1 and y_2 be the same with both methods (with and without τ lepton impact parameter). The result is depicted in Fig. 6. Only $\sim 52\%$ of events are accepted, but the parity effect grows by $\sim 107\%$ in comparison to Fig. 4.

The events can also be treated differently, according to the number of solutions; see Fig. 2. If the same cut is imposed only for $\tau^\pm$ leptons with double solutions, while for

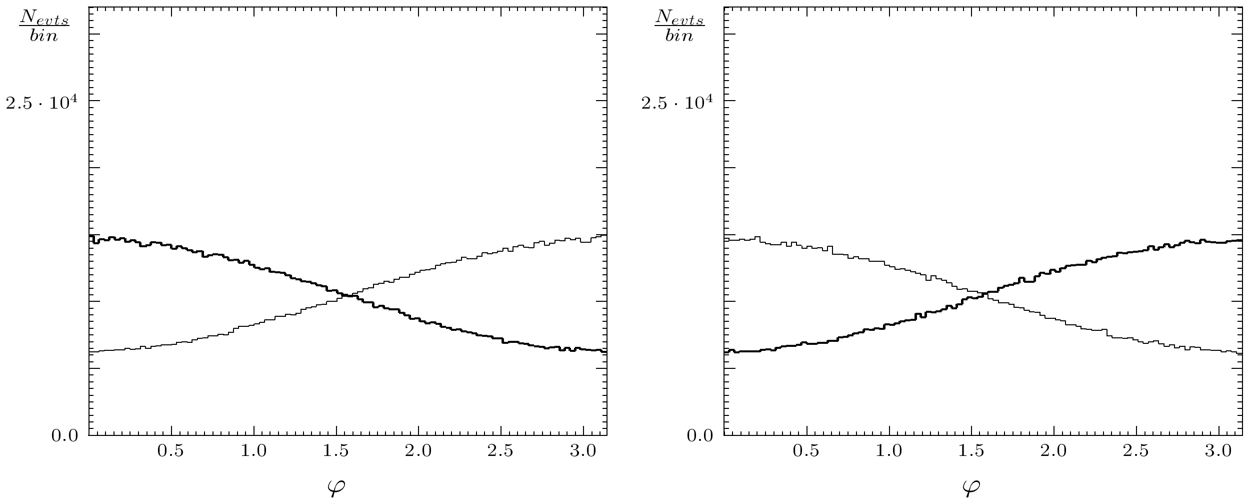

Figure 6: *The $\rho^+\rho^-$ decay products' acoplanarity distribution angle, φ, in the rest frame of the $\rho^+\rho^-$ pair. Only events where the signs of the energy differences y1 and y2 are the same whether calculated using the method described in Ref. [8] or with the help of the τ impact parameter are taken. The thick line corresponds to a scalar Higgs boson, the thin line to a pseudoscalar one.*

single-solution events the $\tau^\pm$ rest frames obtained with the help of the impact parameter are used alone, the number of accepted events turns out to be $\sim 72\%$, but the parity effect is larger by only $\sim 67\%$ than in Fig. 4. This result is depicted in Fig. 7.

Other cuts are also envisageable, but the two proposed here are easy to implement and lead to an acceptable loss of statistics, together with a sizeable increase of parity effect so that there is not much room for further improvements.

6 Summary

We have found that the measurement of the impact parameter can be helpful in the determination of the Higgs boson parity at a future linear collider, using the $H/A \to \tau^+\tau^-$; $\tau^\pm \to \rho^\pm\bar{\nu}_\tau(\nu_\tau)$ cascade decay. Thanks to the improved method the effect of parity is enhanced by $\sim 107\%$ at the cost of losing only $\sim 48\%$ events, which represents a gain of a factor of ~ 1.5 in sensitivity with respect to the analysis without impact parameter.

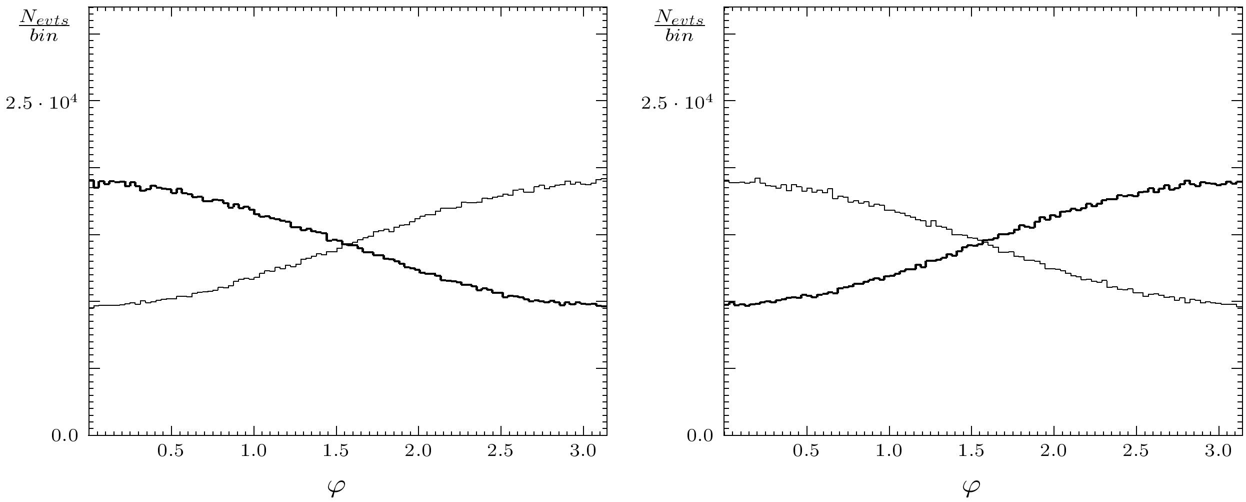

Figure 7: *he $\rho^+\rho^-$ decay products' acoplanarity distribution angle, φ, in the rest frame of the $\rho^+\rho^-$ pair, with cuts depending on the position of the pion with respect to the τ cone. The selection is as in Fig. 6, but in the case of single-solution events (Fig. 2 case A), the result from the old method is ignored. The thick line corresponds to a scalar Higgs boson, the thin line to a pseudoscalar one.*

This correspond to $\sim 4.5\sigma$ separation between scalar and pseudoscalar for the 120 GeV Standard Model Higgs and other assumptions similar to the one taken in Ref. [8].

We have also checked, that reasonable variations in the assumptions of detection uncertainties in measurement of charged and neutral π's as well as of the τ^+ and τ^- impact parameters, hardly change the sensitivity of our method at all. This is because the dominant uncertainty is saturated by the twofold ambiguity as explained in Fig. 2. We are afraid that a reconstruction of the Higgs boson momentum to precision better than the τ mass would be needed to resolve the ambiguity. This seems unrealistic for several reasons, e.g. the beamstrahlung effect.

Probably, a fit to the multidimensional distribution might be more efficient than the simple cuts applied in this work. We believe, however, that such studies should be done only when the detector properties have been understood better. Finally, we expect that further improvements, e.g. involving the $\tau^\pm \to a_1^\pm \nu_\tau$ decay mode, may turn out to be helpful for the measurement of the Higgs boson parity.

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