Recent Results on Sudakov Logarithms in Electroweak Processes

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Recent results for the asymptotic behavior of fermion scattering amplitudes in the Sudakov limit are discussed including next-to-leading logarithmic (NLL) and NNLL corrections. These are used for the analysis of the dominant electroweak corrections to the fermion-antifermion pair production in e^+e^- annihilation at high energy. The importance of angular dependent as well as subleading terms is emphasized.

1. Introduction

Four fermion processes are generally considered as benchmark processes at high energy colliders, with electron positron annihilation into muon or quark pairs at LEP and the Drell Yan process at hadron colliders as characteristic examples. Within the presently accessible energy region, typically up to 200 GeV, radiative corrections are dominated by the shift in the W and Z masses as parametrized by the ρ parameter and by the running of the coupling constant. Vertex corrections and box diagrams involving gauge bosons are generally of minor importance. In the TeV region, accessible at future colliders like the LHC or TESLA, this picture changes drastically. A new class of effects starts to become relevant and rapidly dominant which are generally denoted as double logarithmic corrections and which were first observed by Sudakov in the context of quantum electrodynamics for reactions with a tight cut on the radiated energy of the photons. For electroweak interactions large negative corrections arise from the exchange of gauge bosons which remain uncompensated if one restricts the analysis to exclusive final states, consisting e.g. of a fermion antifermion pair only. The discussion of double logarithmic corrections is fairly straightforward for a theory with massive gauge bosons only. An important complication arises from the presence of massless photons in the final state. Events with soft and hard photon radiation are normally included in the sample – whence a "semi-inclusive" definition of the cross section is closest to the the experimental analysis.

One loop corrections to the four fermion process are available since long (see e.g. [1]). In this case the separation of photonic and weak corrections is still possible and, employing axial gauge, the leading logarithms can be trivially attributed to fermion self energy diagrams with virtual W or Z boson emission [2].

Results for the leading logarithmic corrections in higher orders have been presented in [2,3]. In [4], which includes NLL corrections and angular dependent terms, it was demonstrated that the approach developed in [2], where axial gauge was adopted and both real and virtual photon emission neglected, is numerically nearly identical to the results of [3] where virtual photon corrections and real photon emmission are included – a consequence of the smallness of the weak mixing angle, $\sin^2\theta_W = 0.23$. The analysis of subleading logarithms for the form factor, for the four fermion scattering in a spontaneously broken $SU(2)$ gauge theory and, last not least, for the Standard Model with W, Z and the massless photon was performed in [4].

Before presenting a brief review (an extended review of this field can also be found in [5]) of the techniques and some recent results let us emphasize the importance of subleading corrections. In the energy region relevant at LHC or TESLA, i.e. around 1 to 2 TeV, this is already evident from the one-loop vertex correction

$$|F_B + \delta F|^2 \approx F_B \left(1 + 2\frac{\alpha_W}{\pi} C_F \rho(s/M^2)\right), \qquad (1)$$

$$\rho(u) = -\frac{1}{4}\ln^2 u + \frac{3}{4}\ln u + \frac{\pi^2}{12} - \frac{7}{8}.$$

For the timelike region and for characteristic values of the coupling, energy and mass of $\alpha_W/\pi = 10^{-2}$, $C_F = 3/4$ and $s/M^2 = 10^2$ one finds

$$2\frac{\alpha_W}{\pi}C_F\rho \; = \; 1.5\times 10^{-2}(-5.30+3.45-0.05) \quad (2)$$

Large compensations between leading and subleading terms are observed and, in fact, this pattern will reappear for the Standard Model as discussed below.

The calculation of the leading logarithmic terms in [3] is based on evolution equations that govern the dynamics of the amplitudes in the Sudakov limit. In [4] and [6] this approach was applied to the next-to-leading logarithmic and the NNLL analysis respectively of the Abelian form factor and the four fermion amplitude in the $SU(N)$ gauge theory. Functions that enter the evolution equations in the next-to-leading logarithmic approximation were evaluated by using, as an input, asymptotic expansions of one-loop diagrams. The solution of these equations lead to a summation of the leading and subleading Sudakov logarithms. The expansion of one-loop diagrams through the so-called generalized strategy of regions [7,8] identifies in a systematic way the nature of various contributions and the origin of logarithms.

