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Three aspects of tau decays

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ABSTRACT

Three topics of relevance for τ-decays will be presented: i) the possibility to search for CP violation in the Cabbibo suppressed channels $\tau \to \nu K\pi$ and $\nu K\pi\pi$, ii) recent partial results on the semileptonic decay rate in α_s^4 accuracy, and iii) the possibility to determine the strange quark mass in inclusive Cabbibo suppressed decays.

1 Introduction

The analysis of τ-lepton production and decay offers the unique possibility for a multitude of precision measurements, in the context of electroweak interactions as well as for QCD [1]. One of the most precise determination of the vector and axial parts of the Z coupling to leptons originates from the analysis of Z decays to τ leptons and fairly restrictive limits on anomalous charged current couplings have been set by studies of the leptonic decays. First steps towards the precise determination of the strange quark mass in Cabbibo-suppressed decays have been undertaken. Last not least, one of the most precise determinations of the strong coupling constant is derived from the semileptonic decay rate. From the experimental side the final LEP results are expected soon, and the same is true for the CLEO data. The yield of τ-leptons at B-factories is enormous: The integrated luminosity of 100 fb^{-1} which will soon be collected both at BaBar and BELLE, corresponds to about hundred million pairs for each experiment. In principle this huge sample could open the door to qualitatively new precision studies of the lepton, and thus give important information on this member of the third quark-lepton-family. In this contribution we will discuss three characteristic examples for the enormous potential of the τ-lepton:

1. Properly chosen angular distributions in the decay modes $\tau \to \nu K\pi$ or $\nu K\pi\pi$ allow to test for CP violating scalar couplings the of the type $(\bar{\nu}\tau)(\bar{s}u)$ mediated by charged Higgs decay as discussed in section 2.

2. The determination of α_s from the decays rate has been one of the central themes in τ physics. At present the error in α_s is largely determinated by theoretical uncertainties, in particular by the present ignorance on the behaviour of the perturbative series beyond the α_s^3 term. Recently a subset of the α_s^4 terms has become available. We discuss the magnitude of these terms and the implications for the full α_s^4 result.

3. During the past years the determination of the strange quark mass has been addressed in the various theoretical and experimental analyses. The Cabbibo suppressed decay rate $\Gamma(\tau \to \nu s\bar{u})$ depends on m_s and the quadratic term in the expansion in m_s^2/M_τ^2 has been evaluated in pQCD up to order α_s^2, and for some moments even up to α_s^3. The large coefficients of the series in α_s lead to a fairly poor convergence and shed doubt on the validity of the entire approach. In section five we argue, on the basis of the newly calculated terms of order $\alpha_s^3 n_f m_s^2$ (together with those of order $\alpha_s^3 n_f^2 m_s^2$), that the complete α_s^3 term exhibits again a very large coefficient indicating poor convergence of the complete series.

2 CP-violation in τ -decays

2.1 Preliminaries

CP violation has been experimentally observed only in the K meson system. The effect can be explained by a nontrivial complex phase in the CKM flavour mixing matrix [2]. However, the fundamental origin of this CP violation is still unknown. In particular the CP properties of the third fermion family are largely unexplored. Production and decay of τ leptons might offer a particularly clean laboratory to study these effects. In this contribution, we investigate the effects of CP violation [3] to be observed in semileptonic τ decays which could arise in a framework outside the mechanism prosposed by Kobayashi and Maskawa. We show that the structure function formalism of Ref. [4] allows for a systematic analysis of possible CP violation effects in the two and three meson channels. Special emphasis is put on the $\Delta S = 1$ transition $\tau \to K\pi\nu_\tau$ where possible CP violating signals from multi Higgs boson models would be signaled by a nonvanishing difference between the structure functions $W_{SF}[\tau^- \to (K\pi)^-\nu_\tau]$ and $W_{SF}[\tau^+ \to (K\pi)^+\nu_\tau]$. Such a measurement is possible for unpolarized single τ's without reconstruction of the τ rest frame and without polarized incident e^+e^- beams. It is shown that this CP violation requires both nonvanishing hadronic phases and CP violating phases in the Hamiltonian, where the hadronic phases arise from the interference of complex Breit-Wigner propagators, whereas the CP violating phases could arise from an exotic charged Higgs boson. An additional independent test of CP violation in the two meson case is possible, but would require the knowledge of the full kinematics and τ polarization.

