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Two-loop electroweak corrections at high energies†

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Abstract:
We discuss two-loop leading and angular-dependent next-to-leading logarithmic electroweak virtual corrections to arbitrary processes at energies above the electroweak scale. The relevant Feynman diagrams involving soft-collinear gauge bosons $\gamma, \mathrm{Z}, \mathrm{W}^{\pm}$ have been evaluated in eikonal approximation. We present results obtained from the analytic evaluation of massive loop integrals. To isolate mass singularities we used the Sudakov method and alternatively the sector decomposition method in the Feynman-parameter representation.

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We discuss two-loop leading and angular-dependent next-to-leading logarithmic electroweak virtual corrections to arbitrary processes at energies above the electroweak scale. The relevant Feynman diagrams involving soft-collinear gauge bosons $\gamma, \mathrm{Z}, \mathrm{W}^{\pm}$ have been evaluated in eikonal approximation. We present results obtained from the analytic evaluation of massive loop integrals. To isolate mass singularities we used the Sudakov method and alternatively the sector decomposition method in the Feynman-parameter representation.

1. Introduction

The main task of future colliders such as the LHC or an $\mathrm{e}^+\mathrm{e}^-$ Linear Collider (LC) is the investigation of the origin of electroweak symmetry breaking and the exploration of the limits of the Electroweak Standard Model. In order to disentangle effects of physics beyond the Standard Model, the inclusion of QCD and electroweak radiative corrections into the theoretical predictions is crucial.

In the energy range of future colliders, i.e. at energies above the electroweak scale, $\sqrt{s} \gg M_\mathrm{W}$, the electroweak corrections are enhanced by large logarithmic contributions [1] of the type

$$\alpha^N \log^M \left(\frac{s}{M_\mathrm{W}^2}\right), \quad M > 0. \tag{1}$$

The leading logarithms (LL), also known as Sudakov logarithms [2], correspond to $M = 2N$, the next-to-leading logarithms (NLL) to $M = 2N-1$, etc. The logarithmic dependence on various kinematic invariants $r = s, t, u, \ldots$ gives rise to subleading logarithms that involve ratios of invariants, and which we denote as angular-dependent logarithms

$$\alpha^N \log^{M-L} \left(\frac{s}{M_\mathrm{W}^2}\right) \log^L \left(\frac{|r|}{s}\right), \quad M - L > 0. \tag{2}$$

We will consider the kinematical region $|r| \gg M_\mathrm{W}^2$, where all invariants are much larger than the electroweak scale. The general form of electroweak logarithmic corrections is complicated by the hierarchy of mass scales $m_\mathrm{t} \sim M_\mathrm{H} \sim M_\mathrm{Z} \sim M_\mathrm{W} \gg m_f \gg \lambda$, with heavy masses at the electroweak scale, light-fermion masses m_f, and the photon mass λ as infrared regulator. As a consequence all logarithms of the large ratios M_W/m_f and M_W/λ have to be taken into account.

All the above logarithmic terms constitute the singular part of the corrections in the massless limit. They result either as mass singularities from soft/collinear emission of virtual or real particles off initial or final-state particles, or as remnant of ultraviolet singularities after parameter renormalization.

At the one-loop level, it has been proven that for processes that are not mass-suppressed at high energies the electroweak logarithms are universal, and general results have been given [3] and applied to gauge-boson pair production at the LHC [4]. These results are in agreement with various explicit diagrammatic calculations for many $2 \to 2$ scattering processes [5–9]. The approximate size of the one-loop electroweak LL and NLL for a typical $2 \to 2$ cross section is

$$\begin{aligned}\frac{\delta\sigma_{1,\mathrm{LL}}}{\sigma_0} &\simeq -\frac{\alpha}{\pi s_\mathrm{W}^2} \log^2 \left(\frac{s}{M_\mathrm{W}^2}\right) \simeq -26\%, \\ \frac{\delta\sigma_{1,\mathrm{NLL}}}{\sigma_0} &\simeq +\frac{3\alpha}{\pi s_\mathrm{W}^2} \log \left(\frac{s}{M_\mathrm{W}^2}\right) \simeq 16\%,\end{aligned} \tag{3}$$

at $\sqrt{s} = 1\,\mathrm{TeV}$, with $1 - s_\mathrm{W}^2 = c_\mathrm{W}^2 = M_\mathrm{W}^2/M_\mathrm{Z}^2$. The LL and NLL have similar size and opposite sign resulting in large cancellations.

