Two-Loop Bosonic Electroweak Corrections to the Muon Lifetime and M_Z–M_W Interdependence

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Abstract

The two-loop electroweak bosonic correction to the muon lifetime and the M_W–M_Z interdependece is computed using analytical methods of asymptotic expansion. Combined with previous calculations this copmletes the full two-loop correction to Δr in the Standard Model. The shift to the prediction of W-boson mass due to two-loop bosonic corrections does not exceed 1 MeV for the range of the Higgs mass from 100 to 1000 GeV.

The Fermi constant G_F plays an important role in the precision tests of the Standard Model. In the theory G_F can be related to other precision observables: electroweak coupling constant α and masses of electroweak gauge bosons M_Z and M_W. Other parameters enter expression through quantum corrections. Usualy one inverts this relation to predict M_W and confront it with experimental value $M_W^{\rm exp}$. The current error (39 MeV) of $M_W^{\rm exp}$ will be drasticaly reduced at future colliders. In fact, at LHC the experimental error can be reduced to 15 MeV [1] and at Linear Collider even down to 6 MeV [2]. Therefore much efforts have been spent to reduce the error of theoretical prediction.

Fermi constant is defined as a coupling constant in low energy four fermion effective lagrangian describing the decay of the muon

$$\mathcal{L}_F = \mathcal{L}_{\rm QED} + \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu\right] \left[\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e\right] \tag{1}$$

where e and μ are electron and muon fields, ν_e and ν_μ are corresponding neutrinos and G_F is Fermi constant. From Lagrangian (1) one gets the value of the muon lifetime

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)(1+\Delta q)\,, \tag{2}$$

[1] E-mail: onish@particle.uni-karlsruhe.de

[2] Supported by DFG-Forschergruppe *"Quantenfeldtheorie, Computeralgebra und Monte-Carlo-Simulation"* (contract FOR 264/2-1)

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[4] Supported by BMBF under grant No 05HT9VKB0.

where factor Δq describes all quantum corrections in the low energy effective theory (i.e. QED corrections). At the one-loop order these corrections have been computed long time ago [3]. Recently also the two-loop results for Δq has been obtained [4]. Taking them into account, the error in G_F nowdays is dominated by the experimental error of τ_μ measurement[5].

In order to relate G_F to M_W one can use matching condition between effective theory (1) and the Standard Model which requires that value of τ_μ does not depend on whether it is evaluated in the Fermi theory or in the full Standard Model. This brings us to the relation

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1+\Delta r), \tag{3}$$

where e is the electric charge, $s_W = \sin\theta_W$ is the SM mixing angle and M_W is W-boson mass. Factor $e^2/8s_W^2 m_W^2$ corresponds to the matching at the three level, while the quantity $\Delta r = \Delta r^{1-\text{loop}} + \Delta r^{2-\text{loop}} + \ldots$ is defined to absorb quantum corrections, coming from matching procedure at higher loop orders. At the one-loop level it was first evaluated in [5]. Further improvement of Δr was achived in [6] where the leading and subleading large t-quark corrections were computed at $O(\alpha^2)$ order. Other two-loop fermionic corrections were considered in a series of papers [7].

The aim of this work is to compute the remained $O(\alpha^2)$ pure bosonic contribution in order to complete the full $O(\alpha^2)$ approximation to Δr. As this work was in progress Ref. [8] has appeared where this missing contribution has been computed numericaly.

Our approach is complementary to that of [8]. We use the semianalytic methods of asymptotic expansion [9] in two different regimes: 1) large Higgs expansion and 2) expansion in difference of masses $M_H^2 - M_Z^2$. In addition we consider also $s_W^2 = \sin^2\theta_W$ as a small parameter in the spirit of [10].

The details of our calculation will be presented in a separate publication [11] and here we only sketch the most important moments.

