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QCD SUM RULES - A WORKING TOOL FOR HADRONIC PHYSICS

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QCD sum rules are overviewed with an emphasize on the practical applications of this method to the physics of light and heavy hadrons.

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1. Introduction

Imagine a big birthday cake for Arkady Vainshtein, each candle on that cake corresponding to one of his outstanding contributions to the modern particle theory. I think, a very bright and illuminating candle should then mark QCD sum rules.

The renown papers introducing QCD sum rules [1] have been published by Shifman, Vainshtein and Zakharov in 1979. The method, known also under the nickname of SVZ or ITEP sum rules, very soon became quite popular in the particle theory community, especially in Russia. Not only experienced theorists, but also many students of that time contributed to the development of this field with important results. It was indeed a lot of fun to start with an explicit QCD calculation in terms of quark-gluon Feynman diagrams and end up estimating dynamical characteristics of real hadrons. The flexibility and universality of the sum rule method allowed one to go from one interesting problem to another, describing, in the same framework, very different hadronic objects, from pions and nucleons to charmonium and B mesons. Nowadays, QCD sum rules are still being

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actively used, providing many important applications and representing an important branch on the evolution tree of approximate QCD methods.

In this short overview, I start, in Sect. 2, from explaining the basic idea of sum rules which is rooted in quantum mechanics. After that, in Sect. 3, I outline the SVZ sum rule derivation in QCD. Some important applications and extensions of the method are listed in Sect. 4. Furthermore, in Sect. 5 I demonstrate how QCD sum rules are used to calculate the soft contributions to the pion form factor. The light-cone version of sum rules is introduced. Many interesting applications of QCD sum rules remain outside this survey, some of them can be found in recent reviews [2,3].

2. SVZ sum rules in quantum mechanics

To grasp the basic idea of the QCD sum rule method it is sufficient to consider a dynamical system much simpler than QCD, that is quantum mechanics of a nonrelativistic particle in the potential $V(r)$. The latter has to be smooth enough at small distances and confining at large distances. The spherically-symmetrical harmonic oscillator $V(r) = m\omega^2 r^2/2$ is a good example. Evidently, having defined the potential, one is able to solve the problem *exactly* e.g., by means of the Schrödinger equation, $H\psi_n(\vec{x}) = E_n\psi_n(\vec{r})$, with the Hamiltonian $H = \vec{p}\,^2/2m + V(r)$, obtaining the wave functions $\psi_n(\vec{r})$ and energies E_n of all eigenstates, $n = 0, 1, \dots$.

As demonstrated in [4], it is possible to use an alternative procedure allowing one to calculate *approximately* the energy E_0 and the wave function at zero, $\psi_0(0)$, of the lowest level. The starting object is the time-evolution operator, or the Green's function of the particle $G(\vec{x}_2, \vec{x}_1; t_2 - t_1)$, taken at $\vec{x}_1 = \vec{x}_2 = 0$ and written in terms of the standard spectral representation:

$$G(\vec{x}_2 = 0, \vec{x}_1 = 0; t_2 - t_1) = \sum_{n=0}^{\infty} |\psi_n(0)|^2 e^{-iE_n(t_2-t_1)} \,. \tag{1}$$

Performing an analytical continuation of the time variable to imaginary values: $t_2 - t_1 \to -i\tau$, one transforms Eq. (1) into a sum over decreasing exponents:

$$G(0,0;-i\tau) \equiv M(\tau) = \sum_{n=0}^{\infty} |\psi_n(0)|^2 e^{-E_n\tau} \,. \tag{2}$$

The function $M(\tau)$ has a *dual* nature depending on the region of the variable τ. At small τ, the perturbative expansion for $M(\tau)$ is valid, and it is sufficient to retain a few first terms:

$$M^{pert}(\tau) = M^{free}(\tau)\left(1 - 4m\int\limits_0^\infty rdrV(r)e^{-2mr^2/\tau} + O(V^2) + ...\right)\,, \quad (3)$$

where $M^{free}(\tau) = (\frac{m}{2\pi\tau})^{3/2}$ is the Green's function of the free particle motion. Using QCD terminology, we may call the behavior of $M(\tau)$ at small τ "asymptotically free" having in mind that it is approximated by a universal, interaction-free particle motion. Equating (2) and (3) we obtain

$$\sum_{n=0}^{\infty} |\psi_n(0)|^2 e^{-E_n\tau} \simeq M^{pert}(\tau)\,, \quad (4)$$

a typical *sum rule* which is valid at small τ, relating the sum over the bound-state contributions to the result of the perturbative expansion. Note that the latter includes certain "nonperturbative" or "long-distance" effects too, namely the subleading terms containing the interaction potential V.

At large τ one has a completely different picture. In the spectral representation (2) the entire sum over excited levels dies away exponentially with respect to the lowest level contribution:

$$\lim_{\tau\to\infty} M(\tau) = |\psi_0(0)|^2 e^{-E_0\tau}\,. \quad (5)$$

Thus, at large τ one encounters a typical "confinement" regime, because the lowest level parameters determining $M(\tau)$ essentially depend on the long-distance dynamics (in this case determined by $V(r)$ at large r).

An important observation made in [4] is that at intermediate values of τ both descriptions (3) and (5) are approximately valid (see Fig. 1). It is therefore possible to retain only the lowest-level contribution in the sum rule (4) allowing one to estimate both E_0 and $|\psi_0(0)|$, without actually solving the Schrödinger equation.

To further improve the quality of this determination, $M^{pert}(\tau)$ can be rewritten in a form of the integral

$$M^{pert}(\tau) = \int\limits_0^\infty \rho^{pert}(E)e^{-E\tau}dE\,, \quad (6)$$

resembling the spectral representation, so that the positive definite function $\rho^{pert}(E)$ can be called perturbative spectral density. The integral in Eq. (6) is then splitted into two parts, introducing some threshold energy $E_{th} > E_0$

and the sum over all excited states $n \geq 1$ in Eq. (4) is approximated by an integral over $\rho^{pert}(E)$ starting from this threshold:

$$\sum_{n=1}^{\infty} |\psi_n(0)|^2 e^{-E_n \tau} \simeq \int\limits_{E_{th}}^{\infty} \rho^{pert}(E) e^{-E\tau} dE \,. \tag{7}$$

The latter equation can be called a "duality" relation having in mind duality between the asymptotic-freedom regime and the spectral sum. The integral (7) is then subtracted from both sides of Eq. (4) leading to the sum rule for the lowest level:

$$|\psi_0(0)|^2 e^{-E_0 \tau} = \int\limits_{0}^{E_{th}} \rho^{pert}(E) e^{-E\tau} dE \,. \tag{8}$$

Note that one could make use of the sum rule relations similar to Eq. (4) in an opposite way. Imagine that the interaction potential is unknown but we have a possibility to measure, for a set of low levels, their wave functions at zero separations and energies experimentally. The sum rule (4) could then be used to extract or at least constrain the potential $V(r)$.