Assuming that at high energies the symmetric phase of the electroweak theory can be used, resummations of the electroweak logarithms have been proposed based on techniques and results known from QCD. Fadin *et al.* [10] have resummed the LL by means of the infrared evolution equation. Kühn *et al.* have applied results from QCD to resum the logarithmic corrections to massless 4-fermion processes, $e^+e^- \to f\bar{f}$ up to the NLL [11] and even to the NNLL [12]. It was found that at 1 TeV there is no clear hierarchy between LL, NLL and NNLL, and that the angular-dependent logarithms are important. Melles has proposed a resummation of the NLL for arbitrary processes [13], which relies on the prescription of matching a symmetric $\mathrm{SU}(2) \times \mathrm{U}(1)$ theory with QED at the electroweak scale. Recently also an extension of this resummation to the angular-dependent NLL has been proposed [14].

All these resummations amount to exponentiations of the electroweak logarithms. The approximate size of the resulting two-loop LL and NLL for typical $2 \to 2$ processes at $\sqrt{s} = 1\,\mathrm{TeV}$ is

$$\begin{aligned} \frac{\delta\sigma_{2,\mathrm{LL}}}{\sigma_0} &\simeq +\frac{\alpha^2}{2\pi^2 s_\mathrm{w}^4}\log^4\left(\frac{s}{M_\mathrm{W}^2}\right) \simeq 3.5\%, \\ \frac{\delta\sigma_{2,\mathrm{NLL}}}{\sigma_0} &\simeq -\frac{3\alpha^2}{\pi^2 s_\mathrm{w}^4}\log^3\left(\frac{s}{M_\mathrm{W}^2}\right) \simeq -4.2\%, \end{aligned} \quad (4)$$

and it is clear that in view of the precision objectives of a LC below the per-cent level these two-loop logarithms must be under control.

All the above resummation prescriptions result from matching a *symmetric* $\mathrm{SU}(2) \times \mathrm{U}(1)$ theory and QED at the electroweak scale, assuming that other effects related to spontaneous symmetry breaking may be neglected at high energies. This assumption needs to be checked by explicit diagrammatic two-loop calculations based on the electroweak Lagrangian, where all nontrivial effects related to *spontaneous symmetry breaking* are taken into account, in particular (i) the large gap $M_\mathrm{W} \sim M_\mathrm{Z} \gg \lambda$ in the gauge sector, (ii) the mixing between the gauge-group eigenstates B, W^3 resulting into the mass eigenstates γ, Z, and (iii) the presence of longitudinal gauge bosons as physical asymptotic states.

The resummation of the two-loop LL has been checked for the massless fermionic form factor in Refs. [15,16] and for arbitrary processes in the massive Coulomb gauge in Ref. [17]. The resummation of the NLL has so far not been confirmed by explicit electroweak two-loop calculations.

A subset of the NLL is furnished by the angular-dependent logarithms of type (2) with $M = 2N$, $L = 1$. These contributions are numerically important [8,11,12]. At one-loop order, in the t' Hooft–Feynman gauge, they result only from diagrams where a gauge boson is exchanged between two external lines. Similarly, the angular-dependent NLL at two-loop order can be traced back to a relatively small set of Feynman diagrams. This allows us to perform a diagrammatic calculation of the two-loop angular-dependent NLL for arbitrary processes. The relevant massive two-loop integrals have been evaluated in eikonal approximation, and the logarithms have been obtained analytically using two independent methods: the first one goes back to Sudakov [2], the other one uses sector decomposition of Feynman-parameter integrals [18–20]. A detailed description of this calculation can be found in Ref. [21]. Here we summarize the main ingredients and results.

2. Feynman diagrams

In the following we consider electroweak processes[1] $\varphi_{i_1}(p_1)\ldots\varphi_{i_n}(p_n) \to 0$, involving n arbitrary mass-eigenstate particles. The kinematical invariants are denoted as $r_{kl} = (p_k + p_l)^2 \simeq 2p_k p_l$, and the matrix elements as

$$\mathcal{M} \equiv \mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}}(p_1,\ldots,p_n). \quad (5)$$

We restrict ourselves to matrix elements that are not mass-suppressed at high energies. In this case global gauge invariance implies

$$\sum_{k=1}^{n} \mathcal{M}\, I^a(k) = \mathcal{O}\left(\frac{M^2}{s}\right)\mathcal{M}, \quad a = \gamma, \mathrm{Z}, \mathrm{W}^\pm, \quad (6)$$

where the gauge couplings $I^a(k)$ act as (transposed) matrices on the external-legs k of the matrix element[2].