2.2 CP Violating Signals in the $\tau \to K\pi\nu_\tau$ Decay Mode

Transitions from the vacuum to two pseudoscalar mesons h_1 and h_2 are induced through vector and scalar currents only. Expanding these hadronic matrix elements along the set of independent momenta $(q_1 - q_2)_\beta$ and $Q_\beta = (q_1 + q_2)_\beta$ we define ($T_{\alpha\beta} = g_{\alpha\beta} - (Q_\alpha Q_\beta)/Q^2$)

$$\langle h_1(q_1)h_2(q_2)|\bar{u}\gamma_\beta d|0\rangle = (q_1 - q_2)^\alpha \, T_{\alpha\beta}\; F(Q^2) \,+\, Q_\beta \, F_S(Q^2) \qquad (1)$$

$$\langle h_1(q_1)h_2(q_2)|\bar{u}d|0\rangle = F_H(Q^2). \qquad (2)$$

The representation of the hadronic amplitude $\langle h_1 h_2 | \bar{u}\gamma_\beta d | 0\rangle$ corresponds to a decomposition into spin one and spin zero contributions, *e.g.* the vector form factor $F(Q^2)$ corresponds to the $J^P = 1^-$ component of the weak charged current, and the scalar form factor $F_S(Q^2)$ to the $J^P = 0^+$ component. A fictitious scalar Higgs exchange contribution is proportional to $\eta_S F_H$. The general amplitude for the $\Delta S = 1$ decay (where $h_1 \equiv K, h_2 \equiv \pi$)

$$\tau^-(l,s) \to \nu(l',s') + h_1(q_1,m_1) + h_2(q_2,m_2) \,, \tag{3}$$

can thus be written as

$$\mathcal{M} = \sin\theta_c \frac{G}{\sqrt{2}} \bar{u}(l',s')\,\gamma_\alpha(1-\gamma_5)\,u(l,s)\,\Big[(q_1-q_2)_\beta\, T^{\alpha\beta}\, F \,+\, Q^\alpha\, \tilde{F}_S\Big] \tag{4}$$

$$\text{with} \qquad \tilde{F}_S = F_S \,+\, \frac{\eta_S}{m_\tau}\, F_H \tag{5}$$

In Eq. (4) s denotes the polarization four-vector of the τ lepton. The complex parameter η_S in Eq. (5) transforms like

$$\eta_S \quad \stackrel{\rm CP}{\longrightarrow} \quad \eta_S^* \tag{6}$$

and thus allows for the parametrization of possible CP violation. Up to the small isospin breaking terms, induced for example by the small quark mass difference, CVC implies the vanishing of F_S for the two pion ($h_1 \equiv \pi^-, h_2 \equiv \pi^0$) case. For the transition $\tau \to K\pi\nu$ the $J = 1$ form factor F is dominated by the $K^*(892)$ vector resonance contribution. The scalar form factor F_S is expected [5] to receive a sizable resonance contribution ($\sim 5\%$ to the decay rate) from the $K_0^*(1430)$ with $J^P = 0^+$. The corresponding τ^+ decay is obtained from Eq. (4) through the substitutions

$$(1-\gamma_5) \;\to\; (1+\gamma_5), \qquad\qquad \eta_S \;\to\; \eta_S^*. \tag{7}$$

Reaction (3) is most easily analyzed in the hadronic rest frame [4] $\vec{q}_1 + \vec{q}_2 = 0$. After integration over the unobserved neutrino direction, the differential decay rate in the rest frame of $h_1 + h_2$ is given by [5, 4]

$$\begin{aligned} d\Gamma(\tau^- \to 2h\nu_\tau) \quad &= \quad \left\{\bar{L}_B W_B + \bar{L}_{SA} W_{SA} + \bar{L}_{SF} W_{SF} + \bar{L}_{SG} W_{SG}\right\} \\ & \frac{G^2}{2m_\tau} \sin^2\theta_c \frac{1}{(4\pi)^3} \frac{(m_\tau^2 - Q^2)^2}{m_\tau^2}\; |\vec{q}_1|\; \frac{dQ^2}{\sqrt{Q^2}}\, \frac{d\cos\theta}{2}\, \frac{d\alpha}{2\pi}\, \frac{d\cos\beta}{2}\,. \end{aligned} \tag{8}$$