- G_F is nothing but the Wilson coefficient function of corresponding operator in the low energy four fermion effective theory. Therefore it depends only on the "hard" physics and is insensitive to low energy field modes in loops. The simplest way to compute such a quantity would be the use of factorization theorem (if it exists)—the theorem allowing one to separate "hard" and "soft" physics[6]. Such a theorem for the construction of low energy effective theories is known for a long time (see e.g. [12]) and is based on the euclidean structure of large mass expansion.

[5]The present value is $G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2} and possible future experiments could reduce the error by an order of magnitude.

[6]In our case by "soft" we understand energies and momenta of order of muon mass m_μ, while "hard" means scales $\gg m_\mu$

- Using the technique mentioned above the problem is reduced to evaluation of two-loop bubble diagrams.
- All calculations have been performed in a general R_ξ gauge with three arbitrary gauge parameters. By explicit calculation we checked that contribution to Δr is gauge invariant.
- In order to have explicit gauge invariant expressions also at the level of bare quantities we explicitly include tadpole diagrams. We have checked that applying this prescripture all bare parameters and all counterterms for masses and coupling constant are explicitely gauge invariant.
- We performed renormalization in two different schemes: $\overline{\mathrm{MS}}$ and on-shell scheme. For the transformation from one scheme to another one needs the results of [10] where the transformation formulae for gauge boson masses were given to two loops.

To obtain two-loop bosonic contribution to $\triangle r$ a number of two-loop diagrams of the order of 10^4 should be evaluated. To automate calculation process computer algebra system FORM [13] was used. To obtain the FORM readable input we make use of input generator DIANA [14].

Let us now present the results of calculation. Analytical formulae for $\Delta r^{(2)}_{\mathrm{bos}}$ in the on-shell scheme are given in Appendix in two different regimes of expansion. In Fig. 1 results of expansions

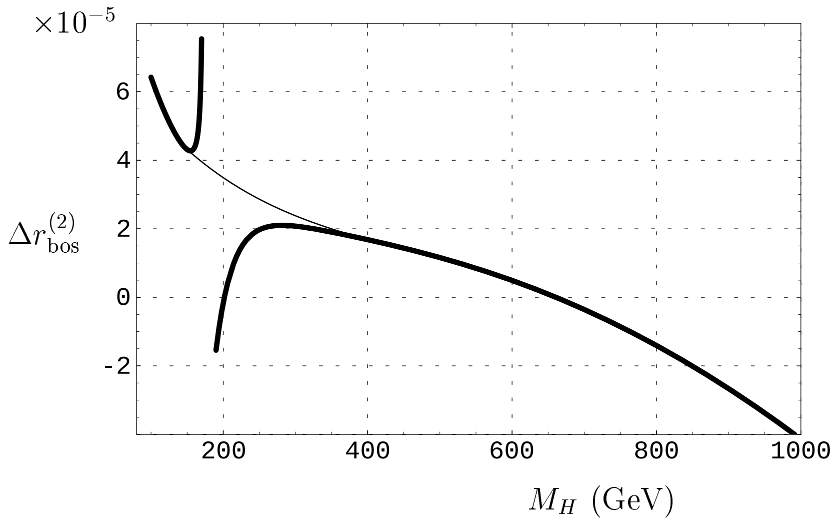

Figure 1: Dependence of $\Delta r^{(2)}_{\mathrm{bos}}$ on Higgs mass M_H (see explanation in the text). Parameter choice for the pole masses: $M_Z = 91.1876\,\mathrm{GeV}$ and $M_W = 80.423\,\mathrm{GeV}$.

are ploted as a function of on-shell Higgs mass M_H. The left bold curve represents [4/4] Pade approximant based on expansion of Δr in mass difference $(M_H - M_Z)$. Another bold curve is a sum of 4 coefficients of large Higgs expansion. In the intermediate region 155 GeV $< M_H <$ 350 GeV both approximations fail to converge. This is due to the presence of (pseudo)threshold singularities at $M_H = 2M_W$ or $M_H = 2M_Z$. Such singularity appears for example in diagram of Fig. 2. To get a result for Higgs mass values in the region 155–350 GeV we made simple polynomial interpolation.