Interestingly, quantum mechanics may also serve as a model for more complicated patterns of nonperturbative interactions, in which case sum rules have to be treated with care. An example presented in [5] is a po-

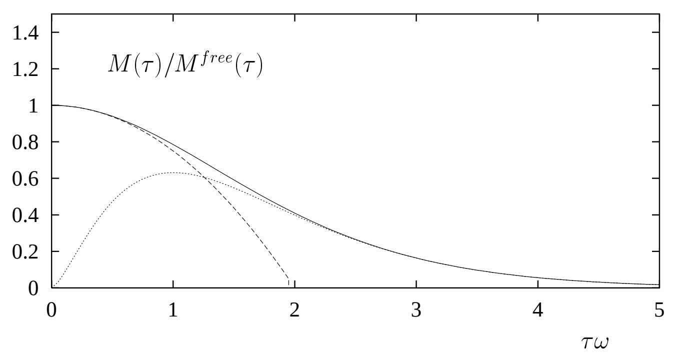


Figure 1. The analytically continued Green's function $M(\tau)$ for a particle in the oscillator $V(r) = m\omega^2 r^2/2$, normalized to $M^{free}(\tau)$ and plotted as a function of $\tau\omega$. The exact solution (solid) is compared with the perturbative calculation including the first order in V correction (dashed) and with the contribution of the lowest bound state (dotted).

tential containing two terms: $V(r) = V_0[(r/r_4)^4 + (r/r_{11})^{11}]$ with $r_{11} \ll r_4$, that is, a sharp short-range confining potential combined with a broader one. At $\tau \to 0$, in the perturbative expansion of $M(\tau)$ the correction due to the second term in the potential is much smaller than the correction associated with the first term. However, if one ignores the "nonperturbative effect" related to the short-distance scale r_{11}, the resulting sum rule simply reproduces the lowest level in the potential $V(r) = V_0(r/r_4)^4$. In reality, the physical picture is quite different because it is the $\sim r^{11}$ part of the potential which mainly determines the formation of bound states. The important lesson drawn from this example is: if for some reason a short-distance nonperturbative effect is missing and/or ignored, the sum rule does not work (or, in other words, duality is violated).

Interestingly, the sum rule approach in quantum mechanics can be generalized to calculate more complicated characteristics such as the amplitudes of electric-dipole transitions between the lowest S and P levels in a given nonrelativistic potential [6]. One has to construct a three-point correlation function:

$$\widetilde{M}(\tau_1,\tau_2) = \Big\{ \int dt_3 d\vec{x}_3 \frac{\partial}{\partial|\vec{x}_2|} G(\vec{x}_2, \vec{x}_3; -i\tau_2 - t_3) \times (\vec{e}\cdot\vec{v}_3) G(\vec{x}_3, \vec{x}_1; t_3 - (-i\tau_1)) \Big\}_{\vec{x}_{1,2}=0}, \tag{9}$$

where $\vec{v} = i(H\vec{x} - \vec{x}H)$ is the quantum-mechanical velocity operator, and $\vec{e}\cdot\vec{v}$ is the operator corresponding to the dipole radiation of a photon with polarization $\vec{e}$. The correlator (9) corresponds to the propagation of a particle in P wave (below threshold or in imaginary time) from point 2 to point 3 where a dipole photon is radiated and then further propagation to point 1 in S wave. Calculating Eq. (9) perturbatively and matching it to the double spectral sum over P and S levels, one gets a sum rule which, at intermediate values of the two variables $\tau_{1,2}$ is well approximated by the contributions of the three lowest E1 transition amplitudes ($1P \to 1S, 2S \to 1P, 2P \to 2S$).

The sum rule approach considered here is, of course not very important for quantum mechanics itself, but as we shall see in a moment, serves as a very convenient prototype for an analogous method in QCD, in the theory where no exact solution is so far available.

3. SVZ Sum rules in QCD

We now move from the safe haven of nonrelativistic quantum mechanics to QCD, a complicated theory with a rich pattern of quark-gluon and gluon-gluon interactions. At short distances, due to asymptotic freedom the theory can still be resolved. One considers a quasi-free quark propagation with calculable perturbative corrections. However, at large distances, $r \sim 1/\Lambda_{QCD}$, the QCD perturbation theory becomes inapplicable and the confinement phenomenon takes over, driven by the quark-gluon fluctuations in the QCD vacuum. As a result, quarks build coherent bound states, hadrons. In general, it is not possible to describe QCD interactions with a potential. Nevertheless, qualitatively, the pattern of quark-antiquark forces in QCD, with asymptotic freedom at small distances and formation of bound states at large distances, is very similar to the quantum-mechanical motion in the confining, oscillator-type potential considered in the previous section. It is therefore not surprising that sum rules [1] analogous to the quantum-mechanical ones exist also in QCD.

The starting object in QCD analogous to the Green's function $G(0,0,t)$ is the correlation function describing an evolution of a colorless quark-antiquark pair emitted and absorbed by external currents. A "classical example" considered in [1] is the correlation of two $j^\rho_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2$ quark currents with the ρ^0 meson quantum numbers (isospin 1, $J^P = 1^-$):

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|T\{j^\rho_\mu(x), j^\rho_\nu(0)\}|0\rangle\,. \tag{10}$$

The dispersion relation (Källen-Lehmann representation) for this correlation function contains a sum over all intermediate hadronic states, a direct analog of the spectral representation (1) :

$$\Pi_{\mu\nu}(q) = \sum_h \frac{\langle 0|j^\rho_\mu|h\rangle\langle h|j^\rho_\nu|0\rangle}{m_h^2 - q^2} + \text{subtractions}\,. \tag{11}$$

Note that, for brevity, I wrote the above relation in a very schematic way, including the excited ρ resonances and the continuum states with ρ quantum numbers in one discrete sum.

