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TOWARDS DEEP-INELASTIC STRUCTURE FUNCTIONS AT THREE LOOPS

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We report on the status of the calculation of deep-inelastic structure functions at three loops in perturbative QCD. The method employed allows to calculate the Mellin moments of structure functions analytically as a general function of N. As an illustration, we present the leading fermionic contributions to the non-singlet anomalous dimension of F_2 at three loops and, as a new result, to the non-singlet coefficient function of F_2 at three loops.

1. Introduction

Today, structure functions in inclusive deep-inelastic scattering are extremely well measured quantities. As a consequence, they offer the possibility for very precise determinations of the strong coupling α_s and the parton distribution functions. The high statistical accuracy of the present and upcoming experimental measurements demands analyses to next-to-next-to leading order (NNLO) of perturbative QCD for the structure functions F_2, F_3 and F_L.

However, the complete NNLO corrections are not fully available yet. The two-loop coefficient functions of F_2, F_3 and F_L have been calculated [1, 2]. For the three-loop anomalous dimensions only a finite number of fixed Mellin moments [3] are presently available. In addition, some information about leading fermionic contributions [4, 5] and the small-x limit [6] exists.

In the following, we briefly report on the status of the calculation of the coefficient functions and the anomalous dimensions to three loops in perturbative QCD. Furthermore, we present results for the leading fermionic contributions to the non-singlet structure function F_2.

2. Method

We employ the optical theorem and the operator product expansion (OPE) to calculate the deep-inelastic structure functions in Mellin space analytically [7, 8] as general functions of N. For the N-th Mellin moment of F_2 we can write

$$F_2^N(Q^2) \;=\; \int_0^1 dx\, x^{N-2} F_2(x,Q^2) \;=\; \sum_{j=\alpha,\mathrm{q},\mathrm{g}} C_{2,j}^N\left(\frac{Q^2}{\mu^2},\alpha_s\right) A_{\mathrm{P},N}^j\left(\mu^2\right)\,, \quad (1)$$

where $C_{2,j}^N$ denote the coefficient functions and $A_{\mathrm{P},N}^j$ the spin averaged hadronic matrix elements of singlet operators O^{q}, O^{g} and non-singlet operators O^α, $\alpha = 1,2,\ldots,(n_f^2-1)$, of leading twist. Both, the coefficient functions and the renormalized operator matrix elements in eq.(1) satisfy renormalization group equations governed by the same anomalous dimensions γ_{jk}. The anomalous dimensions determine the scale evolution of deep-inelastic structure functions.

The calculation of the coefficient functions $C_{2,j}^N$ and anomalous dimensions γ_{jk} at a given order in perturbation theory amounts to the determination of the N-th moment of all contributing Feynman diagrams with external partons of momentum p with $p^2 = 0$ and photons of momentum q with $q^2 = -Q^2$. To achieve this task, we apply the following strategy [2, 9]. We set up a hierarchy among all diagrams depending on the number of p-dependent propagators. We define basic building blocks (BBB) as diagrams in which the parton momentum p flows only through a single line in the diagram, while composite building blocks (CBB) denote all diagrams with more than one p-dependent propagator.

Then, with the help of integration-by-parts [10] and scaling identities [2] we determine reduction schemes that map the CBB's of a given topology to the BBB's of the same topology or to the CBB's of a simpler topology. Subsequently, we use reduction identities that express the BBB's of a given topology in terms of simpler topologies. Working in Mellin space, the reduction equations often involve explicitly the parameter N of the Mellin moment. Sometimes, one encounters difference equations in N for the N-th moment $F(N)$ of a diagram,

$$\begin{aligned} a_0(N)F(N) + a_1(N)F(N-1) &+ \\ \ldots + a_n(N)F(N-n) + G(N) &\;=\; 0\,, \end{aligned} \quad (2)$$

where $G(N)$ denotes the N-th Mellin moment of simpler diagrams. First order difference equations can be solved at the cost of one sum over Γ-functions in dimensional regularization. We use $D = 4-2\epsilon$. The Γ-functions can be expanded in ϵ and the sum can be solved to any order in ϵ in terms of harmonic sums [11, 12]. Higher order difference equations could be solved constructively. On the mathematical side, the approach to calculate Mellin moments of structure functions relies on particular mathematical concepts [13], such as harmonic sums [11, 12] and our ability to set up and solve the difference equations as nested sums in N.

3. Leading fermionic contributions

To illustrate the method, we discuss the leading fermionic contributions to the non-singlet structure function. At three loops, they are proportional to n_f^2, with n_f being the number of massless fermions. These contributions form a gauge-invariant subset, but do not involve yet any genuine three-loop topologies. Therefore, in the sense of the reduction strategy sketched above, the n_f^2-terms are easier to calculate.

The result for the n_f^2-contribution to the non-singlet anomalous dimension $g_{\rm qq}^{(2),\rm ns}$ at three loops is known from the work of Gracey [4]. It is given by,

$$\begin{aligned} g_{\rm qq}^{(2),\rm ns} &= C_F n_f^2 \left(\frac{17}{9} + \frac{32}{9}\frac{1}{N+1} - \frac{88}{27}\frac{1}{(N+1)^2} + \frac{8}{9}\frac{1}{(N+1)^3} - \frac{32}{9}\frac{1}{N} \right. \\ & \left. + \frac{88}{27}\frac{1}{N^2} - \frac{8}{9}\frac{1}{N^3} - \frac{16}{27}S_1(N) - \frac{80}{27}S_2(N) + \frac{16}{9}S_3(N) \right), \qquad (3) \end{aligned}$$

with $C_F = (N^2-1)/(2N)$, which is 4/3 for QCD.