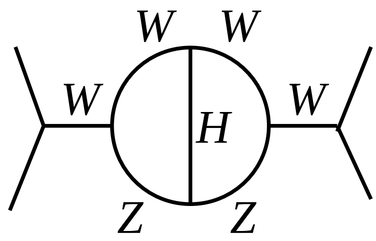

Figure 2: Example of a diagram that has singularity near $M_H = 2M_W$.

Our comparision with numerical curve from Ref. [8] shows a perfect agreement between two calculations. The difference for 155 GeV $< M_H <$ 350 GeV does not exceed 5% while for M_H below 155 GeV and above 350 GeV it is much smaller.

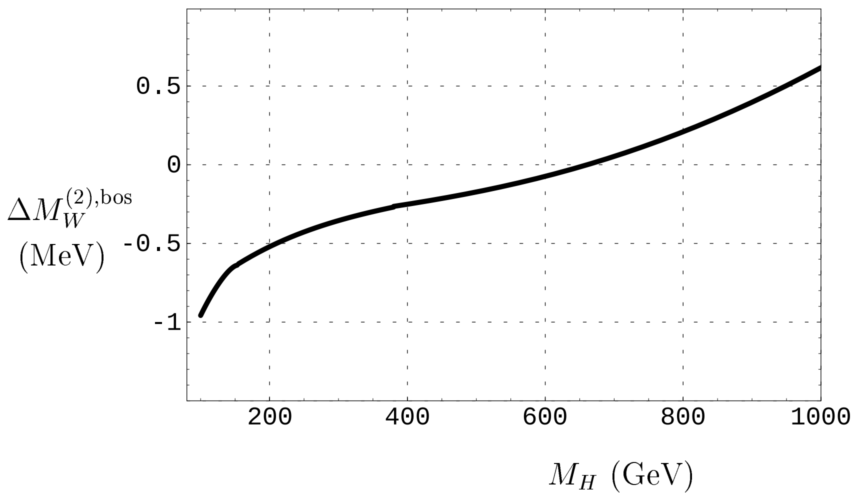

Figure 3: Shift ΔM_W of the predicted mass of W-boson due to two-loop bosonic correction to Δr.

Let us now turn to the shift of M_W mass which is caused by $\Delta r^{(2)}_{\mathrm{bos}}$. It is plotted in Fig. 3. For the broad region of M_H from 100 to 1000 GeV this correction does not exeed 1 MeV and appears to be insignificant. So, we can see, that the uncertainty in M_W prediction at the moment is dominated by the uncertainty in experimental value of t-quark pole mass.

In conclusion the bosonic contributions to Δr has been computed using methods of asymtotic expansions and low energy factorization theorem. The gauge invariance of $\Delta r^{(2)}_{\mathrm{bos}}$ has been proved explicitely. By inclusion of Higgs-tadpole diagrams it has been checked that all mass- and charge-counterterms are gauge invariant.

Acknowledgements: Authors would like to thank M. Kalmykov, M. Awramik, M. Czakon, K. Chetyrkin and J. Kühn for fruitful discussions. We are grateful to M. Tentyukov for his help with input generator DIANA.

A Appendix

In this Appendix we present several first coefficients of expansion of $\Delta r^{(2)}_{\rm bos}$ in two different regimes: $M_H^2 \to \infty$ and $M_H^2 \to M_Z^2$. For simplicity the following notation for numerical constants is introduced

$$l_2 = \log 2, \quad l_3 = \log 3, \quad \zeta_n = \zeta(n), \quad S_1 = \frac{\pi}{\sqrt{3}},$$
$$S_2 = \frac{4}{9}\frac{\mathrm{Cl}_2(\pi/3)}{\sqrt{3}} \approx 0.2604341376\ldots.$$

The formulae below describe $\Delta r^{(2)_{\rm bos}}$ in the on-shell scheme with the definition of the electroweak mixing angle

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}\,,$$

where M_W and M_Z are pole masses.