2. The Abelian form factor in the Sudakov limit

Let us first analyze the (vector) form factor which determines the amplitude of the fermion scattering in the external Abelian field. In Born approximation

$$\mathcal{F}_B = \bar{\psi}(p_2)\gamma_\mu\psi(p_1). \quad (3)$$

We consider the limit $s=(p_1-p_2)^2 \to -\infty$ with on-shell massless fermions, $p_1^2 = p_2^2 = 0$, and gauge bosons with a small non-zero mass $M^2 \ll -s$. For convenience $p_{1,2} = (Q/2,0,0,\mp Q/2)$ so that $2p_1p_2 = Q^2 = -s$. The asymptotic behavior of the form factor is obtained from

$$\mathcal{F} = F_0(\alpha(M^2))\exp\Big\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x} \times\Big[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\Big]\Big\}\mathcal{F}_B. \quad (4)$$

The NLL approximation includes all the terms of the form $\alpha^n\log^{2n-m}(Q^2/M^2)$ with $m=0,\ 1$. This requires the one-loop values of $\gamma(\alpha)$ and $\zeta(\alpha)+\xi(\alpha)$ and using the one-loop running of α in $\gamma(\alpha)$. The NNLL approximation includes all the terms of the form $\alpha^n\log^{2n-m}(Q^2/M^2)$ with $m=0,\ 1,\ 2$. In this case $\gamma(\alpha)$ is required up to $\mathcal{O}(\alpha^2)$, $\zeta(\alpha)$, $\xi(\alpha)$ and $F_0(\alpha)$ up to $\mathcal{O}(\alpha)$ together with the two-loop running of α in $\gamma(\alpha)$ and one-loop running of α in $\zeta(\alpha)$.

The expansion by regions consists of the following steps: (i) consider various regions of a loop momentum k and expand, in every region, the integrand in Taylor series with respect to the parameters that are there considered small, (ii) integrate the expanded integrand over the whole integration domain of the loop momenta, (iii) put to zero any scaleless integral. In step (ii) dimensional regularization with $d=4-2\epsilon$ space-time dimensions is used to handle the divergences. The following regions are of relevance in this context:

$$\begin{aligned}
\textit{hard}\ (\mathrm{h}):&\quad k\sim Q,\\
\textit{1-collinear}\ (\mathrm{1c}):&\quad k_+\sim Q,\ k_-\sim M^2/Q,\ \underline{k}\sim M,\\
\textit{2-collinear}\ (\mathrm{2c}):&\quad k_-\sim Q,\ k_+\sim M^2/Q,\ \underline{k}\sim M,\\
\textit{soft}\ (\mathrm{s}):&\quad k\sim M.
\end{aligned} \quad (5)$$

with $k_\pm = k_0 \pm k_3$, $\underline{k} = (k_1,k_2)$.

From the one-loop result and the hard part in two-loop approximation one finds

$$\begin{aligned}
\gamma^{(1)} &= -2C_F\,,\quad \zeta^{(1)} = 3C_F\,,\quad \xi^{(1)} = 0\,,\\
F_0^{(1)} &= -C_F\left(\frac{7}{2}+\frac{2\pi^2}{3}\right)\,,\\
\gamma^{(2)} &= -2C_F\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]\,,
\end{aligned} \quad (6)$$

with $\gamma = \left(\frac{\alpha}{4\pi}\right)\gamma^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2\gamma^{(2)} + \ldots$ and similarly for ζ, ξ and F_0.

Let us also consider the two-loop corrections. The LL, NLL and NNLL approximations are given respectively by

$$\mathcal{F}^{(2)}_{LL} = \frac{1}{8}(\gamma^{(1)})^2\ln^4\left(\frac{Q^2}{M^2}\right)\mathcal{F}_B\,, \quad (7)$$

$$\mathcal{F}^{(2)}_{NLL} = \frac{1}{2}\left(\zeta^{(1)}-\frac{1}{3}\beta_0\right)\gamma^{(1)}\ln^3\left(\frac{Q^2}{M^2}\right)\mathcal{F}_B\,, \quad (8)$$

$$\mathcal{F}^{(2)}_{NNLL} = \frac{1}{2}\Big(\gamma^{(2)} + \left(\zeta^{(1)} - \beta_0\right)\zeta^{(1)} + F_0^{(1)}\gamma^{(1)}\Big)\ln^2\left(\frac{Q^2}{M^2}\right)\mathcal{F}_B\,. \quad (9)$$

The relatively small coefficient of the LL terms and the large coefficient of the NNLL terms in the form factor are clearly indicative of the importance of the NNLL corrections and, as we will see, reflect the general structure of the logarithmically enhanced electroweak corrections.

3. The four fermion amplitude

Let us now investigate the four fermion scattering at fixed angles in the limit when all the invariant energy and momentum transfers of the process are far larger than the gauge boson mass, $|s| \sim |t| \sim |u| \gg M^2$. The analysis of the four fermion amplitude is complicated by the extra kinematical variable and the presence of different "color" and Lorentz structures. The complete basis consists of four independent chiral amplitudes, each of them of two possible color structures.