As a new result, we give here the n_f^2-contribution to the non-singlet coefficient function $c_{2,\rm qq}^{(3),\rm ns}$ at three loops for the flavour class, where both photons couple to the external quark. Strictly speaking, the three-loop coefficient functions contribute in a perturbative expansion only at next-to-next-to-next-to-leading order (NNNLO). However, the result illustrates nicely that our method will not only provide the anomalous dimensions, which are proportional to the single pole in ϵ in dimensional regularization, but also the coefficient functions which are determined by the finite terms, since our approach is (at least in principle) not limitated to a given order in ϵ.

In terms of harmonic sums up to weight four, our result for the n_f^2-contribution to $c_{2,\rm qq}^{(3),\rm ns}$ reads

$$\begin{aligned} c_{2,\rm qq}^{(3),\rm ns} &= \\ C_F n_f^2 & \left(-\frac{9517}{486} - \frac{8}{9}\zeta_3 + \frac{36748}{729}\frac{1}{N+1} + \frac{16}{27}\frac{\zeta_3}{N+1} - \frac{4384}{81}\frac{1}{(N+1)^2} \right. \end{aligned}$$

$$
\begin{aligned}
&+\frac{2360}{81}\frac{1}{(N+1)^3}-\frac{184}{27}\frac{1}{(N+1)^4}+\frac{16}{3}\frac{S_1(N+1)}{(N+1)^3}-\frac{544}{27}\frac{S_1(N+1)}{(N+1)^2}\\
&-\frac{32}{9}\frac{S_{1,1}(N+1)}{(N+1)^2}+\frac{32}{9}\frac{S_2(N+1)}{(N+1)^2}+\frac{2240}{81}\frac{S_1(N+1)}{N+1}+\frac{272}{27}\frac{S_{1,1}(N+1)}{N+1}\\
&+\frac{16}{9}\frac{S_{1,1,1}(N+1)}{N+1}-\frac{16}{9}\frac{S_{1,2}(N+1)}{N+1}-\frac{272}{27}\frac{S_2(N+1)}{N+1}-\frac{16}{9}\frac{S_{2,1}(N+1)}{N+1}\\
&+\frac{16}{9}\frac{S_3(N+1)}{N+1}-\frac{11170}{729}\frac{1}{N}-\frac{16}{27}\frac{\zeta_3}{N}+\frac{1204}{81}\frac{1}{N^2}-\frac{992}{81}\frac{1}{N^3}+\frac{184}{27}\frac{1}{N^4}\\
&-\frac{16}{3}\frac{S_1(N)}{N^3}+\frac{232}{27}\frac{S_1(N)}{N^2}+\frac{32}{9}\frac{S_{1,1}(N)}{N^2}-\frac{32}{9}\frac{S_2(N)}{N^2}-\frac{644}{81}\frac{S_1(N)}{N}\\
&-\frac{104}{27}\frac{S_{1,1}(N)}{N}-\frac{16}{9}\frac{S_{1,1,1}(N)}{N}+\frac{16}{9}\frac{S_{1,2}(N)}{N}+\frac{104}{27}\frac{S_2(N)}{N}\\
&+\frac{16}{9}\frac{S_{2,1}(N)}{N}-\frac{16}{9}\frac{S_3(N)}{N}+\frac{8714}{729}S_1(N)+\frac{32}{27}S_1(N)\zeta_3+\frac{940}{81}S_{1,1}(N)\\
&+\frac{232}{27}S_{1,1,1}(N)+\frac{32}{9}S_{1,1,1,1}(N)-\frac{32}{9}S_{1,1,2}(N)-\frac{232}{27}S_{1,2}(N)\\
&-\frac{32}{9}S_{1,2,1}(N)+\frac{32}{9}S_{1,3}(N)-\frac{860}{27}S_2(N)-\frac{536}{27}S_{2,1}(N)-\frac{64}{9}S_{2,1,1}(N)\\
&+\frac{64}{9}S_{2,2}(N)+\frac{2440}{81}S_3(N)+\frac{32}{3}S_{3,1}(N)-\frac{368}{27}S_4(N)\Bigg)\,. \qquad (4)
\end{aligned}
$$

This result agrees with the one of the fixed Mellin moment calculation [3] for $N=2,\ldots,14$. It may be used to improve approximations [14] to the full functional form of $c_{2,\mathrm{qq}}^{(3),\mathrm{ns}}$, although the n_f^2-terms are numerically not the most dominant contribution.

4. Conclusions

The present approach based on the OPE, to calculate the Mellin moments of structure functions allows for the calculation of the complete coefficient functions and the anomalous dimensions at three loops.

As a next step, one can consider the subleading fermionic contributions at three loops proportional to n_f. The determination of these terms already relies on major parts of the complete reduction scheme as it requires the calculation of several genuine three-loop topologies of the Benz type as well as the calculation of two-loop topologies with a self-energy insertion. The results for the n_f-contribution to the non-singlet anomalous dimension will be presented elsewhere.

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