Subleading Shape Functions and the Determination of V_{ub}

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The endpoint region of the inclusive lepton energy spectrum in $B \to X_u \ell \bar{\nu}$ can be used for a determination of V_{ub} by comparing it to the endpoint region of the inclusive photon spectrum of $B \to X_s \gamma$. We consider the subleading twist contributions to this V_{ub} determination.

A precise determination of the CKM matrix element $|V_{ub}|$ is one of the most important measurements in the current quest to overconstrain the unitarity triangle to test the standard model. It is currently believed that the theoretically cleanest determination of $|V_{ub}|$ comes from the inclusive rate for $B \to X_u \ell \bar{\nu}$, which may be calculated model-independently via an operator product expansion in terms of local operators [1]. Such a measurement is hampered by the experimental necessity to suppress the overwhelming charm background, which usually requires tight kinematic cuts. With a few exceptions [2], such cuts invalidate the local OPE usually used to calculate the inclusive branching ratio, and a twist expansion has to be performed instead [3]. At leading order in this twist expansion, and ignoring perturbative corrections, the lepton energy spectrum in inclusive $B \to X_u \ell \bar{\nu}$ decay is given by

$$\frac{d\Gamma}{dE_\ell} = \frac{2\Gamma_0}{m_b} \int\limits_{-\infty}^{m_b - 2E_\ell} d\omega\, f(\omega)\,,\ \Gamma_0 = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3}\,, \quad (1)$$

where $f(\omega)$ is the nonperturbative light cone structure function of the B meson

$$f(\omega) = \frac{1}{2m_B} \langle B | \bar{h}\, \delta(in \cdot D + \omega)\, h | B \rangle\,. \quad (2)$$

Since this function is a property of the B meson, it is process independent and can thus be measured elsewhere. The best way to measure this structure function is from the photon energy spectrum of the inclusive decay $B \to X_s \gamma$ [4]. Up to perturbative and subleading twist corrections, this spectrum is directly proportional to the structure function,

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha |C_7^{\rm eff}|^2 m_b^5}{32\pi^4} f(E_\gamma)\,. \quad (3)$$

Thus, combining data on $B \to X_s \gamma$ with data from $B \to X_u \ell \bar{\nu}$, one can eliminate the dependence on the structure function and therefore determine $|V_{ub}|$ with no model dependence at leading order [3–5]. The relation is

$$\left| \frac{V_{ub}}{V_{tb} V_{ts}^*} \right|^2 = \frac{3\alpha}{\pi} |C_7^{\rm eff}|^2 \frac{\Gamma_u(E_c)}{\Gamma_s(E_c)} + \cdots \quad (4)$$

where the elipses denote terms of order $1/m_b$ and α_s and

$$\Gamma_u(E_c) \equiv \int_{E_c}^{m_B/2} dE_\ell \frac{d\Gamma_u}{dE_\ell}$$

$$\Gamma_s(E_c) \equiv \frac{2}{m_b} \int_{E_c}^{m_B/2} dE_\gamma (E_\gamma - E_c) \frac{d\Gamma_s}{dE_\gamma}\,. \quad (5)$$

In this talk I describe the work done in collaboration with C. Bauer and M. Luke [6,7] (see also [8]), where we calculate the subleading twist corrections to Eq. (4). While perturbative corrections to this relation have been studied in detail [4,5], much less is known about nonperturbative corrections suppressed by powers of $\Lambda_{\rm QCD}/m_b$. In [6] we introduced a formalism which allows us to parametrize these subleading twist corrections in terms of subleading shape functions. The subleading twist corrections to the photon energy spectrum in $B \to X_s \gamma$ read

$$\frac{m_b}{\Gamma_0^s} \frac{d\Gamma}{dE_\gamma} = (4E_\gamma - m_b) F(m_b - 2E_\gamma) + \frac{1}{m_b} \left[h_1(m_b - 2E_\gamma) + H_2(m_b - 2E_\gamma) \right] \quad (6)$$

where

$$\Gamma_0^s = \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha |C_7^{\rm eff}|^2 m_b^5}{32\pi^4} \quad (7)$$

and the functions F, h_1 and H_2 are defined in (14) below.

The subleading twist corrections to the lepton energy spectrum in $B \to X_u \ell \bar{\nu}$ may be calculated in much the same way as the photon spectrum for $B \to X_s \gamma$. The matching calculation is performed by expanding full QCD matrix elements to subleading twist and identifying the result with non-local operators introduced in [6]. The calculation is standard and uses the optical theorem to obtain the lepton energy spectrum from the imaginary part of a forward scattering matrix element. This matrix element is rewritten using $p_b^\mu = m_b v^\mu + k^\mu$ and $Q^\mu = m_b v^\mu - p_\ell^\mu$ and the light cone vectors n^μ, $\bar{n}^\mu$ ($n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$, $2v = n + \bar{n}$), defined by $p_\ell^\mu = E_\ell \bar{n}^\mu$. The endpoint region is defined by

$$\bar{n} \cdot Q = m_b \quad n \cdot Q = m_b - 2E_\ell \sim O(\Lambda_{\mathrm{QCD}})$$

and the matrix element is expanded in powers of $1/m_b$, assuming $k^\mu \sim O(\Lambda_{\mathrm{QCD}})$.