First we give result of expansion in variable $z = M_Z^2/M_H^2$. The leading $O(M_H^2)$ term for the heavy Higgs boson can be evaluated exactly. Up to the orders $O(z^3)$ and $O(s_W^6)$ we have

$$\begin{aligned}
\Delta r^{(2)}_{\rm bos} = &\left(\frac{\alpha}{4\pi\, s_W^2}\right)^2 \left\{ \frac{M_H^2}{M_W^2}\left(\frac{243}{32}S_2 - \frac{33}{32}S_1 - \frac{29}{864}\zeta_2 + \frac{49}{576}\right)\right.\\
&+\tfrac{7069829}{20736} + \tfrac{4131}{32}S_2S_1 + \tfrac{72171}{320}S_2 z + \tfrac{95067}{160}S_2 z^2 + \tfrac{3853287}{2240}S_2 z^3 + \tfrac{259443}{128}S_2 - 95 S_1 z L - \tfrac{75341}{960}S_1 z\\
&-\tfrac{29439}{80}S_1 z^2 L - \tfrac{1519543}{3200}S_1 z^2 - \tfrac{7468}{5}S_1 z^3 L - \tfrac{34374449}{16800}S_1 z^3 - \tfrac{125}{4}S_1 L - \tfrac{303659}{384}S_1 + \tfrac{1089}{8}S_1^2\\
&-\tfrac{41441}{1440}\zeta_2 z - \tfrac{4031}{30}\zeta_2 z^2 - \tfrac{1837867}{3360}\zeta_2 z^3 + 66\zeta_2 l_3 + \tfrac{17305}{576}\zeta_2 - \tfrac{1515}{16}\zeta_3 + \tfrac{42481}{320}zL - \tfrac{83}{48}zL^2\\
&+\tfrac{5940941}{86400}z + \tfrac{6319637}{17280}z^2 L - \tfrac{70567}{2880}z^2 L^2 + \tfrac{599726311}{1296000}z^2 + \tfrac{11836033}{10800}z^3 L - \tfrac{92423}{720}z^3 L^2\\
&+\tfrac{47950477091}{25401600}z^3 + \tfrac{98125}{1728}L + \tfrac{41}{288}L^2\\
&+s_W^2\left(-\tfrac{4850423}{2304} - \tfrac{14733}{32}S_2S_1 - \tfrac{40167}{160}S_2 z - \tfrac{22023}{80}S_2 z^2 + \tfrac{44091}{56}S_2 z^3 - \tfrac{772245}{128}S_2\right.\\
&+\tfrac{11653}{48}S_1 z L + \tfrac{248651}{1440}S_1 z + \tfrac{168033}{160}S_1 z^2 L + \tfrac{13792549}{9600}S_1 z^2 + \tfrac{58457}{12}S_1 z^3 L + \tfrac{72817723}{10080}S_1 z^3\\
&-66 S_1 l_3 + \tfrac{6541}{96}S_1 L + \tfrac{3844355}{1152}S_1 - \tfrac{8349}{16}S_1^2 + 96 l_2\zeta_2 + \tfrac{47807}{720}\zeta_2 z + \tfrac{3561}{10}\zeta_2 z^2 + \tfrac{4466227}{2520}\zeta_2 z^3\\
&-264\zeta_2 l_3 - \tfrac{44117}{192}\zeta_2 + \tfrac{5653}{16}\zeta_3 - \tfrac{373487}{960}zL + \tfrac{203}{72}zL^2 - \tfrac{7519453}{28800}z - \tfrac{108967649}{86400}z^2 L\\
&\left.+\tfrac{189529}{2880}z^2L^2 - \tfrac{5103328117}{2592000}z^2 - \tfrac{1265074333}{302400}z^3 L + \tfrac{122819}{288}z^3L^2 - \tfrac{11560206263}{1296000}z^3 - \tfrac{24641}{192}L + \tfrac{115}{288}L^2\right)\\
&+s_W^4\left(+\tfrac{94100843}{20736} + \tfrac{11109}{32}S_2S_1 + \tfrac{32847}{160}S_2 z + \tfrac{19761}{40}S_2 z^2 - \tfrac{675}{112}S_2 z^3 + \tfrac{535053}{128}S_2 - \tfrac{2539}{16}S_1 z L\right.
\end{aligned}$$