The Borel transformation, $\hat{B}\{1/(m_h^2 - q^2)\} \to \exp(-m_h^2/M^2)$, converts the hadronic representation (11) into a sum over decreasing exponents,

$$\hat{B}\Pi_{\mu\nu} = \sum_h \langle 0|j^\rho_\mu|h\rangle\langle h|j^\rho_\nu|0\rangle e^{-m_h^2/M^2}\ , \tag{12}$$

i.e., the inverse Borel variable $1/M^2$ plays essentially the same role as the auxiliary variable τ in the quantum-mechanical case. Another very important virtue of the Borel transformation is that it kills subtraction terms in the dispersion relation.

At large spacelike momentum transfers $q^2 < 0$, $Q^2 \equiv -q^2 \gg \Lambda^2_{QCD}$ (corresponding to large $M \gg \Lambda_{QCD}$ after Borel transformation) the quark-antiquark propagation described by the correlation function (10) is highly virtual, the characteristic times/distances being $x_0 \sim |\vec{x}| \sim 1/\sqrt{Q^2}$. One can then benefit from asymptotic freedom and calculate the correlation function in this region perturbatively. The corresponding diagrams up to $O(\alpha_s)$ are depicted in Fig. 2.

As first realized in [1], there are additional important effects due to the interactions with the vacuum quark and gluon fields. The latter have typically long-distance ($\sim \Lambda_{QCD}$) scales and, in first approximation, can be replaced by static fields, the *vacuum condensates*. An adequate framework to include these effects in the correlation function was developed in a form of the Wilson operator product expansion (OPE). The Borel transformed answer for the correlation function (10) reads:

$$\hat{B}\Pi^{OPE}_{\mu\nu} = \hat{B}\Pi^{pert}_{\mu\nu} + \sum_{d=3,4,\ldots} \hat{B}C^{d}_{\mu\nu}\langle 0|O_d|0\rangle\,, \tag{13}$$

where the first term on the r.h.s. corresponds to the perturbative diagrams in Fig. 2, whereas the sum contains the contributions of vacuum condensates, ordered by their dimension d. Diagrammatically, these contributions are depicted in Fig. 3. The terms with $d \le 6$ contain the vacuum aver-

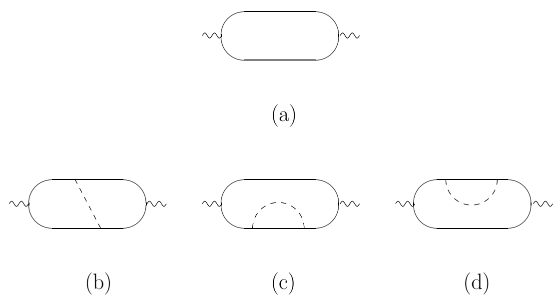


Figure 2. Diagrams determining the perturbative part of the correlation function (10): the free-quark loop (a) and the $O(\alpha_s)$ corrections (b,c,d). Solid lines denote quarks, dashed lines gluons, wavy lines external currents.

ages of the operators $O_3 = \bar{q}q$, $O_4 = G^a_{\mu\nu}G^{a\mu\nu}$, $O_5 = \bar{q}\sigma_{\mu\nu}(\lambda^a/2)G^{a\mu\nu}q$, $O_6 = (\bar{q}\Gamma_r q)(\bar{q}\Gamma_s q)$, and $O^G_6 = f_{abc}G^a_{\mu\nu}G^{b\,\nu}_{\sigma}G^{c\sigma\mu}$, where $q = u, d, s$ are the light-quark fields, $G^a_{\mu\nu}$ is the gluon field strength tensor, and $\Gamma_{r,s}$ denote various combinations of Lorentz and color matrices. Importantly, to compensate the growing dimension of the operator O_d, the Wilson coefficients $C^d_{\mu\nu}$ contain increasing powers of $1/Q^2$. Correspondingly, $\hat{B}C^d_{\mu\nu}$ contain powers of $1/M^2$, making it possible at large M^2 to retain in the r.h.s. of

Eq. (13) only a few first condensates. Thus, at $M^2 \sim 1$ GeV^2 it is practically possible to neglect all operators with $d > 6$.

Equating at large M^2 the hadronic representation to the result of the OPE calculation we obtain the desired sum rule:

$$\sum_h \langle 0|j^\rho_\mu|h\rangle\langle h|j^\rho_\nu|0\rangle e^{-m_h^2/M^2} = \hat{B}\Pi^{pert}_{\mu\nu} + \sum_{d=3,4,..} \hat{B}C^d_{\mu\nu}\langle 0|O_d|0\rangle\,. \tag{14}$$

The explicit form of this relation is [1] :

$$\begin{aligned} & f_\rho^2 e^{-m_\rho^2/M^2} + \{\text{excited,continuum } \rho \text{ states}\} \\ &= M^2\Big[\frac{1}{4\pi^2}\left(1+\frac{\alpha_s(M)}{\pi}\right) + \frac{(m_u+m_d)\langle\bar{q}q\rangle}{M^4} \\ &\quad + \frac{1}{12}\frac{\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\rangle}{M^4} - \frac{112\pi}{81}\frac{\alpha_s\langle\bar{q}q\rangle^2}{M^6}\Big]\,, \end{aligned} \tag{15}$$

where the decay constant of the ρ meson is defined in the standard way, $\langle\rho^0 \mid j^\rho_\nu \mid 0\rangle = (f_\rho/\sqrt{2})m_\rho\epsilon^{(\rho)*}_\nu$. In obtaining this relation the four-quark vacuum densities are factorized into a product of quark condensates. The quark-gluon and three-gluon condensates have very small Wilson coefficients and are neglected. The strong coupling α_s is taken at the scale M which is the characteristic virtuality of the loop diagrams after the Borel transformation. A more detailed derivation of this sum rule can be found, e.g. in the review [3]. The QCD vacuum condensates were recently discussed in [7].

In full analogy with quantum mechanics, there exists a SVZ region of intermediate M^2 where the ρ meson contribution alone saturates the l.h.s. of the sum rule (15). To illustrate this statement numerically, in Fig. 4 the experimentally measured f_ρ (obtained from the $\rho^0 \to e^+e^-$ width) is compared with the same hadronic parameter calculated from Eq. (15)

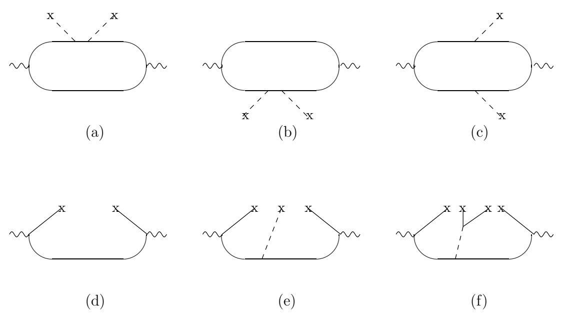


Figure 3. Diagrams corresponding to the gluon (a,b,c,), quark (d), quark-gluon (e) and four-quark (f) condensate contributions to the correlation function (10). The crosses denote the vacuum fields.

where all contributions of excited and continuum states are neglected. One indeed observes a good agreement in the region $M^2 \sim 1$ GeV^2.

