Two-Loop Electroweak Corrections to Δr

A. Onishchenko[ab*] and O. Veretin[b†]

[a] State Research Center, Institute for High Energy Physics,
Protvino, Moscow Region, 142284 Russia Mathematics and Computer Science Division

[b] Institut für Theoretische Teilchenphysik,
Universität Karlsruhe, D-76128 Karlsruhe, Germany

The two-loop electroweak bosonic correction to the muon lifetime is computed using methods of asymptotic expansion. Combined with previous calculations this completes the full two-loop correction to Δr in the Standard Model.

The Fermi constant G_F plays an important role in the precision tests of the Standard Model. Theoretically G_F can be related to other precision observables: the electroweak coupling constant α and the masses of electroweak gauge bosons M_Z and M_W. Other parameters enter this expression through quantum corrections. Usually one inverts this relation in order to predict M_W through M_Z which is measured much more accurate. This $M_Z - M_W$ interdependence can be then confronted with experimental value $M_W^{\rm exp}$. The current error (39 MeV) of $M_W^{\rm exp}$ will be drastically reduced at future colliders. In fact, at LHC the experimental error can be reduced to 15 MeV [1] and at Linear Collider even down to 6 MeV [2]. Therefore much efforts have been spent to reduce the error of the theoretical prediction.

The one-loop correction to G_F is known since long ago [3] along with the leading two-loop [4] corrections. Large two-loop contributions from fermionic loops have been calculated in [5]. The current prediction is affected by two types of uncertainties. First, apart from the still unknown Higgs boson mass, two input parameters introduce large errors. The current knowledge of the top quark mass results in an error of about 30 MeV [6], which should be reduced by LHC to 10 MeV and by a linear collider even down to 1.2 MeV. The inaccuracy of the knowledge of the running of the fine structure constant up to the M_Z scale, $\Delta\alpha(M_Z)$, introduces a further 6.5 MeV error. Second, several higher order corrections are unknown. In fact the last unknown correction at the $\mathcal{O}(\alpha^2)$ order has been calculated only recently in [7],[8] and [9]. This contribution comes from diagrams with no closed fermion loops.

Fermi constant is defined as the coupling constant in the low energy four fermion effective Lagrangian describing the decay of the muon

$$\mathcal{L}_F = \mathcal{L}_{\rm QED} + \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu\right] \left[\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e\right] \tag{1}$$

where e and μ are electron and muon fields, ν_e and ν_μ are the corresponding neutrinos and G_F is the Fermi constant. From the Lagrangian (1) one gets the following value for the muon lifetime

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)(1+\Delta q)\,, \tag{2}$$

where the factor Δq describes all the quantum corrections in the low energy effective theory (i.e. QED corrections). At one-loop order these corrections have been computed a long time ago [10]. Recently also the two-loop result for Δq has been obtained [11]. Taking it into account, the error of G_F is nowadays dominated by the experimental error of τ_μ measurement[3].

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[3]The present value is $G_F = 1.16637(1)\times 10^{-5}$ GeV^{-2} and

In order to relate G_F to M_W one can use the matching condition between effective theory (1) and the Standard Model, which requires that the value of τ_μ does not depend on whether it is evaluated in the Fermi theory or in the full Standard Model up to operators of higher dimensions, i.e.

$$A^{\mathrm{SM}} = \frac{G_F}{\sqrt{2}} \langle \mu | O_F | e \nu_\mu \bar{\nu}_e \rangle + \mathcal{O}\left(\frac{m_\mu^4}{M_W^4} \right) . \qquad (3)$$

This equation however gets quantum corrections. In fact in the l.h.s. of (3) there appear contributions coming from both short and long distance. It the r.h.s. these contributions should be separately absorbed in G_F and the matrix element $\langle O_F \rangle$ respectively. In order to make this in the most easiest and elegant way the factorization theorem is used.

In order to separate the short and long distance dynamics at the level of a single Feynmans diagram we use the large mass expansion procedure [12]. Namely, given the graf F, then asymptotically

$$F \sim \sum_{H \subseteq F} S \cdot T(H) , \qquad (4)$$

where the sum runs over all "hard" subgraphs H of the diagram F; S is a "soft" subgraph obtained from F by shrinking H to a point and T stands for the Taylor expansion (before integration!) of H with respect to all "soft" parameters. The exact rules for construction of hard subgraphs are discussed in details in [12].

At the level of Feynman amplitude the situation is more involved, since each diagram has its own subgraphs. However it is possible to rearrange the action of asymptotic expansion (4) in the sum of all diagrams such that it exponentiates. Such rearrangement is based on the combinatorial properties of R-operation and the special structure of the expansion (4). The rigorous prove of factorization theorem can be found in [13].

In order to demonstrate the idea we consider a simple example. In Fig. 1 the contibution of ladder topologies to muon decay amplitude A^{SM}

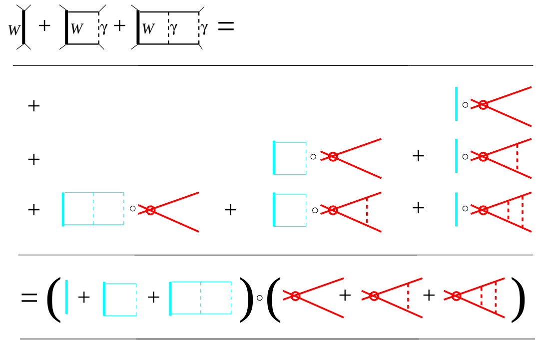

Figure 1. Illustration of the factorization theorem.

is depicted. One can notice however that this sum can be writen as a product of two factors. One corresponds to short distance Wilson coefficient function (G_F) and another to long distance matrix element. All other topologies can be considered similary. As a result of this procedure, the evaluation of G_F is reduced to computaion of bubble diagrams only.

If we want to use the on-shell scheme, the mass counterterms for W and Z bosons are needed. They are given by the on-shell selfenergy diagrams. This is the most difficult part of the calculation. However in this case the asymptotic expansion can be applied. In particular, the missing two-loop contributions to the mass counterterms have been completed resently in [14] (see also this proceeding).

Therefore we have all ingredients to obtain the complete two-loop electroweak correction to G_F. Explicit results are given in [7–9]. In order to perform automatic calculation the simpbolic program FORM [15] and the diagram generator DIANA [16] have been used. From the result of calculation, in particular, it was found that the to the prediction of M_W induced by the two-loop bosonic correction does not exceed 1 MeV for the broad range of the Higgs boson mass from 100 to 1000 GeV.

In conclusion, recent calculation of the two-loop bosonic corrections to Δr performed by two

possible future experiments could reduce the error by an order of magnitude.

independent groups has been reviewed. We concidered some details of the matching onto the Fermi theory. The framework for the evaluation of the Fermi constant G_F based on the low energy factorisation theorem has been constructed. It allows one to compute G_F as a Wilson coefficient in a simple manner. This approach is general and is also applicable to other low energy quantities.

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