The leading twist contribution is the first term of this expansion. It can be written as a convolution of a Wilson coefficient function $C_0(\omega)$ with an operator $O_0(\omega)$

$$\int d\omega\, C_0(\omega) O_0(\omega) = \int d\omega\, C_0(\omega) \bar{h} \delta(in \cdot D + \omega) h \quad (8)$$

and one finds at tree level

$$C_0(\omega) = 2\,\Gamma_0\,\theta(m_b - 2E_\ell - \omega)\,, \quad (9)$$

leading to Eq. (1).

At subleading twist there are five nonlocal operators contributing to B meson decays [6]

$$\begin{aligned}
O_1^\mu(\omega) &= \bar{h}_v \left\{ iD^\mu, \delta(in\cdot D + \omega) \right\} h_v \\
P_2^{\mu\alpha}(\omega) &= \bar{h}_v s^\alpha \left[iD^\mu, \delta(in\cdot D + \omega) \right] h_v \\
O_T(\omega) &= i \int d^4y\, \frac{dt}{2\pi}\, e^{-i\omega t} T\left(\bar{h}_v(0) h_v(t) O_{1/m}(y) \right) \\
O_3(\omega) &= \int d\omega_1 d\omega_2\, \frac{\delta(\omega - \omega_1) - \delta(\omega - \omega_2)}{\omega_1 - \omega_2} \\
&\left[h_v \delta(in\cdot D + \omega_2)(iD_\perp)^2 \delta(in\cdot D + \omega_1) h_v \right] \\
P_4^\alpha(\omega) &= g \int d\omega_1 d\omega_2\, \frac{\delta(\omega - \omega_1) - \delta(\omega - \omega_2)}{\omega_1 - \omega_2} \\
&\left[h_v s^\alpha \delta(in\cdot D + \omega_2) \epsilon_\perp^{\mu\nu} G_{\mu\nu}^\perp \delta(in\cdot D + \omega_1) h_v \right],
\end{aligned}$$

where $s^\alpha = P_+ \gamma^\alpha \gamma^5 P_+$ and $O_{1/m}$ are the subleading contributions to the Lagrangian.

To match onto these operators we expand the full theory calculations to one order higher in k^μ / m_b and $n \cdot Q / m_b$. The result for the coefficients for the subleading twist operators $O_1^\mu(\omega)$ and $O_3(\omega)$ is

$$C_1^\mu(\omega) = \frac{\Gamma_0}{m_b} n^\mu \theta(m_b - 2E_\ell - \omega) \quad (10)$$

$$C_3(\omega) = -\frac{2\Gamma_0}{m_b} \theta(m_b - 2E_\ell - \omega)\,. \quad (11)$$

The result for the the coefficient $C_3(\omega)$ agrees with the prediction from reparameterization invariance, which relates this coefficient to the leading order Wilson coefficient to all orders in perturbation theory [9]. For the operators $P_2^{\mu\alpha}(\omega)$ and $P_4^\alpha(\omega)$ we obtain the matching coefficients

$$D_2^{\mu,\alpha}(\omega) = -\frac{\Gamma_0}{m_b} i\epsilon_\perp^{\mu\alpha} \theta(m_b - 2E_\ell - \omega) \quad (12)$$

$$D_4^\alpha(\omega) = \frac{3\Gamma_0}{m_b} n^\alpha \theta(m_b - 2E_\ell - \omega)\,. \quad (13)$$

Finally we have to parametrize the matrix elements of these operators between B meson states. We define

$$\begin{aligned}
\langle H | O_0(\omega) | H \rangle &= 2m_B\, f(\omega) \\
\langle H | O_1^\mu(\omega) | H \rangle &= -4m_B\, \omega f(\omega)\, (v^\mu - n^\mu) \\
\langle H | O_3(\omega) | H \rangle &= 2m_B\, G_2(\omega) \\
\langle H | O_T(\omega) | H \rangle &= 2m_B\, t(\omega) \\
\langle H | P_2^{\alpha,\mu}(\omega) | H \rangle &= -2m_B\, i\epsilon_\perp^{\mu\alpha} h_1(\omega) \\
\langle H | P_4^\alpha(\omega) | H \rangle &= 4m_B\, (v^\alpha - n^\alpha) H_2(\omega)
\end{aligned} \quad (14)$$