For the definition and discussion of the angles and leptonic coefficients $\bar{L}_X$ in Eq. (8) we refer the reader to Ref. [3]. The coefficients $\bar{L}_X$ contain all α, β and θ angular and τ-polarization dependence. The hadronic structure functions W_X, $X \in \{B, SA, SF, SG\}$, depend only on Q^2 and the form factors F and $\tilde{F}_S$ of the hadronic current. One has [5, 4, 3]:

$$\begin{aligned} W_B[\tau^-] &= 4(\vec{q}_1)^2\,|F|^2 & \qquad W_{SF}[\tau^-] &= 4\sqrt{Q^2}|\vec{q}_1|\,\mathrm{Re}\left[F\tilde{F}_S^*\right] \\ W_{SA}[\tau^-] &= Q^2\,|\tilde{F}_S|^2 & \qquad W_{SG}[\tau^-] &= -4\sqrt{Q^2}|\vec{q}_1|\,\mathrm{Im}\left[F\tilde{F}_S^*\right] \end{aligned} \tag{9}$$

The hadronic structure functions $W_X[\tau^+]$ are obtained by the replacement $\eta_S \to \eta_S^*$ in $\tilde{F}_S$ in Eqs. (9,5). CP conservation implies that all four structure functions are identical for τ^+ and τ^-. With the ansatz for the form factors formulated in Eq. (4) CP violation can be present in W_{SF} and W_{SG} only and requires complex η_S.

As demonstrated in Ref. [3] W_{SF} can be measured in e^+e^- annihilation experiments in the study of single unpolarized τ decays even if the τ rest frame cannot be reconstructed. In this respect the result differ from earlier studies of the two meson modes where either polarized beams and reconstruction of the full kinematics [6] or correlated fully reconstructed τ^- and τ^+ decays were required [7]. The determination of W_{SG}, however, requires the knowledge of the full τ kinematics and τ polarization [3] which is possible with the help of vertex detectors.

The crucial observation made in Ref. [3] is that one can measure the following CP-violating differences

$$\Delta W_{SF} = \frac{1}{2}\left(W_{SF}[\tau^-] - W_{SF}[\tau^+]\right) \qquad \Delta W_{SG} = \frac{1}{2}\left(W_{SG}[\tau^-] - W_{SG}[\tau^+]\right) \tag{10}$$

under the above mentioned conditions. The hadronic structure functions $W_X[\tau^-]$ and $W_X[\tau^+]$ differ only in the phase os η_S and one obtains

$$\Delta W_{SF} \;=\; 4\sqrt{Q^2}|\vec{q}_1|\,\frac{1}{m_\tau}\,\mathrm{Im}\,(FF_H^*)\;\;\mathrm{Im}\,(\,\eta_S)\;. \tag{11}$$

In essence the measurement on ΔW_{SF} analyses the difference in the correlated energy distribution of the mesons h_1 and h_2 from τ^+ and τ^- decay in the laboratory. As already mentioned, ΔW_{SF} is observable for single τ^+ and τ^- decays without knowledge of the τ rest frame. Any nonvanishing experimental result for ΔW_{SF} would be a clear signal of CP violation. Note that a nonvanishing

ΔW_{SF} requires nontrivial hadronic phases (in addition to the CP violating phases η_S) in the form factors F and F_H. Such hadronic phases in F (F_H) originate in the $K\pi\nu_\tau$ decay mode from complex Breit Wigner propagators for the K^* (K_0^*) resonance. Sizable effects of these hadronic phases are expected in this decay mode [5].

Once the τ rest frame is known and a preferred direction of polarization exists one may also determine ΔW_{SG} which is theoretically given by

$$\Delta W_{SG} \;=\; 4\sqrt{Q^2}|\vec{q}_1| \;\frac{1}{m_\tau}\; \mathrm{Re}\,(FF_H^*)\;\; \mathrm{Im}\,(\eta_S)\;. \tag{12}$$

(This preferred direction of polarization could be substituted by the analysis of correlations between τ^+ and τ^+ decay modes in one and the same event.) Any observed nonzero value of ΔW_{SF} or ΔW_{SG} would signal a true CP violation. Eqs.(11) and (12) show that the sensitivity to CP violating effects in ΔW_{SF} and ΔW_{SG} can be fairly different depending on the hadronic phases. Whereas ΔW_{SF} requires nontrivial hadronic phases ΔW_{SG} is maximal for fixed η_S in the absence of hadronic phases.