[1] As a convention, all particles φ_{i_k} and their momenta p_k are assumed to be incoming. The corresponding $2 \to n-2$ processes are easily obtained by crossing symmetry.

[2] Details concerning our notation can be found in Ref. [3].

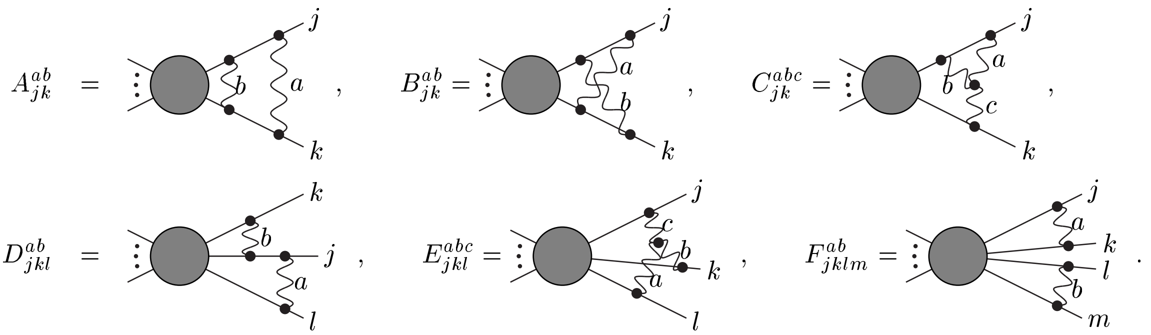

Figure 1. Two-loop diagrams with soft-collinear gauge bosons $a, b, c = \gamma, \mathrm{Z}, \mathrm{W}^{\pm}$ exchanged between external legs $j, k, l, m = 1, \dots, n$.

In the t' Hooft–Feynman gauge, the leading mass singularities originate from diagrams with soft-collinear virtual gauge bosons coupling to external particles. The relevant two-loop diagrams are depicted in Fig. 1, where the soft-collinear gauge bosons are exchanged between two, three, or four of the n on-shell external legs.

Each loop integral has to be evaluated for all different mass assignments that occur in the electroweak model. For the internal lines we have the cases $a, b, c = \gamma, \mathrm{Z}, \mathrm{W}^{\pm}$ and for the external masses we assume $M_{\mathrm{W}} \gtrsim m_{\mathrm{ext}} \gg \lambda$.

The Feynman diagrams are evaluated in eikonal approximation, i.e. by neglecting mass terms and the momenta of the soft gauge bosons everywhere in the numerators, apart from the momenta in the couplings of three soft gauge bosons. In the massless limit, longitudinal gauge bosons have to be substituted by the corresponding would-be Goldstone Bosons using the Goldstone-Boson equivalence theorem.

3. Loop integrals in logarithmic approximation

In the evaluation of the loop integrals we include the LL and the angular-dependent NLL, and we use $m_{\mathrm{t}} \simeq M_{\mathrm{H}} \simeq M_{\mathrm{Z}} \simeq M_{\mathrm{W}}$, i.e. we neglect logarithms of ratios of heavy masses. Two analytical methods have been used: the Sudakov method [2] and sector decomposition [18–20], which permits to factorize overlapping ultraviolet or mass singularities in Feynman-parameter integrals. Here we only sketch the main steps of the second method applied to a generic two-loop massive integral.

Step 1: The integral is written in Feynman parametrization and the denominator is split into polynomials according to the hierarchy of scales $s \gg r \gg M^2 \gg \dots \gg \lambda^2$ in the diagram[3]

$$\begin{aligned} I &= \int_{[0,1]^n} \mathrm{d}\vec{x}\, \frac{f(\vec{x})}{[D(\vec{x})]^e}, \\ D(\vec{x}) &= sP_s(\vec{x}) + rP_r(\vec{x}) + \dots + \lambda^2 P_\lambda(\vec{x}). \end{aligned} \tag{7}$$

These polynomials have various zeros of the form

$$P(\vec{x}) = \sum_{i=1}^{m} x_i P_i(\vec{x}) = 0, \tag{8}$$

at $x_1 = \dots = x_m = 0$, which give rise to mass singularities.

