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Radiative return at e^+e^- factories. *

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The energy dependence of the electron - positron hadronic cross section can be measured not only by a straightforward energy scan, but also by means of the radiative return method. To provide extensive comparisons between theory and experiment a Monte Carlo event generator is an indispensable tool. We have developed such a generator called PHOKHARA, which simulates $e^+e^- \to$ mesons + photon(s) processes. In this paper we present its latest tests and upgrades.

1. INTRODUCTION AND PHOKHARA UPGRADES

In view of the precision of the recent measurements of the muon anomalous magnetic moment $a_\mu \equiv (g-2)_\mu/2$ at BNL [1], hadronic contributions are crucial for the interpretation of this measurement, in particular for the isolation of the electroweak or of non-Standard Model physics contributions [2]. Their understanding becomes even more important, as it seems [3] that the e^+e^- annihilation data are not consistent with the τ decay data. A new a_μ measurement, which is under way, will challenge the theoretical predictions even more.

An important ingredient and the dominant source of uncertainties in the theoretical prediction for the muon anomalous magnetic moment is the hadronic vacuum polarization [4]. It is in turn related via dispersion relations to the cross section for electron–positron annihilation into hadrons $\sigma_{had} = \sigma(e^+e^- \to hadrons)$ and in some cases via isospin symmetry to the decay width $\tau \to \nu_\tau + hadrons$. This quantity plays an important role also in the evolution of the electromagnetic coupling α_{QED} from the Thompson limit to high energies [4,5]. The interpretation of improved measurements at high energy colliders such as LEP, Tevatron, the LHC or TESLA therefore depends significantly on the precise knowledge of σ_{had}.

The feasibility of using tagged photon events at high luminosity electron–positron storage rings, such as the ϕ-factory, DAΦNE, CLEO-C or B-factories, to measure σ_{had} over a wide range of energies has been proposed and studied in detail in [6–8] (see also [9,10]). The machine is operating at a fixed energy of the e^+e^- centre-of-mass system (cms) and the initial state radiation (ISR) is used to reduce the invariant mass of the hadronic system.

The radiation of photons from the hadronic system (the final state radiation, FSR) should be considered as a background and can be suppressed by choosing suitable kinematical cuts, or controlled by the simulation, once a suitable model for this amplitude has been adopted. One finds that selecting events with the tagged photons close to the beam axis and well separated from the hadrons indeed reduces FSR drastically. As demonstrated in Fig. 1, the FSR contribution

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to the total cross section can be easily reduced to the 1% level. The model dependence of the FSR in the case of the $\pi^+\pi^-$ final state can be controlled by the same experiment through studies of the forward–backward asymmetry of the angular charged pion distribution. The asymmetry comes from FSR–ISR interference and integrates to zero for 'charge blind' configurations. As a result it does not contributes to rates in Fig. 1. It can be used, however, to calibrate the FSR amplitude and more detailed tests of its model dependence.

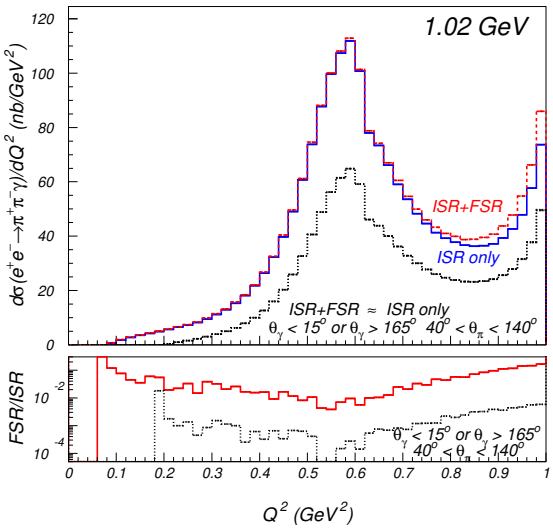

Figure 1. The role of the cuts in the suppression of the FSR contributions to the cross section. Results from the PHOKHARA generator. No cuts (upper curves) and suitable cuts applied (lower curves).

When running at higher energies, the FSR is suppressed with respect to the ISR by the different behaviour of various propagators and form factors relevant to the problem. In practice it means that no special angular cuts are needed to suppress the FSR contribution when running at high energies.

The suppression of the FSR overcomes the problem of its model dependence, which must be taken into account in a completely inclusive measurement [11].

Preliminary experimental results using this method have been presented recently by the KLOE collaboration at DAΦNE [12–15]. Large event rates were also observed by the BaBar collaboration [16].

In the first version of the newly developed Monte Carlo program PHOKHARA [17] we have considered the full next-to-leading order (NLO) QED corrections to the ISR in the annihilation process $e^+e^- \to \gamma + hadrons$, for the case where the photon is observed under a non-vanishing angle relative to the beam direction. The virtual and soft photon corrections were presented in [18] and the contribution of the emission of a second hard photon in [17]. The final hadronic state was limited to the $\pi^+\pi^-$, with the hadronic current modeled as in [19], and the final state emission was not included. The program allowed also for the generation of $\mu^+\mu^-\gamma(\gamma)$ final states, again limited to the emission of photon(s) from the initial leptons.