$$
\begin{aligned}
&-\tfrac{172439}{960}S_1 z-\tfrac{8525}{8}S_1 z^2 L-\tfrac{1135551}{640}S_1 z^2-\tfrac{32914}{5}S_1 z^3 L-\tfrac{189343717}{16800}S_1 z^3+242 S_1 l_3-\tfrac{635}{32}S_1 L\\
&-\tfrac{1857047}{384}S_1+\tfrac{4719}{8}S_1^2-192 l_2\zeta_2-\tfrac{33067}{720}\zeta_2 z-\tfrac{23417}{80}\zeta_2 z^2-\tfrac{5087309}{2520}\zeta_2 z^3+288\zeta_2 l_3+\tfrac{256511}{576}\zeta_2\\
&-\tfrac{17687}{48}\zeta_3+\tfrac{2467333}{8640}zL+\tfrac{11}{12}zL^2+\tfrac{165415279}{518400}z+\tfrac{136065731}{86400}z^2 L-\tfrac{12427}{576}z^2 L^2+\tfrac{590467441}{216000}z^2\\
&+\tfrac{90743033}{12096}z^3 L-\tfrac{116609}{360}z^3 L^2+\tfrac{139434492997}{9072000}z^3+\tfrac{21407}{576}L+\tfrac{115}{288}L^2\Bigg)\\
&+s_W^6\Bigg(-\tfrac{29707045}{10368}-\tfrac{709}{16}S_2 S_1+\tfrac{41379}{320}S_2 z+\tfrac{121301}{640}S_2 z^2+\tfrac{623873}{840}S_2 z^3-\tfrac{77817}{64}S_2+\tfrac{2221}{108}S_1 zL\\
&+\tfrac{1428847}{25920}S_1 z+\tfrac{110077}{240}S_1 z^2 L+\tfrac{45009703}{43200}S_1 z^2+\tfrac{2489969}{540}S_1 z^3 L+\tfrac{4336498723}{453600}S_1 z^3-\tfrac{589}{3}S_1 l_3\\
&+\tfrac{1739}{864}S_1 L+\tfrac{1769833}{648}S_1-\tfrac{5071}{24}S_1^2+\tfrac{31091}{1440}\zeta_2 z+\tfrac{109387}{960}\zeta_2 z^2+\tfrac{99685}{84}\zeta_2 z^3-\tfrac{664}{9}\zeta_2 l_3\\
&-\tfrac{366221}{864}\zeta_2+\tfrac{10357}{72}\zeta_3-\tfrac{36923}{960}zL+\tfrac{11}{12}zL^2-\tfrac{46220713}{259200}z-\tfrac{35333761}{43200}z^2 L+\tfrac{12311}{2880}z^2 L^2\\
&-\tfrac{2086198469}{1036800}z^2-\tfrac{227753417}{33600}z^3 L+\tfrac{9103}{80}z^3 L^2-\tfrac{658300265597}{42336000}z^3-\tfrac{5257}{1728}L+\tfrac{115}{288}L^2\Bigg),
\end{aligned}
\tag{4}
$$

where $L=\log(M_Z^2/M_H^2)$.