2.3 Three Meson Decays

The structure function formalism [4] allows also for a systematic analysis of possible CP violation effects in the three meson case. Some of these effects have already been briefly discussed in Ref. [8]. The $K\pi\pi$ and $KK\pi$ decay modes with nonvanishing vector and axial vector current are of particular importance for the detection of possible CP violation originating from exotic intermediate vector bosons. This would be signalled by a nonvanishing difference between the structure functions $W_X(\tau^-)$ and $W_X(\tau^+)$ with $X \in \{F,G,H,I\}$. A difference in the structure functions with $X \in \{SB,SC,SD,SE,SF,SG\}$ can again be induced through a CP violating scalar exchange.

3 Towards Order α_s^4 Accuracy in τ-decays

The dependence of the τ-decay rate on the strong coupling α_s has been used for a determination of α_s at lower energies, with the most recent results of $0.334 \pm 0.007_{exp} \pm 0.021_{theo}$ and $0.3478 \pm 0.009_{exp} \pm 0.019_{theo}$ by the ALEPH [12] and OPAL [13] collaborations, which after evolution up to higher energies agree remarkably well with determinations based on the hadronic Z decay rate. In view of the relatively large value of $\alpha_s(M_\tau)$ estimates for the yet unknown terms of higher orders play an important role in current determinations of α_s at low energies. This is in contrast to high energy measurements, where the uncertainty from terms of order α_s^4 of 0.001 to 0.002 is somewhat less or at most comparable to to the present experimental errors [14].

The situation is even more problematic for the determination of the strange quark mass from the Cabbibo suppressed τ decays. Perturbative QCD corrections affecting the m_s^2 term are extremely large and contributions from increasing powers of α_s are barely decreasing, which casts doubts on our ability to extract a reliable result for m_s from this (in principle) clean and straightforward measurement [15, 16, 17, 18].

Partial results of order α_s^4 for the absorptive part of the massless vector and scalar correlators have been obtained recently [19], namely terms proportional to n_f^2, where n_f denotes the number of massless fermion species. These allow test two popular optimization schemes — based on the principles of "minimal sensitivity" (PMS) and of fastest apparent convergence [21, 22] (FAC) — which have been used to predict yet uncalulated higher order terms [23, 24].

It will be demonstrated that the predictions of both schemes for the coefficient of order $n_f^2\alpha_s^4$ are in reasonable agreement with our calculations, which are then used to predict the complete fixed order (FO) and the "contour improved" (CI) [25, 26] $\mathcal{O}(\alpha_s^4)$ contributions to τ decay rate. Employing the four-loop QCD beta function in combination with improved α_s^4 terms a rough estimate even for $\mathcal{O}(\alpha_s^5)$ terms can be obtained. The results lead to fairly stable values for α_s consistent with current analysises.

Unfortunately, no essential decrease of the difference between the values $\alpha_s(M_\tau)$ as obtained with FI and CI approaches correspondingly is observed after including higher order α_s^4 and α_s^5 corrections, assuming for the moment that these estimates are indeed correct. On the other hand, the theoretical uncertainty assigned to $\alpha_s(M_\tau)$ *within* each method does decrease significantly.

The implication of this approach for the extraction of m_s from Cabbibo suppressed decays will be investigated along the same lines. New results, based on Refs. [19, 20] will be presented for the terms of order $m_s^2 n_f \alpha_s^3$ in the total rate. In this case the agreement between FAC/PMS predictions and our results is quite encouraging and naturally suggests the use of the former as a reliable prediction for the complete $m_s^2\alpha_s^3$ term. Following an approach discussed in [24], even a rough estimate $m_s^2\alpha_s^4 n_f^2$ terms can be obtained from these considerations.

However, the rapid increase of the coefficients indicates, that inherent uncertainty of the present m_s determinations will be worsened by the higher orders. As we will see, the situation is somewhat better for the spin one contribution if considered separately.