Step 2: In order to separate overlapping singularities, we decompose the sector $[0,1]^m$ into m subsectors Ω_j with $x_j > x_{i \neq j}$, and in each subsector Ω_j we perform variable transformations $x_i \to x_j x_i'$, which remap $\Omega_j \to [0,1]^m$ and permit

[3]To extract the angular-dependent logarithms $\log(s/r)$ with $r = t, u, \dots$, we compute the integrals in the euclidean region in various limits of the type $s \gg t = u$, $s = t \gg u$, etc., where we separate the energy scales in various ways.

to factorize the variable x_j in

$$P(\vec{x}) = \left[P_j(\vec{x}) + \sum_{i\neq j} x_i' P_i(\vec{x}) \right] x_j. \tag{9}$$

Step 3: Recursive application of step 2 permits to factorize all zeros at all scales, until the denominator assumes the form

$$\begin{aligned} D(\vec{x}) &= \left\{ \left[s\hat{P}_s(\vec{x}) \prod_k x_k + r\hat{P}_r(\vec{x}) \right] \prod_l x_l \right. \\ &\quad \left. + \ldots + \lambda^2 \hat{P}_\lambda(\vec{x}) \right\} \prod_m x_m, \end{aligned} \tag{10}$$

where all Feynman parameters that give rise to mass singularities are factorized and the polynomials $\hat{P}$ are non-vanishing. This allows for a simple power counting of the logarithmically divergent integrations.

Step 4: In leading-logarithmic approximation the polynomials $\hat{P}$ can be treated as constants $\hat{P}(\vec{x}) \simeq \hat{P}(\vec{0})$, and all logarithms of ratios of scales can be extracted by analytical integration of the singular parameters.

4. Results

As a basis for the presentation of our two-loop results, we recall the one-loop results for LL and angular-dependent NLL given in Ref. [3].

4.1. One-loop results

At one-loop level we have

$$\mathcal{M}_1 = \mathcal{M}_0(1 + \delta_{\mathrm{EW}}). \tag{11}$$

The most symmetric form to write the electroweak logarithmic corrections consists in splitting them into $\delta_{\mathrm{EW}} = \delta_{\mathrm{sew}} + \delta_{\mathrm{sem}}$, with

$$\begin{aligned} \delta_{\mathrm{sew}} &= \delta_{\mathrm{EW}}|_{\lambda=M_{\mathrm{W}}}, \\ \delta_{\mathrm{sem}} &= \delta_{\mathrm{EW}} - \delta_{\mathrm{EW}}|_{\lambda=M_{\mathrm{W}}}. \end{aligned} \tag{12}$$

The symmetric electroweak part (sew) corresponds to the case when the photon mass λ equals the electroweak scale and reads

$$\begin{aligned} \delta_{\mathrm{sew}} &= \frac{\alpha}{4\pi} \sum_{k=1}^{n} \left\{ -\frac{1}{2} C^{\mathrm{ew}}(k) \log^2 \frac{s}{M_{\mathrm{W}}^2} \right. \\ &\quad \left. + \sum_{l\neq k} \sum_{a=\gamma,\mathrm{Z},\mathrm{W}^\pm} I^a(k) I^{\bar{a}}(l) \log \frac{|r_{kl}|}{s} \log \frac{s}{M_{\mathrm{W}}^2} \right\}, \end{aligned} \tag{13}$$

where $C^{\mathrm{ew}} = Y^2/(4c_{\mathrm{W}}^2) + C^{\mathrm{SU(2)}}/s_{\mathrm{W}}^2$ represents the electroweak Casimir operator. The remaining part is a subtracted electromagnetic (sem) contribution originating from the fact that $\lambda \ll M_{\mathrm{W}}$,

$$\begin{aligned} \delta_{\mathrm{sem}} &= \frac{\alpha}{4\pi} \sum_{k=1}^{n} \left\{ -\frac{1}{2} Q^2(k) \left[2\log \frac{s}{m_k^2} \log \frac{M_{\mathrm{W}}^2}{\lambda^2} \right. \right. \\ &\quad \left. \left. - \log^2 \frac{M_{\mathrm{W}}^2}{m_k^2} \right] + \sum_{l\neq k} Q(k)Q(l) \log \frac{|r_{kl}|}{s} \log \frac{M_{\mathrm{W}}^2}{\lambda^2} \right\}. \end{aligned} \tag{14}$$