In the case when Higgs mass is close to that of Z-boson we introduce variable $y=(M_H^2-M_Z^2)/M_Z^2$. To order $O(y^6)$ and $O(s_W^6)$ the result reads

$$
\begin{aligned}
&\Delta r_{\rm bos}^{(2)}=\left(\frac{\alpha}{4\pi\, s_W^2}\right)^2\Bigg\{\\
&+\tfrac{20659}{48}+\tfrac{305}{4}S_2 S_1 y+\tfrac{99}{32}S_2 S_1 y^2+\tfrac{23}{8}S_2 S_1 y^3+\tfrac{3}{8}S_2 S_1 y^4+\tfrac{75}{4}S_2 S_1+\tfrac{5201}{32}S_2 y-\tfrac{3595}{192}S_2 y^2\\
&-\tfrac{12373}{864}S_2 y^3-\tfrac{1937}{288}S_2 y^4+\tfrac{2149}{216}S_2 y^5-\tfrac{24179}{2430}S_2 y^6+\tfrac{59629}{32}S_2-\tfrac{4}{3}S_1 l_3 y+\tfrac{5}{3}S_1 l_3 y^2\\
&-\tfrac{1}{3}S_1 l_3 y^3+\tfrac{31}{162}S_1 l_3 y^4-\tfrac{41}{810}S_1 l_3 y^5+\tfrac{67}{2430}S_1 l_3 y^6+6 S_1 l_3-\tfrac{959}{18}S_1 y+\tfrac{8587}{216}S_1 y^2\\
&-\tfrac{3491}{720}S_1 y^3+\tfrac{113201}{12960}S_1 y^4-\tfrac{11777083}{1632960}S_1 y^5+\tfrac{21795877}{1959552}S_1 y^6-\tfrac{10375}{12}S_1+\tfrac{3271}{216}S_1^2 y-\tfrac{823}{81}S_1^2 y^2\\
&+\tfrac{5615}{972}S_1^2 y^3-\tfrac{33611}{5832}S_1^2 y^4+\tfrac{22141}{4374}S_1^2 y^5-\tfrac{531793}{104976}S_1^2 y^6+\tfrac{2909}{18}S_1^2+\tfrac{44}{9}\zeta_2 l_3 y-\tfrac{4}{3}\zeta_2 l_3 y^2\\
&+\tfrac{4}{9}\zeta_2 l_3 y^3+62\zeta_2 l_3-\tfrac{2489}{144}\zeta_2 y-\tfrac{4477}{864}\zeta_2 y^2-\tfrac{16531}{1296}\zeta_2 y^3+\tfrac{151487}{23328}\zeta_2 y^4-\tfrac{1990873}{233280}\zeta_2 y^5\\
&+\tfrac{16998781}{5598720}\zeta_2 y^6+\tfrac{7151}{144}\zeta_2-\tfrac{371}{18}\zeta_3 y+\tfrac{5}{16}\zeta_3 y^2-\tfrac{35}{36}\zeta_3 y^3-\tfrac{1}{12}\zeta_3 y^4-\tfrac{403}{6}\zeta_3\\