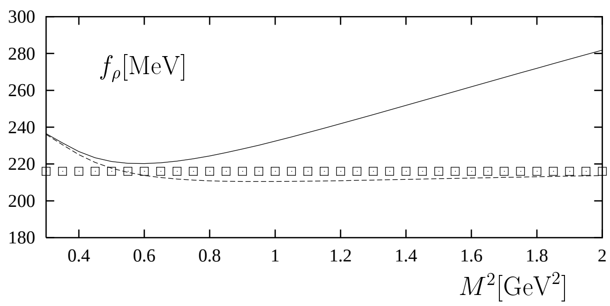


Figure 4. The ρ meson decay constant calculated from the sum rule (15) neglecting all excited and continuum states (solid), as a function of the Borel parameter, in comparison with the experimental value (boxes). The dashed curve corresponds to an improved calculation , where the sum over excited and continuum states is estimated using quark-hadron duality with a threshold $s_0^\rho = 1.7$ GeV^2.

An important step to improve the sum rule (14) is to use the *quark-hadron duality* approximation. The perturbative contribution to the correlation function (the sum of Fig. 2 diagrams) is represented in the form of a dispersion integral and splitted into two parts:

$$\hat{B}\Pi^{pert}_{\mu\nu} = \int\limits_0^{s_0} \rho^{pert}_{\mu\nu} e^{-s/M^2} + \int\limits_{s_0}^{\infty} \rho^{pert}_{\mu\nu} e^{-s/M^2} \,. \tag{16}$$

The sum over excited state and continuum contributions in Eq. (14) is approximated by the second integral over the perturbative spectral density $\rho^{pert}_{\mu\nu}$. This integral is then subtracted from both parts of Eq. (14). Correspondingly Eq. (15) is modified: the l.h.s. contains only the ρ term , and, on the r.h.s., the perturbative contribution has to be multiplied by a factor $(1 - e^{-s_0/M^2})$. The numerical result obtained from the duality improved SVZ sum rule is also shown in Fig. 4. Quark-hadron duality can independently be checked for the channels with sufficient experimental information on excited hadronic states, such as the J/ψ and ρ channels. For ψ resonances one of the first analyses of that type has been done in [8].

Importantly, not all correlation functions lead to valid QCD sum rules. The response of the QCD vacuum to the quarks and gluons "injected" by external currents crucially depends on the quantum numbers and flavour content of the current. After all, that is the main reason why hadrons are not alike [5]. In certain channels, e.g. for the correlation functions of spin zero light-quark currents, specific short-distance nonperturbative effects related to instantons are present (the so called "direct instantons")[5,9]. These effects remain important even at comparatively large M^2 and are not accountable in a form of OPE. The subtle duality balance between "quasiperturbative" OPE and resonances is destroyed in such cases. Due to instantons, it is not possible, for example, to calculate the pion parameters using correlators with pseudoscalar $\bar{u}\gamma_5 d$ currents. Models based on the instanton calculus have to be invoked (see ,e.g.[10]).

The QCD procedure outlined in this section has indeed many similarities with the sum rule derivation in quantum mechanics. To make the analogy more transparent, in the following table I put together the main points of the two sum rule approaches: in quantum mechanics and in QCD.

Quantum mechanics	QCD
Particle in a smooth confining potential	quark-antiquark pair in QCD vacuum
Green's function $G(0,0,t)$	Correlation function $\Pi_{\mu\nu}(q^2)$
Spectral representation	Dispersion relation in q^2
Analytical continuation $to - i\tau$	Borel transformation $q^2 \to M^2$
Perturb. expansion in powers of V	OPE (Condensate expansion)
matching the lowest level to $M^{pert}(\tau)$	matching the lowest hadron to $\hat{B}\Pi(M^2)$
duality of the quasifree-motion and the spectral sum	quark-hadron duality
extracting $V(r)$ from exp. known spectral sum	extracting condensates, $m_{u,d,s,c,b}$ from exp. known spectral density
sum rule does not work if the short-distance part of the potential is ignored	sum rules do not work in the channels with direct instantons
3-point sum rules for E1-transition amplitudes	3-point sum rules for hadronic matrix elements

4. Applying and extending the method

4.1. *Baryons*

Following very successful applications of QCD sum rules in the mesonic channels [1], the next essential step was to extend the method to the baryonic sector [11,12]. Correspondingly, the correlators of specially constructed quark currents with baryon quantum numbers were considered. A well known example is the Ioffe current with the proton quantum numbers:

$$J^N(x) = \epsilon_{abc}(u^{aT}(x)\hat{C}\gamma_\mu u^b(x))\gamma_5\gamma^\mu d^c(x)\,, \tag{17}$$

where a, b, c are color indices and $\hat{C}$ is the charge conjugation matrix. From the QCD sum rule for the correlator $\langle 0|J_N(x)J_N^\dagger(0)|0\rangle$ an approximate formula can be obtained,

$$m_N \simeq [-(2.0)(2\pi)^2\langle 0|\bar{q}q|0\rangle(\mu = 1\mathrm{GeV})]^{1/3}\,, \tag{18}$$

relating the nucleon mass and the quark condensate density. Thus, QCD sum rules unambiguously confirm the fundamental fact that $\sim 99\%$ of the baryonic mass in the Universe is due to the vacuum condensates.

4.2. *Quark mass determination*

The sum rule relations similar to Eq. (14) are widely used to extract the fundamental QCD parameters, not only the condensates themselves but also the quark masses. One needs sufficient experimental data on hadronic parameters in a given channel (masses and decay constants of ground and excited states, experimentally fitted ansätze for continuum states) in order to saturate the hadronic part of the sum rule.