4.2. Two-loop results

Detailed results for the individual two-loop diagrams in Fig. 1 are presented in Ref. [21]. These diagrams have to be combined as follows

$$\begin{aligned} \delta\mathcal{M}_2 &= \sum_{a,b} \left\{ \sum_{j,k} \left[\frac{1}{2} \left(A_{jk}^{ab} + B_{jk}^{ab} \right) + \sum_c C_{jk}^{abc} \right] \right. \\ &\quad \left. + \sum_{j,k,l} \left[D_{jkl}^{ab} + \frac{1}{6} \sum_c E_{jkl}^{abc} \right] + \frac{1}{8} \sum_{j,k,l,m} F_{jklm}^{ab} \right\}, \end{aligned}$$

taking into account all sums over virtual gauge bosons $a, b, c = \gamma, \mathrm{Z}, \mathrm{W}^\pm$ and external legs[4] $j, k, l, m = 1, \ldots, n$, with appropriate symmetry factors. These sums can be simplified by means of (6), and it turns out that the result corresponds to the exponentiation of the one-loop corrections (13),(14) in the form

$$\mathcal{M}_2 = \mathcal{M}_0 \exp(\delta_{\mathrm{sew}}) \exp(\delta_{\mathrm{sem}}), \tag{15}$$

where the symmetric electroweak and the subtracted electromagnetic parts exponentiate separately.

5. Discussion

Our result confirms the exponentiation of the electroweak LL obtained with the infrared evolution equation [10] and in the Coulomb gauge [17]. The subset of diagrams (A-C) has been evaluated also in the special case of massless external particles and agreement has been found with the form-factor calculation of Refs. [15,16]. The exponentiation of the electroweak angular-dependent

[4]In the sums only the contributions from different external legs $j \neq k$, etc. have to be considered.

NLL agrees with the results of Ref. [11] for massless fermionic processes, and Ref. [14] for arbitrary processes. These results were obtained using matching conditions at the electroweak scale.

In the following we discuss the idea of matching by applying it to our result. If we set $\lambda = M_{\mathrm{W}}$ or $s = M_{\mathrm{W}}^2$, we obtain

$$\begin{aligned} \lambda = M_{\mathrm{W}} &\Rightarrow \delta_{\mathrm{sem}} = 0,\ \mathcal{M}_2 = \exp\left[\delta_{\mathrm{sew}}\right]\mathcal{M}_0, \\ s = M_{\mathrm{W}}^2 &\Rightarrow \delta_{\mathrm{sew}} = 0,\ \mathcal{M}_2 = \exp\left[\delta_{\mathrm{sem}}\right]\mathcal{M}_0, \end{aligned} \quad (16)$$

i.e. we observe exponentiation within a symmetric SU(2) $\times$ U(1) theory ($\lambda = M_{\mathrm{W}}$) and QED ($s = M_{\mathrm{W}}^2$). This provides a simple consistency check of our result. However, we stress that the matching conditions (16) are not sufficient in order to determine the interference terms $\delta_{\mathrm{sew}}\delta_{\mathrm{sem}}$ between electroweak and electromagnetic contributions, which vanish at both matching points. These two-loop terms are fixed by our full electroweak calculation ($s \gg M_{\mathrm{W}}^2 \gg \lambda^2$) and are crucial in order to predict the ordering of the two exponentials in (15), which is non-trivial at the level of the angular-dependent NLL, since

$$[\delta_{\mathrm{sew}}, \delta_{\mathrm{sem}}] = \mathcal{O}\left[\log\frac{r_{kl}}{s}\log^2\frac{s}{M_{\mathrm{W}}^2}\log\frac{M_{\mathrm{W}}^2}{\lambda^2}\right]. \quad (17)$$

The fact that δ_{sem} appears on the right-hand side in (15) means that the subtracted electromagnetic corrections are only sensitive to the electromagnetic charges of the external particles.

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