Radiative corrections proportional to (αm_e^2) relevant to configurations with photons emitted at very low ($\simeq m_e/\sqrt{s}$) angles relative to the beam direction were calculated in [20] and are included in the new version of PHOKHARA [21]. The leading order corrections proportional to m_e^2 are typically of the order of a few per cent [22], while the non-leading ones are of order of 0.1%, as seen from Fig.2. They will be important when the precision of the measurement will be below 1%. Their effect depends on Q^2 and thus affects the Q^2 distribution from which the hadronic cross section is extracted.

Another new feature of the PHOKHARA event generator is the inclusion of the four-pion hadronic final states ($2\pi^+2\pi^-$ and $2\pi^0\pi^+\pi^-$). The description of the hadronic current in that case is based on the paper [23], with changes described in [8]. The comparison with the Monte Carlo, which simulates the same process at leading order [8] and includes additional collinear radiation through structure function (SF) techniques, shows typical difference of order of 1% as seen in Fig. 3 (a similar behaviour can be observed for $2\pi^0\pi^+\pi^-$ final state). The non-leading

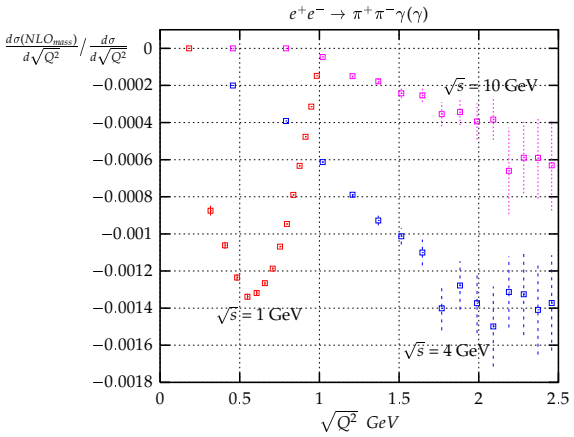

Figure 2. The relative contributions of the non-leading mass corrections to the differential cross section at $\sqrt{s} = 1$ GeV, 4 GeV and 10 GeV.

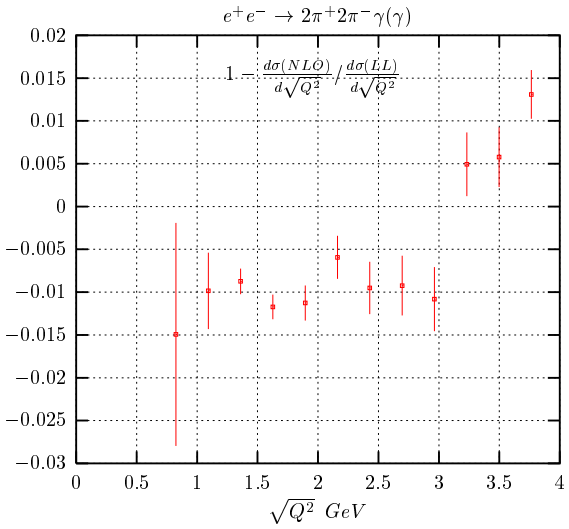

Figure 3. The relative non-leading contributions to the differential cross section at $\sqrt{s} = 4$ GeV. NLO - full next-to-leading result, LL - leading logarithmic approximation.

contributions to the cross section of the four-pion final states are of the expected size and of the same order as for the two-pion final state [17].

The program now includes also the contributions from the final state emitted photons together with the ISR–FSR interference calculated at the lowest order for $\pi^+\pi^-$ and $\mu^+\mu^-$ final states, while for the four-pion final states the FSR contribution is not taken into account.

2. TESTS OF PHOKHARA

An obvious and one of the most important tasks in the construction of a Monte Carlo event generator is to demonstrate that its technical accuracy is much better than the desired physical accuracy. The tests that were performed for the previous version of PHOKHARA [17] are still valid, within the limitations of this version, which was applicable for non-vanishing photon angles.

In the first step we demonstrate the independence of total and differential cross sections of the separation parameter ϵ (called w in [17]) between soft and hard photon regions. The soft photon contribution is calculated analytically, while the additional hard photon is treated via Monte Carlo simulation. The parameter that specifies the separation between the two regions of the phase space has to be kept small enough to validate the soft photon approximation and large enough to avoid negative weights. We performed the tests for a $\pi^+\pi^-$ hadronic final state in [17], while for one of the four-pion modes the results are collected in Figs. 4 and 5. From Fig. 4 it is clear that the choice $\epsilon = 10^{-3}$ is still too big, whereas Fig. 5 demonstrates the stability of the results between $\epsilon = 10^{-4}$ and $\epsilon = 10^{-5}$. This also proves that the Monte Carlo integration works well in the soft photon region. The Monte Carlo integration of the part of the program that simulates one hard large-angle photon emission was tested in [17] against a Gauss numerical integration. As shown in Fig. 6 a technical precision of the program at the level of 10^{-4} was demonstrated. Analytical results exist for the differential (in Q^2) cross section integrated over the whole angular range of the photon(s) for both one and two emitted photons [24]. The comparison with PHOKHARA can be found in Fig. 7,