The ratios of the light (u, d, s) quark masses are predicted from the QCD chiral perturbation theory [13]:

$$\frac{m_u}{m_d} = 0.553 \pm 0.043, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8, \quad \frac{2m_s}{m_u + m_d} = 24.4 \pm 1.5\,, \tag{19}$$

(there is also a more recent estimate $m_u/m_d = 0.46 \pm 0.09$ [14]). QCD sum rules offer a unique opportunity to estimate the individual masses of u, d, s quarks. To illustrate the continuous efforts in this direction, let me mention one recent determination of the strange quark mass [15], based on the correlation function of the derivatives of the strangeness-changing vector current $j_\mu = \bar{s}\gamma_\mu q$, $q = u, d$:

$$\Pi^s(q) = i\int d^4x e^{iqx}\langle 0|T\{\partial_\mu j^\mu(x)\partial_\nu j^{\dagger\nu}(0)\}|0\rangle\,. \tag{20}$$

The OPE answer for Π^s is proportional to $(m_s - m_q)^2 \simeq m_s^2$ turning this correlator into a very convenient object for the m_s extraction. Furthermore, the recent progress in the multiloop QCD calculations allows to reach the $O(\alpha_s^3)$ accuracy in the perturbative part of Π^s. An updated analysis of kaon S wave scattering on π, η, η' is used to reproduce the hadronic spectral density. The sum rule yields [15] for the running mass in the $\overline{\mathrm{MS}}$ scheme: $m_s(2\mathrm{GeV}) = 99 \pm 16$ MeV, in a good agreement with the recent lattice

QCD estimates. Using the ratios (19) one obtains $m_u(2\text{GeV}) = 2.9 \pm 0.6$ MeV and $m_d(2\text{GeV}) = 5.2 \pm 0.9$ MeV. The earlier work on predicting the light-quark masses from QCD sum rules is summarized in [3,16].

The charmed quark mass determination was one of the first successful applications of the QCD sum rule approach [17,1]. The correlation function of two $\bar{c}\gamma_\mu c$ currents (in other words, the charm contribution to the photon polarization operator) was matched to its hadronic dispersion relation, where the imaginary part is simply proportional to the $e^+e^- \to charm$ cross section including ψ resonances and the open charm continuum. The lowest power moments of this sum rule at $q^2 = 0$ are well suited for m_c determination because nonpertrbative effects are extremely small. Replacing $c \to b$, $\psi \to \Upsilon$ and open charm by open beauty one obtains analogous sum rule relations for the b quark [18]. In recent years the mainstream development in the heavy quark mass determination went in another direction, employing the higher moments which are less sensitive to the experimental input above the open flavour threshold. These moments, however, demand careful treatment of Coulomb corrections [19] which is only possible in the nonrelativistic QCD (the current status of this field is reviewed in [20]). Recent precise measurements of the $e^+e^- \to hadrons$ cross section on one side and a substantial progress in the calculation of perturbative diagrams on the other side, allowed to reanalyze with a higher precision the low moments of the original SVZ sum rules for quarkonia with the following results [21] for the $\overline{\text{MS}}$ masses: $m_c(m_c) = 1.304 \pm 0.027$ GeV, $m_b(m_b) = 4.209 \pm 0.05$ GeV. Another subset of charmonium sum rules (higher moments at fixed large $q^2 < 0$) was employed in [22], with a prediction for m_c in agreement with the above.

4.3. *Calculation of the B meson decay constant*

Having determined the condensates and quark masses from a set of experimentally proven QCD sum rules for light-quark and quarkonium systems one has an exciting possibility to predict the unknown hadronic characteristics of B meson. In the amplitudes of exclusive weak B decays the hadronic matrix elements are multiplied by poorly known CKM parameters, such as V_{ub}. QCD sum rule calculations may therefore provide a useful hadronic input for extraction of CKM parameters from data on exclusive B decays. Importantly, the theoretical accuracy of the sum rule determination can be estimated by varying the input within allowed intervals.

One of the most important parameters involved in B physics is the B meson decay constant f_B defined via the matrix element $\langle 0|\bar{u}i\gamma_5 b|B\rangle$. The calculation of f_B using QCD sum rules has a long history, a detailed review and relevant references can be found, e,g. in [3,23], I only mention the very first papers [17,24]. One usually employs the SVZ sum rule for the two-point correlator of $\bar{b}i\gamma_5 q$ currents. I will not write down this sum rule explicitly. It looks very similar to the one for f_ρ discussed in sect. 3, in a sense that the sum rule contains a (duality subtracted) perturbative part and condensate terms. The expressions for the Wilson coefficients are in this case much more complicated, especially the radiative corrections to the heavy-light loop diagrams. The recent essential update of the sum rule for f_B is worked out in [25] taking into account the $O(\alpha_s^2)$ corrections to the heavy-light loop recently calculated in [26] and treating the b quark mass in $\overline{\mathrm{MS}}$ scheme. The result is (for $m_b(m_b) = 4.21 \pm 0.05$ GeV): $f_B = 210 \pm 19$ MeV and $f_{B_s} = 244 \pm 21$ MeV, in a good agreement with the most recent lattice QCD determination (including dynamical sea-quark effects). I think, the example of f_B determination demonstrates that QCD sum rules indeed provide a reliable analytical tool for the hadronic B physics.

4.4. *Hadronic amplitudes*

To complete this short survey of QCD sum rule applications, it is important to mention that this method allows to calculate various hadronic amplitudes involving more than one hadron. Let me consider, as a generic example, a calculation of the hadronic matrix element $\langle h_f(p+q)|j|h_i(p)\rangle$ of a certain quark current j with a momentum transfer q. The convenient starting object in this case is the three-point correlation function depending on two independent 4-momenta:

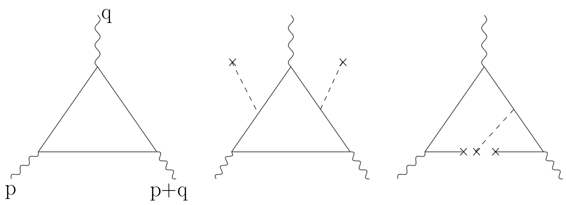
$$T_{fi}(p,q) = (i)^2 \int d^4x\, d^4y\, e^{-i(px+qy)} \langle 0|T\{j_f(0) j(y) j_i(x)\}|0\rangle \,. \qquad (21)$$

As a next step, one writes down a double dispersion relation, in the variables p^2 and $(p+q)^2$ at fixed q^2, expressed in a form similar to Eq. (11):

$$T_{fi}(p,q) = \sum_{h_f} \sum_{h_i} \frac{\langle 0|j_f|h_f\rangle \langle h_f|j|h_i\rangle \langle h_i|j_i|0\rangle}{(m_{h_f}^2 - (p+q)^2)(m_{h_i}^2 - p^2)} + \text{subtractions}\,, \qquad (22)$$

where the double sum includes all possible transitions between the states with h_i and h_f quantum numbers. Two independent Borel transformations in p^2 and $(p+q)^2$ applied to Eq. (22) enhance the ground-state term containing the desired matrix element and allow to get rid of subtraction terms:

$$\hat{B}_1 \hat{B}_2 T_{fi} = \sum_{h_f} \sum_{h_i} \langle 0|j_f|h_f\rangle \langle h_f|j|h_i\rangle \langle h_i|j_i|0\rangle e^{-m_{h_f}^2/M_2^2 - m_{h_i}^2/M_1^2}\,, \qquad (23)$$



