Determination of α_s and the charm and bottom quark masses from recent measurements of $R(s)$

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Recent experimental data for the total cross section $\sigma(e^+e^- \to \text{hadrons})$ are compared with the up-to-date theoretical prediction of perturbative QCD for those energies where perturbation theory is reliable. This allows for a determination of the strong coupling $\alpha_s^{(4)}(5\text{ GeV}) = 0.235 \pm 0.047$. The precise data from the charm threshold region, when combined with the recent evaluation of moments with three loop accuracy, lead to a direct determination of the short distance $\overline{\text{MS}}$ charm quark mass $m_c(m_c) = 1.304(27)$. Applying the same approach to the bottom quark we obtain $m_b(m_b) = 4.191(51)$ GeV.

1. Introduction

During the past years new and more precise data for $\sigma(e^+e^- \to \text{hadrons})$ have become available in the low energy region between 2 and 10 GeV. At the same time increasingly precise calculations have been performed in the framework of perturbative QCD (pQCD), both for the cross section as a function of the center-of-mass energy $\sqrt{s}$, including quark mass effects, and for its moments which allow for a precise determination of the quark mass. A fresh look at the determination of α_s and the evaluation of the charm quark mass with the help of sum rules is thus an obvious task. We will concentrate on low moments as suggested by the ITEP group long ago [1]. This is a natural route to determine directly a short distance mass, say $m_c(m_c)$ or even $m_c(3\text{ GeV})$ in the $\overline{\text{MS}}$ scheme [1,2]. In the present work we concentrate on the mass in the $\overline{\text{MS}}$ scheme, $m_c(\mu)$, and adopt $\mu = 3$ GeV as our default value, a scale characteristic for the present problem and sufficiently high to ensure convergence of the perturbative series. As already discussed in [1], fixed order pQCD is adequate, if only low moments are used in the analysis.

2. α_s from the continuum region

As stated in the Introduction, we distinguish two energy regions: first, the continuum region where pQCD and local duality are expected to give reliable predictions for the hadronic cross section and, second, the charm threshold region starting from the D meson threshold at 3.73 GeV up to approximately 5 GeV, where the cross section exhibits rapid variations, plus the J/Ψ and Ψ' resonances. The former will be mainly sensitive to the value of α_s. The latter will be used to evaluate moments and to determine the charm quark mass.

The most recent theoretical results for $R(s)$ based on NNLO pQCD, including quark mass effects, were employed in [3]. Assigning a fully correlated systematic error of 4.3% to the BES data [4] they find

$$\alpha_s^{(3)}(3\text{ GeV}) = 0.369^{+0.047+0.123}_{-0.046-0.130}\,, \qquad (1)$$

from the BES data below 3.73 GeV and

$$\alpha_s^{(4)}(4.8\text{ GeV}) = 0.183^{+0.059+0.053}_{-0.064-0.057}\,, \qquad (2)$$

from the point at 4.8 GeV. The uncertainties refer to the uncorrelated (statistical) and correlated errors, respectively. The same analysis is applicable to the MD-1 results. From their average value

$R((8.9\text{ GeV})^2) = 3.578 \pm 0.021 \pm 0.140$ as quoted in their summary one obtains

$$\alpha_s^{(4)}(8.9\text{ GeV}) = 0.193^{+0.017+0.127}_{-0.017-0.107}\,, \tag{3}$$

From the CLEO value $R((10.52\text{ GeV})^2) = 3.56 \pm 0.01 \pm 0.07$ one obtains

$$\alpha_s^{(4)}(10.52\text{ GeV}) = 0.186^{+0.008+0.061}_{-0.008-0.057}\,. \tag{4}$$

Combining the results of Eqs. (1), (2), (3) and (4) by assuming uncorrelated systematic errors and evolving them to a common scale of 5 GeV leads to

$$\alpha_s^{(4)}(5\text{ GeV}) = 0.235^{+0.047}_{-0.047}\,. \tag{5}$$

Evolving this value up to M_Z

$$\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}\,, \tag{6}$$

good agreement with other determinations [5] is observed.