&+\tfrac{2783}{288}y-\tfrac{43603}{1728}y^2+\tfrac{194641}{15552}y^3-\tfrac{88369}{17280}y^4+\tfrac{1105799}{155520}y^5-\tfrac{129457217}{24494400}y^6\\
&+s_W^2\Bigg(-\tfrac{166967}{72}-\tfrac{605}{4}S_2 S_1 y+\tfrac{441}{32}S_2 S_1 y^2+\tfrac{13}{16}S_2 S_1 y^3+\tfrac{3}{8}S_2 S_1 y^4-\tfrac{445}{4}S_2 S_1\\
&-\tfrac{17111}{96}S_2 y-\tfrac{31285}{576}S_2 y^2+\tfrac{21419}{864}S_2 y^3-\tfrac{3611}{1728}S_2 y^4+\tfrac{9079}{1296}S_2 y^5-\tfrac{137423}{38880}S_2 y^6-\tfrac{45355}{8}S_2\\
&+\tfrac{14}{3}S_1 l_3 y-S_1 l_3 y^2+\tfrac{35}{81}S_1 l_3 y^3+\tfrac{7}{54}S_1 l_3 y^4+\tfrac{11}{810}S_1 l_3 y^5-\tfrac{91}{21870}S_1 l_3 y^6-\tfrac{238}{3}S_1 l_3\\
&+\tfrac{24007}{216}S_1 y-\tfrac{106343}{4320}S_1 y^2-\tfrac{92377}{77760}S_1 y^3+\tfrac{10434679}{816480}S_1 y^4+\tfrac{13120697}{1632960}S_1 y^5+\tfrac{403256701}{29393280}S_1 y^6+\tfrac{31450}{9}S_1\\
&-\tfrac{8869}{324}S_1^2 y+\tfrac{15737}{1296}S_1^2 y^2-\tfrac{29293}{5832}S_1^2 y^3+\tfrac{80429}{34992}S_1^2 y^4-\tfrac{5701}{1944}S_1^2 y^5+\tfrac{338213}{157464}S_1^2 y^6-\tfrac{62045}{108}S_1^2\\
&+96 log2\zeta_2-\tfrac{68}{9}\zeta_2 l_3 y+\tfrac{4}{3}\zeta_2 l_3 y^2+\tfrac{4}{9}\zeta_2 l_3 y^3-\tfrac{2260}{9}\zeta_2 l_3-\tfrac{409}{432}\zeta_2 y-\tfrac{52787}{2592}\zeta_2 y^2\\
&+\tfrac{137425}{11664}\zeta_2 y^3-\tfrac{532225}{23328}\zeta_2 y^4-\tfrac{5168077}{1399680}\zeta_2 y^5-\tfrac{969117317}{40310784}\zeta_2 y^6-\tfrac{9565}{36}\zeta_2+\tfrac{707}{18}\zeta_3 y-\tfrac{65}{16}\zeta_3 y^2\\
&-\tfrac{37}{72}\zeta_3 y^3-\tfrac{1}{12}\zeta_3 y^4+\tfrac{4789}{18}\zeta_3-\tfrac{24917}{864}y+\tfrac{247739}{5184}y^2-\tfrac{1085639}{155520}y^3+\tfrac{4592603}{777600}y^4
\end{aligned}
$$