(a) (b) (c)

Figure 5. Contributions to the 3-point correlation function (21): (a) perturbative, zeroth order in α_s; (b)-(c) some nonperturbative corrections.

where M_1 and M_2 are the Borel variables corresponding to p^2 and $(p+q)^2$, respectively. On the other hand, the correlator (21) can be computed, in terms of perturbative and condensate contributions:

$$\hat{B}_1\hat{B}_2T^{OPE}_{fi}=\int ds\, ds'\rho^{pert}_{fi}(s,s',q^2)e^{-s'/M_2^2-s/M_1^2}+\sum_{d=3,4,..}\hat{B}_1\hat{B}_2C^{fi}_d\langle 0|O_d|0\rangle\,. \quad (24)$$

In the above, the perturbative contribution calculated from the diagram in Fig. 5a is represented in a convenient form of double spectral representation with a spectral density ρ^{pert}_{fi}. The Wilson coefficients C^{fi}_d are calculated from the diagrams exemplified in Fig. 5b,c. Importantly, one does not introduce new parameters/inputs in this calculation, benefiting from the universality of quark/gluon condensates. The above expansion is valid at large spacelike external momenta: $|p^2|, |(p+q)^2| \gg \Lambda^2_{QCD}$, far from the hadronic thresholds in the corresponding channels. Accordingly, the squared momentum transfer is also kept large, $Q^2 = -q^2 \gg \Lambda^2_{QCD}$, at least for the currents containing light quarks [a]. Equating the hadronic dispersion relation to the OPE result at large M_1^2, M_2^2 and invoking quark-hadron duality one obtains the sum rule for the matrix element:

$$f_i f_f\langle h_f|j|h_i\rangle e^{-m^2_{h_f}/M_2^2-m^2_{h_i}/M_1^2} = \int\limits_{R(s_0,s_0')} ds\; ds'\rho^{pert}_{fi}(s,s',q^2)e^{-s'/M_2^2-s/M_1^2}$$
$$+\sum_d \hat{B}_1\hat{B}_2C^d_{fi}\langle 0|O_d|0\rangle\,, \quad (25)$$

where $f_i = \langle h_i|j_i|0\rangle$, $f_f = \langle 0|j_f|h_f\rangle$ are the decay constants of the initial and final hadrons. The latter are calculable from two-point sum rules, or

[a] Hadronic matrix elements at $q^2 = 0$, e.g., the nucleon magnetic moment can also be calculated within QCD sum rule approach, using the external (background) field technique [27], with additional vacuum condensates induced by external, non-QCD fields.

simply known from experiment. In the above, $R(s_0, s'_0)$ is the quark-hadron duality domain in the s, s' plane, s_0, s'_0 are the corresponding thresholds. Using 3-point correlators, the sum rules for charmonium radiative transitions have been derived in [28]. Another important application [29] is the pion e.m. form factor discussed in more detail in the next section.

5. QCD sum rules and the pion form factor

One of the celebrated study objects in hadronic physics is the pion electromagnetic form factor $F_\pi(q^2)$ determining the pion matrix element $\langle\pi(p+q)|j_\mu^{em}|\pi(p)\rangle = F_\pi(q^2)(2p+q)_\mu$ of the quark e.m. current $j_\mu^{em} = e_u\bar{u}\gamma_\mu u + e_d\bar{d}\gamma_\mu d$.

At very large values of the spacelike momentum transfer $Q^2 \equiv -q^2 \to \infty$ the form factor is determined by the perturbative QCD factorization [30]:

$$F_\pi(Q^2) = \frac{8\pi\alpha_s f_\pi^2}{9Q^2}\left|\int_0^1 du \frac{\varphi_\pi(u)}{1-u}\right|^2 , \tag{26}$$

obtained by the convolution of distribution amplitudes (DA) $\varphi_\pi(u)$ of the initial and final pions (see the definition below) with the $O(\alpha_s)$ quark-gluon hard-scattering amplitude. At finite Q^2, the major problem is to estimate the "soft", $O(\alpha_s^0/Q^4)$ part of this form factor. It corresponds to an overlap of end-point configurations of the quark-parton momenta in the initial and final pions, so that the large momentum is transferred without a hard gluon exchange (the so called Feynman mechanism).

The first model-independent estimate of the soft contribution to the pion form factor was provided by QCD sum rules [29]. The three-point correlator (21) was used, with j, j_i and j_f replaced by j_μ^{em}, $j_{\nu 5}$ and $j_{\rho 5}^\dagger$, respectively, where $j_{\nu 5} = \bar{u}\gamma_\nu\gamma_5 d$ is the axial-vector current generating the pion state from the vacuum: $\langle\pi(p)|j_{\nu 5}^{(\pi)}|0\rangle = -if_\pi p_\nu$. The calculation based on OPE and condensates is valid at sufficiently large Q^2, practically at $Q^2 \sim$ 1 GeV^2. The resulting sum rule for the form factor written in the form (25) has a rather compact expression:

$$f_\pi^2 F_\pi(Q^2) = \int\limits_{R(s_0^\pi)} ds\, ds' \rho^{pert}(s,s',Q^2)e^{-\frac{s+s'}{M^2}}$$
$$+ \frac{\alpha_s}{12\pi M^2}\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle + \frac{208\pi}{81M^4}\alpha_s\langle\bar{q}q\rangle^2\left(1+\frac{2Q^2}{13M^2}\right) , \tag{27}$$

where the perturbative spectral density is

$$\rho^{pert}(s,s',Q^2) = \frac{3Q^4}{4\pi^2}\frac{1}{\lambda^{7/2}}\left[3\lambda(\sigma+Q^2)(\sigma+2Q^2) - \lambda^2 - 5Q^2(\sigma+Q^2)^3\right] , \tag{28}$$