3. Charm quark mass determination from the threshold region

The optimal method for the determination of the charm quark mass is based on the direct comparison of theoretical and experimental moments of the charm quark contribution to the photon polarization function. In the limit of small momentum the latter can be cast into the form [6]

$$\Pi_c(q^2) = Q_c^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n z^n\,, \tag{7}$$

with $z = q^2/(4m_c^2)$ where $m_c = m_c(\mu)$ is the $\overline{\text{MS}}$ charm quark mass at the scale μ. The perturbative series for the coefficients $\bar{C}_n$ up to $n = 8$ is known analytically [7,6] up to order α_s^2. We define the moments through

$$\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s) = \mathcal{M}_n^{\text{th}} = \frac{9}{4} Q_c^2 \left(\frac{1}{4m_c^2}\right)^n \bar{C}_n\,. \tag{8}$$

The charm quark mass can be obtained from

$$m_c(\mu) = \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\text{exp}}}\right)^{1/(2n)}\,. \tag{9}$$

Figure 1. $m_c(m_c)$ for $n = 1, 2, 3$ and 4. For each value of n the results from left to right correspond the inclusion of terms of order α_s^0, α_s^1 and α_s^2 in the coefficients $\bar{C}_n$. Note, that for $n = 3$ and $n = 4$ the errors can not be determined with the help of Eq. (9) in those cases where only the two-loop corrections of order α_s are included into the coefficients $\bar{C}_n$ as the equation cannot be solved for $m_c(3\text{ GeV})$.

The analysis of the experimental moments with $n = 1, \ldots, 4$ leads to the results displayed in Fig. 1.

The moment with $n = 1$ is evidently least sensitive to non-perturbative contributions from condensates, to the Coulombic higher order effects, the variation of μ and the parametric α_s dependence. Hence

$$m_c(m_c) = 1.304(27)\text{ GeV}\,. \tag{10}$$

is adopted as the final result. Using the two- and three-loop relation [8–10] between the pole- and the $\overline{\text{MS}}$-mass this corresponds to

$$M_c^{(2-\text{loop})} = 1.514(34)\text{ GeV}\,,$$
$$M_c^{(3-\text{loop})} = 1.691(35)\text{ GeV}\,. \tag{11}$$

4. The bottom quark mass

The same approach is also applicable to the determination of m_b. Again a significant improvement of the stability of the prediction by inclusion of the NNLO terms is observed. As final result one finds

$$m_b(m_b) = 4.191(51)\text{ GeV}\,, \tag{12}$$

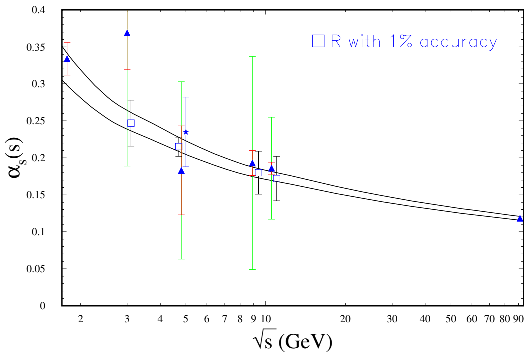

Figure 2. α_s as a function of $\sqrt{s}$. Results from τ and Z decays and those extracted in Section 2 are shown. The two error bars on the data points indicate the statistical (inner) and systematical (outer) uncertainty. For the combined result (indicated by a star) at $\sqrt{s} = 5.0$ GeV only the error after adding the statistical and systematical uncertainty in quadrature is shown. For illustration $\delta\alpha_s$ reduced to the error as expected from $\delta R/R = 1\%$ are also shown (slightly displaced in order to make the presentation more visible).

well consistent with evaluations based on the analysis of bottonium and high spectral moments [11–16]. The $\overline{\rm MS}$ result of Eq. (12) corresponds to a pole mass of [8–10]

$$\begin{aligned} M_b^{(2-\text{loop})} &= 4.651(57)\ \text{GeV}\,, \\ M_b^{(3-\text{loop})} &= 4.819(57)\ \text{GeV}\,, \end{aligned} \tag{13}$$

using $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ accuracy, respectively.

5. Perspectives [17]

Precise measurements of the R-ratio in the charm- and bottom-threshold region are well within reach at CLEO. The values of α_s as derived from present measurements in Section 2 and the impact of a reduction of the relative error down to 1% are demonstrated in Fig. 2 [17].

The quark mass determinations are at present strongly affected by the relatively large error in the electronic widths of the narrow charmonium and bottomium resonances and by the poorly measured threshold region. More precise measurements might well lead to quark mass determinations with $\delta m_Q \approx 15$ MeV and 30 MeV for charm and bottom, respectively.

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