$$
\begin{aligned}
&+\tfrac{35899}{326592}y^5+\tfrac{2705739767}{342921600}y^6\Big)+s_W^4\Big(+\tfrac{5908523}{1296}-\tfrac{55}{2}S_2S_1y\\
&+\tfrac{231}{32}S_2S_1y^2-\tfrac{81}{16}S_2S_1y^3-\tfrac{3}{8}S_2S_1y^4+\tfrac{317}{2}S_2S_1-\tfrac{20417}{432}S_2y-\tfrac{70549}{1728}S_2y^2+\tfrac{162193}{2592}S_2y^3\\
&-\tfrac{41531}{7776}S_2y^4+\tfrac{53273}{2430}S_2y^5-\tfrac{10142657}{699840}S_2y^6+\tfrac{1132651}{288}S_2+\tfrac{1}{3}S_1l_3y-\tfrac{41}{27}S_1l_3y^2+\tfrac{83}{81}S_1l_3y^3\\
&+\tfrac{1}{27}S_1l_3y^4+\tfrac{238}{3645}S_1l_3y^5-\tfrac{217}{10935}S_1l_3y^6+245S_1l_3-\tfrac{6461}{1440}S_1y-\tfrac{293137}{77760}S_1y^2+\tfrac{8558191}{489888}S_1y^3\\
&+\tfrac{21579127}{699840}S_1y^4+\tfrac{36284071}{979776}S_1y^5+\tfrac{2526114757}{44089920}S_1y^6-\tfrac{1559051}{324}S_1+\tfrac{19085}{1944}S_1^2y-\tfrac{605}{162}S_1^2y^2+\tfrac{311}{729}S_1^2y^3\\
&-\tfrac{58837}{13122}S_1^2y^4+\tfrac{61837}{26244}S_1^2y^5-\tfrac{1014359}{314928}S_1^2y^6+\tfrac{5426}{9}S_1^2-192log2\zeta_2-\tfrac{32}{9}\zeta_2l_3y+\tfrac{8}{3}\zeta_2l_3y^2\\
&+\tfrac{2536}{9}\zeta_2l_3-\tfrac{10121}{324}\zeta_2y+\tfrac{43439}{2592}\zeta_2y^2-\tfrac{13853}{486}\zeta_2y^3-\tfrac{16024969}{559872}\zeta_2y^4-\tfrac{997982197}{16796160}\zeta_2y^5-\tfrac{27329221217}{403107840}\zeta_2y^6\\
&+\tfrac{21139}{48}\zeta_2+\tfrac{79}{9}\zeta_3y-\tfrac{173}{48}\zeta_3y^2+\tfrac{9}{8}\zeta_3y^3+\tfrac{1}{12}\zeta_3y^4-\tfrac{2897}{9}\zeta_3+\tfrac{746443}{15552}y\\
&+\tfrac{214469}{155520}y^2-\tfrac{3867907}{1166400}y^3+\tfrac{7035509}{979776}y^4+\tfrac{2931335759}{171460800}y^5+\tfrac{10819102807}{493807104}y^6\Big)\\
&+s_W^6\Big(-\tfrac{5582263}{1944}-\tfrac{319}{8}S_2S_1y-\tfrac{57}{16}S_2S_1y^2-\tfrac{139}{8}S_2S_1y^3-\tfrac{9}{4}S_2S_1y^4-\tfrac{637}{8}S_2S_1\\
&-\tfrac{659}{12}S_2y+\tfrac{69397}{3456}S_2y^2+\tfrac{1926995}{15552}S_2y^3+\tfrac{27433}{3888}S_2y^4+\tfrac{333811}{9720}S_2y^5-\tfrac{29405383}{2099520}S_2y^6-\tfrac{521903}{432}S_2\\
&-\tfrac{181}{81}S_1l_3y-\tfrac{125}{81}S_1l_3y^2+\tfrac{299}{243}S_1l_3y^3-\tfrac{401}{2187}S_1l_3y^4+\tfrac{31}{243}S_1l_3y^5-\tfrac{353}{10935}S_1l_3y^6-\tfrac{586}{3}S_1l_3\\
&-\tfrac{536677}{38880}S_1y+\tfrac{27541403}{1088640}S_1y^2+\tfrac{4479889}{136080}S_1y^3+\tfrac{386672993}{4199040}S_1y^4+\tfrac{3657923837}{29393280}S_1y^5+\tfrac{84929297903}{387991296}S_1y^6\\
&+\tfrac{109883}{40}S_1-\tfrac{6535}{5832}S_1^2y-\tfrac{45707}{8748}S_1^2y^2-\tfrac{11707}{13122}S_1^2y^3-\tfrac{646097}{157464}S_1^2y^4+\tfrac{506321}{236196}S_1^2y^5-\tfrac{2664919}{708588}S_1^2y^6\\
&-\tfrac{17078}{81}S_1^2-\tfrac{8}{9}\zeta_2l_3y+\tfrac{8}{3}\zeta_2l_3y^2-\tfrac{8}{9}\zeta_2l_3y^3-\tfrac{704}{9}\zeta_2l_3+\tfrac{139991}{5832}\zeta_2y-\tfrac{355457}{46656}\zeta_2y^2-\tfrac{926609}{17496}\zeta_2y^3\\
&-\tfrac{559304957}{5038848}\zeta_2y^4-\tfrac{18490951031}{100776960}\zeta_2y^5-\tfrac{1383711806891}{4837294080}\zeta_2y^6-\tfrac{281321}{648}\zeta_2+\tfrac{343}{36}\zeta_3y-\tfrac{29}{24}\zeta_3y^2+\tfrac{163}{36}\zeta_3y^3\\
&+\tfrac{1}{2}\zeta_3y^4+\tfrac{5581}{36}\zeta_3+\tfrac{437711}{25920}y-\tfrac{15188507}{777600}y^2-\tfrac{25783}{2449440}y^3+\tfrac{9642347291}{342921600}y^4+\tfrac{82083463913}{1371686400}y^5\\
&+\tfrac{1659175944397}{18517766400}y^6\Big)\Big\}\,. \qquad (5)
\end{aligned}
$$

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