Due to reparametrization invariance and the general structure of the $1/m_b$ corrections we find that we only can have the combination

$$F(\omega) = f(\omega) + \frac{1}{2m_b} \left[t(\omega) - 2\,G_2(\omega) \right]\,. \quad (15)$$

which is a universal function for any decay rate. Putting all this information together, we therefore find for the lepton energy spectrum in $B \to X_u \ell \bar{\nu}$

$$\begin{aligned}
\frac{d\Gamma}{dE_\ell} &= \frac{4\,\Gamma_0}{m_b} \int d\omega\, \theta(m_b - 2E_\ell - \omega) \\
&\left[F(\omega) \left(1 - \frac{\omega}{m_b} \right) - \frac{1}{m_b} h_1(\omega) + \frac{3}{m_b} H_2(\omega) \right]
\end{aligned} \quad (16)$$

up to terms of order $\Lambda_{\rm QCD}^2/m_b^2$. We can now use these results to get the $O(\Lambda_{\rm QCD}/m_B)$ corrections to the determination of $|V_{ub}|$ given in Eq. (4). We find for the observables defined in Eq. (5)

$$\Gamma_u(E_c) = \frac{4\,\Gamma_0}{m_b}\int_{E_c}^{m_B/2} dE \int_{-\infty}^{m_b-2E} d\omega$$
$$\left[F(\omega)\left(1-\frac{\omega}{m_b}\right) - \frac{1}{m_b}h_1(\omega) + \frac{3}{m_b}H_2(\omega)\right] \quad (17)$$
$$\Gamma_s(E_c) = \frac{2\Gamma_0^s}{m_b}\int_{E_c}^{m_B/2} dE \int_{-\infty}^{m_b-2E} d\omega$$
$$\left[F(\omega)\left(1-\frac{2\omega}{m_b}\right) + \frac{1}{m_b}h_1(\omega) + \frac{1}{m_b}H_2(\omega)\right] \quad (18)$$

where Γ_0^s and Γ_0 are defined in Eqs. (7) and (1). This finally allows us to obtain the correction to the determination of V_{ub}

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right| = \left(\frac{3\alpha}{\pi}|C_7^{\rm eff}|^2\frac{\Gamma_u(E_c)}{\Gamma_s(E_c)}\right)^{\frac{1}{2}}(1+\delta(E_c))\,, \quad (19)$$

where $\delta(E_c)$ can be calculated form (17) and (18).

To get a quantitative estimate for these corrections, we have to model the subleading structure functions. Using a simple model for the leading order structure function given in [10], we take the subleading functions as the derivative of the leading-order shape function, normalized such that they reproduce the known first moments.

$$f_{\rm mod}(\omega) = \frac{32}{\pi^2\Lambda}\left(1+\frac{\omega}{\Lambda}\right)^2 \exp\left[-\frac{4}{\pi}\left(1+\frac{\omega}{\Lambda}\right)^2\right] \theta\left(1+\frac{\omega}{\Lambda}\right) \quad (20)$$

where $\Lambda = m_B - m_b$. This gives

$$F_{\rm mod}(\omega) = f_{\rm mod}(\omega) + \frac{\lambda_1 - 9\lambda_2}{6\,m_b} f'_{\rm mod}(\omega) \quad (21)$$
$$h_{1\,{\rm mod}} = \lambda_2\, f'_{\rm mod}(\omega) \qquad H_{2\,{\rm mod}} = -\lambda_2\, f'_{\rm mod}(\omega)$$

and allows us to compute the correction $\delta(E_c)$, which is plotted in Fig. 1.

The dashed line in Fig. 1 indicates a typical cut, and the corrections in this case are of the order of 15% and hence are sizable. Since this result is obtained using a model, the conservative estimate is to assign an uncertainty of this size to the determination of V_{ub}. However, it has been argued [11] that the situation may be not as bad, since the first moments of the subleading functions are known and that the situation may be improved including this information.

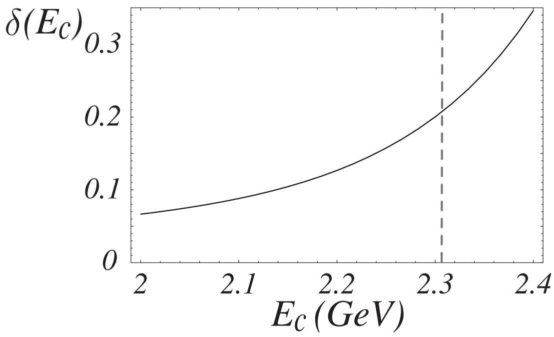

Figure 1. $\delta(E_c)$ as a function of E_c.

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