The collinear divergences in the hard part of the virtual corrections and the corresponding "collinear" logarithms are known to factorize. They are responsible, in particular, for the double logarithmic contribution, depend only on the properties of the external on-shell particles but not on a specific process and are the same as for the form factor $\mathcal{F}$ discussed in the previous section. Let us denote by $\tilde{\mathcal{A}}$ the amplitude with the collinear logarithms factored out. It contains the logarithms of "soft" nature corresponding to the soft divergences of the hard region contribution and the renormalization group logarithms. It can be represented as a vector in the color/chiral basis and satisfies the evolution equation

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}\,, \quad (10)$$

where $\chi(\alpha)$ is the matrix of the soft anomalous dimensions. It can be extracted from the coefficient in front of the single pole of the hard contribution to $\tilde{\mathcal{A}}$. The matrix elements of $\chi(\alpha)$ depend on the scattering angle. These angular dependent terms as well as those in the starting values of $\tilde{\mathcal{A}}$ discussed below are important for the calculation of NLL (see also [9]) and NNLL terms in cross sections and asymmetries. The solution of eq. (10) reads

$$\tilde{\mathcal{A}} = \sum_i \tilde{\mathcal{A}}_{0i}(\alpha(M^2))\exp\left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\chi_i(\alpha(x))\right], \quad (11)$$

where $\chi_i(\alpha)$ are eigenvalues of $\chi(\alpha)$ and $\tilde{\mathcal{A}}_{0i}(\alpha)$ are Q-independent eigenvectors of $\chi(\alpha)$ which determine the initial conditions for the evolution equation at $Q = M$.

To illustrate the significance of the subleading contributions let us again discuss the standard model inspired example considered in the previous section. For the annihilation process one has to make the analytical continuation to the Minkowskian region of negative $Q^2 = -s$. After integrating the differential cross section over all angles one obtains the result with logarithmic accuracy, e.g. for the case of $SU(2)_L$ group with $n_f = 6$ and one scalar boson

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{80}{3}\ln\left(\frac{s}{M^2}\right) - \left(\frac{25}{9} + 3\pi^2\right)\right]\sigma_B\,,$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{449}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{4855}{9} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_B\,, \quad (12)$$

if initial and final state fermions have the same isospin. Again we observe a relatively small coefficient of the LL and a large coefficient of the NNLL terms.

4. Electroweak processes

To analyze the electroweak correction to the process $f'\bar{f}' \to f\bar{f}$ the approximation with the W and Z bosons of the same mass M, the Higgs boson of the mass $M_H \sim M$ and massless quarks and leptons is employed. A fictitious photon mass λ has to be introduced to regularize the infrared divergences. The NNLL approximation is not sensitive to the fine details of the mass generation because we need only the hard part of the potentially dangerous self-energy insertion to the gauge boson propagator.

The photon is, however, massless and the corresponding infrared divergent contributions should be accompanied by the real soft photon radiation integrated to some resolution energy ω_{res} to get an infrared safe cross section independent on an auxiliary photon mass. To study the virtual corrections in the limit of the vanishing photon mass the general approach of the infrared evolution equations developed in [3] (see also references therein) is particularly useful.

Explicit expressions for the NNLL results are given in [6]. Numerically, for the reaction $e^+e^- \to \mu^+\mu^-$ as a typical example, one obtains a correction factor

$$
\begin{aligned}
R^{\mu^+\mu^-} &= 1 - 1.39\, L(s) + 10.12\, \ell(s) - 31.33\, a \\
&\quad +1.42\, L^2(s) - 18.43\, L(s)\ell(s) + 99.89\, \ell^2(s)
\end{aligned} \tag{13}
$$

with $a = g^2/16\pi^2 = 2.69{\cdot}10^{-3}$, $\ell(s) = a \ln s/M^2$, $L(s) = a \ln^2 s/M^2$. To get the infrared safe result for the semi-inclusive cross sections one has to include the standard QED corrections due to soft photon emission and the pure QED virtual corrections.

In view of the large subleading terms the evaluation of two-loop terms linear in the logarithm or even of the constant terms seems desirable. As a first step the fermionic corrections to the form factor, i.e. terms resulting from the inclusion of massless fermions in loop insertions in the boson propagator, have been calculated for the Abelian form factor [10]:

$$
\begin{aligned}
F_{2,n_f} &= n_f \frac{g^2}{16\pi^2}\left\{ -\frac{4}{9}\ln^3\left(\frac{-s}{M^2}\right) + \frac{38}{9}\ln^2\left(\frac{-s}{M^2}\right) \right.\\
&\quad \left. -\frac{34}{9}\ln\left(\frac{-s}{M^2}\right) + \frac{16}{27}\pi^2 + \frac{115}{9}\right\}
\end{aligned} \tag{14}
$$

The oscillating pattern of the signs in subleading terms continues, and the linear logarithm as well as the constant terms are sizeable. These results indicate the need for the evaluation of two-loop corrections for realistic reactions, including the linear logarithm and the constant term.

In addition to four fermion processes, which are clearly the benchmark reactions at high energies, also other reactions have been investigated in the high energy limit. In the case of gauge boson pair production LL and NLL terms have been evaluated by several authors [11]. Exclusive production of transversly and longitudinally polarized W is suppressed by $\exp(-C_A \ln^2 s/M^2)$ and $\exp(-C_F \ln^2 s/M^2)$ respectively, as expected from the equivalence theorem. The implications of these results for gauge boson pair production at hadron colliders were studied in [12]. As noted in [13], enhanced electroweak corrections are also present in inclusive boson fusion processes, a consequence of the lack of compensation between virtual and real emission due to the non-Abelian charges of the incoming particles.

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