We start with the well-known representation of the tau-lepton hadronic rate as the contour integral along a circle C of the radius $|s| = M_\tau^2$

$$R_\tau = 6i\pi \int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\Pi^{[2]}(s) - \frac{2}{M_\tau^2}\Pi^{[1]}(s)\right]. \tag{13}$$

Here $\Pi^{[1]}$ and $\Pi^{[2]}$ are proper flavour combinations of the polarization operators appearing in the following decomposition of the correlators of vector and axial vector currents of light quarks proportional to $g_{\mu\nu}$ and $q_\mu q_\nu$ respectively.

The full tau-lepton hadron rate R_τ can be presented as a sum of spin 1 and spin 0 parts, viz.

$$\begin{aligned}
R_\tau^{(1)} &= 6i\pi \int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{[1]}(0)/s\right], \\
R_\tau^{(0)} &= 6i\pi \int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\Pi^{(0)}(s) - \Pi^{[1]}(0)/s\right].
\end{aligned} \tag{14}$$

where

$$\Pi^{(1)} = -\Pi^{[1]}/q^2, \quad \Pi^{(0)} = \Pi^{[2]} + \Pi^{[1]}/q^2. \tag{15}$$

and the contribution of the singularity at the origin (proportional to $\Pi^{[1]}(0)$) has to be included. A nonvanishing value of $\Pi^{[1]}(0)$ is a nonperturbative constant.

On the other hand, the unknown constant drops out if one considers moments

$$R_\tau^{(1,0)k,l}(s_0) = \int_0^{s_0} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{dR_\tau^{(1,0)}}{ds}, \tag{16}$$

with $k \geq 0, \quad l \geq 1$.

The integral in eq. (13) is, obviously, insensitive, to the Q^2-independent terms in the polarization functions $\Pi_0^{[1]}/Q^2$ and $\Pi_2^{[1]}$. This means that without loss of generality we may deal with the corresponding (Adler) D-functions, viz.

$$D_0^{[1]}(Q^2) \equiv \frac{1}{4}Q^2 \frac{\mathrm{d}}{\mathrm{d}\,Q^2}\Pi_0^{[1]}/Q^2, \quad D_2^{[1]}(Q^2) \equiv -\frac{1}{2}Q^2 \frac{\mathrm{d}}{\mathrm{d}\,Q^2}\Pi_2^{[1]}.$$

An important property of the functions $D_0^{[1]}, D_2^{[1]}$ and $\Pi_2^{[2]}$ is their scale independence, which implies that they are directly related to measurements.

Using the technique described in [19] the leading and subleading (in n_f) terms of the next order in the perturbative series for $D_0^{[1]}$, $D_2^{[1]}$ are obtained:

$$
\begin{aligned}
D_0^{[1]} &= 1+\frac{\alpha_s(Q^2)}{\pi}+\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2(-0.115295n_f^2+1.98571) \\
&+ \left(\frac{\alpha_s(Q^2)}{\pi}\right)^3(0.0862069n_f^3-4.21585n_f+18.2427) \qquad (17)\\
&+ \left(\frac{\alpha_s(Q^2)}{\pi}\right)^4(-0.0100928n_f^3+1.87525n_f^2+d_{0,1}^{[1]4}n_f+d_{0,0}^{[1]4})
\end{aligned}
$$

$$
\begin{aligned}
D_2^{[1]} &= 1+\frac{\alpha_s(Q^2)}{\pi}1.66667+\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2(-0.573629n_f+2.5668) \\
&+ \left(\frac{\alpha_s(Q^2)}{\pi}\right)^3(0.16463n_f^2-14.3097n_f+148.978) \qquad (18)\\
&+ \left(\frac{\alpha_s(Q^2)}{\pi}\right)^4(-0.0156289n_f^3+6.06677n_f^2+d_{2,1}^{[1]4}n_f+d_{2,0}^{[1]4}) \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_2^{[2]} &= -4m_s^2\left(1+\frac{\alpha_s(Q^2)}{\pi}\frac{7}{3}+\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2(23.5094-1.30879n_f)\right. \\
&+ \left.\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3(0.464663n_f^2-32.0843n_f+k_{2,0}^{[2]3})\right) \qquad (20)
\end{aligned}
$$

The result for the α_s^4 terms in $D_0^{1]}$ has been already given in [19], the coefficients of the α_s^4 and α_s^3 terms in in $D_2^{[1]}$ and $\Pi_2^{[2]}$ respectively have been obtained in [20].