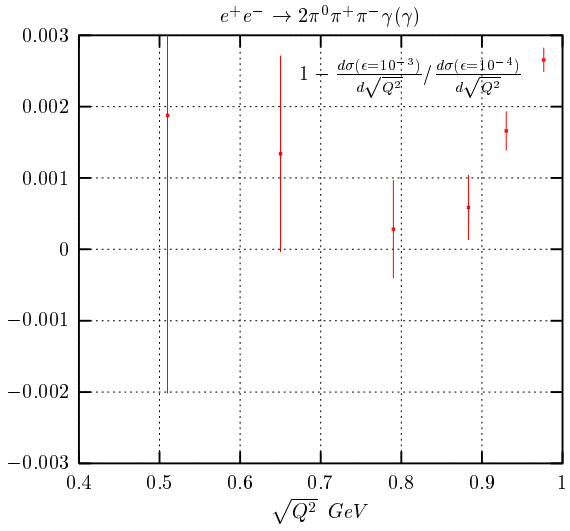

Figure 4. The relative difference of the differential cross sections for two different values of the separation parameter ϵ.

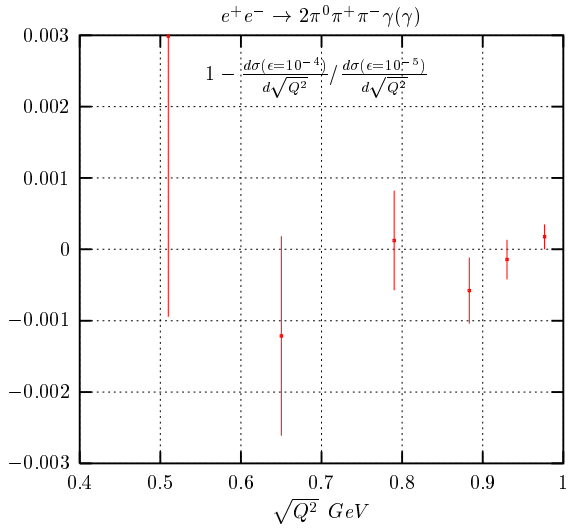

Figure 5. The relative difference of the differential cross sections for two different values of the separation parameter ϵ.

demonstrating again an excellent technical precision also for the two-photon final state. The results presented in Fig. 7 refer to the sum of virtual and hard corrections to the $e^+e^- \to \pi^+\pi^-\gamma$ cross section, while more detailed tests can be found in [21].

3. CONCLUSIONS

The PHOKHARA Monte Carlo event generator was upgraded, allowing for simulation in the small photon angles region. Besides the $\pi^+\pi^-$ hadronic final state, its present version includes also $2\pi^+2\pi^-$ and $2\pi^0\pi^+\pi^-$ final states. For the $\pi^+\pi^-$ and $\mu^+\mu^-$ final states, the FSR photonic contributions were implemented at the lowest order, including ISR–FSR interference. Further upgrades are in progress.

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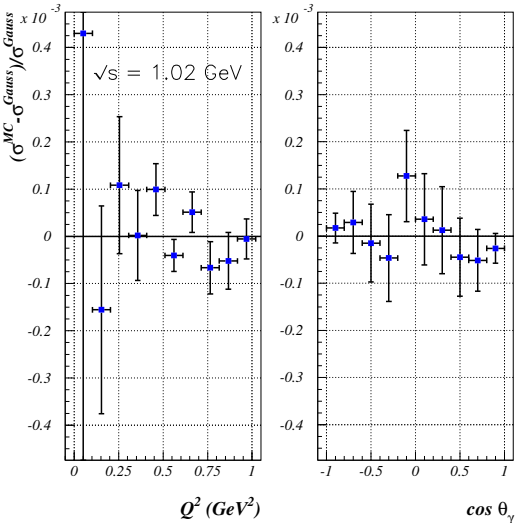

Figure 6. The relative difference between differential cross sections obtained by the PHOKHARA Monte Carlo generator (MC) and a Gauss numerical integration (Gauss).

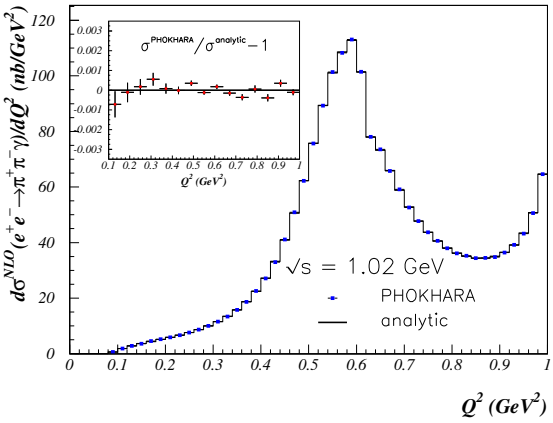

Figure 7. A comparison between PHOKHARA and analytical [24] results.

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