with $\lambda = (\sigma + Q^2)^2 - 4ss'$ and $\sigma = s + s'$. In Eq. (27) the condensates up to $d = 6$ are included, $m_\pi = 0$, $M_1 = M_2 = M$ and $s_0 = s'_0 = s_0^\pi \simeq 0.7$ GeV^2. The duality threshold is inferred from the two-point sum rule for the axial-vector channel [1]. At $Q^2 = 1 \div 3$ GeV^2, the form factor predicted from the sum rule agrees with the experimental data. E.g., compare $F_\pi(Q^2 = 1\mathrm{GeV}^2) \simeq 0.3$ predicted from Eq. (27) with the most accurate CEBAF data [31] shown in Fig. 7 below. The good agreement indicates that the soft mechanism is the most important one in this region and that the $O(\alpha_s)$ hard scattering effect which should dominate at infinitely large Q^2 is still a small correction. (The latter corresponds to the gluon exchanges added to the diagram of Fig. 5a). At large Q^2 the perturbative part of the sum rule (27) has a $\sim 1/Q^4$ behavior, in full accordance with our expectation for the soft, end-point contribution to the form factor. However, the condensate contributions to $F_\pi(Q^2)$ are either Q^2-independent or grow $\sim Q^2/M^2$. A careful look at one of the relevant diagrams in Fig. 5c reveals the reason of this anomalous behavior. Using local (static field) condensate approximation, one implicitly neglects the momenta of vacuum quark/gluon fields. The external large momentum p is carried by a single quark, which, after the photon absorption, propagates with the momentum $p+q$, so that the contribution of this diagram is q^2 independent. Therefore, the truncated local condensate expansion is not an adequate approximation to reproduce the large Q^2 behavior of the pion form factor.

A possibility to calculate $F_\pi(Q^2)$ including both soft and hard scattering effects at large Q^2 [32,33], is provided by the light-cone sum rule (LCSR) approach [34] combining the elements of the theory of hard exclusive processes [30] with the SVZ procedure.

One starts with introducing a vacuum-to-pion correlation function

$$F_{\mu\nu}(p,q) = i \int d^4x\, e^{-iqx} \langle 0 | T\{ j_{\mu5}(0) j_\nu^{\mathrm{em}}(x)\} | \pi(p)\rangle \ , \tag{29}$$

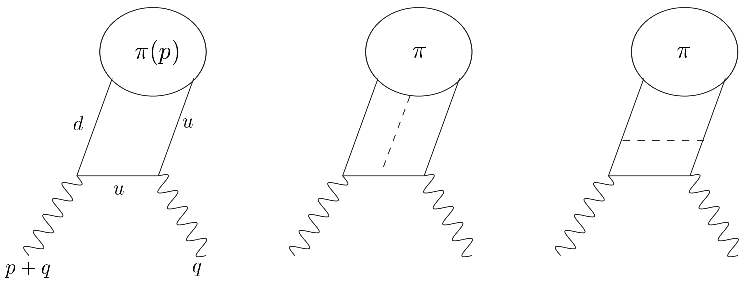
where one of the pions is put on-shell, $p^2 = m_\pi^2$, and the second one is replaced by the generating current $j_{\mu5}$. For this correlator a dispersion relation is written, in full analogy with Eq. (11):

$$F_{\mu\nu}(p,q) = \sum_h \frac{\langle 0 | j_{\mu5} | h\rangle \langle h | j_\nu^{\mathrm{em}} | \pi(p)\rangle}{m_h^2 - (p+q)^2} + \mathrm{subtractions}\,. \tag{30}$$

The lowest pion-state term $(h = \pi)$ in the hadronic sum,

$$F^{(\pi)}_{\mu\nu}(p,q) = \frac{i f_\pi F_\pi(Q^2)(p+q)_\mu (2p+q)_\nu}{m_\pi^2 - (p+q)^2}\,, \tag{31}$$

contains the desired form factor.



(a) (b) (c)

Figure 6. Light-cone expansion of the correlation function (29).

At large spacelike momenta, $Q^2, |(p+q)^2| \gg \Lambda^2_{QCD}$, the correlation function (29) is dominated by small values of the space-time interval x^2, allowing one to expand the product of two currents around the light-cone $x^2 = 0$. The leading-order contribution is obtained from the diagram in Fig. 6a and consists of two parts: (1) the short-distance amplitude involving the virtual quark propagating between the points x and 0, and (2) the vacuum-to-pion matrix element of a nonlocal quark-antiquark operator, $\langle 0|\bar{u}(x)\gamma_\rho\gamma_5 d(0)|\pi\rangle$. This matrix element contains long-distance effects and is therefore not directly calculable. On the other hand, being expanded near $x^2 = 0$ it can be resolved in terms of universal distribution functions:

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|\pi(p)\rangle = -ip_\mu f_\pi \int_0^1 du\, e^{-iup\cdot x}\left(\varphi_\pi(u,\mu) + x^2 g_1(u,\mu)\right)$$
$$+ f_\pi\left(x_\mu - \frac{x^2 p_\mu}{p\cdot x}\right)\int_0^1 du\, e^{-iup\cdot x} g_2(u,\mu) + \dots, \quad (32)$$

where the terms up to $O(x^2)$ are shown explicitly. This expansion contains normalized light-cone distribution amplitudes (DA): $\varphi_\pi(u,\mu)$, $g_1(u,\mu)$, $g_2(u,\mu)$, ... , and the scale μ reflects the logarithmic dependence on x^2. Importantly, the power moments of DA, e.g., $M_n(\mu) = \int_0^1 du\; u^n\varphi_\pi(u,\mu)$, are related to the vacuum-to-pion matrix elements of local quark-antiquark operators with a definite *twist* (dimension minus Lorentz spin). For that reason, φ_π is called twist 2 DA, and, correspondingly, g_1 and g_2 are of twist 4. Thus, in the light-cone OPE one deals with a completely different pattern of long-distance effects, as compared with the local OPE considered in Sect. 3. Instead of a set of universal vacuum condensates, there is a set of DA for a given light meson, each of DA representing a series of matrix elements.

Actually, the twist 2 DA φ_π was originally introduced in the QCD analysis of hard exclusive hadronic processes [30], see e.g., Eq. (26). Some of its properties are well understood, in particular, the following expansion can be written:

$$\varphi_\pi(u,\mu) = 6u(1-u)\left(1+\sum_{n=1} a_{2n}(\mu)C_{2n}^{3/2}(2u-1)\right), \tag{33}$$

based on the approximate conformal symmetry of QCD with light quarks. In the above, C_{2n} are Gegenbauer polynomials and the coefficients $a_{2n}(\mu)$ determine the deviation of φ_π from its asymptotic form $6u(1-u)$. Due to the perturbative evolution, $a_{2n}(\mu)$ are logarithmically suppressed at large μ. The low-scale values of a_{2n} (and of similar coefficients for other DA) have to be considered a nonperturbative input.