4 Massless case: phenomenological implications

Both FAC (Fastest Apparent Convergence) and PMS (Principle of Minimal Sensitivity) methods have been used to "predict" (at least roughly) the magnitude of higher order corrections. For the function $D_0^{[1]}$ the result is known since long from the work by Kataev and Starshenko [23].

n_f	$d_3^{\rm exact}$	$d_3^{\rm FAC}$	$d_3^{\rm PMS}$	$d_4^{\rm FAC/PMS}$	$d_5^{\rm FAC}$
3	6.371	5.604	6.394	27 ± 16	145 ± 100
4	2.758	4.671	5.264	8 ± 28	40 ± 160
5	-0.68	3.762	4.160	-8 ± 44	-3 ± 230

Table 1: Estimates for the coefficients $d_3 = d_0^{[1]3}$ and $d_4 = d_0^{[1]4}$ in the function $D_0^{[1]}$ based on FAC and PMS optimizations. The estimate for of $d_0^{[1]4}$ employs the exact value of $d_0^{[1]3}$. The last column contains the FAC predictions for the coefficient $d_5^{\rm FAC}$ which was obtained assuming the value for $d_0^{[1]4}$ as given in the fifth column; the corresponding uncertainties have been estimated as described in the text.

From the predictions corresponding to $n_f = 3, 4$ and 5 one easily restores the FAC/PMS prediction for the n_f dependence of the α_s^4 term in $D_0^{[1]}$

$$d_0^{[1]4} = 127.58 - 44.211 n_f + 3.6439 n_f^2 - 0.0100928 n_f^3. \tag{21}$$

It is also interesting to compare the FAC/PMS predictions for the n_f dependence of coefficient $d_0^{[1]3}$ with the exact result given in eq. (17). The results of both estimates are

$$d_0^{[1]3}(FAC) = 8.54 - 1.013 n_f + 0.0116 n_f^2, \tag{22}$$

$$d_0^{[1]3}(PMS) = 9.93 - 1.23 n_f + 0.0125 n_f^2. \tag{23}$$

The comparison of the complete α_s^3 and partial α_s^4 results with FAC and PMS estimates leads to following observations:

- Starting from α_s^3, the terms of order $\alpha_s^3 n_f^2$ and $\alpha_s^4 n_f^3$ are numerically quite small (at least for $n_f \leq 6$) and, thus, should have a negligibly small influence on the coefficients of the α_s expansion. On the other hand, the subleading in n_f term, say, of order $\alpha_s^3 n_f$ is comparable in size with the term of order $\alpha_s^3 n_f^0$. Similarly, the $\alpha_s^4 n_f^3$ term is significantly smaller than the $\alpha_s^4 n_f^2$ one, whereas the $\alpha_s^4 n_f$ and $\alpha_s^4 n_f^0$ terms are expected to be of similar magnitude.

- In general FAC/PMS methods correctly reproduce sign and order of magnitude of the higher order coefficients. The agreement is getting better when the coefficients happen to be large, as is the case for the quadratic mass corrections (see below).

- Taken separately, the FAC/PMS estimates of the coefficients of the n_f expansion could deviate rather strongly from the true result. However, for a given n_f, the deviation in the predicted value of the full $O(\alpha_s^n)$ term tends to be significantly smaller than what could be expected from summing individual terms of the n_f expansion. In addition, for the particular point $n_f = 3$ very good agreement is observed.

These coefficients can be used to predict R_τ in the massless limit first in fixed order perturbation theory ($a = \alpha_s(M_\tau)/\pi$)

$$R_\tau^{FOPT} = 3\left(1 + a + 5.202a + 26.37a^3 + a^4\ (105 \pm 16) + a^5(138 \pm 213 \pm 100)\right). \tag{24}$$

Here the first uncertainty in the α_s^5 term comes from that of $d_0^{[1]4}$ while the second is our estimation [20] of the error in the very coefficient $d_0^{[1]5}$. Similarly, we can use "contour improved" formulae [25, 26] to get

$$R_\tau^{CI} = 3\left(1 + 1.364a + 2.54a^2 + 9.71a^3 + a^4(35 \pm 20) + a^5(138 \pm 95)\right). \tag{25}$$