The correlation function (29) calculated from the light-cone OPE represents a convolution of the pion DA and short-distance (hard scattering) amplitudes:

$$F_{\mu\nu}(p,q) = 2if_\pi p_\mu p_\nu \int\limits_0^1 du \frac{u\varphi_\pi(u,\mu)}{(1-u)Q^2 - u(p+q)^2} + \dots . \tag{34}$$

For simplicity, only the leading order, twist 2 contribution with the relevant kinematical structure is shown. The ellipses denote the $O(\alpha_s)$ corrections (one of the diagrams is presented in Fig. 6c) and the higher twist contributions suppressed by powers of the denominator. Physically, the higher-twist corrections take into account the transverse momentum of the quark-antiquark state (e.g., the twist 4 terms in the expansion (32)) and the contributions of higher Fock states in the pion wave function (such as the quark-antiquark-gluon DA contributing via the diagram in Fig. 6b.). These two effects are related via QCD equations of motion. More details on the pion DA of higher twists can be found in [35]. The factorization scale μ in Eq. (34) effectively separates the large virtualities ($> \mu^2$) in the hard scattering amplitude from the small ones ($< \mu^2$) in the pion DA.

Equating the dispersion relation (30) to the OPE result (34) at large spacelike $(p+q)^2$, one extracts the form factor $F_\pi(Q^2)$ applying the standard elements of the QCD sum rule technique, the Borel transformation in the variable $(p+q)^2$ and the quark-hadron duality. The latter reduces to a simple replacement of the lower limit in the u-integration in Eq. (34), $0 \to$

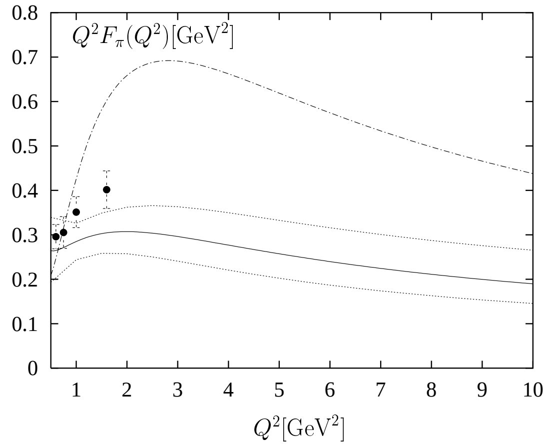


Figure 7. The pion e.m. form factor calculated from LCSR in comparison with the CEBAF data shown with points. The solid line corresponds to the asymptotic pion DA, dashed lines indicate the estimated overall theoretical uncertainty ; the dash-dotted line is calculated with the CZ model of the pion DA.

$Q^2/(s_0^\pi + Q^2)$. The resulting sum rule [32,33] is:

$$F_\pi(Q^2) = \int\limits_{Q^2/(s_0^\pi+Q^2)}^{1} du\, \varphi_\pi(u,\mu) e^{-\frac{(1-u)Q^2}{uM^2}} + F_\pi^{(tw2,\alpha_s)}(Q^2) + F_\pi^{(tw4,6)}(Q^2)\,, \quad (35)$$

where the leading-order twist 2 part is shown explicitly. The $1/Q^4$ behavior of Eq. (35) corresponds to the soft end-point mechanism, provided that in the $Q^2 \to \infty$ limit the integration region shrinks to the point $u = 1$. The $O(\alpha_s)$ part of this sum rule was calculated in [33] and is indeed small at $Q^2 \sim 1$ GeV^2. Importantly, in $F_\pi^{(\alpha_s)}(Q^2)$ one recovers the $\sim 1/Q^2$ asymptotic term corresponding to the hard perturbative mechanism, with a coefficient which, in the adopted approximation, coincides with the one in Eq. (26). The higher twist contributions to the sum rule (35) manifest the same $\sim 1/Q^4$ behavior as the leading twist. Altogether, we seem to achieve the goal. The pion form factor obtained from LCSR contains both the hard-scattering and soft (end-point) contributions, with a proper asymptotic behavior at large Q^2.

The updated LCSR prediction for $F_\pi(Q^2)$ [36] is shown in Fig. 7. One important practical use of this result is to estimate/constrain the nonasymptotic coefficients a_{2n} by fitting the sum rule (35) to the experimental data on the pion form factor. However, currently there are no sufficient data at Q^2 >1 GeV^2 to constrain complicated patterns of nonasymptotic coefficients. Considering simple ones, one finds that, e.g. the asymptotic DA $\varphi_\pi(u) = 6u(1-u)$ is not excluded, whereas the CZ -model [37] seems to be disfavored by data. Assuming that only $a_2 \neq 0$ and neglecting all other coefficients yields [36] the following range $a_2(1\,\mathrm{GeV}) = 0.24 \pm 0.14 \pm 0.08$, where the first error reflects the estimated theoretical uncertainty and the second one corresponds to the experimental errors.

Other LCSR applications to the physics of hard exclusive processes include the $\gamma^*\gamma\pi$ form factor[38], the kaon e.m. form factor[36], and the first attempt to calculate the nucleon form factors [39].

Furthermore, an important task of LCSR is to provide B physics with various heavy-to-light hadronic matrix elements. In particular, the sum rule for the $B \to \pi$ form factor can be obtained from a correlation function very similar to the one depicted in Fig. 6, if one replaces the virtual light quark by a b quark. The calculable short-distance part will then change considerably, but the long-distance part remains essentially the same, determined by the set of pion DA. The sum rule predictions for the $B \to \pi$ [40] and $B \to \rho$ [41] form factors are already used to extract $|V_{ub}|$ from the widths of $B \to \pi(\rho) l\nu_l$ decays. One can also employ LCSR to estimate the hadronic amplitudes for $B \to \pi\pi$ and similar decays beyond factorization [42]. A summary of the sum rule applications to the heavy flavour physics can be found in [43].

6. Conclusion

For more than twenty years, QCD(SVZ) sum rules serve as a virtual laboratory for studying the transition from short- to long-distance QCD. The practical use of this analytical method is twofold. On one hand, using sum rules for experimentally known hadronic quantities, QCD parameters such as quark masses are extracted. On the other hand, the sum rules are employed to predict unknown hadronic parameters, for example f_B, with a controllable accuracy. Finally, the example of the pion form factor calculation demonstrates that the light-cone version of QCD sum rules has a large potential in describing exclusive hadronic transitions.

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