It is of interest to see in detail which accuracy in the determination of $\alpha_s(M_\tau)$ one could achieve assuming that the term of order α_s^4 and α_s^5 are known. It is natural to introduce the quality δ_P as follows:

$$R_{\tau S=0} = \frac{\Gamma(\tau \to h_{S=0}\nu)}{\Gamma(\tau \to l\bar{\nu}\nu)} = |V_{ud}|^2 S_{EW} R_\tau, \tag{26}$$

with (in obvious notataion) $R_\tau = 3(1 + \delta_P + \delta_{EW} + \delta_{NP})$ The first term here is the parton result, the second stands for pQCD effects. The non-perturbative corrections represented by δ_{NP} happens to be rather small and will be neglected in the following. The factor $S_{EW} = 1.0194$ is the electroweak correction which collects the large logarithmic terms [27], while $\delta_{EW} = 0.001$ is an additive not logarithmically enhanced electroweak correction [28]. Using for definiteness the result of ALEPH one arrives at $\delta_P^{exp} = 0.203 \pm 0.007$.

Unfortunately, there is no unique way to assign a theoretical uncertainty $\delta\alpha_s$ to the obtained value of $\alpha_s(M_\tau)$. In the literature one finds several suggestions. Let us consider them in turn.

$d_0^{[1]4}$	α_s^4	$d_0^{[1]5}$	α_s^5
27	0.327 ± 0.012	145	0.326 ± 0.004
	0.351 ± 0.005		0.349 ± 0.0015
43	0.326 ± 0.014	245	0.321 ± 0.006
	0.347 ± 0.0065		0.343 ± 0.002
11	0.329 ± 0.011	45	0.332 ± 0.0095
	0.355 ± 0.0035		0.355 ± 0.001

Table 2: The predicted value of $\alpha_s(M_\tau)$ in dependence of chosen values for the coefficients $d_0^{[1]4}$, $d_0^{[1]5}$. The third and the forth columns differ in the the number of terms in the perturbative series included. The upper value of α_s is the one predicted within FOPT, the lower corresponds CIPT. The uncertainty in the value of α_s corresponds to changing the normalization point μ as follows $\mu^2/M_\tau^2 = 0.4 - 2$.

1. $\delta\alpha_s$ is a half of the shift in α_s induced by the last computed term in the PT (that is by the one of order α_s^3 at present).

2. $\delta\alpha_s$ is equal to the change in α_s caused by varying the normalization point μ around M_τ, typically within the range of $1.1 - -2.5$ GeV.

   The corresponding results read (terms of order α_s^4 and higher in eq. (24) and in the D-function have been set to zero)

$$\alpha_s^{FOPT} = 0.340 \pm (0.027|0.033), \tag{27}$$

$$\alpha_s^{CIPT} = 0.358 \pm (0.011|0.016). \tag{28}$$

   Here the first(second) value in brackets correspond to the use of the first(second) suggestion for the error estimation.

3. $\delta\alpha_s$ is equal to the change in α_s caused by the uncertainity in the predicted (that is not yet completely known) higher order terms in the perturbative series for R_τ;

4. $\delta\alpha_s$ is a half of the difference in the $\alpha_s(M_\tau)$ as obtained within FOPT and CIPT. This difference comes from different suggestions about the behaviour of PT in higher orders.

In order to quantify the error estimates according to 2. – 4. we have shown in Table 2 the results for α_s obtained with various choices for $d_0^{[1]4}$, $d_0^{[1]5}$ and μ. For the plausible values of $d_0^{[1]4}$ and $d_0^{[1]5}$ we observe an significant decrease of the μ dependence after inclusion of additional terms in the α_s series. However, as discussed before, the large difference between FOPT and CIPT results remains unaffected if α_s^4 terms and even α_s^5 based on FAC estimate are included.

5 Quadratic mass corrections

A comparison of estimates and the exact results reveals for the mass corrections a picture qualitatively similar to the massless case but with some modifications. Let us just present the final result of thsese considerations. In a fixed order calculation one obtaines for the mass correction to the total rate

$$\begin{aligned}\delta^{00}_{us,2} &= -8\frac{m_s^2}{M_\tau^2}(1+0.567+0.520+0.521+0.592) \\ &= -8\frac{m_s^2}{M_\tau^2}(3.2\pm 0.6) \end{aligned} \tag{29}$$

where we have displayed separately the terms of increasing order in α_s. However, one must admit that terms of order α_s, α_s^2, α_s^3 and α_s^4 are all of comparable size, casting doubt on the stability of the series!

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